μ-deformed extinction of the Bose-Einstein condensate dark matter model

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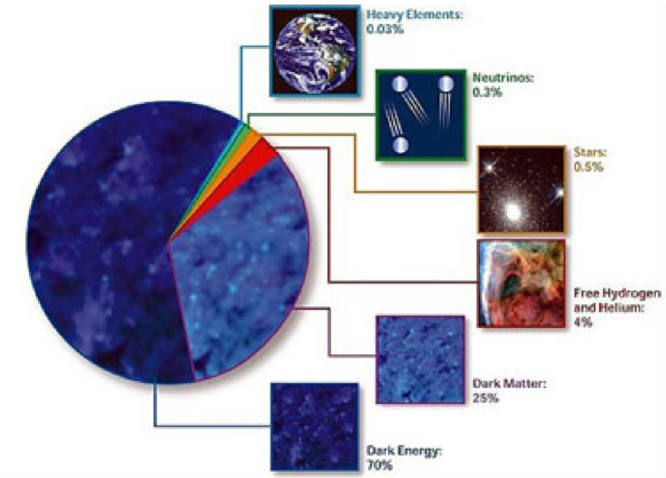
Standard Cosmological Model

Cosmological observation which are in remarkable agreement with ΛCDM:

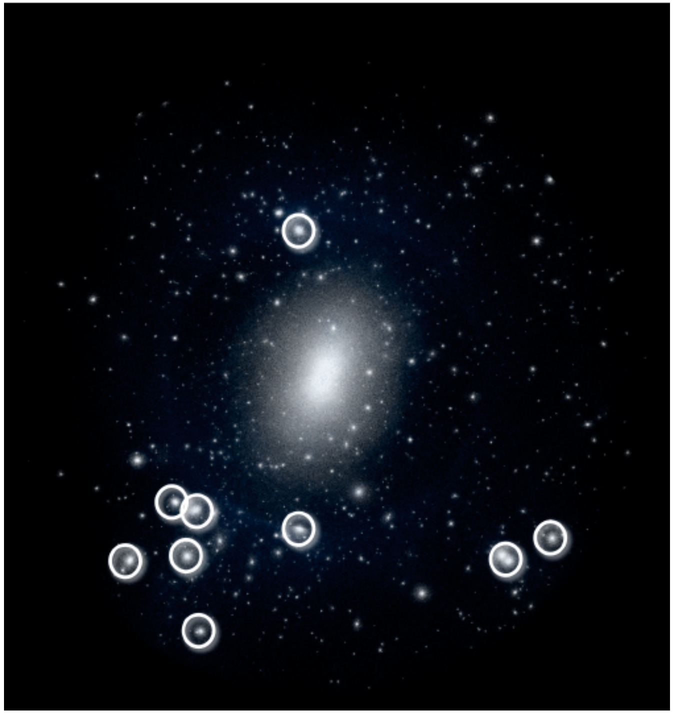


- Prediction of CMB
- Spectrum of CMB
- Large-scale structures
- Abundance of light elements
- Matter power spectrum

Cold dark matter

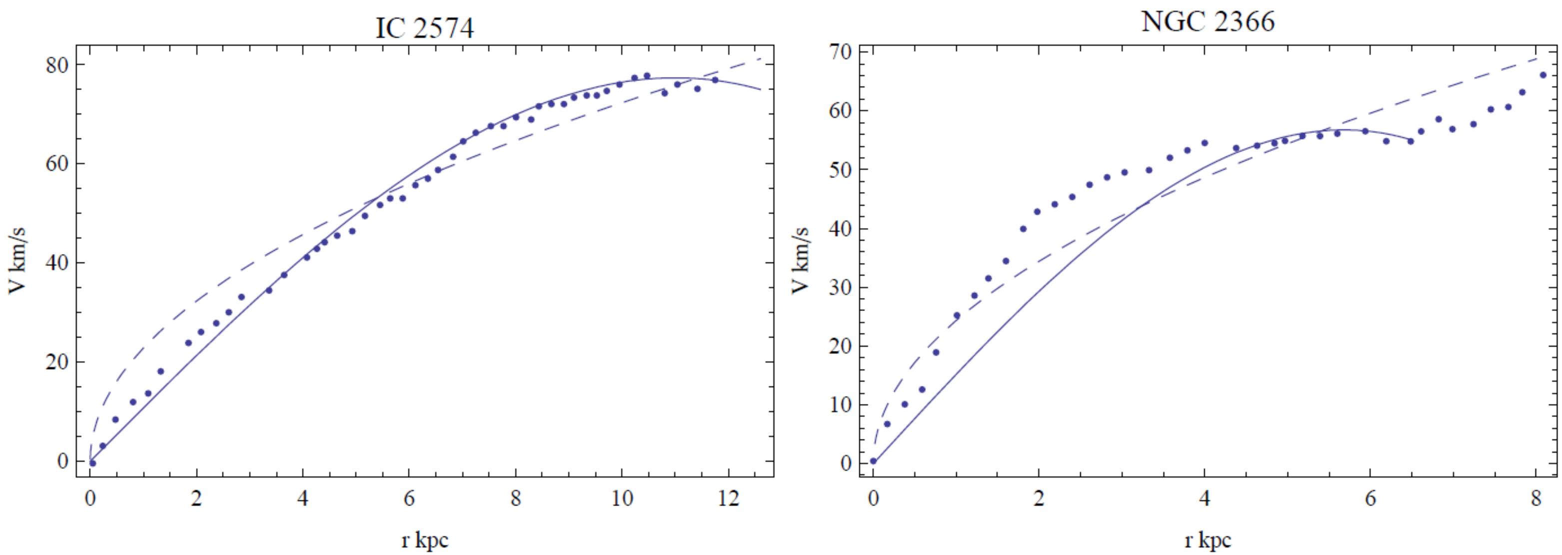
paradigm problems on the small scales

- Core-cusp problem
- Diversity problem
- Missing satellite problem



Bose-Einstein condensate dark matter model

could provide a solution of core-cusp problem



(T. Harko, 2011)

Extinction od BEC DM model via deformation of Bose-gas thermodynamics

Elements of μ - calculus: μ-derivative and μ-bracket

$$\mathcal{D}_x^{(\mu)} x^n = [n]_\mu x^{n-1}, \quad [n]_\mu \equiv \frac{n}{1+\mu n}$$

- total number of particles of standard gas

$$N = z \frac{d}{dz} \ln Z$$

- total number of particles of μ-deformed gas

$$N^{(\mu)} = z \mathcal{D}_z^{(\mu)} \ln Z$$

- grand partition function of μ-deformed gas

$$\ln Z^{(\mu)} = \left(z \frac{d}{dz} \right)^{-1} N^{(\mu)}$$

μ-deformed Bose gas extinction

1)total number of particles:

$$N^{(\mu)} = \frac{V}{\lambda^3} g_{3/2}^{(\mu)}(z) + g_0^{(\mu)}(z)$$

2)μ-polylogarithm generalization of polylogarithm function:

$$g_l^{(\mu)}(z) = \sum_{n=1}^{\infty} \frac{[n]_\mu}{n^{l+1}} z^n$$

3)grand canonical partition function:

$$\ln Z^{(\mu)} = \frac{V}{\lambda^3} g_{5/2}^{(\mu)} + g_1^{(\mu)}$$

Ruppeiner geometry

1) Metric components:

$$G_{\beta\beta} = \frac{\partial^2 \ln Z}{\partial \beta^2} = -\left(\frac{\partial U}{\partial \beta}\right)_\gamma$$

$$G_{\beta\gamma} = \frac{\partial^2 \ln Z}{\partial \beta^2} = -\left(\frac{\partial N}{\partial \beta}\right)_\gamma$$

$$G_{\gamma\gamma} = \frac{\partial^2 \ln Z}{\partial \beta^2} = -\left(\frac{\partial N}{\partial \gamma}\right)_\beta$$

2) Christoffel symbols of the second kind and Riemann tensor:

$$\Gamma_{\lambda\mu\nu} = \frac{1}{2}(\ln Z)_{,\lambda\mu\nu}$$

$$R_{\lambda\mu\nu\rho} \equiv g^{\kappa\tau}\left(\Gamma_{\kappa\lambda\rho}\Gamma_{\tau\mu\nu} - \Gamma_{\kappa\lambda\nu}\Gamma_{\tau\mu\rho}\right)$$

Geometrical approach to μ-Bose gas model

$$G_{\beta\beta} = \frac{15}{4}\frac{V}{\lambda^3\beta^2}\, g^{(\mu)}_{\frac{5}{2}}(z)$$

$$G_{\beta\gamma} = \frac{3}{2}\frac{V}{\lambda^3\beta}\, g^{(\mu)}_{\frac{3}{2}}(z)$$

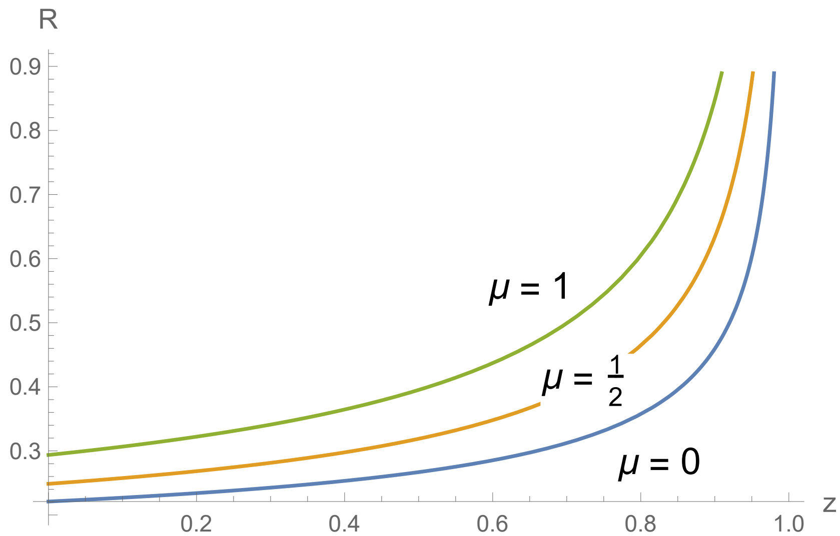
$$G_{\gamma\gamma} = \frac{V}{\lambda^3}\, g^{(\mu)}_{\frac{1}{2}}(z) + g^{(\mu)}_{-1}(z)$$

Scalar curvature of the 2-dimensional thermodynamic space:

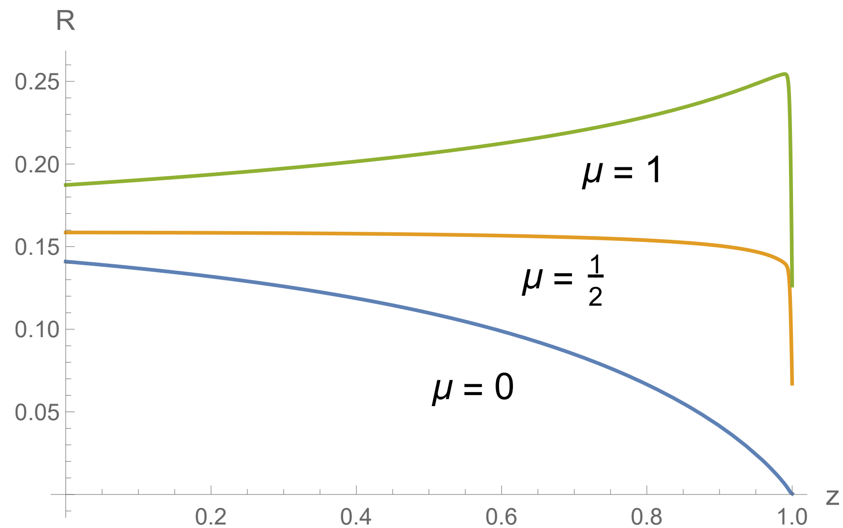
$$R = \frac{5}{2}\frac{\left(5g^{(\mu)}_{\frac{3}{2}}g^{(\mu)}_{\frac{3}{2}}g^{(\mu)}_{-1} - 7g^{(\mu)}_{\frac{5}{2}}g^{(\mu)}_{\frac{1}{2}}g^{(\mu)}_{-1} + 2g^{(\mu)}_{\frac{3}{2}}g^{(\mu)}_{\frac{5}{2}}g^{(\mu)}_{-2} + \frac{V}{\lambda^3}\,\mathcal{W}\right)}{\left(5g^{(\mu)}_{\frac{5}{2}}g^{(\mu)}_{-1} + \frac{V}{\lambda^3}\left(5g^{(\mu)}_{\frac{5}{2}}\,g^{(\mu)}_{\frac{1}{2}} - 3g^{(\mu)}_{\frac{3}{2}}\,g^{(\mu)}_{\frac{3}{2}}\right)\right)^2}$$

$$\mathcal{W} = \left(2g^{(\mu)}_{\frac{3}{2}}g^{(\mu)}_{\frac{5}{2}}g^{(\mu)}_{-\frac{1}{2}} - 4g^{(\mu)}_{\frac{5}{2}}g^{(\mu)}_{\frac{1}{2}}g^{(\mu)}_{\frac{1}{2}} + 2g^{(\mu)}_{\frac{3}{2}}g^{(\mu)}_{\frac{3}{2}}g^{(\mu)}_{\frac{1}{2}}\right)$$

Phase transitions in μ-Bose gas



$V/\lambda^3 \gg 1$



$V/\lambda^3 \sim 1$ and $V/\lambda^3 \ll 1$

V - volume of gas, $\lambda = \sqrt{\frac{2\pi\hbar^2}{mkT}}$ – thermal wavelength

Thermodynamical properties of μ-Bose gas

- The critical temperature of μ-deformed Bose gas:

$$T_c^{(\mu)} = \frac{2\pi\hbar^2/mk}{\left(v g_{3/2}^{(\mu)}(1)\right)^{2/3}}$$

- The ratio of critical temperatures of μ-deformed and usual Bose-gas:

$$\frac{T_c^{(\mu)}}{T_c} = \left(\frac{2.61}{g_{3/2}^{(\mu)}(1)}\right)^{2/3}$$

- Seconde virial coefficient in compassion with usual Bose gas:

$$V_2^{(\mu)} - V_2^{Bose} = 2^{-5/2}\frac{\mu^2}{1+2/\mu} > 0$$

Dark Matter halo parameters

1) radius of galactic DM halo:

$$R = \pi \sqrt{\frac{\hbar^2 a}{G m^3}}$$

2) total DM halo mass:

$$M^{(\mu)} = \frac{\pi}{6} m g_{3/2}^{(\mu)}(1) f^3$$

the dimensionless factor

$$f = \sqrt{\frac{2\pi a k T}{G m^3}} \gg 1$$

Summary

In the presented work, we extend condensate DM models by introducing for that role an analog of Bose gas whose particles obey deformed statistics.

The main advantages of presented model, as compared with classical BEC Dark Matter model are:

- wider range of temperatures relevant for the condensate state
- better agreement of predicted halo mass with observational data