Annihilation and bremsstrahlung channels in kinetics of the electron-positron plasma created from vacuum in a strong electric field

S.O. Pirogov[1]

Saratov State University
Department of Theoretical Physics
[1]1st-year master student

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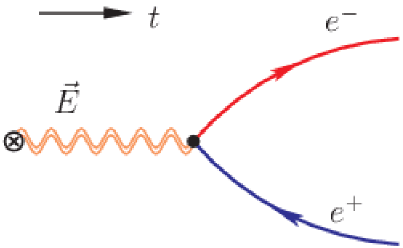
Outline

Preliminaries

The Schwinger effect



Sauter-Heisenberg-Euler-Schwinger formula

$$\varpi = \frac{ce^2E^2}{4\pi^3\hbar^2}\exp^{-\frac{E_{cr}}{E}},\ where\ is\ E_{cr} = \frac{\pi m^2 c^3}{e\hbar}$$

- Necessary value: $E_{cr} = \frac{m^2}{e} \backsimeq 1.3 \cdot 10^{16}\ \frac{V}{cm}$

General equations for the problem

Examples I

Standard QED Lagrangian:

$$\mathcal{L} = \mathcal{L}_{qc} + \mathcal{L}_I,$$

$$\mathcal{L}_I = -e\overline{\psi}\gamma^\mu \hat{A}_\mu \psi \ , \ \mathcal{L}_{qc} = \frac{i}{2}(\overline{\psi}\gamma^\mu D_\mu \psi - (D^*_\mu \overline{\psi})\gamma^\mu \psi) - m\overline{\psi}\psi$$

4-potential of an external electric field in the Hamiltonian gauge:

$$A^\mu(t) = (0, 0, 0, A(t))$$

General equations for the problem

Examples II

Nonstationary spinor basis:

$$u_1^+(\mathbf{p},t) = B(\mathbf{p})[\omega_+, 0, P^3, P_-]$$
$$u_2^+(\mathbf{p},t) = B(\mathbf{p})[0, \omega_+, P_+, -P^3]$$
$$v_1^+(-\mathbf{p},t) = B(\mathbf{p})[-P^3, -P_-, \omega_+, 0]$$
$$v_2^+(-\mathbf{p},t) = B(\mathbf{p})[-P_+, P^3, 0, \omega_+]$$

Diagonal form of the fermionic Hamiltonian:

$$H_f(t) = \sum_\alpha \int d^3p\, \omega(\mathbf{p},t)[a_\alpha^+(\mathbf{p},t)a_\alpha(\mathbf{p},t) - b_\alpha(-\mathbf{p},t)b_\alpha^+(-\mathbf{p},t)]$$

Photon sector
The basic equations

Hamiltonian of the interaction with the quantized field:

$$H_{int}(t) = e(2\pi)^{-3/2} \sum_{\alpha\beta} \int d^3p_1 d^3p_2 \frac{d^3k}{\sqrt{2k}} \delta(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{k}) \cdot$$

$$\cdot : ([\overline{u}u]^r_{\beta\alpha} a^+_\alpha a_\beta + [\overline{u}\upsilon]^r_{\beta\alpha} a^+_\alpha b^+_\beta + [\overline{\upsilon}u]^r_{\beta\alpha} b_\alpha a_\beta + [\overline{\upsilon}\upsilon]^r_{\beta\alpha} b_\alpha b^+_\beta) A_r(\mathbf{k}, t) :$$

Heisenberg-like equations of motion:

$$\dot{a}_\alpha = -i\omega a_\alpha - U^{(1)}_{\alpha\beta} a_\beta - U^{(2)}_{\alpha\beta} b^+_\beta - ie(2\pi)^{-3/2} \int d^3p_1 \frac{d^3k}{\sqrt{2k}} \delta(\mathbf{p} - \mathbf{p}_1 + \mathbf{k}) \cdot$$

$$\cdot (a_\beta [\overline{u}u]^r_{\beta\alpha} + b^+_\beta [\overline{u}\upsilon]^r_{\beta\alpha}) A_r(\mathbf{k}, t)$$

Photon sector
The first equation of the BBGKY hierarchy

Photon correlation function:

$$F_{rr'}(\mathbf{k},\mathbf{k}',t) = \left\langle A_r^{(+)}(\mathbf{k},t)A_{r'}^{(-)}(\mathbf{k}',t)\right\rangle$$

It's equation of motion:

$$\dot{F}_{rr'}(\mathbf{k},\mathbf{k}',t) = ie(2\pi)-3/2\sum_{\alpha,\beta}\int d^3p_1d^3p_2[-\frac{1}{\sqrt{2k}}\delta(\mathbf{p}_1-\mathbf{p}_2+\mathbf{k})\cdot$$

$$\cdot([\overline{u}v]^r_{\alpha\beta}\left\langle a^+_\alpha b^+_\beta A^{(-)}_{r'}\right\rangle+\dots)+\frac{1}{\sqrt{2k'}}\delta(\mathbf{p}_1-\mathbf{p}_2+\mathbf{k}')([\overline{v}u]^{r'}_{\alpha\beta}\left\langle b_\alpha a_\beta A^{(+)}_r\right\rangle+$$

$$[\overline{u}u]^{r'}_{\alpha\beta}\left\langle a^+_\alpha a_\beta A^{(+)}_r\right\rangle+\dots)]+i(k-k')F_{rr'}(\mathbf{k},\mathbf{k}',t)$$

One-photon channel
Annihilation channel

The truncation procedure:

$$\left\langle b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)A_r^\pm(\mathbf{k},t)\right\rangle = \left\langle b_\alpha(-\mathbf{p}_1,t)a_\beta(\mathbf{p}_2,t)\right\rangle\left\langle A_r^\pm(\mathbf{k},t)\right\rangle = 0$$

Equation of motion for "the pink" correlator:

$$(\frac{\partial}{\partial t}+i\omega(\mathbf{p}_1,t)+i\omega(\mathbf{p}_2,t)-ik)\left\langle b_\alpha a_\beta A_r^{(+)}\right\rangle = S^r_{\alpha\beta}+U^r_{\alpha\beta}+$$

$$+ie(2\pi)^{-3/2}\int d^3p'\frac{d^3k'}{\sqrt{2k'}}[\delta(\mathbf{p'}-\mathbf{p}_1+\mathbf{k'})([\overline{u}v]^{r'}_{\alpha\beta'}\left\langle a^+_{\beta'}a_\beta A_{r'}A_r^{(+)}\right\rangle+\ldots)$$

$$-\delta(\mathbf{p}_2-\mathbf{p'}+\mathbf{k'})([\overline{u}u]^{r'}_{\beta'\beta}\left\langle b_\alpha a_{\beta'}A_{r'}A_r^{(+)}\right\rangle+\ldots)]$$

One-photon channel
Bremsstrahlung channel

Equation of motion for "the red" correlator:

$$(\frac{\partial}{\partial t} - i\omega(\mathbf{p}_1, t) + i\omega(\mathbf{p}_2, t) - ik) \left\langle a_\alpha^+ a_\beta A_r^{(+)} \right\rangle = S_{\alpha\beta}^{r\ (rad)} + U_{\alpha\beta}^{r\ (rad)} +$$

$$+ie(2\pi)^{-3/2} \int d^3p' \frac{d^3k'}{\sqrt{2k'}} [\delta(\mathbf{p}_1 - \mathbf{p'} + \mathbf{k'})([\overline{u}v]_{\alpha\beta'}^{r'+} \left\langle b_{\beta'} a_\beta A_{r'} A_r^{(+)} \right\rangle + \ldots)$$

$$-\delta(\mathbf{p}_2 - \mathbf{p'} + \mathbf{k'})([\overline{u}u]_{\beta'\beta}^{r'} \left\langle a_\alpha^+ a_{\beta'} A_{r'} A_r^{(+)} \right\rangle + \ldots)]$$

Summary

- Good ideas of simplifying at the frames of supplied issue
- Equations for two main processes
- Overview for further work
  - Solutions of correlators for the Photon correlation function
  - Which type of truncation is better?
  - More complications with another components of A^{μ}

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