Contribution of nontrivial angular momenta in bound state energy of triton in Bethe-Salpeter-Faddeev approach

S. A. Yurev, S. G. Bonbarenko, V. V. Burov

Joint Institute for Nuclear Research

Laboratory of Theoretical Physics

JINR

DUBNA

26.04.2018

object

Three-nucleon systems:

Bound states ^{3}He(ppn) T(^{3}H)(nnp)

Collisions (pD→pD,pD→ppn)

Elastic electron scattering on ^{3}He: (e^{3}He →e^{3}He)

JLab exp:
E04018 (OLD)
E1214009 at 12 GeV (NEW)

goal

The study of these systems at relativistic energies

method of research

nonrelativistic case | relativistic case

2 particles

Lippmann-Schwinger equation → Bethe-Salpeter equation

$$p = (p_1, p_2)$$

$$t(p,p') = V(p,p') + \int dp'' V(p,p'') G(p'') t(p'',p')$$

3 particles

Faddeev equation → Relativistic Faddeev equation

Bethe-Salpeter-Faddeev

$$p = (p_1, p_2, p_3)$$

$$T^{(i)}(p,p') = t^{(i)}(p,p') + \int dp'' t^{(i)}(p,p'') G(p'') [T^{(j)}(p'',p') + T^{(k)}(p'',p')]$$

Relativistic Faddeev equation

Two-particle t-matrix

$$T^{(i)}(k_1, k_2, k_3; k_1', k_2', k_3') = t_i(k_1, k_2, k_3; k_1', k_2', k_3') +$$

$$+ \int dk_1'' dk_2'' dk_3'' \, t_i(k_1, k_2, k_3; k_1'', k_2'', k_3'') G(k_1'', k_2'', k_3'') \times$$

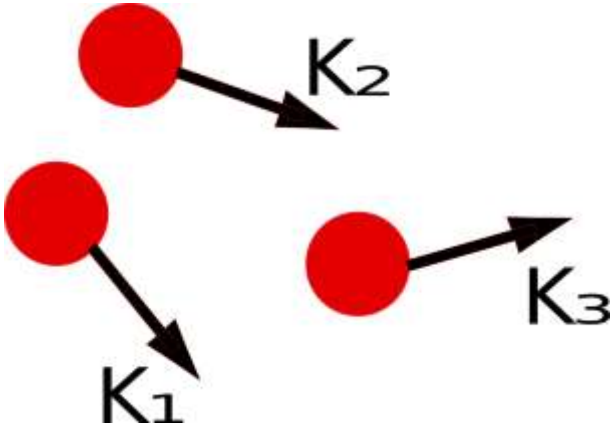
$$\times [T^{(j)}(k_1'', k_2'', k_3''; k_1', k_2', k_3') + T^{(k)}(k_1'', k_2'', k_3''; k_1', k_2', k_3')]$$

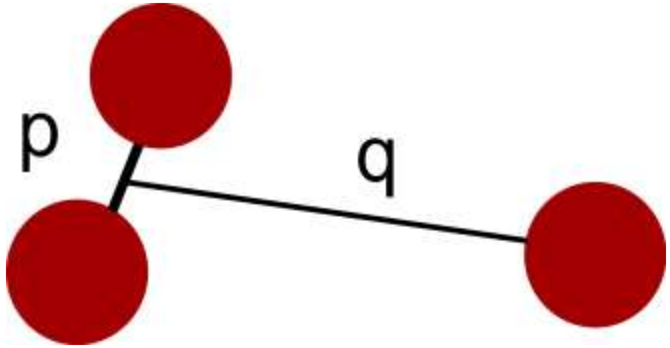
Components of the full three-particle t matrix $T = T^1 + T^2 + T^3$

Two-particle propagator

$$G_i = (k_j^2 - m_n^2 + i\epsilon)^{-1}(k_k^2 - m_n^2 + i\epsilon)^{-1}$$

Jacobi variables





$$k_1, k_2, k_3 \Rightarrow p, q, P$$

$$p_i = \frac{1}{2}(k_j - k_n), q_i = \frac{1}{3}P - k_i, P = k_1 + k_2 + k_3$$

$$\int dk_1 dk_2 dk_3 \Rightarrow \int dpdq$$

Relativistic Faddeev equation

Two-particle t-matrix

$$T^{(i)}(p_i, q_i; p_i', q_i'; P) = t^{(i)}(p_i, q_i; p_i', q_i'; P) +$$

$$\int dp_i'' dq_i'' t^{(i)}(p_i, q_i; p_i', q_i'; P) G(p'', q'', P) \times$$

$$\times [T^{(j)}(p_j'', q_j''; p_j', q_j'; P) + T^{(k)}(p_k'', q_k''; p_k', q_k'; P)]$$

Components of the full three-particle t matrix $T = T^1 + T^2 + T^3$

Two-particle propagator

The Bethe-Salpeter equation

equation for the relativistic system of two particles

$$p = \frac{1}{2}(k_2 - k_1)$$

$$P = k_2 + k_1$$

Two-particle t-matrix

$$t(p;p';P) = V(p;p';P) + \int dp'' V(p;p'';P) G(p'',P) t(p'';p';P)$$

Potential of NN interaction

Two-particle propagator

Separable potential of nucleon-nucleon interaction

$$V(p,p') = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p')$$

g - form factor of potential

N - rank of potential

Yamaguchi functions for form factor of potential

S state

$$g_Y^{[S]}(p_0,p) = \frac{1}{-p_0^2 + p^2 + \beta_0^2 - i\epsilon}$$

P state

$$g_Y^{[P]}(p_0,p) = \frac{\sqrt{|-p_0^2+p^2|}}{(-p_0^2 + p^2 + \beta_1^2 - i\epsilon)^2}$$

D state

$$g_Y^{[D]}(p_0,p) = \frac{C(-p_0^2+p^2)}{(-p_0^2 + p^2 + \beta_2^2 - i\epsilon)^2}$$

In the case of a separable NN potential, the two-particle t-matrix will also have a separable form

$$V(p,p') = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p')$$

Bethe-Salpeter equation

$$t = V + \int VGt$$

$$t(p,p') = \sum_{ij=1}^{N} \tau_{ij}(s) g_i(p) g_j(p')$$

$$\tau(s)_{ij} = [\frac{1}{\lambda_{ij}} - \underbrace{\frac{i}{4\pi^3}\int_{-\infty}^{\infty} dk^0 \int_0^{\infty} k^2 dk \quad g_i(k) g_j(k) G(k^0,k;s)}_{h(s)}]^{-1}$$

$$T_l(\bar{p}) = -\frac{8\pi\sqrt{s}}{\bar{p}} e^{i\delta_L(\bar{p})} sin\delta_L(\bar{p})$$

$$\delta_L = atan(\frac{h_{Im}}{\lambda^{-1} + h_{Re}})$$

$$\bar{p}ctg\delta(\bar{p}) = -1/a + \frac{r}{2}\bar{p}^2$$ (S state)

condition of bound state:

$$det([\tau^{-1}(s)]_{ij}) = 0 \qquad h(s) = \lambda^{-1}$$

$$\sqrt{s} = 2m - E_{BS}$$

r - effective range

E_{BS} - bound state energy

a - scattering length

δ - scattering phase shift

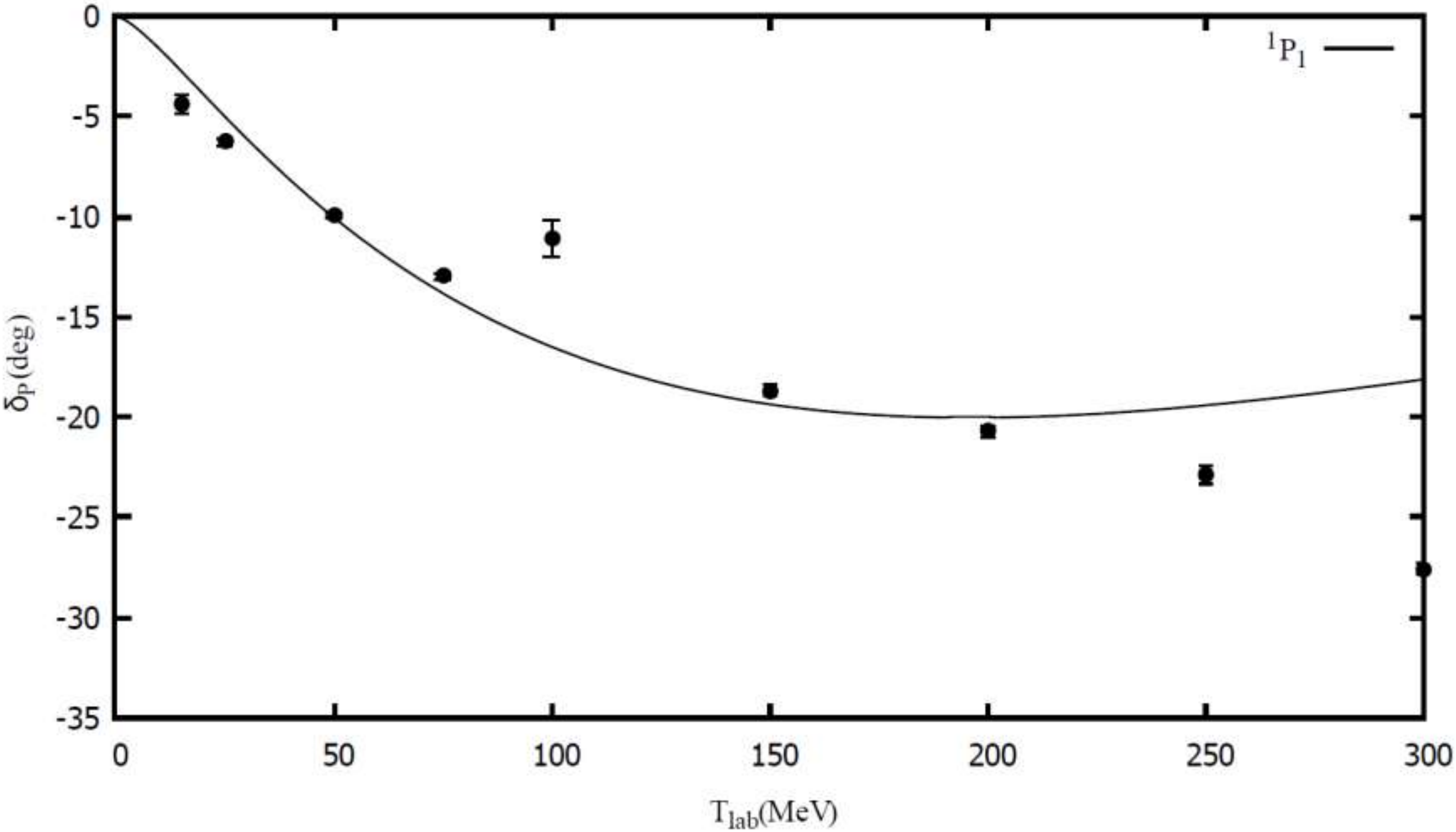
Parameters for the Yamaguchi potential

Physics of Particles and Nuclei Letters 15(4) (2018).

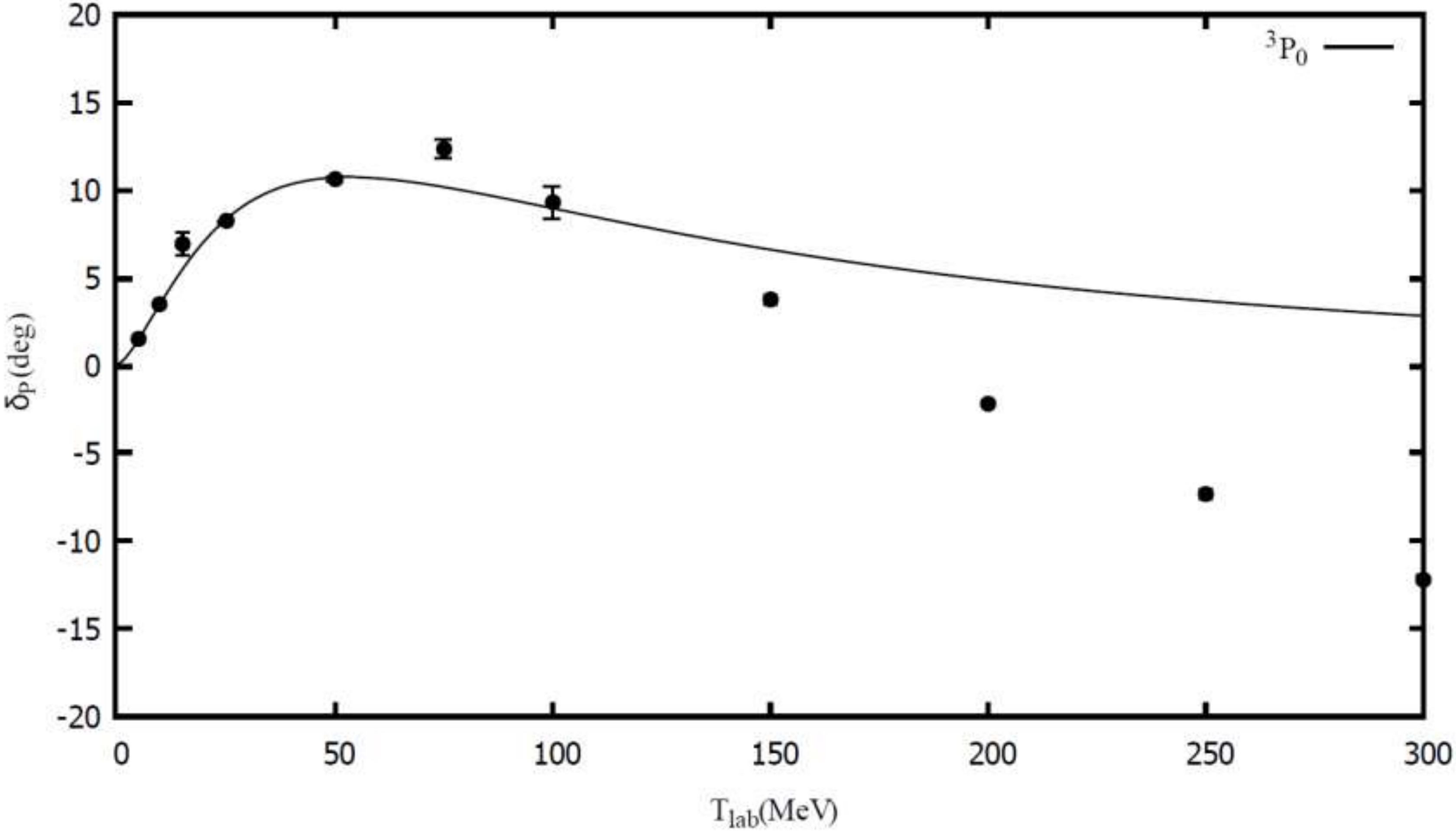
	Exp. from [7]	$^1S_0$
λ (GeV^4)		-1.12087
β_0 (GeV)		0.228302
a_L (fm)	-23.748	-23.753
r_L (fm)	2.75	2.75

	Exp. from [7]	$^3S_1 -^3 D_1$ ($p_d = 4\%$)	$^3S_1 -^3 D_1$ ($p_d = 5\%$)	$^3S_1 -^3 D_1$ ($p_d = 6\%$)
λ (GeV^4)		-1.83756	-1.57495	-1.34207
β_0 (GeV)		0.251248	0.246713	0.242291
C_2		1.71475	2.52745	3.46353
β_2 (GeV)		0.294096	0.324494	0.350217
a_L (fm)	5.424	5.454	5.454	5.453
r_L (fm)	1.756	1.81	1.81	1.80

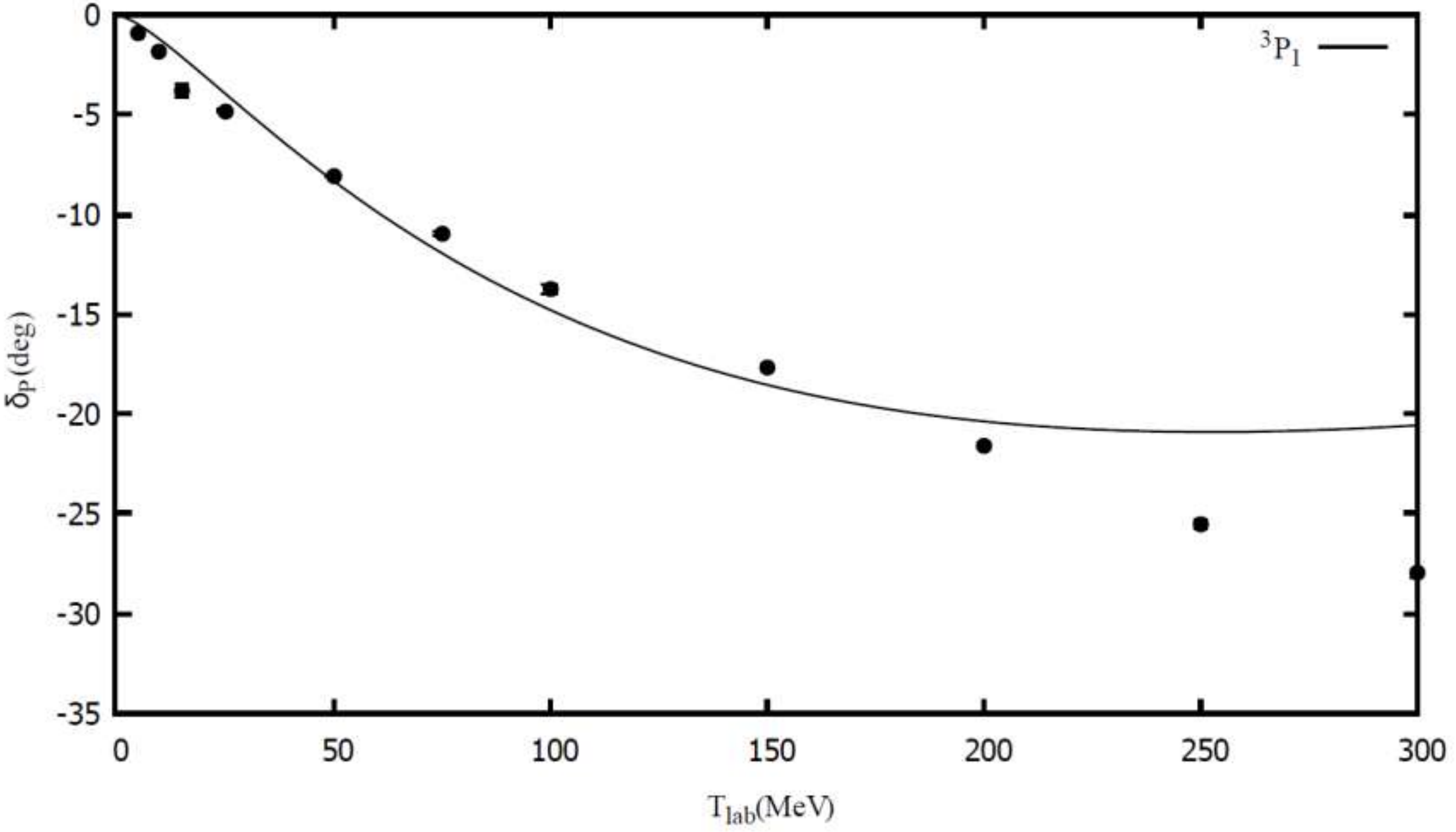
The 1P1 channel phase shifts.



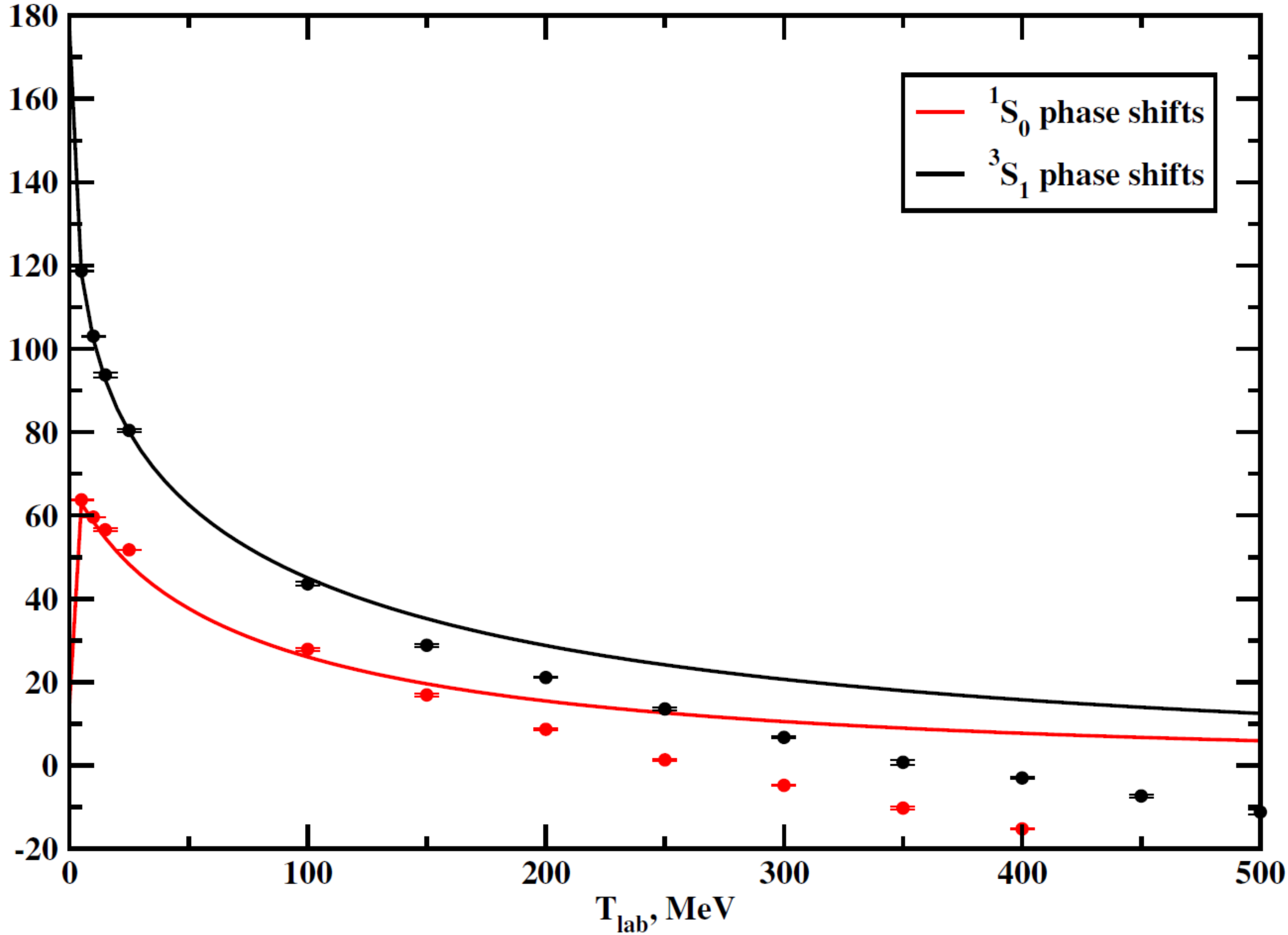
The 3P0 channel phase shifts.



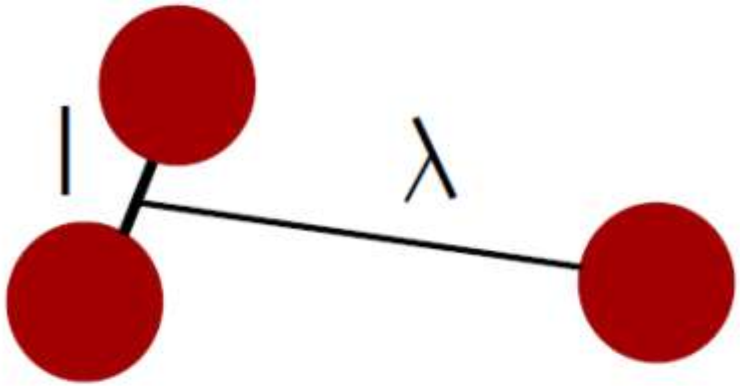
The 3P1 channel phase shifts.



Phases for Yamaguchi



Partial-wave decomposition



$$\Psi(\mathbf{p},\mathbf{q};s)=\sum_{l\lambda LM}\Psi_{l\lambda L}(p,q;s)\mathcal{Y}_{l\lambda LM}(\mathbf{p},\mathbf{q})$$

где

$$\mathcal{Y}_{l\lambda LM}(\mathbf{p},\mathbf{q})=\sum_{m\mu}Y_{lm}(\mathbf{p})Y_{\lambda\mu}(\mathbf{q})$$

$$t(\mathbf{p},\mathbf{p}')=\sum_{lm}t_l(p,p')Y_{lm}(\mathbf{p})Y_{lm}(\mathbf{p}')$$

The system of integral equations for a three-body system

$$V(p,p') = \sum_{ij=1}^{N} \lambda_{ij} g_i(p) g_j(p')$$

$$t(p,p') = \sum_{ij=1}^{N} \tau_{ij}(s) g_i(p) g_j(p')$$

Bethe-Salpeter-Faddeev equation

$$\Psi^{(i)} = \int tG[\Psi^{(j)} + \Psi^{(k)}]$$

after partial-wave decomposition

$$\Psi(p,q;s) = \sum_{ij=1}^{N} g_i(p) \tau_{ij}(s) \Phi(q)$$

The system of integral equations for Φ

$$\Phi^{a}_{jl\lambda L}(q_0,q) = -\frac{1}{4\pi^3}\sum_{b}\sum_{kn}\sum_{l'\lambda'}\int_{-\infty}^{\infty} dq_0^{'}\int_0^{\infty} q^{'2}dq^{'}\times$$

$$Z^{ab}_{jkl\lambda l'\lambda' L}(iq_0,q;iq_0^{'},q^{'};s)\frac{\tau^{b}_{knl'\lambda'}[(\frac{2}{3}\sqrt{s}+iq_0^{'})^2-q^{'2}]}{(\frac{1}{3}\sqrt{s}-iq_0^{'})^2-q^{'2}-m^2}\Phi^{b}_{jl'\lambda' L}(q_0^{'},q^{'})$$

$$Z^{ab}_{jkl\lambda l'\lambda' L}(iq_0,q;iq_0^{'},q^{'};s) = \Delta^a_l\Delta^b_{l'}C^{ab}\int_{-1}^{1} dx K^{L}_{l\lambda l'\lambda'}(q,q^{'},x)\times$$

$$\frac{g^a_{jl}(-\frac{1}{2}q_0-q_0^{'},\sqrt{\frac{1}{4}q^2+q'^2+qq'x})\,g^b_{kl'}(q_0+\frac{1}{2}q_0^{'},\sqrt{q^2+\frac{1}{4}q'^2+qq'x}}{(\frac{1}{3}\sqrt{s}+q_0+q_0^{'})^2-(q^2+q'^2+2qq'x)-m^2}$$

Where: a,b = $^{2S+1}L_J$

N- order of separability

λ and l is angular momenta

$$\Delta^a_l = \frac{1}{2}[1+(-1)^{a+l+1}]$$

Spin-isospin structure of the system

C^{ab} a = (S,I) (S,I) = {(1,0);(1,1);(0,1);(0,0)}

$$C^{ab} = C^{(s_A,i_A)(s_B,i_B)} =$$

$$=< (s_1 s_2)s_A, s_3, S|(s_2 s_3)s_B, s_1, S >< (i_1 i_2)i_A, i_3, I|(i_2 i_3)i_B, i_1, I >$$

$$C^{ab} = \frac{1}{4}\begin{pmatrix} 1 & \sqrt{3} & -3 & -\sqrt{3} \\ \sqrt{3} & -1 & \sqrt{3} & -3 \\ -3 & \sqrt{3} & 1 & -\sqrt{3} \\ -\sqrt{3} & -3 & -\sqrt{3} & -1 \end{pmatrix}$$

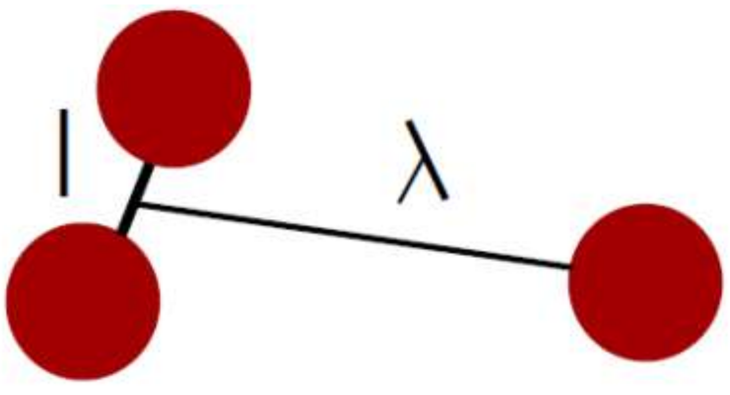
	$^1S_0$	$^3S_1$	$^3D_1$	$^3P_0$	$^1P_1$	$^3P_1$
S	0	1	1	1	0	1
I	1	0	0	1	0	1

The influence of the orbital angular momentum

$$K^{(aa')}_{\lambda\lambda' L}(q, q', \cos\vartheta_{qq'}) = (4\pi)^{3/2}\frac{\sqrt{2\lambda+1}}{2L+1}$$

$$\sum_{mm'} C^{Lm}_{lm\lambda 0} C^{Lm}_{l'm'\lambda' m-m'} Y^{*}_{lm}(\vartheta, 0) Y_{l'm'}(\vartheta', 0) Y_{\lambda' m-m'}(\vartheta_{qq'}, 0)$$

$$\cos\vartheta = (\frac{q}{2} + q'\cos\vartheta_{qq'})/|\frac{\mathbf{q}}{2} + \mathbf{q}'|, \qquad \cos\vartheta' = (q + \frac{q'}{2}\cos\vartheta_{qq'})/|\mathbf{q} + \frac{\mathbf{q}'}{2}|$$



Iterative method for solving integral equations

$$f(x) = \int_a^b K(x, y, s) f(y) dy$$

$$f_0(x) = 1$$

$$f_1(x) = \int_a^b K(x, y, s) f_0(y) dy$$

....

$$f_i(x) = \int_a^b K(x, y, s) f_{i-1}(y) dy$$

bound-state condition

$$\lim_{i \to \infty} \frac{f_i(x, s)}{f_{i+1}(x, s)} = 1$$

$$\sqrt{s} = 3m - E_{bs}$$

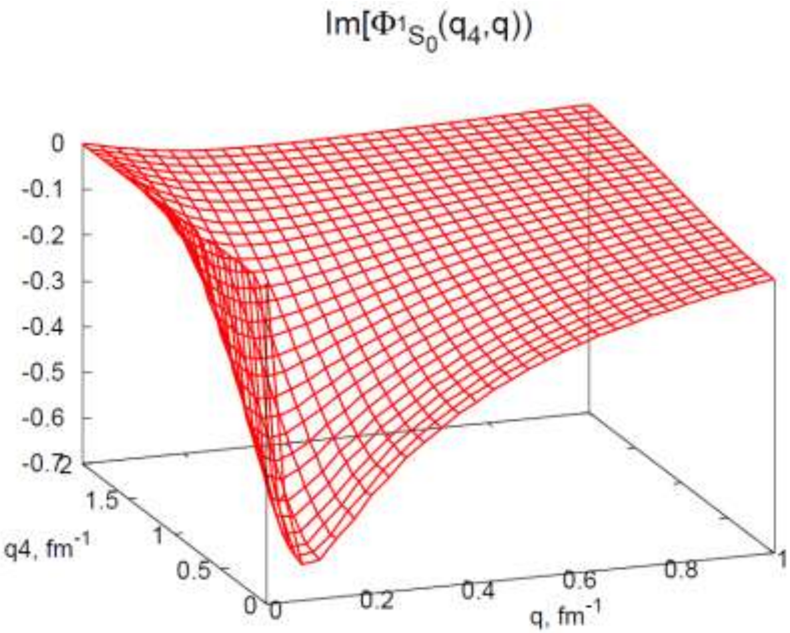
The binding energy of the triton **T (nnp)** in the case of the Yamaguchi potential

Exp.: 8.48 MeV

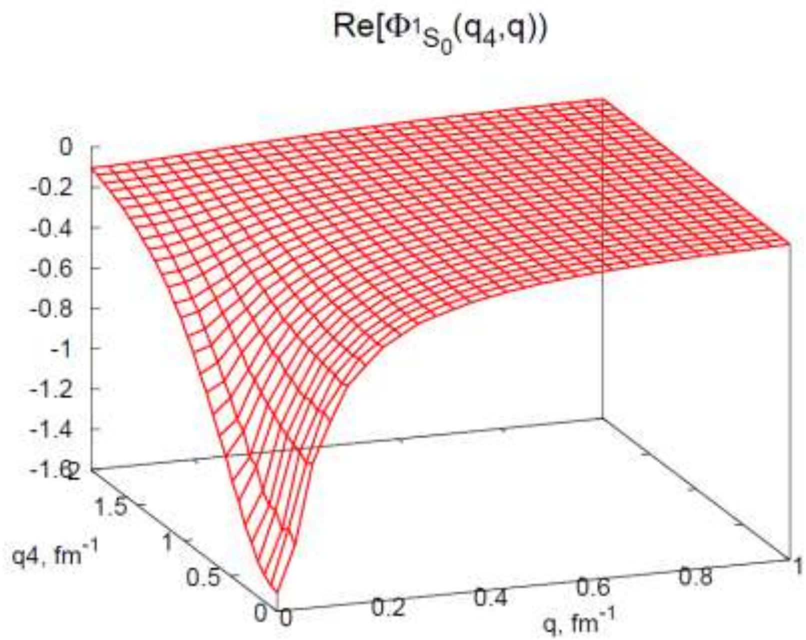
p_D	$^1S_0 -^3 S_1$	$^3D_1$	$^3P_0$	$^1P_1$	$^3P_1$	
4	**9.221**	**9.294**	**9.314**	**9.287**	**9.271**	
	0	0.073	0.020	−0.027	−0.016	0.050
5	**8.819**	**8.909**	**8.928**	**8.903**	**8.889**	
	0	0.090	0.019	−0.025	−0.014	0.070
6	**8.442**	**8.545**	**8.562**	**8.540**	**8.527**	
	0	0.103	0.017	−0.022	−0.013	0.085

1S0 amplitude

Imaginary part

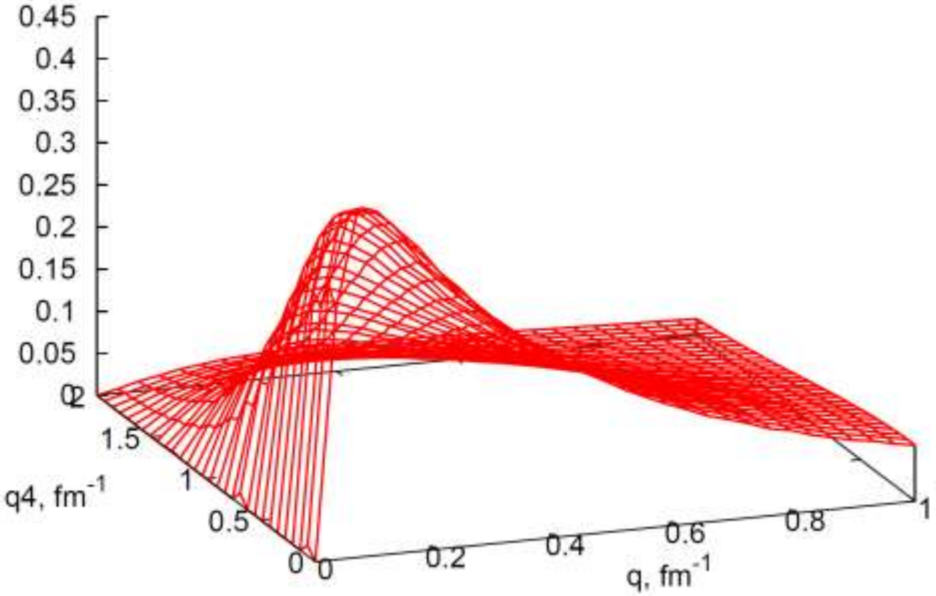


Real part

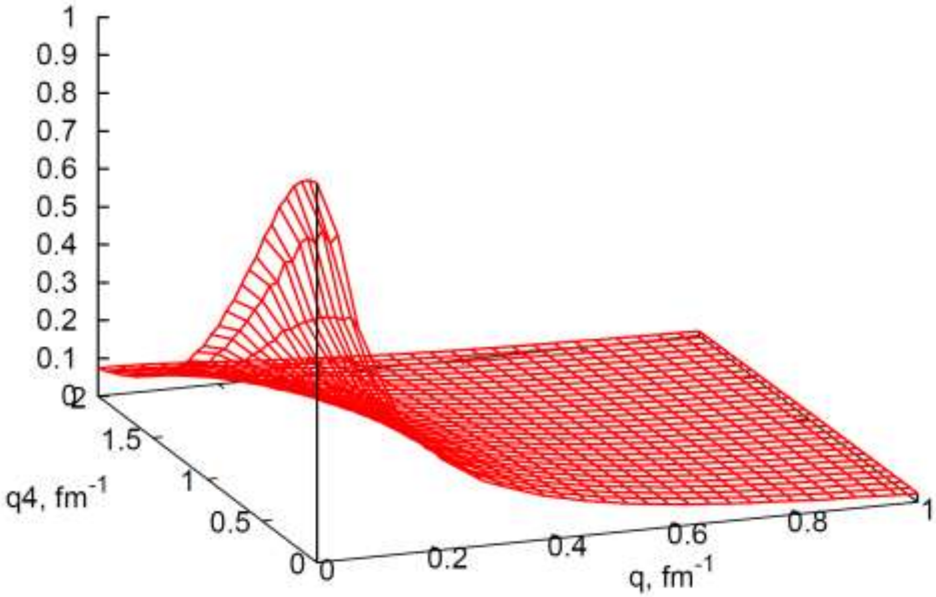


3S1 amplitude

Imaginary part

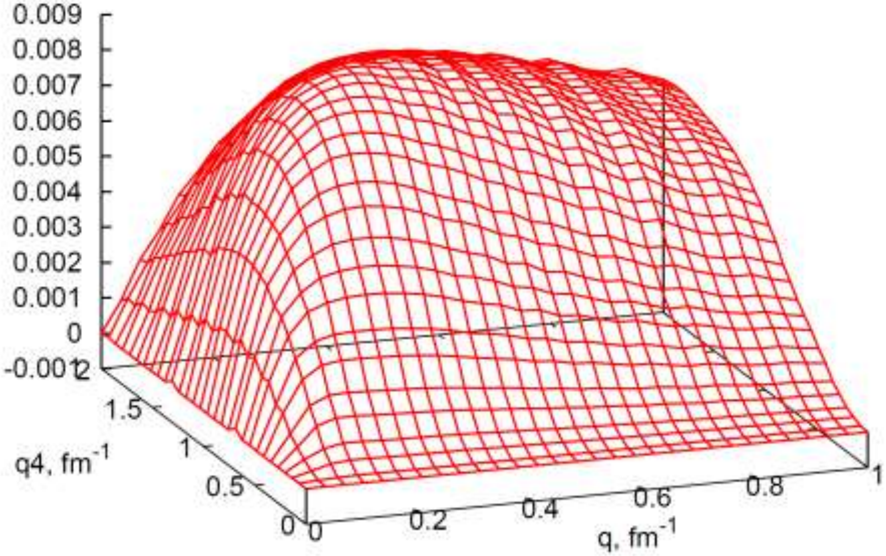


Real part

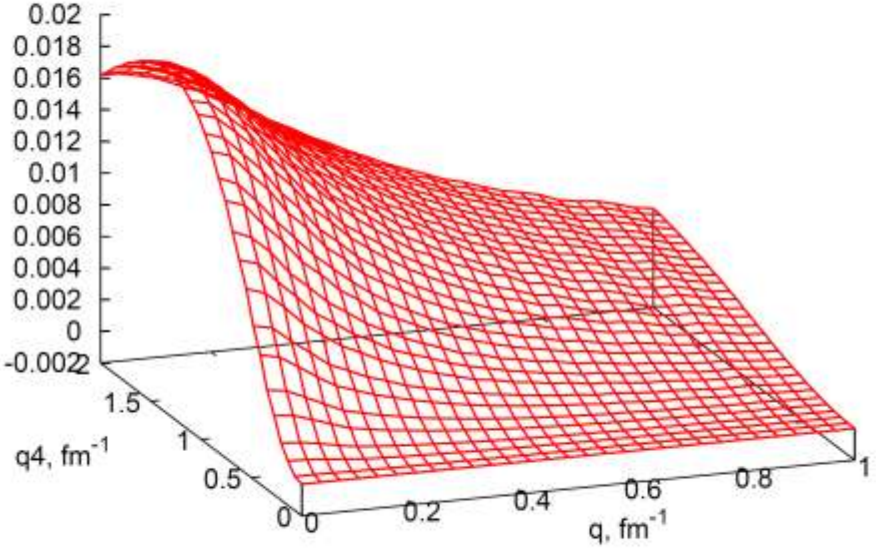


3D1 amplitude

Imaginary part

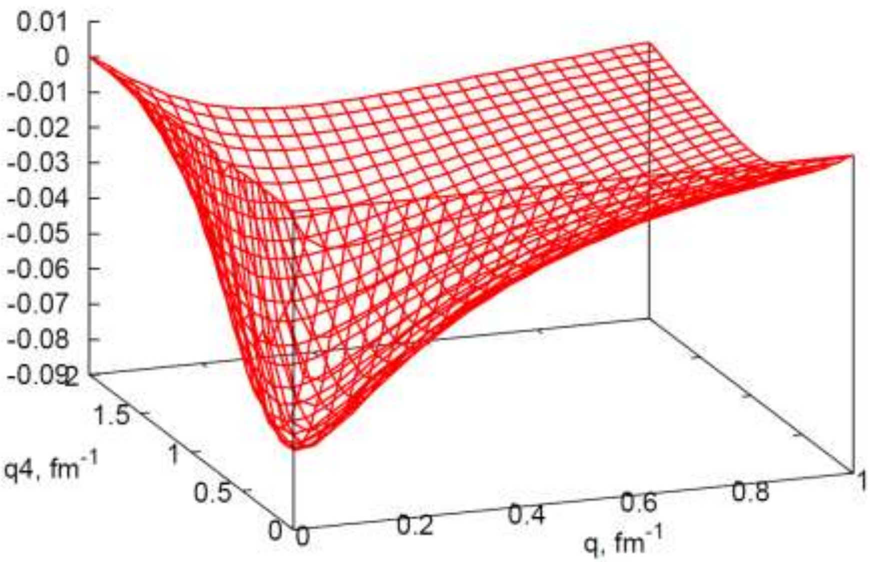


Real part

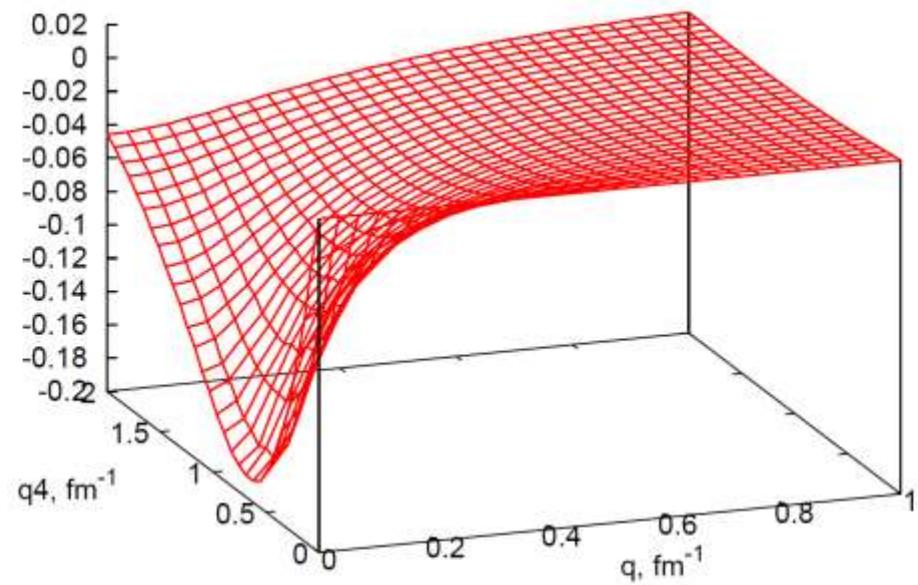


3P0 amplitude

Imaginary part

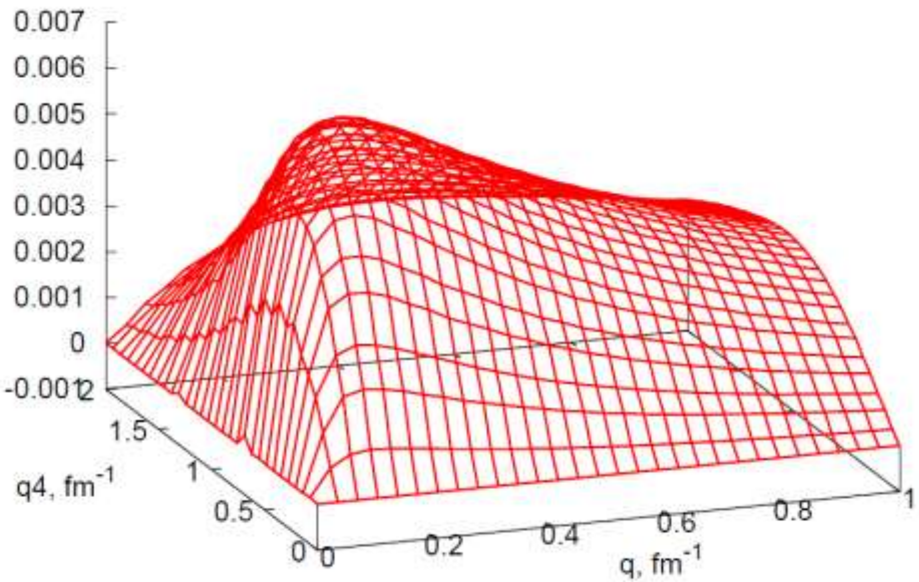


Real part

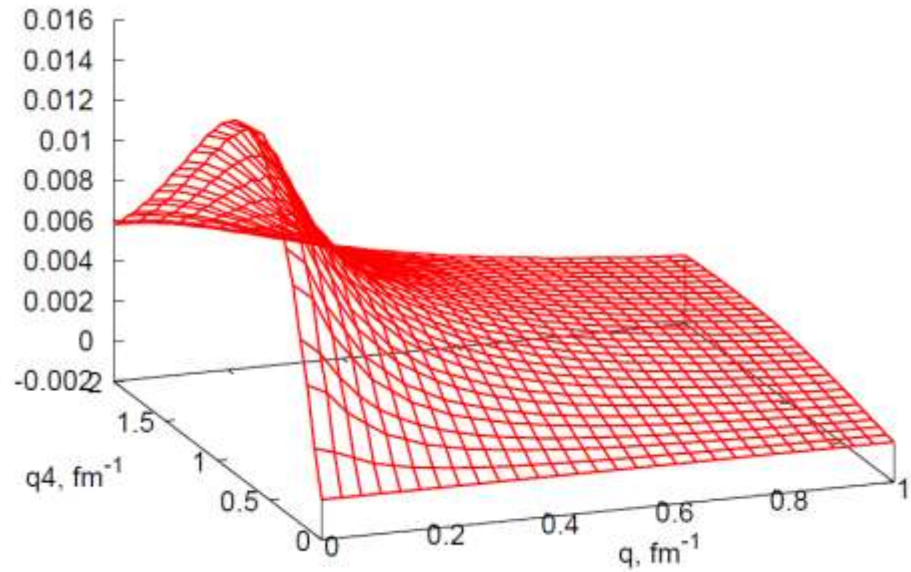


1P1 amplitude

Imaginary part

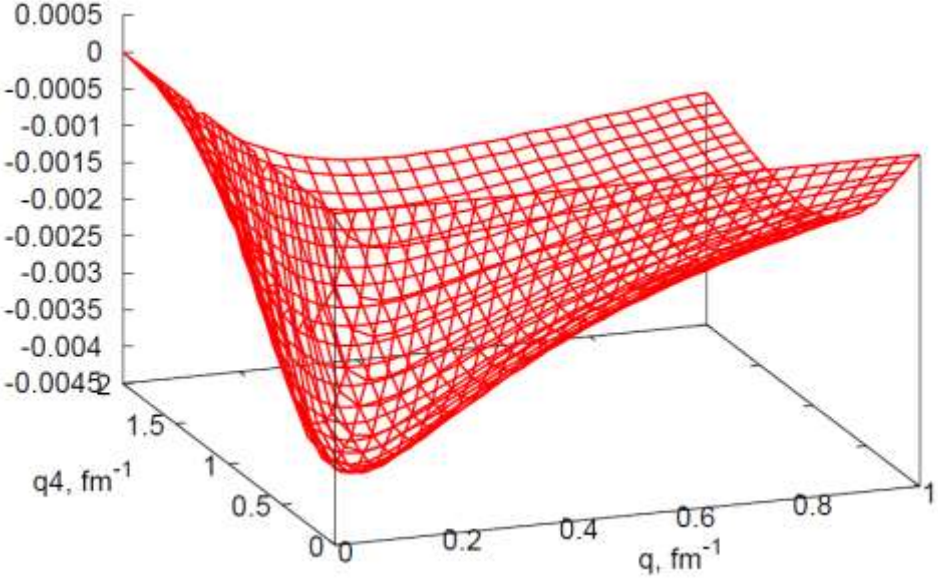


Real part

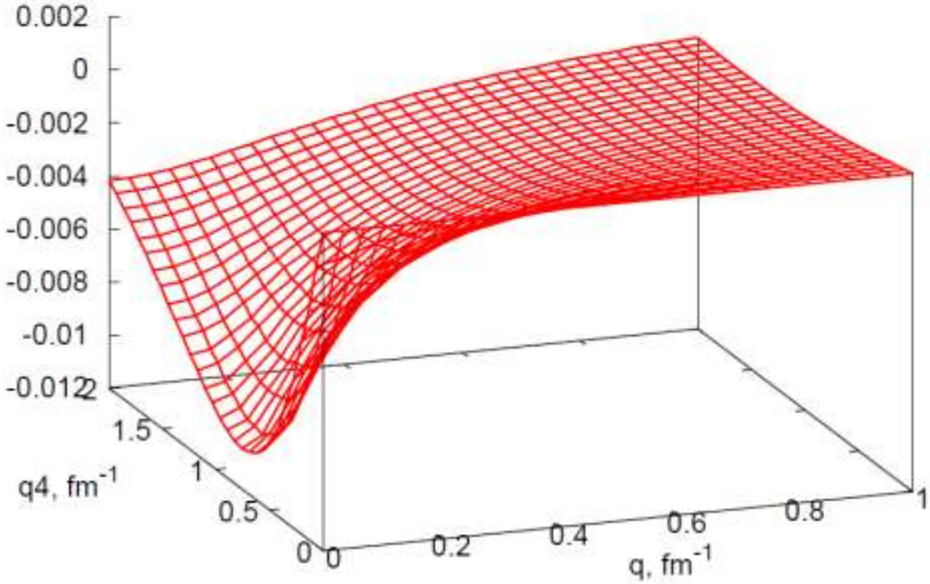


3P1 amplitude

Imaginary part



Real part



Summary

Triton [T = 3H = (nnp)] was investigated.

For this, a relativistic generalization of the Faddeev equation was applied.

As a two-particle t matrix, we used the solution of the Bethe-Salpeter equation.

The potential of NN interaction is taken in a separable form.

The system of integral equations describing T was solved by the iterative method.

The binding energy of T and the amplitudes of its S, P, and D states were calculated.

Form factors of the three-nucleus nucleus

$$2F_{ch}(^3He) = (2F^p_{ch} + F^n_{ch})F_1 - \frac{2}{3}(F^p_{ch} - F^n_{ch})F_2$$

$$F_{ch}(^3H) = (F^p_{ch} + 2F^n_{ch})F_1 + \frac{2}{3}(F^p_{ch} - F^n_{ch})F_2$$

$$F_1(Q) = \int dp_4 \int d\mathbf{p} \int dq_4 \int d\mathbf{q} G_1 G_2 G_3 G_3' \Psi^*_S(p_4, p, q_4, q)\Psi_S(p_4, p, q_4, |\mathbf{q} - \frac{2}{3}\mathbf{Q}|)$$

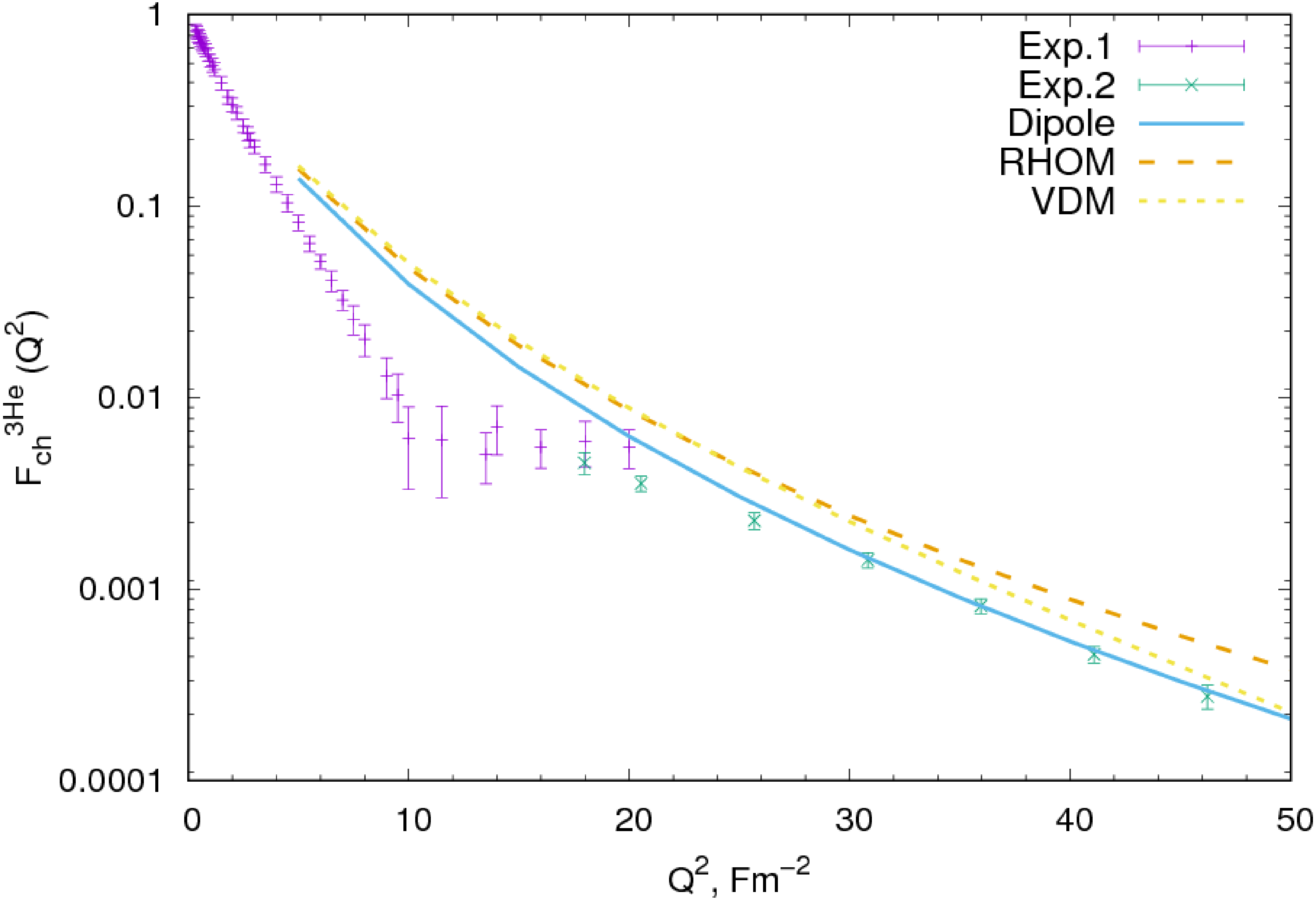
$$= 4\pi^2 \int dp_4 \int dp \int dq_4 \int dq \int_{-1}^{1} d[Cos(\mathbf{p}, \mathbf{q})] \int_{-1}^{1} d[Cos(\mathbf{q}, \mathbf{Q})] p^2 q^2$$

$$G_1 G_2 G_3 G_3' \Psi^*_S(p_4, p, q_4, q)\Psi_S(p_4, p, q_4, |\mathbf{q} - \frac{2}{3}\mathbf{Q}|)$$

$$F_2(Q) = -3 \int dp_4 \int d\mathbf{p} \int dq_4 \int d\mathbf{q} G_1 G_2 G_3 G_3' \Psi^*_S(p_4, p, q_4, q)\Psi_{S'}(p_4, p, q_4, |\mathbf{q} - \frac{2}{3}\mathbf{Q}|)$$

$$= -12\pi^2 \int dp_4 \int dp \int dq_4 \int dq \int_{-1}^{1} d[Cos(\mathbf{q}, \mathbf{Q})] p^2 q^2$$

$$C G_3 G_3' \Psi^*_S(p_4, p, q_4, q)\Psi_{S'}(p_4, p, q_4, \sqrt{q^2 + \frac{4}{9}Q^2 - \frac{4}{3}qQCos(\mathbf{q}, \mathbf{Q})})$$



Exp.1
Exp.2
Dipole
RHOM
VDM
1
0.1
0.01
0.001
0.0001
0
10
20
30
40
50
$F_{ch}^{3He}(Q^2)$
Q^2, Fm^{-2}

Bound state energy of Triton

Experiment: $E_{bs} = 8.48 MeV$

relativistic

Potential	only S -state	with D
GRAZ-II(1)	8.716	8.716
GRAZ-II(2)	8.298	8.298
GRAZ-II(3)	7.894	7.894
Paris-I	7.545	7.545

nonrelativistic

Potential	only S -state	with D
GRAZ-II(1)	8.372	8.334
GRAZ-II(2)	7.964	7.934
GRAZ-II(3)	7.569	7.548