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# Non-Markovian dynamics of fermionic and bosonic oscillators coupled to several heat baths

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# Introduction

- A small quantum mechanical systems are never completely isolated but interact with a large number of degrees of freedom of the surrounding macroscopic environment.
- In practice, a quantum system often couples to multiple reservoirs as is the case of cavity quantum electrodynamics, photon-ion interfaces, ion chain systems etc.
- The interest in considering fermionic baths is growing up due to the possibility of creating and manipulating fermionic systems in condensed matter, atomic and nuclear physics.

# Formalism

Hamiltonian of the whole system

$$H = H_c + \sum_{\lambda=1}^{N_b} H_\lambda + \sum_{\lambda=1}^{N_b} H_{c,\lambda} \,,$$

$$H_c = \hbar\omega a^\dagger a \,,$$

$$H_\lambda = \sum_i \hbar\omega_{\lambda,i} c_{\lambda,i}^\dagger c_{\lambda,i} \,,$$

$$H_{c,\lambda} = \sum_i \alpha_{\lambda,i} (a^\dagger + a)(c_{\lambda,i}^\dagger + c_{\lambda,i}).$$

# Formalism: Non-Markovian Langevin approach

Equations of motion

$$\frac{d}{dt}[a^\dagger(t)+a(t)] = i\omega[a^\dagger(t)-a(t)]$$

$$-i\frac{d}{dt}[a^\dagger(t)-a(t)] = \Omega[a^\dagger(t)+a(t)] + K(t)[a^\dagger(0)+a(0)] + \int_0^t d\tau\, K(t-\tau)\,\frac{d}{d\tau}[a^\dagger(\tau)+a(\tau)] + F(t),$$

$$K(t) = \sum_\lambda K_\lambda(t) = 4\sum_{\lambda,i}\frac{\alpha^2_{\lambda,i}}{\hbar^2\omega_{\lambda,i}}\cos(\omega_{\lambda,i}t),$$

$$F_\lambda(t) \equiv \sum_i[\mathcal{F}^+_{\lambda,i}(t)+\mathcal{F}_{\lambda,i}(t)] = \frac{2}{\hbar}\sum_i \alpha_{\lambda,i}[c^\dagger_{\lambda,i}(0)e^{i\omega_{\lambda,i}t} + c_{\lambda,i}(0)e^{-i\omega_{\lambda,i}t}],$$

$$\Omega = \omega - 4\sum_{\lambda,i}\frac{\alpha^2_{\lambda,i}}{\hbar^2\omega_{\lambda,i}}, \qquad \lambda = 1,\ldots,N_b.$$

# Formalism: Non-Markovian Langevin approach

Analytic expressions for occupation number

$$a^\dagger(t) = a^\dagger(0)A^*(t) + a(0)B(t) + \tilde{F}^\dagger(t):$$

$$A(t) = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2s - i[\Omega + \omega] - isK(s)}{s^2 + \omega\Omega + s\omega K(s)}\right\},$$

$$B(t) = \frac{i}{2}\mathcal{L}^{-1}\left\{\frac{\Omega - \omega + sK(s)}{s^2 + \omega\Omega + s\omega K(s)}\right\},$$

$$\tilde{F}(t) = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{[is + \omega]F(s)}{s^2 + \omega\Omega + s\omega K(s)}\right\}$$

$$= -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{[is + \omega]F_\lambda(s)}{s^2 + \omega\Omega + s\omega K(s)}\right\} \equiv \sum_\lambda \tilde{F}_\lambda(t),$$

$$d(s) \equiv s^2 + \omega\Omega + s\omega K(s) = 0.$$

# Formalism: Non-Markovian Langevin approach

Analytic expressions for occupation number

$$n_a(t) = n_a(0)|A(t)|^2 + [1 + \varepsilon n_a(0)]|B(t)|^2 + I_a(t),$$

$$I_a(t) = \langle\langle \tilde{F}^+(t)\tilde{F}(t)\rangle\rangle = \sum_\lambda \langle\langle \tilde{F}_\lambda^+(t)\tilde{F}_\lambda(t)\rangle\rangle \equiv \sum_\lambda I_a^{(\lambda)}(t).$$

$$\frac{dn_a(t)}{dt} = -2\lambda_a(t)n_a(t) + 2D_a(t).$$

$$\lambda_a(t) = -\frac{1}{2}\frac{d}{dt}\ln[|A(t)|^2 + \varepsilon|B(t)|^2], \quad D_a(t) = \lambda_a(t)[|B(t)|^2 + I_a(t)] + \frac{1}{2}\frac{d}{dt}[|B(t)|^2 + I_a(t)].$$

# The case of ohmic dissipation with lorenzian cutoffs

Spectral densities of heat-bath excitations $\rho_\lambda(w)$

$$\sum_i \cdots \to \int_0^\infty dw \rho_\lambda(w) \cdots$$

$$\frac{\alpha^2_{\lambda,i}}{\hbar^2 \omega_{\lambda,i}} \to \frac{\rho_\lambda(w)\alpha^2_{\lambda,w}}{\hbar^2 w} = \frac{1}{\pi}\alpha_\lambda \frac{\gamma_\lambda^2}{\gamma_\lambda^2 + w^2}.$$

$$K(t) = 2\sum_\lambda \alpha_\lambda \gamma_\lambda e^{-\gamma_\lambda t}.$$

# Solution of non-Markovian Langevin equations

$$\left[s^2 + \omega\Omega + 2s\omega\sum_{\nu=1}^{N_b}\alpha_\nu\gamma_\nu/(s+\gamma_\nu)\right]\prod_{\lambda=1}^{N_b}(s+\gamma_\lambda) = 0.$$

$$I_{\mathrm{a}}^{(\lambda)}(t) = \frac{\alpha_\lambda\gamma_\lambda^2}{\pi}\int_0^\infty dw\frac{w}{\gamma_\lambda^2 + w^2}\{n_a^{(\lambda)}(w)|M(w,t)|^2 + [1+\varepsilon n_a^{(\lambda)}(w)]|N(w,t)|^2\},$$

$$n_a^{(\lambda)}(\omega_{\lambda,i}) = [\exp(\hbar\omega_{\lambda,i}/kT_\lambda) - \varepsilon]^{-1},$$

$$N(w,t) = \sum_{k=0}^{N_0}\xi_k e^{s_k t}(is_k - \omega)\prod_{\lambda=1}^{N_b}(s_k+\gamma_\lambda), \qquad \xi_k = \prod_{i=0,i\neq k}^{N_0}\frac{1}{s_k - s_i},$$

$$M(w,t) = -\sum_{k=0}^{N_0}\xi_k e^{s_k t}(is_k + \omega)\prod_{\lambda=1}^{N_b}(s_k+\gamma_\lambda).$$

$$N_0 = N_b + 2,$$
$$k = 1,\ldots,N_0,$$
$$s_0 = -iw.$$

# Results

## Asymptotic occupation numbers

$$|A(t \to \infty)|^2 = |B(t \to \infty)|^2 = 0,$$

$$n_{\mathrm{a}}(t \to \infty) = \sum_{\lambda=1}^{N_b} \frac{\alpha_\lambda \gamma_\lambda^2}{\pi} \int_0^\infty dw \frac{w}{\gamma_\lambda^2 + w^2} \{[\omega + w]^2 n_{\mathrm{a}}^{(\lambda)}(w) + [\omega - w]^2 [1 + \varepsilon n_{\mathrm{a}}^{(\lambda)}(w)]\} \times \frac{\prod_{\mu=1}^{N_b} (\gamma_\mu^2 + w^2)}{\prod_{k=1}^{N_0} (s_k^2 + w^2)} .$$

In the Markovian weak-coupling limit:

$$n_{\mathrm{a}}(t \to \infty) = n_{\mathrm{a}}^{\mathrm{eff}}(\omega) = \frac{1}{g_0} \sum_{\lambda=1}^{N_b} \alpha_\lambda n_{\mathrm{a}}^{(\lambda)}(\omega),$$ where $g_0 = \sum_{\lambda=1}^{N_b} \alpha_\lambda$.

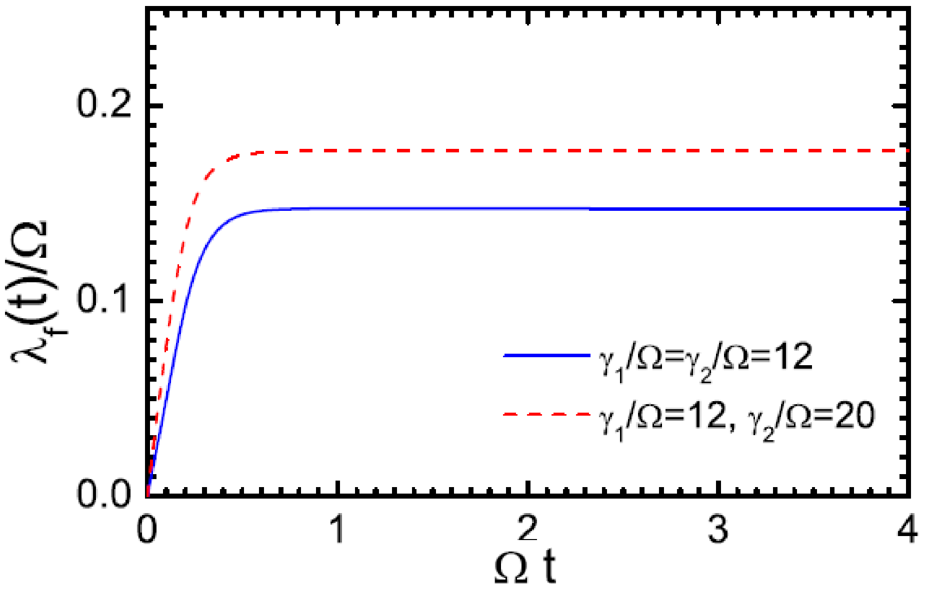
Expanding $n_{\mathrm{a}}^{(\lambda)}(\omega)$ in $\hbar\omega/(kT_\lambda)$, one can deduce that the system coupled to several heat-baths with different temperatures $T_\lambda$ , is equivalent to system coupled to one heat-bath with an effective coupling strength $g_0$, with an effective temperature $T$.

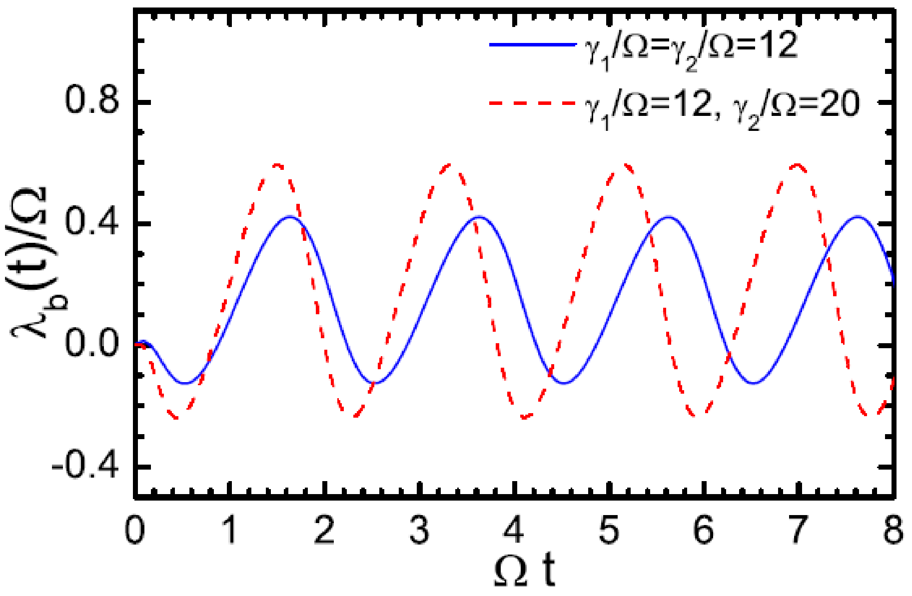
$$\frac{g_0}{T} = \sum_{\lambda=1}^{N_b} \frac{\alpha_\lambda}{T_\lambda}$$ for fermions,

$$g_0 T = \sum_{\lambda=1}^{N_b} \alpha_\lambda T_\lambda$$ for bosons.

# Results

Results of numerical calculations for a system coupled to two heat-baths

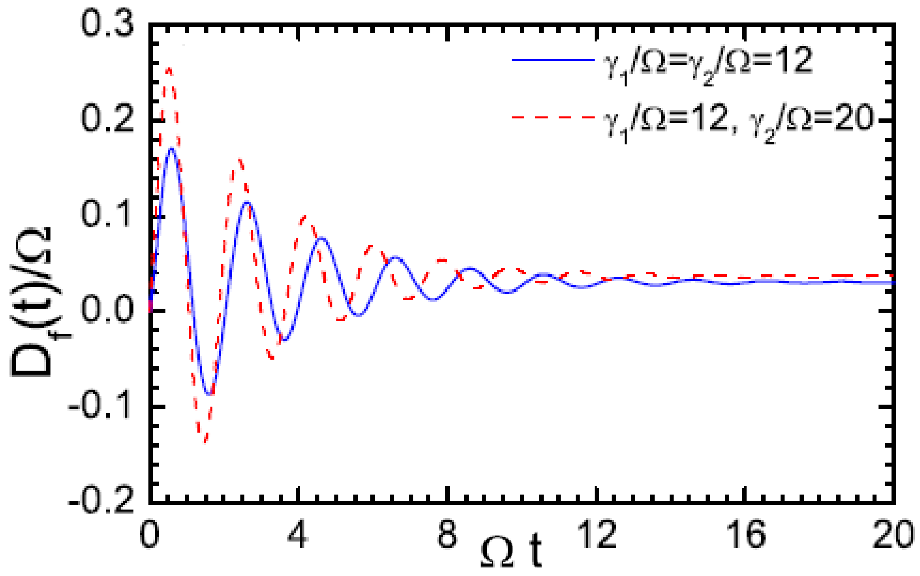


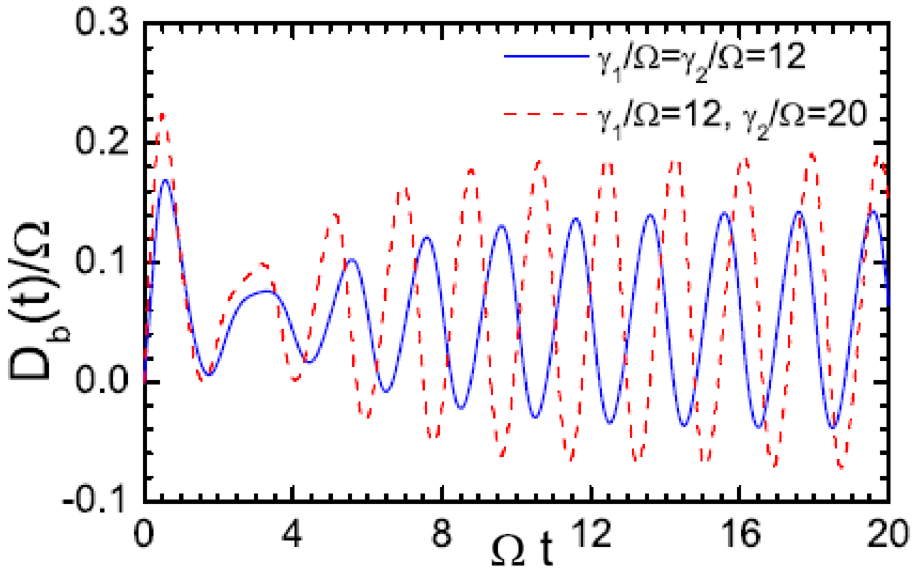




The calculated dependencies of the friction coefficients on time.
Here $\alpha_1 = \alpha_2 = 0.03.$

# Results

Results of numerical calculations for a system coupled to two heat-baths

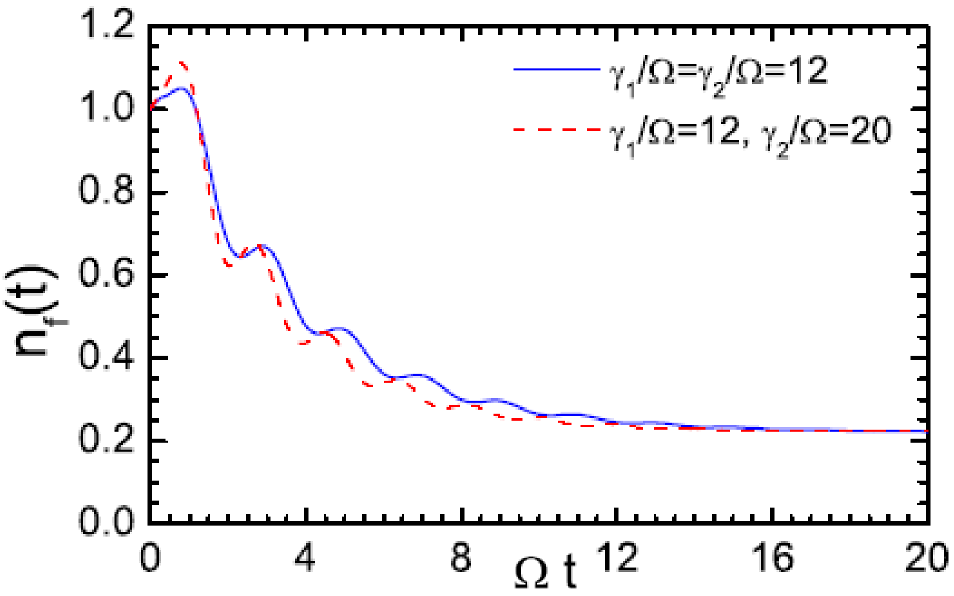


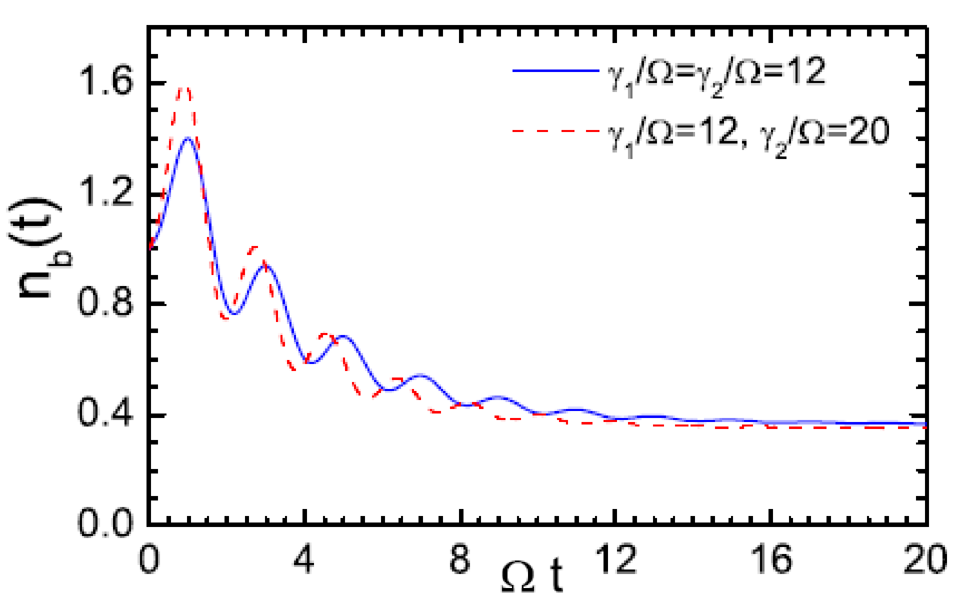




Diffusion coefficients.
Here $\alpha_1 = \alpha_2 = 0.03$, $kT_1/(\hbar\Omega) = kT_2/(\hbar\Omega) = 1.$

# Results

Results of numerical calculations for a system coupled to two heat-baths

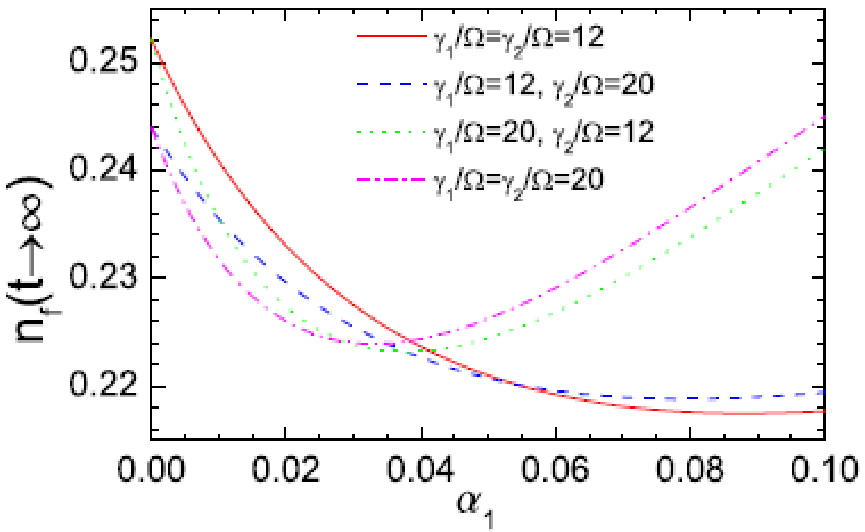


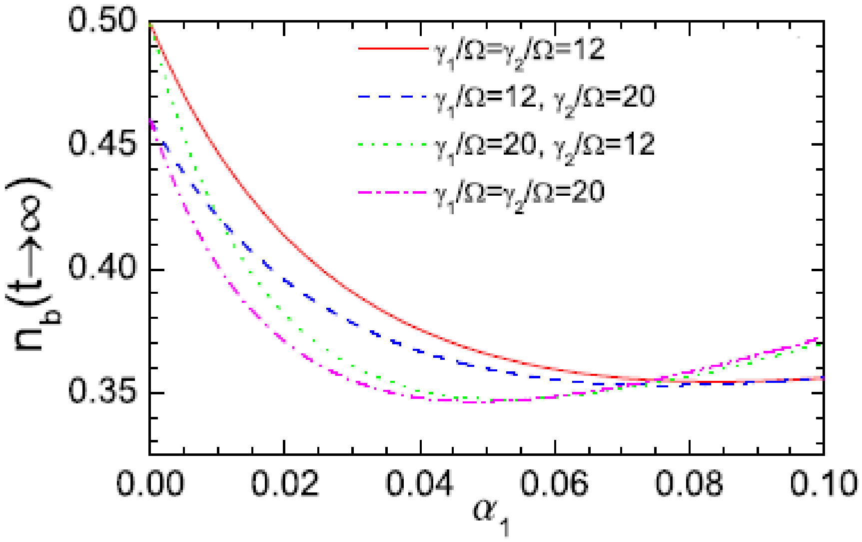




Dependencies of the average occupation numbers on time.
Here $n_a(0) = 1, \alpha_1 = \alpha_2 = 0.03, kT_1/(\hbar\Omega) = kT_2/(\hbar\Omega) = 1.$

# Results

Results of numerical calculations for a system coupled to two heat-baths




Dependencies of the asymptotic collective occupation numbers on coupling constant for different bandwidths. Here $\alpha_2 = 0.01, kT_1/(\hbar\Omega) = kT_2/(\hbar\Omega) = 1.$

# Summary

- The system with two heat baths is analytically reduced into the system with one heat bath.
- The values of the time-dependent occupation number of the bosonic oscillator oscillate with the larger amplitude than those in the case of fermionic oscillator.
- The relaxation time is almost independent of the statistical nature of the baths.
- Two independent non-Markovian bath modify in an non-additive manner the dynamics of a collective subsystem.

# Publications

"Non-Markovian dynamics of fermionic and bosonic systems coupled to several heat baths", Physical Review E, 2018, (Vol. 97, No. 3), pages 032134 1-12; DOI: 0.1103/PhysRevE.97.032134 (Co-authors V. V. Sargsyan, G. G. Adamian, N. V. Antonenko, and D. Lacroix)

Thank you for your attention!

$$\begin{aligned}
\frac{dn_a(t)}{dt} &= \frac{i}{\hbar}\sum_i \alpha_i[a(t)-a^\dagger(t)][b_i^\dagger(t)+b_i(t)] + \frac{i}{\hbar}\sum_j \beta_j[a(t)-a^\dagger(t)][c_j^\dagger(t)+c_j(t)] \\
&= \frac{i}{\hbar}\sum_i \alpha_i[b_i^\dagger(t)a(t)-a^\dagger(t)b_i(t)+a(t)b_i(t)-a^\dagger(t)b_i^\dagger(t)] \\
&+ \frac{i}{\hbar}\sum_j \beta_j[c_j^\dagger(t)a(t)-a^\dagger(t)c_j(t)+a(t)c_j(t)-a^\dagger(t)c_j^\dagger(t)]
\end{aligned}$$

$$\begin{aligned}
b_i^\dagger(t)+b_i(t) &= [e^{i\omega_i t}b_i^\dagger(0)+e^{-i\omega_i t}b_i(0)] + \frac{2\alpha_i}{\hbar\omega_i}(-[a^\dagger(t)+a(t)]+\cos(\omega_i t)[a^\dagger(0)+a(0)] \\
&+ \int_0^t d\tau\,\cos(\omega_i[t-\tau])\frac{d}{d\tau}[a^\dagger(\tau)+a(\tau)]), \\
c_j^\dagger(t)+c_j(t) &= [e^{i\omega_j t}c_j^\dagger(0)+e^{-i\omega_j t}c_j(0)] + \frac{2\beta_j}{\hbar\omega_j}(-[a^\dagger(t)+a(t)]+\cos(\omega_j t)[a^\dagger(0)+a(0)] \\
&+ \int_0^t d\tau\,\cos(\omega_j[t-\tau])\frac{d}{d\tau}[a^\dagger(\tau)+a(\tau)]).
\end{aligned}$$