Novel results for gluon TMDs in nucleon

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Plan

Introduction

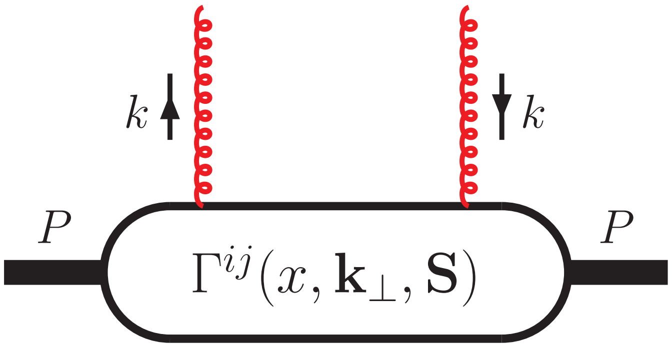
Gluon TMDs in LF QCD

- Using LFWFs for $g + 3q$ Fock component in nucleon we derive gluon TMDs
- TMDs — factorized product of two LFWFs and gluonic matrix encoding information about both T-even and T-odd TMDs
- TDMs obey Mulders-Rodrigues inequalities, small-x and large-x behavior
- New sum rules (SRs) involving TMDs

Summary

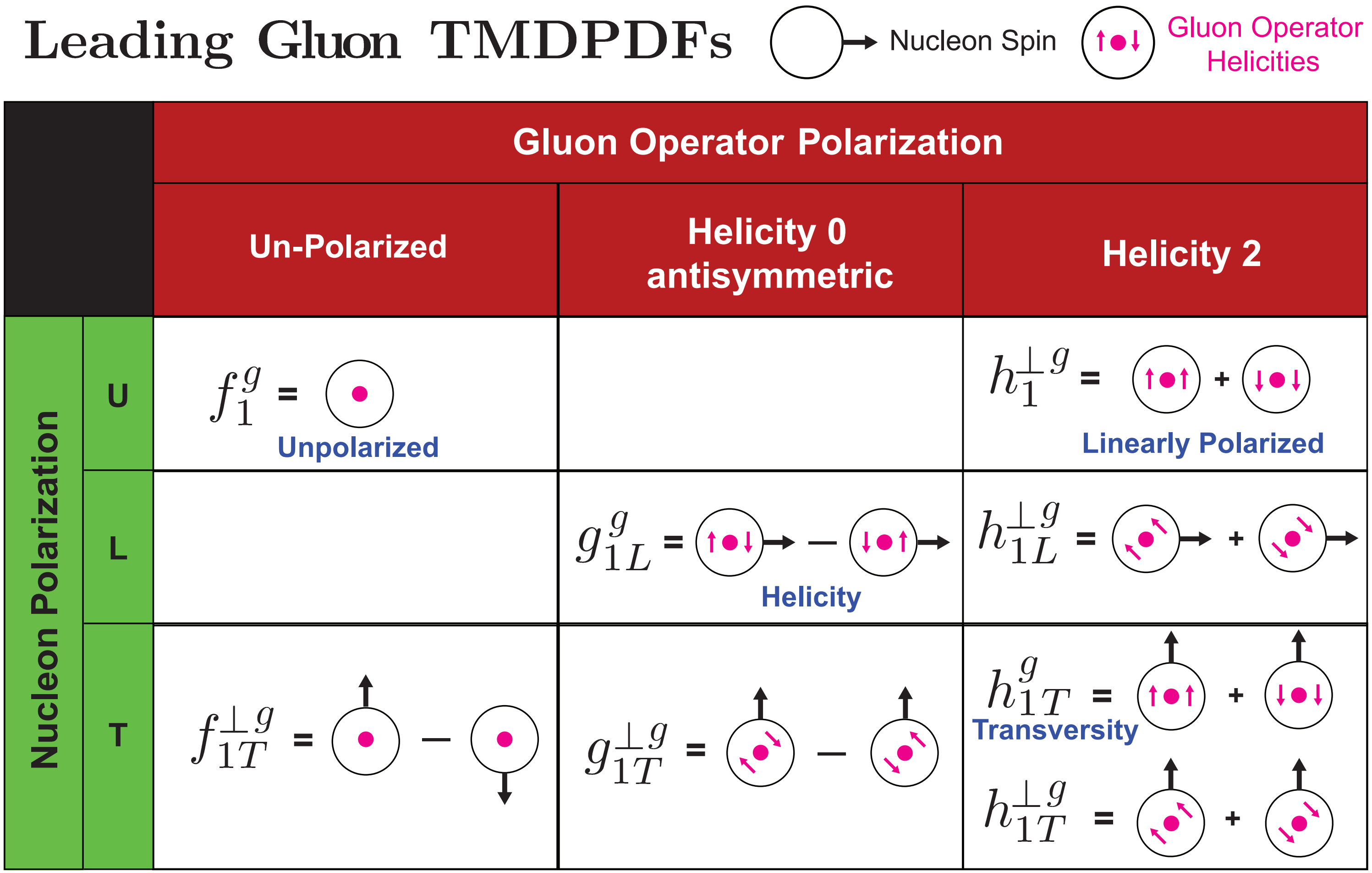
Gluon TMDs in QCD

- Mulders and Rodrigues, PRD63, 094021 (2001):
  Expansion of gluon correlator $\Gamma^{ij}(x, \mathbf{k}_\perp, \mathbf{S})$ in QCD



- i, j – gluon polarization indices; nucleon spin $S^\mu = (0, \mathbf{S})$ and $\mathbf{S} = (S_L, \mathbf{S}_T)$
- $S_L = \cos\theta$ and $\mathbf{S}_T = (\cos\phi\sin\theta, \sin\phi\sin\theta)$
- θ and ϕ – polar and azimuthal angles (orientation of the spin-vector $\mathbf{S}$)
- $k_\perp^\mu = (0, 0, \mathbf{k}_\perp) = \sqrt{\mathbf{k}_\perp^2}\,(0, 0, \cos\phi_k, \sin\phi_k)$ with $k_\perp^2 = -\mathbf{k}_\perp^2$,
  ϕ_k – azimuthal angle (orientation of $\mathbf{k}_\perp$ in the transverse plane).

Gluon TMDs in QCD



Picture taken from R. Boussarie et al. “TMD Handbook,” arXiv:2304.03302 [hep-ph]

Gluon TMDs in QCD

- $U(2)$ group acting in 2D transverse space:
  3 symmetric g_T^{ij}, η_T^{ij}, ξ_T^{ij} and 1 antisymmetric ϵ_T^{ij} transverse tensors

$$g_T^{ij} = -\delta^{ij} = \mathrm{diag}(-1,-1)$$

$$\eta_T^{ij} = \tau_3^{ij}\,\cos 2\phi_k + \tau_1^{ij}\,\sin 2\phi_k = \begin{pmatrix} \cos 2\phi_k & \sin 2\phi_k \\ \sin 2\phi_k & -\cos 2\phi_k \end{pmatrix}$$

$$\xi_T^{ij} = -\tau_3^{ij}\,\sin 2\phi_k + \tau_1^{ij}\,\cos 2\phi_k = \begin{pmatrix} -\sin 2\phi_k & \cos 2\phi_k \\ \cos 2\phi_k & \sin 2\phi_k \end{pmatrix}$$

$$\epsilon_T^{ij} = i\tau_2^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- In Mulders-Rodrigues: 5 tensors for the classification of gluon TMD, including

$$\omega_T^{ij} = 2\left[\xi_T^{ij}\,\mathbf{S}_T\mathbf{k}_\perp + \eta_T^{ij}\,e^{\mathbf{S}_T\mathbf{k}_\perp}\right] = 2\,|\mathbf{S}_T|\,|\mathbf{k}_\perp|\,\begin{pmatrix} -\sin\delta & \cos\delta \\ \cos\delta & \sin\delta \end{pmatrix}\,, \quad \delta = \phi + \phi_k$$

Gluon TMDs in QCD

- Exclusion of ω_T^{ij} from expansion of gluon correlator has several advantages:

  (i) Reduction the number of tensors $(5 \to 4)$ involved in the expansion

  (ii) $\omega_T^{\mu\nu}$ involves transverse spin, while other 4 tensors ($g_T^{\mu\nu}$, $\eta_T^{\mu\nu}$, $\xi_T^{\mu\nu}$, $\epsilon_T^{\mu\nu}$) are manifestly independent on S_T

  (iii) Substitution of $\omega_T^{\mu\nu}$ (linear combination of $\eta_T^{\mu\nu}$ and $\xi_T^{\mu\nu}$) gives separation of T-odd transversity TMDs with L-polarized gluons in T-polarized nucleon:

  - Symmetric transversity TMD $h_{1T}^{+g}(x, \mathbf{k}_\perp^2)$ standing at structure $\xi_T^{\mu\nu}\, \mathbf{S}_T \mathbf{k}_\perp$ symmetric under $\mathbf{S}_T \leftrightarrow \mathbf{k}_\perp$ interchange

  - Antisymmetric transversity TMD $h_{1T}^{-g}(x, \mathbf{k}_\perp^2)$ standing at $\eta_T^{\mu\nu}\, e^{\mathbf{S}_T \mathbf{k}_\perp}$ structure antisymmetric under $\mathbf{S}_T \leftrightarrow \mathbf{k}_\perp$ interchange

Gluon TMDs in QCD

- Mulders-Rodrigues: Gluon correlator $\Gamma^{ij}(x,\mathbf{k}_\perp,\mathbf{S}) = \sum_{P=U,L,T} \Gamma^{ij}_P(x,\mathbf{k}_\perp,\mathbf{S})$

- Γ^{ij}_U (U-polarized nucleon): U-polarized $f_1^g(x,\mathbf{k}_\perp^2)$ and L-polarized $h_1^{\perp g}(x,\mathbf{k}_\perp^2)$

$$\Gamma^{ij}_U = -g_T^{ij}\, f_1^g + \eta_T^{ij}\, h_1^{(1)\perp g}$$

- Γ^{ij}_L (L-polarized nucleon): C-polarized $g_{1L}^g(x,\mathbf{k}_\perp^2)$ and L-polarized $h_{1L}^{\perp g}(x,\mathbf{k}_\perp^2)$

$$\Gamma^{ij}_L(x,\mathbf{k}_\perp,\mathbf{S}) = -i\epsilon_T^{ij}\, S_L\, g_{1L}^g + \xi_T^{ij}\, S_L\, h_{1L}^{(1)\perp g}$$

- Γ^{ij}_T (T-polarized nucleon): U-polarized $f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2)$, C-polarized $g_{1T}^g(x,\mathbf{k}_\perp^2)$ and two L-polarized $h_{1T}^{+g}(x,\mathbf{k}_\perp^2)$ and $h_{1T}^{-g}(x,\mathbf{k}_\perp^2)$

$$\Gamma^{ij}_T = -g_T^{ij}\,\frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\, f_{1T}^{\perp g} - i\epsilon_T^{ij}\,\frac{\mathbf{S}_T\mathbf{k}_\perp}{M_N}\, g_{1T}^g + \xi_T^{ij}\,\frac{\mathbf{S}_T\mathbf{k}_\perp}{M_N}\, h_{1T}^{+g} + \eta_T^{ij}\,\frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\, h_{1T}^{-g}$$

$$\mathrm{TMD}^{(1/2)} = \frac{|\mathbf{k}_\perp|}{M_N}\mathrm{TMD}\,, \quad \mathrm{TMD}^{(n)} = \left[\frac{\mathbf{k}_\perp^2}{2M_N^2}\right]^n \mathrm{TMD}$$

Gluon TMDs in QCD

- Our decomposition

$$\Gamma_T^{ij} = -g_T^{ij}\frac{e^{\mathbf{S}_T \mathbf{k}_\perp}}{M_N} f_{1T}^{\perp g} - i\epsilon_T^{ij}\frac{\mathbf{S}_T \mathbf{k}_\perp}{M_N} g_{1T}^{g} + \xi_T^{ij}\frac{\mathbf{S}_T \mathbf{k}_\perp}{M_N} h_{1T}^{+g} + \eta_T^{ij}\frac{e^{\mathbf{S}_T \mathbf{k}_\perp}}{M_N} h_{1T}^{-g}$$

- Comparison with Mulders-Rodrigues (MR): our L-polarized gluon TMDs $h_{1T}^{\pm g}$ are related to the corresponding MR TMDs – linearity ΔH_T and pretzelosity $\Delta H_T^\perp$

$$h_{1T}^{\pm g}(x,\mathbf{k}_\perp^2) = -\frac{1}{2}\left[\Delta H_T(x,\mathbf{k}_\perp^2) \pm \frac{\mathbf{k}_\perp^2}{2M_N^2}\Delta H_T^\perp(x,\mathbf{k}_\perp^2)\right]$$

- Comparison with Boer et al: analogous sets $(h_{1T}^{g},\, h_{1T}^{\perp g})$ and $(h_1,\, h_{1T},\, h_{1T}^{\perp})$ used in PRL116, 122001 (2016) and JHEP10, 13 (2016)

$$h_{1T}^{+g}(x,\mathbf{k}_\perp^2) = h_{1T}^{g}(x,\mathbf{k}_\perp^2) - h_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) = \frac{1}{2}\left[h_1(x,\mathbf{k}_\perp^2) + \frac{\mathbf{k}_\perp^2}{2M_N^2} h_{1T}^{\perp}(x,\mathbf{k}_\perp^2)\right]$$

$$h_{1T}^{-g}(x,\mathbf{k}_\perp^2) = h_{1T}^{g}(x,\mathbf{k}_\perp^2) = \frac{1}{2}\left[h_1(x,\mathbf{k}_\perp^2) - \frac{\mathbf{k}_\perp^2}{2M_N^2} h_{1T}^{\perp}(x,\mathbf{k}_\perp^2)\right] = \frac{1}{2} h_{1T}(x,\mathbf{k}_\perp^2)$$

Gluon TMDs in QCD

- Full gluon correlation tensor in more compact form:

$$\begin{aligned}\Gamma^{ij}(x,\mathbf{k}_\perp,\mathbf{S}) &= -g_T^{ij}\,F_1^g(x,\mathbf{k}_\perp^2;\mathbf{S}_T) - i\epsilon_T^{ij}\,\mathbf{S}\,\mathbf{G}_1^g(x,\mathbf{k}_\perp^2) \\ &+ \eta_T^{ij}\,H_1^{(\eta)g}(x,\mathbf{k}_\perp^2;\mathbf{S}_T) + \xi_T^{ij}\,\mathbf{S}\,\mathbf{H}_1^{(\xi)g}(x,\mathbf{k}_\perp^2)\end{aligned}$$

where

$$F_1^g(x,\mathbf{k}_\perp^2;\mathbf{S}_T) = f_1^g(x,\mathbf{k}_\perp^2) + \frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) \qquad U-\text{polarized gluon}$$

$$\mathbf{G}_1^g(x,\mathbf{k}_\perp^2) = \left(g_{1L}^g(x,\mathbf{k}_\perp^2),\,\frac{\mathbf{k}_\perp}{M_N}g_{1T}^g(x,\mathbf{k}_\perp^2)\right) \qquad C-\text{polarized gluon}$$

$$H_1^{(\eta)g}(x,\mathbf{k}_\perp^2;\mathbf{S}_T) = h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) + \frac{e^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,h_{1T}^{-g}(x,\mathbf{k}_\perp^2) \qquad L-\text{polarized gluon}$$

$$\mathbf{H}_1^{(\xi)g}(x,\mathbf{k}_\perp^2) = \left(h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2),\,\frac{\mathbf{k}_\perp}{M_N}\,h_{1T}^{+g}(x,\mathbf{k}_\perp^2)\right) \qquad L-\text{polarized gluon}$$

and $\mathbf{S} = (S_L,\mathbf{S}_T)$

Gluon TMDs in QCD

- Following Mulders-Rodrigues expand the gluon tensor in the nucleon spin basis

$$\Gamma^{ii'}(x, \mathbf{k}_\perp, \mathbf{S}) = \sum_{\Lambda,\Lambda'} \rho_{\Lambda'\Lambda}(\mathbf{S})\, \Gamma^{ii'}_{\Lambda\Lambda'}(x, \mathbf{k}_\perp)\,, \qquad \rho(\mathbf{S}) = \frac{1}{2}\Big(\mathbf{1} + \mathbf{S}\,\sigma\Big)$$

- $4 \otimes 4$ matrix $\Gamma^{ii'}_{\Lambda\Lambda'}$ in the gluon-nucleon circular basis

$$\begin{pmatrix} F_1^+ & F_{1T}^+ & H_1^- & \Delta H_{1T} \\ \Big(F_{1T}^+\Big)^\dagger & F_1^- & H_{1T} & H_1^+ \\ \Big(H_1^-\Big)^\dagger & \Big(H_{1T}\Big)^\dagger & F_1^- & F_{1T}^- \\ \Big(\Delta H_{1T}\Big)^\dagger & \Big(H_1^+\Big)^\dagger & \Big(F_{1T}^-\Big)^\dagger & F_1^+ \end{pmatrix}$$

$$F_1^\pm = f_1^g \pm g_{1L}^g\,, \quad F_{1T}^\pm = \pm\frac{|\mathbf{k}_\perp|}{M_N}\, e^{-i\phi_k}\left[g_{1T}^g \pm i f_{1T}^{\perp g}\right],$$

$$H_1^\pm = -e^{-2i\phi_k}\left[h_1^{(1)\perp g} \pm i h_{1L}^{(1)\perp g}\right],$$

$$H_{1T} = \frac{i|\mathbf{k}_\perp|}{M_N}\, e^{-i\phi_k}\left[h_{1T}^{+g} + h_{1T}^{-g}\right], \quad \Delta H_{1T} = \frac{i|\mathbf{k}_\perp|}{M_N}\, e^{-3i\phi_k}\left[h_{1T}^{+g} - h_{1T}^{-g}\right]$$

Gluon TMDs in QCD

- Small-x behavior Boer et al, PRL116, 122001 (2016) and JHEP10, 13 (2016)
- U-polarized tensor

$$x\Gamma_U^{ij}(x,\mathbf{k}_\perp) \xrightarrow{x\to 0} \frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j}{M_N^2}\, e_U(\mathbf{k}_\perp^2) = -g_T^{ij}\frac{\mathbf{k}_\perp^2}{M_N^2}\, e_U(\mathbf{k}_\perp^2) + \eta_T^{ij}\frac{\mathbf{k}_\perp^2}{M_N^2}\, e_U(\mathbf{k}_\perp^2)$$

- Leading to the identity

$$\lim_{x\to 0} x f_1^g(x,\mathbf{k}_\perp^2) = \lim_{x\to 0} x h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) = \frac{\mathbf{k}_\perp^2}{2M_N^2}\, e_U(\mathbf{k}_\perp^2) = e_U^{(1)}(\mathbf{k}_\perp^2)$$

$e_U(\mathbf{k}_\perp^2)$ – scalar function defining U-part of g Wilson loop LF correlator at small x

- L-polarized tensor

$$x\Gamma_L^{ij}(x,\mathbf{k}_\perp) = 0$$

- Leading to the vanishing of the corresponding TMDs:

$$\lim_{x\to 0} x g_{1L}^g(x,\mathbf{k}_\perp^2) = 0\,, \qquad \lim_{x\to 0} x h_{1L}^{\perp g}(x,\mathbf{k}_\perp^2) = 0$$

Gluon TMDs in QCD

- T-polarized tensor

$$x\Gamma_T^{ij}(x,\mathbf{k}_\perp) \quad \xrightarrow{x\to 0} \quad \frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j}{M_N^2}\,\frac{\epsilon_T^{\mathbf{S}_T\mathbf{k}_\perp}}{M_N}\,e_T(\mathbf{k}_\perp^2)$$
$$= \quad \frac{\epsilon_T^{\mathbf{S}_T\mathbf{k}_\perp}}{2M_N}\left[-g_T^{ij}\,\frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_T(\mathbf{k}_\perp^2) + \eta_T^{ij}\,\frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_T(\mathbf{k}_\perp^2)\right],$$

$e_T(\mathbf{k}_\perp^2)$ – scalar function defining T-part of g Wilson loop LF correlator at small x

- It follows

$$\lim_{x\to 0} x f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) \quad = \quad \lim_{x\to 0} x h_{1T}^{-g}(x,\mathbf{k}_\perp^2) = \lim_{x\to 0} x h_1(x,\mathbf{k}_\perp^2)$$
$$= \quad -\frac{\mathbf{k}_\perp^2}{2M_N^2}\lim_{x\to 0} x h_{1T}^{\perp}(x,\mathbf{k}_\perp^2) = \frac{1}{2}\lim_{x\to 0} x h_{1T}(x,\mathbf{k}_\perp^2)$$
$$= \quad \frac{\mathbf{k}_\perp^2}{2M_N^2}\,e_T(\mathbf{k}_\perp^2) = e_T^{(1)}(\mathbf{k}_\perp^2)$$

and

$$\lim_{x\to 0} x h_{1T}^{+g}(x,\mathbf{k}_\perp^2) = 0$$

Gluon TMDs in LF QCD

- Following Brodsky-Hwang-Ma-Schmidt, NPB593, 311 (2001)
- We derive LFWFs $\psi^{\lambda_N}_{\lambda_g;\lambda_X}(x, \mathbf{k}_\perp)$ for bound state of gluon (g) and 3q spectator $X = (uud)$ with helicities $\lambda_N = \uparrow, \downarrow$, $\lambda_g = \pm 1$, $\lambda_X = \pm\frac{1}{2}$:

$$
\begin{aligned}
\psi^{\uparrow}_{+1+\frac{1}{2}}(x, \mathbf{k}_\perp) &= -\Big[\psi^{\downarrow}_{-1-\frac{1}{2}}(x, \mathbf{k}_\perp)\Big]^\dagger = \frac{k^1 - ik^2}{\kappa}\,\varphi^{(2)}(x, \mathbf{k}_\perp)\,, \\
\psi^{\uparrow}_{+1-\frac{1}{2}}(x, \mathbf{k}_\perp) &= +\Big[\psi^{\downarrow}_{-1+\frac{1}{2}}(x, \mathbf{k}_\perp)\Big]^\dagger = \varphi^{(1)}(x, \mathbf{k}_\perp^2)\,, \\
\psi^{\uparrow}_{-1+\frac{1}{2}}(x, \mathbf{k}_\perp) &= -\Big[\psi^{\downarrow}_{+1-\frac{1}{2}}(x, \mathbf{k}_\perp)\Big]^\dagger = -\frac{k^1 + ik^2}{\kappa}\,(1-x)\,\varphi^{(2)}(x, \mathbf{k}_\perp^2)\,, \\
\psi^{\uparrow}_{-1-\frac{1}{2}}(x, \mathbf{k}_\perp) &= \psi^{\downarrow}_{+1+\frac{1}{2}}(x, \mathbf{k}_\perp) = 0\,,
\end{aligned}
$$

$\varphi^{(1,2)}(x, \mathbf{k}_\perp)$ are expressed through the gluon PDF functions $G^{\pm}(x)$ as

$$
\varphi^{(1)} = \frac{4\pi}{\kappa}\sqrt{G^+(x) - \frac{G^-(x)}{(1-x)^2}}\,\exp\Big[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\Big]\,, \qquad \varphi^{(2)} = \frac{4\pi}{\kappa}\,\frac{\sqrt{G^-(x)}}{1-x}\,\exp\Big[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\Big]
$$

$\kappa \sim 350 - 500$ MeV – scale parameter.

Gluon TMDs in LF QCD

- $G^+ = G_{g\uparrow/N\uparrow}$ and $G^- = G_{g\downarrow/N\uparrow}$ – helicity-aligned and antialigned gluon PDFs.
- Gluon unpolarized $G = G^+ + G^-$ and polarized $\Delta G = G^+ - G^-$ PDFs.
- G and ΔG are expressed in terms of derived LFWFs $\psi^{\lambda_N}_{\lambda_g;\lambda_X}(x, \mathbf{k}_\perp)$

$$\begin{pmatrix} G(x) \\ \Delta G(x) \end{pmatrix} = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi^+_{+1+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 + |\psi^+_{+1-\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 \pm |\psi^+_{-1+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 \right]$$

- First calculation in QCD – Brodsky, Schmidt, PLB234, 144 (1990):

$$G^+(x) = N_g\,(1-x)^4\,(1+4x)/x, \quad G^-(x) = N_g\,(1-x)^6/x$$

$N_g = 0.8967$ fixed from 1st moment $\langle x_g \rangle = \int_0^1 dx\, x\, G(x) = (10/21)\, N_g$

- Lattice result: $\langle x_g \rangle = 0.427$ Alexandrou et al, PRD101, 094513 (2020)
- We are not strict to any explicit form of gluon PDFs and one can use results of world data analysis obey very important model-independent constraints

Gluon TMDs in LF QCD

- Gluon correlator $\Gamma_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)$ in LF QCD reads:

$$\Gamma_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) = \sum_{i=1}^{8} \Gamma^{(i)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)$$

$$\Gamma^{(i)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) = \psi^{*\Lambda_1}_{\lambda_1\lambda_X}(x,\mathbf{k}_\perp)\,\frac{G^{\lambda_1\lambda_2;\Lambda_1\Lambda_2;\lambda_X\lambda'_X;(i)}_{\lambda,\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)}{32\pi^3}\,\psi^{\Lambda_2}_{\lambda_2\lambda'_X}(x,\mathbf{k}_\perp)$$

  where $G^{(i)}$ are interaction kernels including both T-even and T-odd structures

- T-odd TMDs contain loop functions $R_{\mathrm{TMD}}(x,\mathbf{k}_\perp^2)$ encoding $g-3q$ rescattering

- Factorization

$$\psi^\dagger(x,\mathbf{k}_\perp)\int d^2\mathbf{k}'_\perp\, F_{\mathrm{TMD}}(x,\mathbf{k}_\perp,\mathbf{k}'_\perp)\,\psi(x,\mathbf{k}'_\perp) = \psi^\dagger(x,\mathbf{k}_\perp)\,R_{\mathrm{TMD}}(x,\mathbf{k}_\perp^2)\,\psi(x,\mathbf{k}_\perp)$$

Gluon TMDs in LF QCD

- Tensorial structures

$$G^{(1)} = \delta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,\delta_{\lambda_1\lambda_2}\,\delta_{\Lambda_2\Lambda_1}\,\delta_{\lambda_X\lambda'_X}$$

$$G^{(2)} = \tau^3_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,\tau^3_{\lambda_1\lambda_2}\,\sigma^3_{\Lambda_2\Lambda_1}\,\delta_{\lambda_X\lambda'_X}$$

$$G^{(3)} = \tau^3_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,\tau^3_{\lambda_1\lambda_2}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda_2\Lambda'}}{M_N}\,\delta_{\lambda_X\lambda'_X}$$

$$G^{(4)} = \eta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,\eta_{\lambda_1\lambda_2}\,\delta_{\Lambda_2\Lambda_1}\,\delta_{\lambda_X\lambda'_X}$$

$$G^{(5)} = \xi_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,(\tau^3\xi)_{\lambda_1\lambda_2}\,\sigma^3_{\Lambda_2\Lambda_1}\,\tau^3_{\lambda_X\lambda'_X}\;iR_{h^g_{1L}}(x,\mathbf{k}^2_\perp)$$

$$G^{(6)} = \xi_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,(\xi\tau^3)_{\lambda_1\lambda_2}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda_2\Lambda_1}}{M_N}\,\tau^3_{\lambda_X\lambda'_X}\;iR_{h^{+g}_{1T}}(x,\mathbf{k}^2_\perp)$$

$$G^{(7)} = \eta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\eta_{\lambda_1\lambda_2}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp}\sigma^3)_{\Lambda_2\Lambda_1}}{M_N}\tau^3_{\lambda_X\lambda'_X}\;iR_{h^{-g}_{1T}}(x,\mathbf{k}^2_\perp)$$

$$G^{(8)} = \delta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\delta_{\lambda_1\lambda_2}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp}\sigma^3)_{\Lambda_2\Lambda_1}}{M_N}\tau^3_{\lambda_X\lambda'_X}\;iR_{f^{\perp g}_{1T}}(x,\mathbf{k}^2_\perp)$$

Gluon TMDs in LF QCD

- Tensorial structures generate eight leading-twist gluon TMDs

$$
\begin{aligned}
\delta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,f_1^g(x,\mathbf{k}_\perp^2) &= \Gamma^{(1)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\tau^3_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,g^g_{1L}(x,\mathbf{k}_\perp^2) &= \Gamma^{(2)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\tau^3_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,g^g_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(3)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\eta_{\lambda\lambda'}\,\delta_{\Lambda\Lambda'}\,h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \Gamma^{(4)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\xi_{\lambda\lambda'}\,\sigma^3_{\Lambda\Lambda'}\,h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \Gamma^{(5)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\xi_{\lambda\lambda'}\,\frac{(\sigma\mathbf{k}_\perp)_{\Lambda\Lambda'}}{M_N}\,h^{+g}_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(6)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\eta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\,h^{-g}_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(7)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp) \\
\delta_{\lambda\lambda'}\,\frac{(\epsilon^{\sigma\mathbf{k}_\perp})_{\Lambda\Lambda'}}{M_N}\,f^g_{1T}(x,\mathbf{k}_\perp^2) &= \Gamma^{(8)}_{\lambda\lambda';\Lambda\Lambda'}(x,\mathbf{k}_\perp)
\end{aligned}
$$

Gluon TMDs in LF QCD

- Analytical expressions for the gluon TMDs in terms of LFWFs are:
- T-even TMDs

$$f_1^g(x,\mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\left[\left(\varphi^{(1)}(x,\mathbf{k}_\perp^2)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left[1+(1-x)^2\right]\left(\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right)^2\right]$$

$$g_{1L}^g(x,\mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\left[\left(\varphi^{(1)}(x,\mathbf{k}_\perp^2)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left[1-(1-x)^2\right]\left(\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right)^2\right]$$

$$g_{1T}^g(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,(1-x)$$

$$h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\frac{\mathbf{k}_\perp^2}{M_N^2}\left[\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right]^2(1-x)$$

- T-odd TMDs

$$h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\frac{\mathbf{k}_\perp^2}{M_N^2}\left[\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right]^2(1-x)\,R_{h_{1L}^{(1)\perp g}}(x,\mathbf{k}_\perp^2)$$

$$h_{1T}^{\pm g}(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,R_{h_{1T}^{\pm g}}(x,\mathbf{k}_\perp^2)$$

$$f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,(1-x)\,R_{f_{1T}^{\perp g}}(x,\mathbf{k}_\perp^2)$$

Gluon TMDs in LF QCD

- T-odd in terms of T-even without referring to specific choice of $\varphi^{(1,2)}(x,\mathbf{k}_\perp^2)$

$$h_{1L}^{(1)\perp g}(x,\mathbf{k}_\perp^2) = \frac{f_1^g(x,\mathbf{k}_\perp^2) - g_{1L}^g(x,\mathbf{k}_\perp^2)}{1-x}\, R_{h_{1L}^{(1)\perp g}}(x,\mathbf{k}_\perp^2)$$
$$= h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)\, R_{h_{1L}^{(1)\perp g}}(x,\mathbf{k}_\perp^2)$$

$$h_{1T}^{\pm g}(x,\mathbf{k}_\perp^2) = \frac{g_{1T}^g(x,\mathbf{k}_\perp^2)}{1-x}\, R_{h_{1T}^{\pm g}}(x,\mathbf{k}_\perp^2)$$

$$f_{1T}^{\perp g}(x,\mathbf{k}_\perp^2) = g_{1T}^g(x,\mathbf{k}_\perp^2)\, R_{f_{1T}^{\perp g}}(x,\mathbf{k}_\perp^2)$$

Gluon TMDs in LF QCD

- Next, using our parametrization for the LFWFs one gets

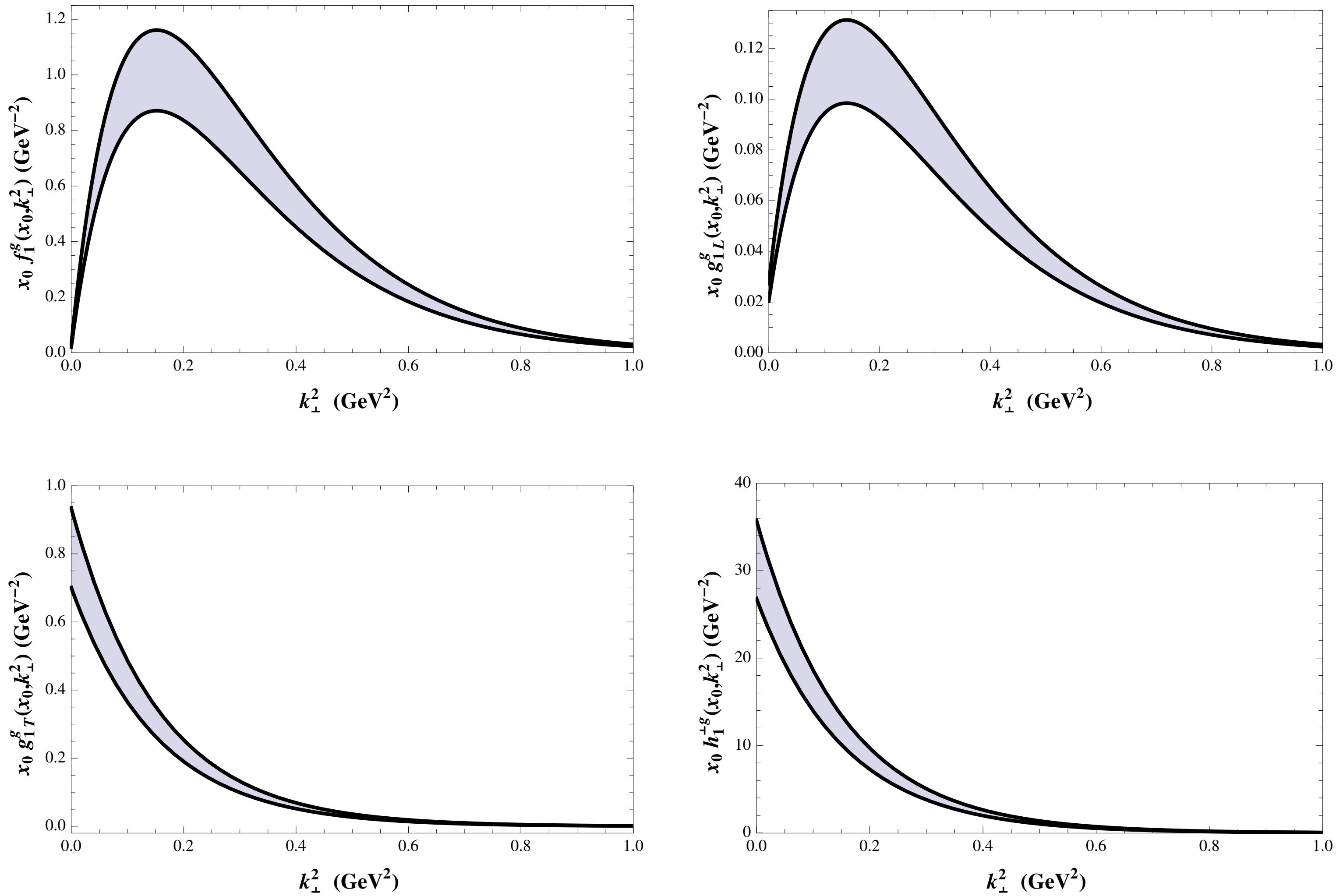
$$
\begin{aligned}
f_1^g(x,\mathbf{k}_\perp^2) &= \frac{1}{\pi\kappa^2}\left[G(x) + G^-(x)\,\alpha_+(x)\left(\frac{\mathbf{k}_\perp^2}{\kappa^2}-1\right)\right]\,\exp\!\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right] \\
g_{1L}^g(x,\mathbf{k}_\perp^2) &= \frac{1}{\pi\kappa^2}\left[\Delta G(x) + G^-(x)\,\alpha_-(x)\left(\frac{\mathbf{k}_\perp^2}{\kappa^2}-1\right)\right]\,\exp\!\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right] \\
g_{1T}^g(x,\mathbf{k}_\perp^2) &= \frac{M_N}{\pi\kappa^3}\,\sqrt{G^2(x)-\Delta G^2(x)}\;\beta(x)\,\exp\!\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right] \\
h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) &= \frac{\mathbf{k}_\perp^2}{\pi\kappa^4}\,\frac{G(x)-\Delta G(x)}{1-x}\;\exp\!\left[-\frac{\mathbf{k}_\perp^2}{\kappa^2}\right]
\end{aligned}
$$

where

$$
\alpha_\pm(x) = \frac{1\pm(1-x)^2}{(1-x)^2}\,, \quad \beta(x) = \sqrt{1-\frac{G^-(x)}{G^+(x)(1-x)^2}}
$$

Gluon TMDs: selected results

- T-even $x\,\mathrm{TMD}(x, \mathbf{k}_\perp^2)$ at $x = 0.1$ and for $\kappa = 380 \pm 30$ MeV
- Very good agreement with Pavia group, Bacchetta et al, EPJC80, 733 (2020)



Nucleon EM Form Factors: selected results

- Nucleon Dirac and Pauli FF $F_{1,2}^N$ ($N = p, n$) are related with valence quark distributions $F_{1,2}^q$ ($q = u, d$) in nucleons as

$$F_i^{p(n)}(Q^2) = \frac{2}{3} F_i^{u(d)}(Q^2) - \frac{1}{3} F_i^{d(u)}(Q^2) \,.$$

- LF representation for the Dirac and Pauli quark FF:
Gutsche, Lyubovitskij, Schmidt, Eur. Phys. J. C **77**, 86 (2017)

$$F_1^q(Q^2) \;=\; \int\limits_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Big[\psi^{+\,*}_{+q}(x, \mathbf{k}'_\perp) \psi^{+}_{+q}(x, \mathbf{k}_\perp) + \psi^{+\,*}_{-q}(x, \mathbf{k}'_\perp) \psi^{+}_{-q}(x, \mathbf{k}_\perp) \Big]$$

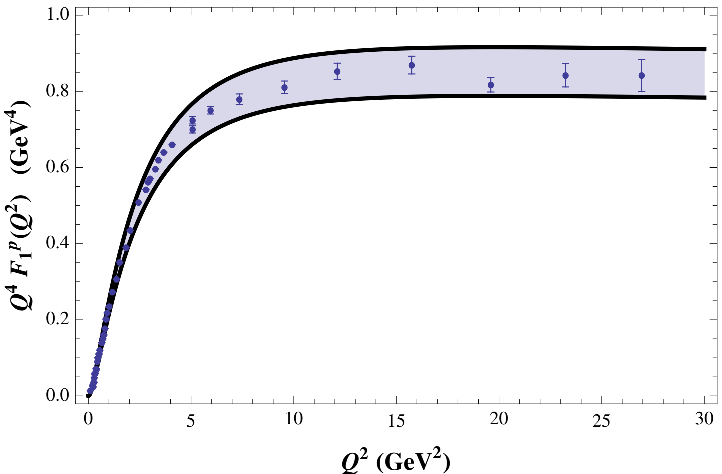
$$F_2^q(Q^2) \;=\; -\frac{2M_N}{q^1 - iq^2} \int\limits_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Big[\psi^{+\,*}_{+q}(x, \mathbf{k}'_\perp) \psi^{-}_{+q}(x, \mathbf{k}_\perp)$$

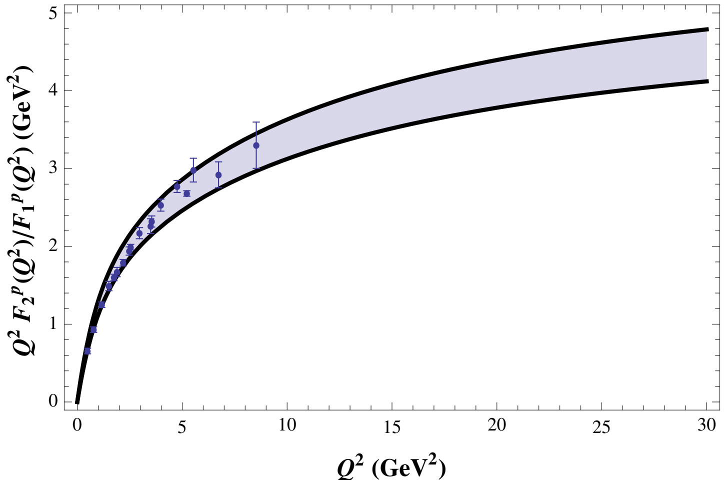
$$+ \quad \psi^{+\,*}_{-q}(x, \mathbf{k}'_\perp) \psi^{-}_{-q}(x, \mathbf{k}_\perp) \Big]$$

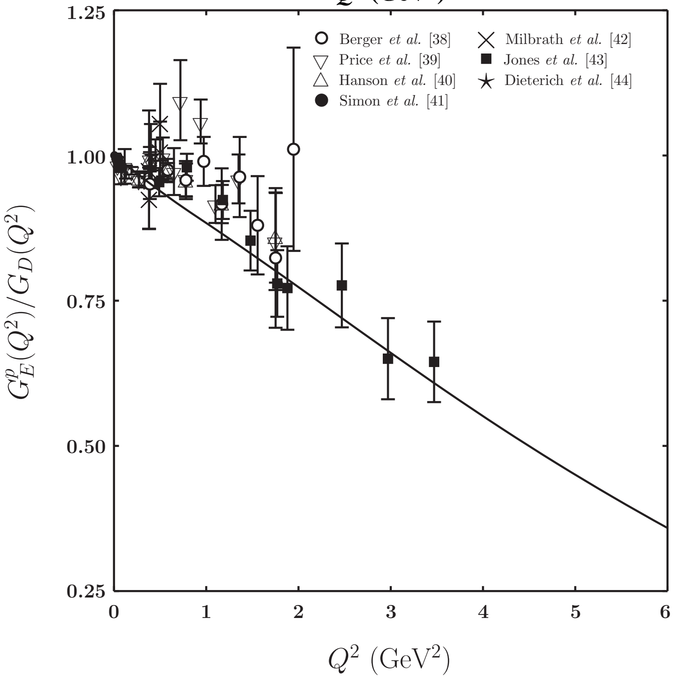
where $\mathbf{k}'_\perp = \mathbf{k}_\perp + \mathbf{q}_\perp (1 - x)$ is the transverse momentum of the recoiled nucleon; $\mathbf{q}_\perp$ is the transverse momentum of the photon.

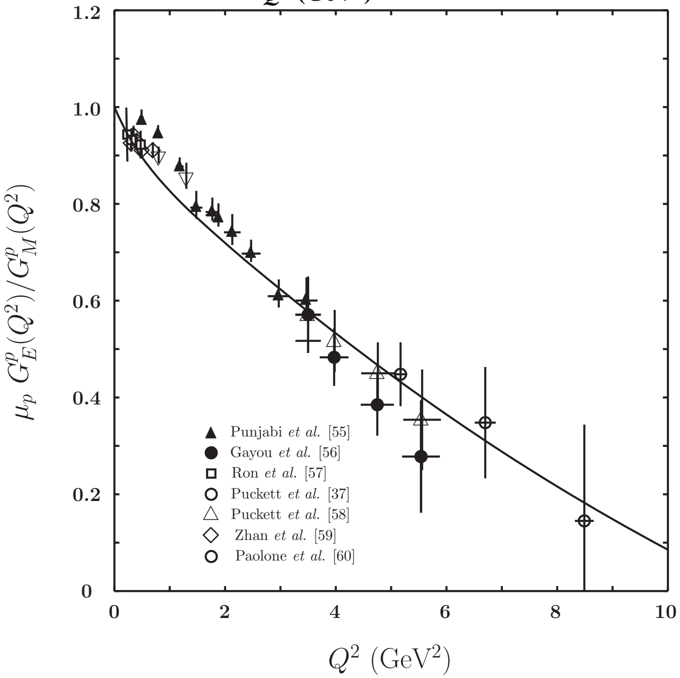
Nucleon EM: selected results

- $Q^4 F_1^p(Q^2)$, ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$, ratio of Sachs FF









Sum Rules for TMDs

- Using analytical expressions for the gluon T-even TMDs in terms of LFWFs

$$f_1^g(x,\mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\left[\left(\varphi^{(1)}(x,\mathbf{k}_\perp^2)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left[1+(1-x)^2\right]\left(\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right)^2\right]$$

$$g_{1L}^g(x,\mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\left[\left(\varphi^{(1)}(x,\mathbf{k}_\perp^2)\right)^2 + \frac{\mathbf{k}_\perp^2}{M_N^2}\left[1-(1-x)^2\right]\left(\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right)^2\right]$$

$$g_{1T}^g(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\varphi^{(1)}(x,\mathbf{k}_\perp^2)\,\varphi^{(2)}(x,\mathbf{k}_\perp^2)\,(1-x)$$

$$h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2) = \frac{1}{8\pi^3}\,\frac{\mathbf{k}_\perp^2}{M_N^2}\left[\varphi^{(2)}(x,\mathbf{k}_\perp^2)\right]^2(1-x)$$

- Two sum rules for T-even TMDs without referring to explicit form of $\varphi^{(1,2)}(x,\mathbf{k}_\perp^2)$

$$\left[f_1^g(x,\mathbf{k}_\perp^2)\right]^2 = \left[g_{1L}^g(x,\mathbf{k}_\perp^2)\right]^2 + \left[g_{1T}^{(1/2)g}(x,\mathbf{k}_\perp^2)\right]^2 + \left[h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)\right]^2$$

$$f_1^g(x,\mathbf{k}_\perp^2) - g_{1L}^g(x,\mathbf{k}_\perp) = (1-x)\,h_1^{(1)\perp g}(x,\mathbf{k}_\perp^2)$$

- Square of the unpolarized TMD = Sum of the squares of three polarized TMDs

- These two SR are derived at α_s^0

Sum Rules for TMDs

- Consistent with Mulders-Rodrigues positivity bounds

$$\sqrt{\left[g_{1L}^{g}(x,\mathbf{k}_\perp^2)\right]^2 + \left[g_{1T}^{(1/2)g}(x,\mathbf{k}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{k}_\perp^2)$$

$$\sqrt{\left[g_{1L}^{g}(x,\mathbf{k}_\perp^2)\right]^2 + \left[h_{1}^{(1)\perp g}(x,\mathbf{k}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{k}_\perp^2)$$

$$\sqrt{\left[g_{1T}^{(1/2)g}(x,\mathbf{k}_\perp^2)\right]^2 + \left[h_{1}^{(1)\perp g}(x,\mathbf{k}_\perp^2)\right]^2} \leq f_1^g(x,\mathbf{k}_\perp^2)$$

Sum Rules for TMDs

- Based on the SR derived for T-even gluon TMDs, we make a conjecture that there should two additional SRs involving T-odd gluon TMDs, valid at α_s and α_s^2

- We conjecture that the derived SRs are consequence of the condition

$$\det\left[\Gamma_{\lambda\lambda';\Lambda\Lambda'}\right] = 0$$

signaling that the gluon TMDs are not independent and are related via SRs

- 3 SRs at orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s^2)$

- From $\det\left[\Gamma_{\lambda\lambda';\Lambda\Lambda'}\right] = 0$ follows the condition

$$\left[R_0 + 2R_1 + R_2\right]\left[R_0 - 2R_1 + R_2\right] = 0$$

- R_0, R_1, R_2 are the combibations of TMDs at orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$

Sum Rules for TMDs

- 3 Sum Rules

$$R_0 = \left[g_{1L}^{g}\right]^2 + \left[g_{1T}^{(1/2)g}\right]^2 + \left[h_{1}^{(1)\perp g}\right]^2 - \left[f_{1}^{g}\right]^2 = 0$$

$$R_1 = f_1^g\, h_{1T}^{(1/2)-g} + g_{1L}^g\, h_{1T}^{(1/2)+g} - g_{1T}^{(1/2)g}\, h_{1L}^{(1)\perp g} - h_1^{(1)\perp g}\, f_{1T}^{(1/2)\perp g} = 0$$

$$R_2 = \left[f_{1T}^{(1/2)\perp g}\right]^2 + \left[h_{1L}^{(1)\perp g}\right]^2 + \left[h_{1T}^{(1/2)+g}\right]^2 - \left[h_{1T}^{(1/2)-g}\right]^2 = 0$$

- 1st SR $R_0 = 0$ is exactly our SR involving four T-even TMDs

- 2nd SR $R_1 = 0$ couples T-even and T-odd TMDs

- 3rd SR $R_2 = 0$ involves only T-odd TMDs

Small-x behavior of TMDs

- Another amazing result: SRs $R_i = 0$ at $x \to 0$ reduce to QCD results (Boer et al)
- In particular, at small x we get:

$$f_1^g = h_1^{(1)\perp g}, \quad h_{1T}^{(1/2)-g} = f_{1T}^{(1/2)\perp g}$$

$$g_{1L}^g = g_{1T}^g = h_{1T}^{(1/2)+g} = 0\,.$$

- Droping vanishing TMDs, the SRs $R_i = 0$ are simplified at small x as

$$\begin{aligned} R_0 &= \left[h_1^{(1)\perp g}\right]^2 - \left[f_1^g\right]^2 = 0 \\ R_1 &= f_1^g\, h_{1T}^{(1/2)-g} - h_1^{(1)\perp g}\, f_{1T}^{(1/2)\perp g} = 0 \\ R_2 &= \left[f_{1T}^{(1/2)\perp g}\right]^2 - \left|h_{1T}^{(1/2)-g}\right]^2 = 0 \end{aligned}$$

- We establish small-x relations/behavior of our TMDs consistent with QCD looking at their expressions at $x \to 0$

Large-x behavior of TMDs

- Large-x scaling: $f_1^g \sim g_{1L}^g \sim (1-x)^4, \quad g_{1T}^{(1/2)g} \sim h_1^{(1)\perp g} \sim (1-x)^5$
- Scaling of $f_{1T}^{(1/2)\perp g}$ and $h_{1L}^{(1)\perp g}$ is similar to $g_{1T}^{(1/2)g}$, up to corresponding loop factors $R_{h_{1L}^{(1)\perp g}}\Big|_{x\to 1}$ and $R_{f_{1T}^{\perp g}}\Big|_{x\to 1}$, which are expected to be constants or power of $(1-x)$
- Scaling of $h_{1T}^{(1/2)\pm g}$ to f_1^g up to corresponding loop factor $R_{h_{1T}^{\pm g}}\Big|_{x\to 1}$, which is expected to be constant or power of $(1-x)$
- At large x SRs are simplified to

$$
\begin{aligned}
R_0 &= \left[g_{1L}^g\right]^2 - \left[f_1^g\right]^2 = 0 \\
R_1 &= f_1^g \left[h_{1T}^{(1/2)-g} + h_{1T}^{(1/2)+g}\right] = 0 \\
R_2 &= \left[h_{1T}^{(1/2)+g}\right]^2 - \left[h_{1T}^{(1/2)-g}\right]^2 = 0
\end{aligned}
$$

- From $R_1 = 0$ and $R_2 = 0$ follows $h_{1T}^{(1/2)+g} = -h_{1T}^{(1/2)-g}$

Summary

- New decomposition of gluon correlator producing TMDs at leading twist
- Clear interpretation of 2 transversity T-odd TMDs with L-polarization of gluons symmetric and antisymmetric under permutation of nucleon S_T and gluon q_T
- Gluon TMDs in LF QCD using LFWFs for $g + 3q$ Fock component in nucleon
- TDMs obey Mulders-Rodrigues inequalities, small-x and large-x behavior
- New Sum Rules involving TMDs
- Our study could serve as useful input for future experimental (SPD experiment) and phenomenological studies of gluon TMDs