Nucleon tomography in momentum space

Review on transverse momentum dependent (TMD) distributions and related topics

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SPD seminar

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Comunidad de Madrid

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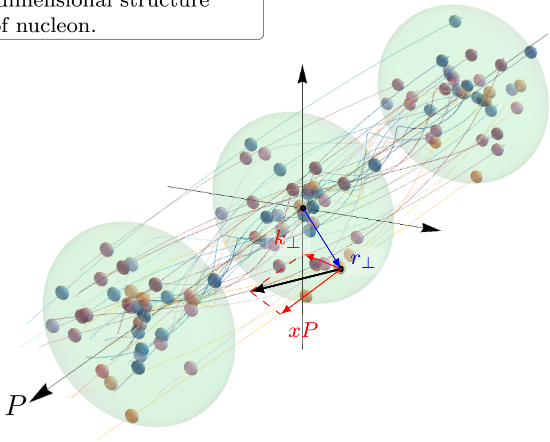
Plan

A general overview of TMD studies, including theory, interpretation, phenomenology.
All topics are covered superficially.
For more details – interrupt and ask questions!

- Part 1
  - General ideology
  - TMD factorization in a nutshell
  - Evolution of TMD distributions
  - Unpolarized TMD distributions (properties)
  - Unpolarized TMD distributions (determination)
- Part 2
  - Zoo of TMD distributions
  - Polarized distributions (properties and determination)
  - Nucleon tomography
  - Problems and perspectives of TMD physics

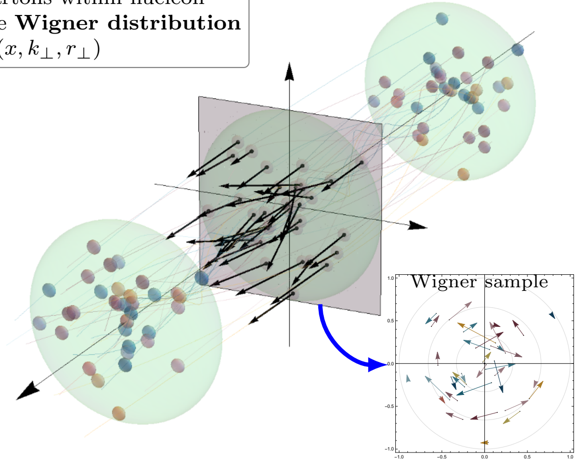
Hadron is a 3D object

Nucleon tomography aims to explore the multi-dimensional structure of nucleon.



Hadron is a 3D object

Complete information about motion of partons within nucleon is encoded in the **Wigner distribution** $W(x, k_\perp, r_\perp)$



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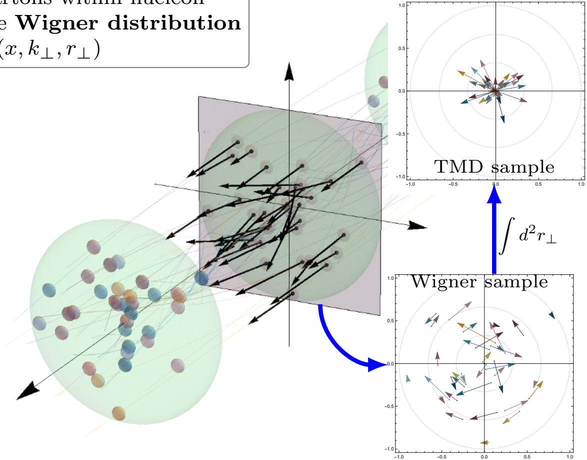
Transverse Momentum Dependent (**TMD**) distribution



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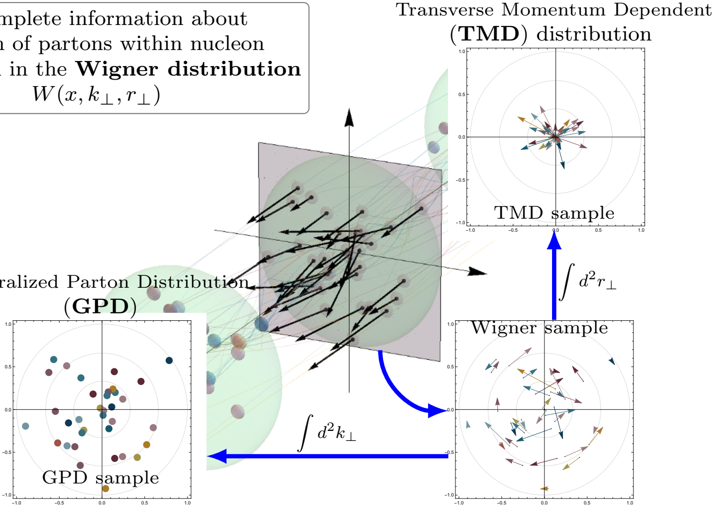
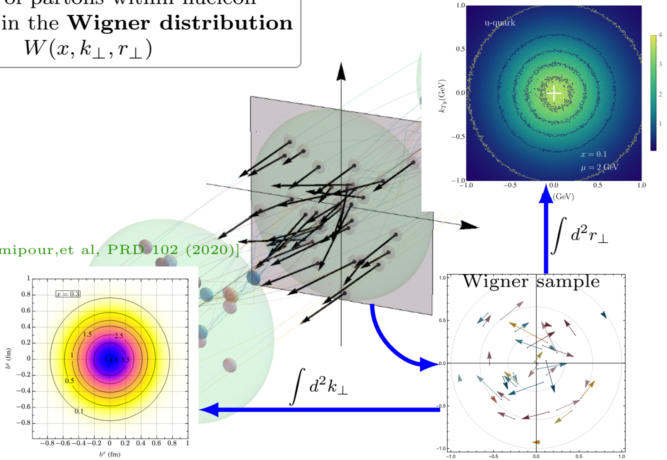
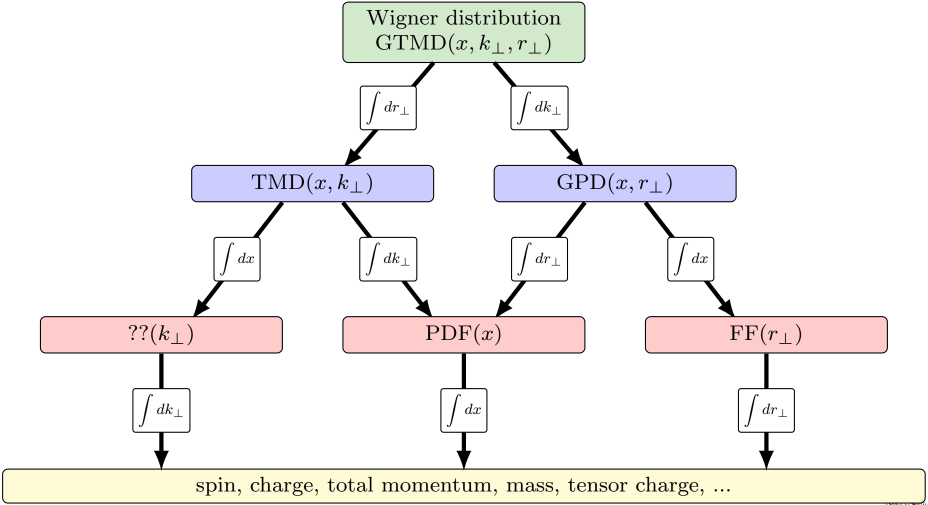


Complete information about
motion of partons within nucleon
is encoded in the **Wigner distribution**
$W(x, k_\perp, r_\perp)$

[Bury, Prokudin, AV, PRL 126 (2021)]

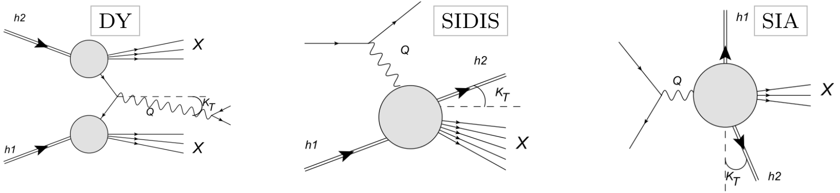
[Hashamipour,et al, PRD 102 (2020)]




Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
??$(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...

Access 3D structure ⇒ process with 3D kinematic ⇒ at least 2 hadron states

Golden processes

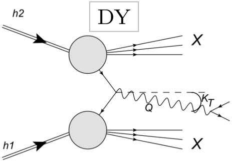


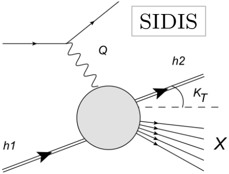
2 hadron states define the "scattering plane"

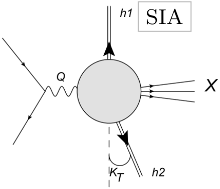
- Invariant mass of the photon $Q^2 \to \infty$
- Transverse momentum of the photon q_T

Access 3D structure ⇒ process with 3D kinematic ⇒ at least 2 hadron states

Golden processes







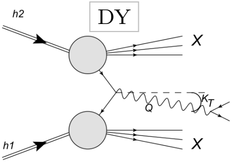
Sources of transverse momentum of photon

- **Perturbative**: from loops and multi-parton interaction $q_T \sim Q \gg \Lambda$
  collinear factorization
- **Non-Perturbative**: from non-colinearity of partons $q_T \sim \Lambda$
  TMD factorization

TMD factorization theorem

$s, Q^2 \to \infty, \qquad$ all other scales (x_1, x_2, q_T) are fixed

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta}) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$



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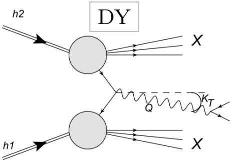

Hard coefficeint function

- Perturbative (known up to N^{4}LO)
- μ is hard-factorization scale ($\mu \sim Q$)

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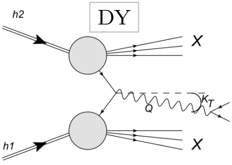

TMD distributions

- Non-Perturbative functions
- One for each hadron (sum over quark-flavors is implied)
- Depend on two scales (μ, ζ)

Nucleon Polarization \ Quark Polarization	Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x, k_T^2)$ Unpolarized		$h_1^\perp(x, k_T^2)$ Boer-Mulders
L		$g_1(x, k_T^2)$ Helicity	$h_{1L}^\perp(x, k_T^2)$ Kozinian-Mulders, "worm" gear
T	$f_{1T}^\perp(x, k_T^2)$ Sivers	$g_{1T}(x, k_T^2)$ Kozinian-Mulders, "worm" gear	$h_1(x, k_T^2)$ Transversity; $h_{1T}^\perp(x, k_T^2)$ Pretzelosity

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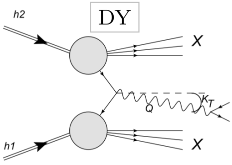

Fourier transform

- TMD factorization is “natural” in position space
- TMD distributions usually defined in position space
- In momentum space

$$\tilde{F}(x, k_T) \simeq \int d^2b e^{i(kb)_T} F(x, b)$$

$$d\sigma \sim \int d^2\mathbf{k}_{1,2} \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2) \tilde{F}(x_1, \mathbf{k}_1) \tilde{F}(x_2, \mathbf{k}_2)$$

TMD factorization theorem

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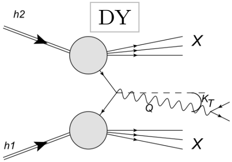


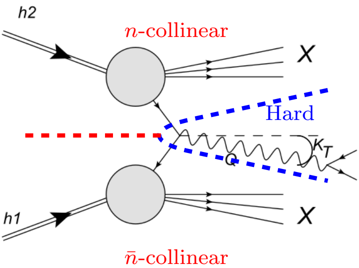
Power corrections

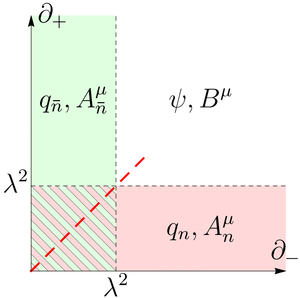
- So far, only theory (known at NLP!)
- Modern frontier..

The leading-power TMD factorization is **proven** at all orders of perturbation theory.
There are several approaches to prove it (each has pros. and cons.)

- Method of regios [Collins' textbook]
- SCET [Becher, Neubert, 2010, Scimemi, Echevarria, Idilbi 2011]
- OPE [AV, Moos, Scimemi, 2021]

Why there are two scales?





To derive TMD factorization one have to distinguish 3 regions

- Hard fields (well-localised interactions)
- $\bar{n}$-collinear fields (belongs to h_1)
- n-collinear fields (belongs to h_2)

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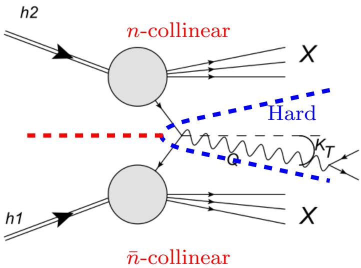

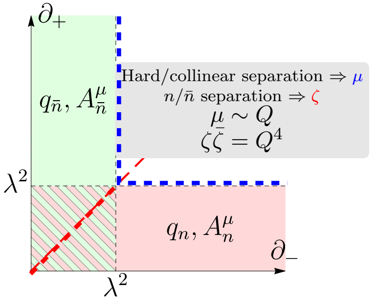

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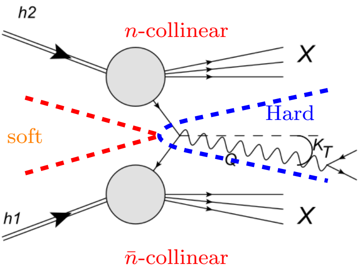

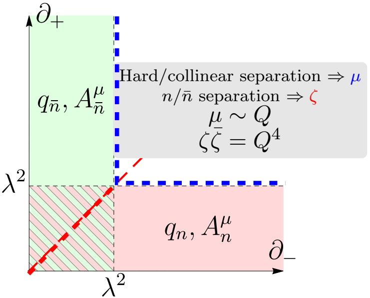

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- Hard fields (well-localised interactions)
- $\bar{n}$-collinear fields (belongs to h_1)
- n-collinear fields (belongs to h_2)
- soft (not necessary)

TMD evolution

same for all TMD-distributions (polarized & unpolarized)

$$\mu^2 \frac{d}{d\mu^2} F(x, \mathbf{b}; \mu, \zeta) = \frac{\gamma_F(\mu,\zeta)}{2} F(x, \mathbf{b}; \mu, \zeta)$$
$$\zeta \frac{d}{d\zeta} F(x, \mathbf{b}; \mu, \zeta) = -\mathcal{D}(\mathbf{b}; \mu) F(x, \mathbf{b}; \mu, \zeta)$$

- γ_F anomalous dimension for hard/collinear separation
  - Usual UV anomalous dimension
  - Perturbative (known up to 4-loops)
- $\mathcal{D}$ Collins-Soper kernel (anomalous dimension for $n/\bar{n}$ separation)
  - also known as "rapidity anomalous dimension"
  - **Non-Perturbative function of b**
- Integrability condition

$$-\frac{d\mathcal{D}(\mathbf{b};\mu)}{d\ln\mu^2} = \frac{1}{2}\frac{d\gamma_F(\mu,\zeta)}{d\ln\zeta} = \frac{\Gamma_{\text{cusp}}(\mu)}{2}$$

TMD evolution

$$\mu^2 \frac{d}{d\mu^2} F(x, \mathbf{b}; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} F(x, \mathbf{b}; \mu, \zeta)$$

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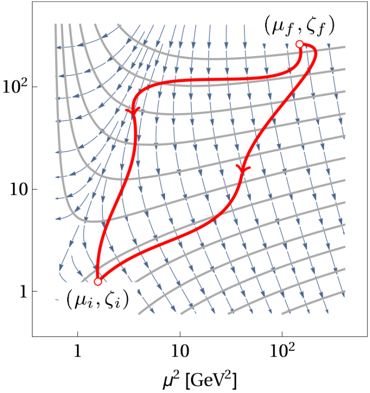


Solution

$$F(x, \mathbf{b}; \mu, \zeta) = R[\mathbf{b}; (\mu, \zeta) \to (\mu_i, \zeta_i)] F(x, \mathbf{b}; \mu_i, \zeta_i)$$

$$R[\mathbf{b}; (\mu, \zeta) \to (\mu_i, \zeta_i)] = \exp\left[\int_P \left(\gamma_F(\mu, \zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mathbf{b}, \mu)\frac{d\zeta}{\zeta}\right)\right]$$

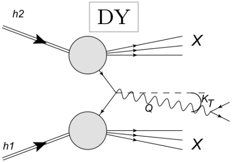
- Path independent

TMD factorization theorem
(practical form)

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta}) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$

Evolution

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) R^2[\mathcal{D}(\mathbf{b})] F(x_1, b) F(x_2, b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$



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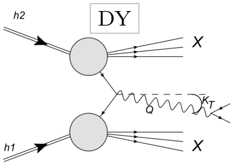
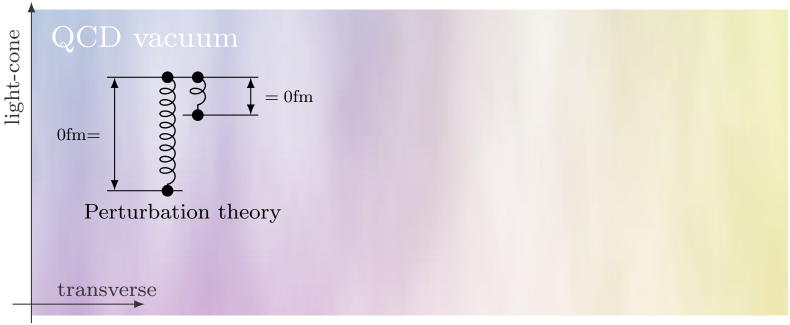


Collins-Soper kernel

- Evolution factor is function of CS kernel
- Universal for all processes
- Universal for all hadrons
- Can be computed with lattice methods

Collins-Soper kernel is about QCD vacuum



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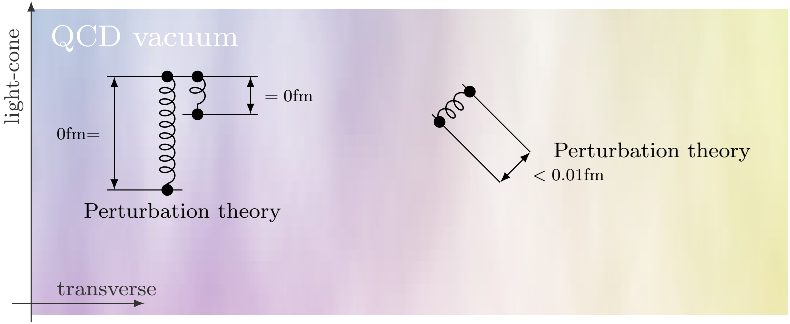

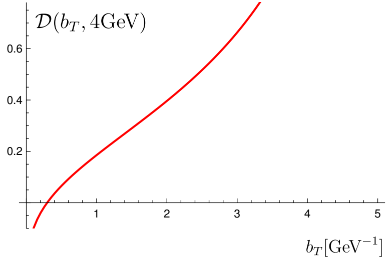

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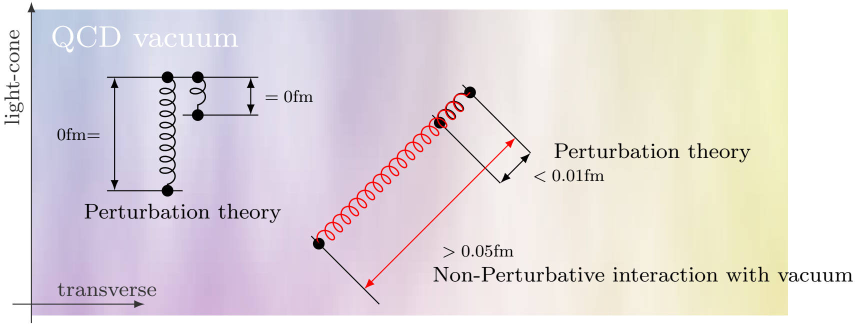
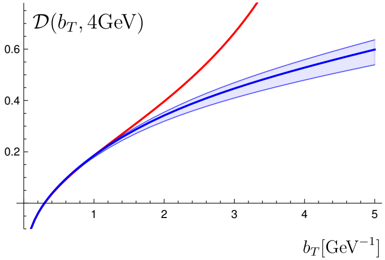
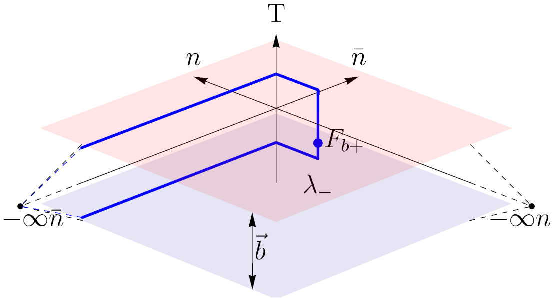




Collins-Soper kernel ∼ Wilson loop

[AV,PRL 125 (2020)]

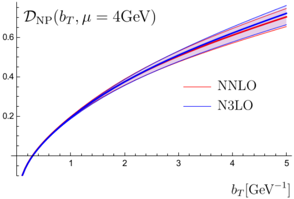


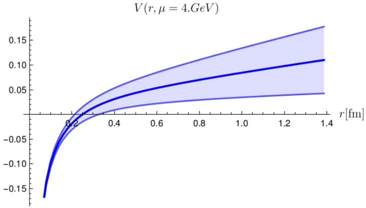
$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \frac{\text{Tr} \int_0^1 d\beta \langle 0|F_{b+}(-\lambda_- n + b\beta) W_{C'}|0\rangle}{\text{Tr}\langle 0|W_{C'}|0\rangle} + Z_{\mathcal{D}}(\mu)$$

Relation to the static potential

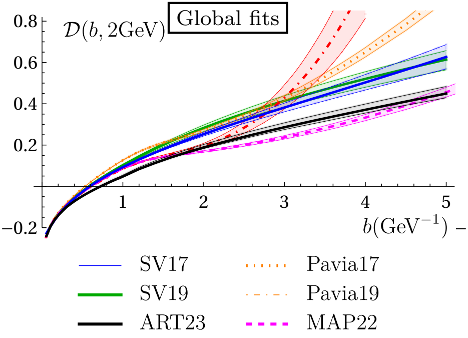
In SVM the potential between two quark sources (confining potential) is [Brambilla,Vairo,hep-ph/9606344]

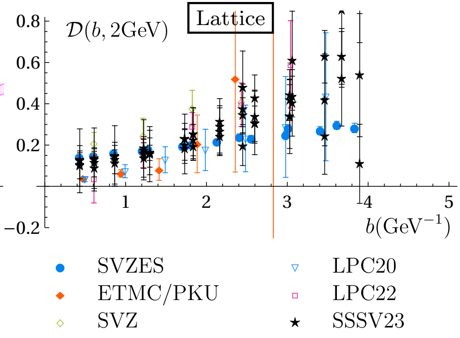
$$V(\boldsymbol{b}) = 2\int_0^{\boldsymbol{b}} d\boldsymbol{y}(\boldsymbol{b}-\boldsymbol{y})\int_0^\infty d\boldsymbol{r}\Delta(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}) + \int_0^{\boldsymbol{b}} d\boldsymbol{y}\boldsymbol{y}\int_0^\infty d\boldsymbol{r}\Delta_1(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}).$$

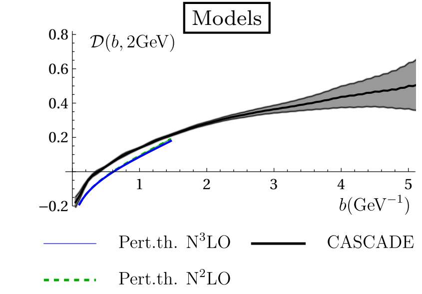




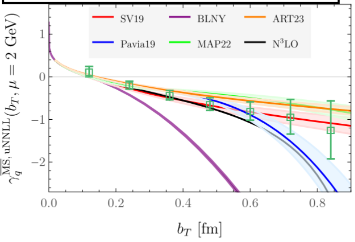
$$V(r) = r\frac{\pi}{4}\mathcal{D}''(0) + \frac{\mathcal{D}'(0)}{2} + \frac{r^2}{2}\int_r^\infty \frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}} + \ldots$$

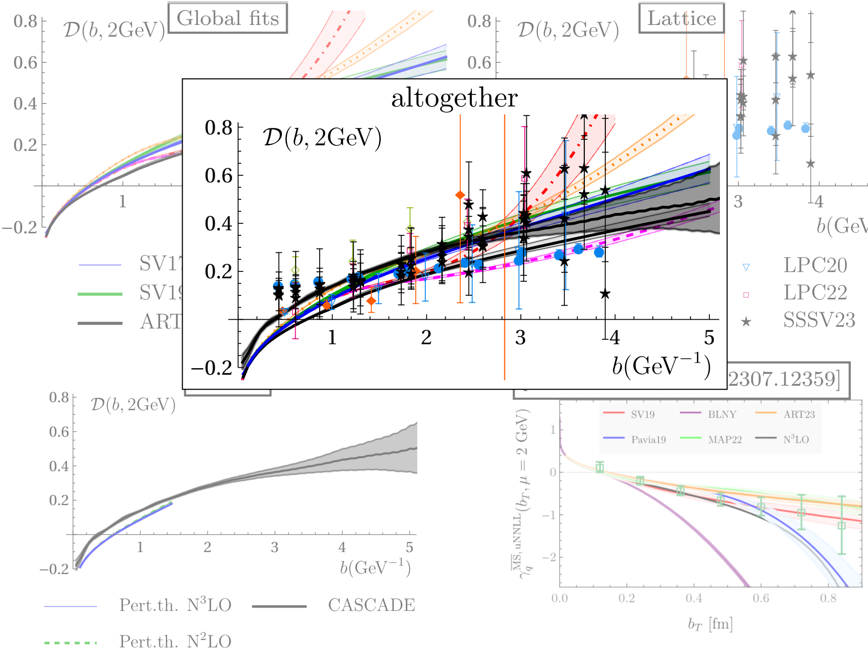


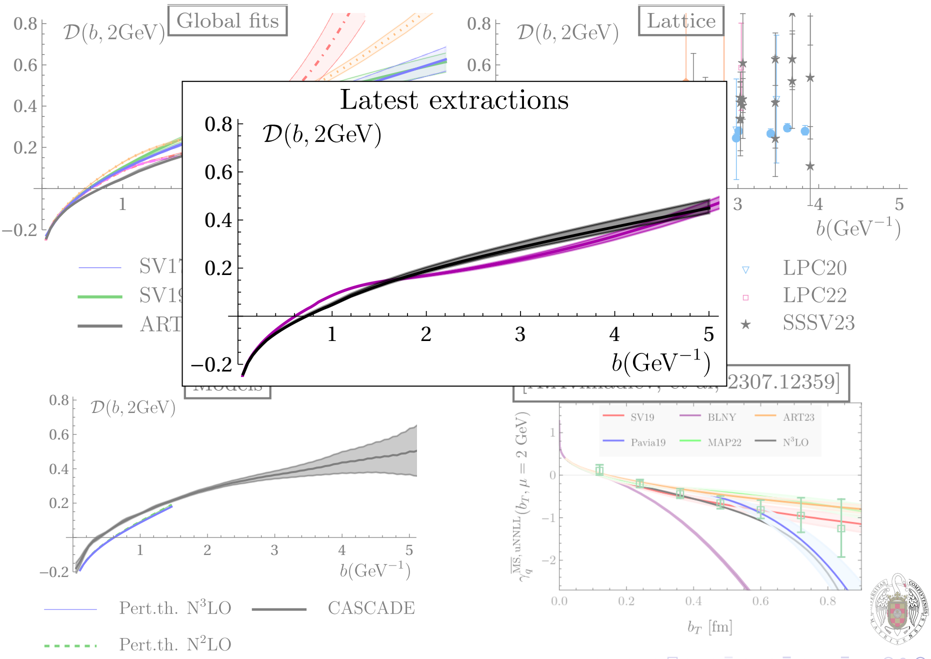




[A.Avkhadiev, et al, 2307.12359]




Global fits
Lattice
altogether
$\mathcal{D}(b, 2\text{GeV})$
$b(\text{GeV}^{-1})$
SV17
SV19
ART
LPC20
LPC22
SSSV23
2307.12359]
SV19
BLNY
ART23
Pavia19
MAP22
N^{3}LO
$\gamma_q^{\overline{\text{MS}},\text{uNNLL}}(b_T, \mu = 2\text{ GeV})$
b_T [fm]
Pert.th. N^{3}LO
CASCADE
Pert.th. N^{2}LO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Latest extractions
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Models
$\mathcal{D}(b, 2\text{GeV})$
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2307.12359]
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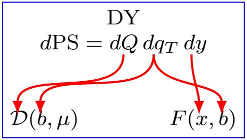
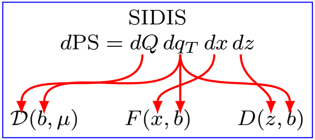
N^{3}LO
$\gamma_q^{\overline{\text{MS}},\text{uNNLL}}(b_T, \mu = 2\text{ GeV})$
b_T [fm]

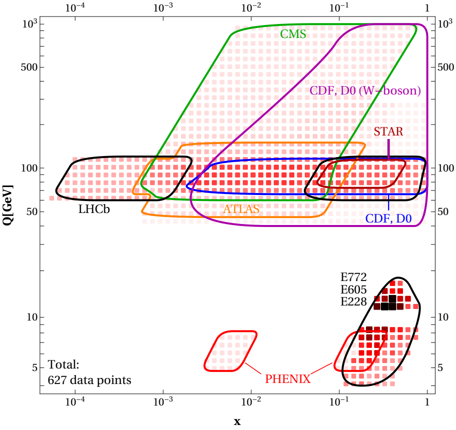
$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) R^2[\mathcal{D}(\mathbf{b})] F(x_1, b) F(x_2, b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$

- Each data-point is a convolution of **three independent nonperturbative** functions
- Each function is responsible for a separate kinematic variable
- Multi-dimensional bining is **essential**





ART23=[Moos,Scimemi,AV,Zurita,2305.07473]
Global extraction of unpolarized TMD & CS-kernel from Drell-Yan data

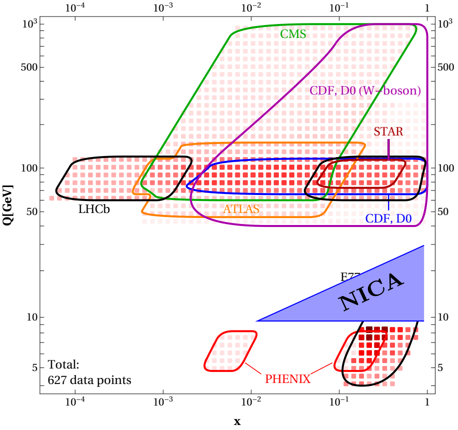


- ATLAS
  - Z-boson at 8 (y-diff.)
  - Z-boson at 13 TeV **(0.1% prec.!)**
- CMS
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
  - Z/γ up to $Q = 1000$GeV
- LHCb
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
- **Further more:**
  - Z-boson at Tevatron
  - W-boson at Tevatron
  - Z-boson at RHIC
  - DY at PHENIX
  - DY at FERMILAB (fix target)

627 data points

ART23=[Moos,Scimemi,AV,Zurita,2305.07473]

Global extraction of unpolarized TMD & CS-kernel from Drell-Yan data



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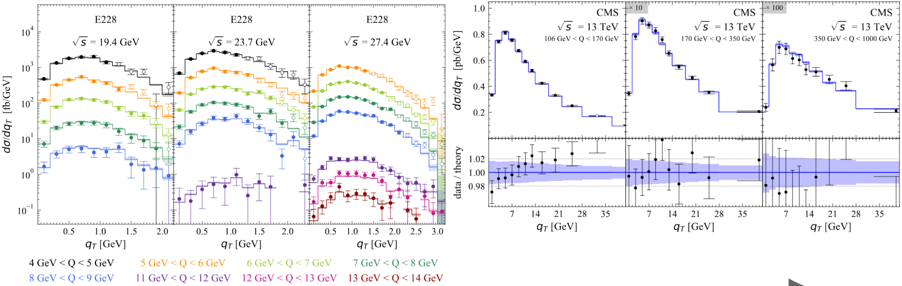
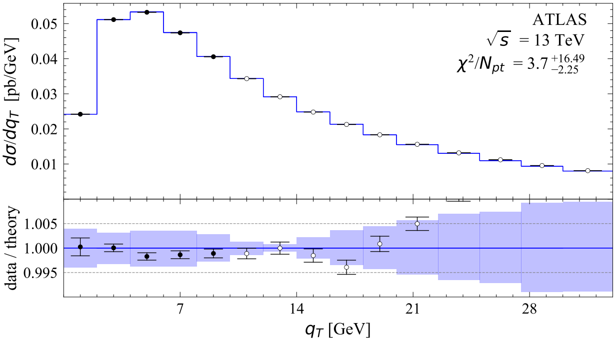
627 data points

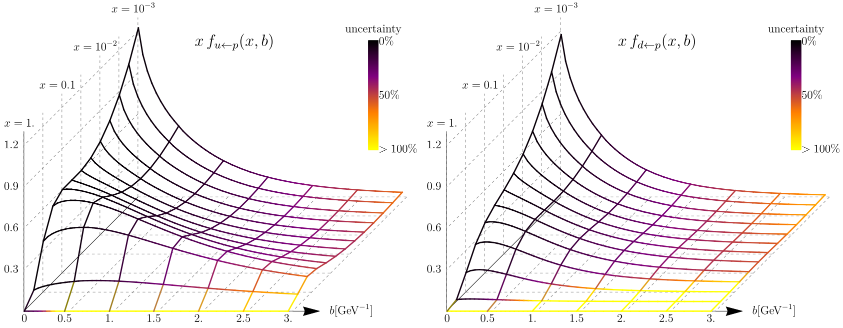


4GeV → 1000GeV

Very presice test of TMD evolution

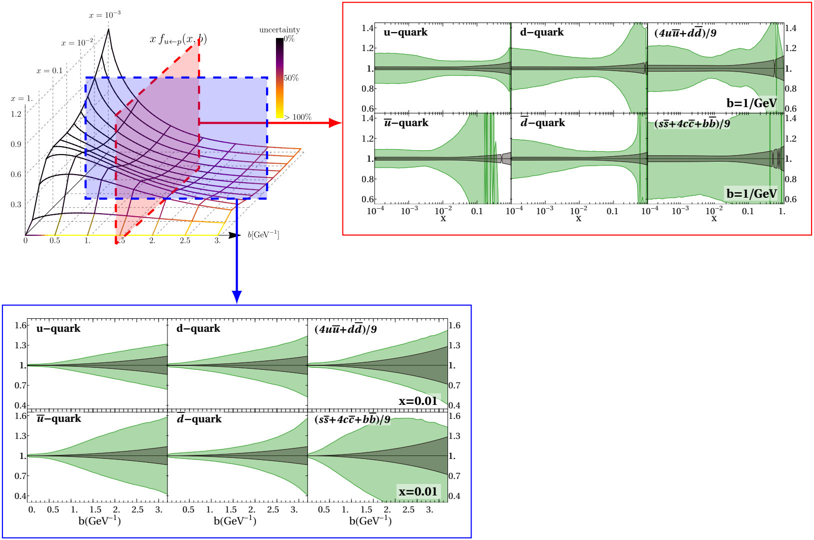


$$\text{TOTAL } (N_{\rm pt} = 627): \quad \chi^2/N_{\rm pt} = 0.96^{+0.09}_{-0.01}$$



Some features of ART23:

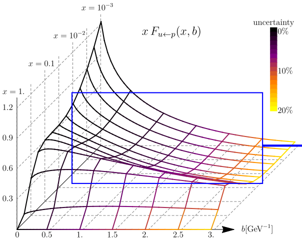
- Hard function and evolution at N^{4}LO
- Matching to PDF at N^{3}LO
- Flavor dependent NP-ansatz
- Consistent inclusion of the PDF uncertainty
- *artemide* (=ART23)

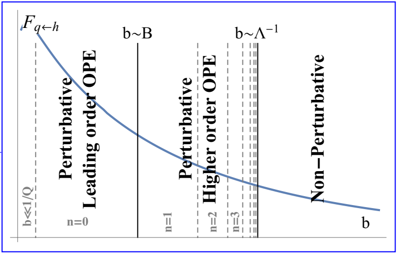


$x\,f_{u\leftarrow p}(x,b)$
uncertainty
0%
50%
> 100%
$x=10^{-3}$
$x=10^{-2}$
$x=0.1$
$x=1.$
b[GeV^{-1}]
u−quark
d−quark
$(4u\bar{u}+d\bar{d})/9$
$\bar{u}$−quark
$\bar{d}$−quark
$(s\bar{s}+4c\bar{c}+b\bar{b})/9$
b=1/GeV
x
x=0.01
b(GeV^{-1})

TMD distributions are nonperturbative 3D functions

However, they match 1D PDFs at $b \to 0$ boundary





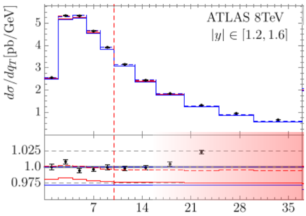
$$F(x,b) = \left[q(x) + \alpha_s\left(p(x)\ln(b^2\mu^2) + ...\right) + \alpha_s^2 ...\right] + b^2 ... + ...$$

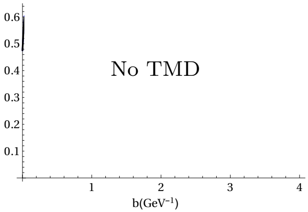
Lead.power OPE up N^{3}LO

Higher power OPE

$$F(x,b) = C(x,b) \otimes q(x) f_{NP}(x,b)$$

Fitting ansatz



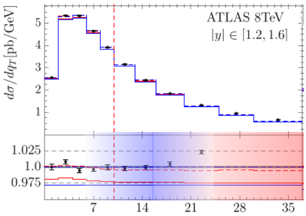


Kinematic ranges:

- Power corrections $q_T \sim Q$

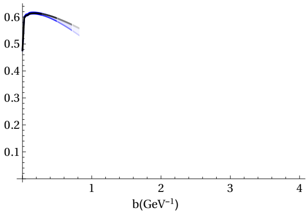
fixed order

$$F(x,b) \sim f(x) \to f(x)\delta(k_T)$$

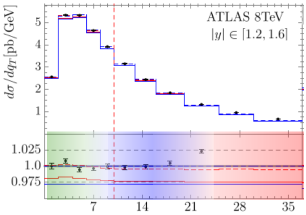


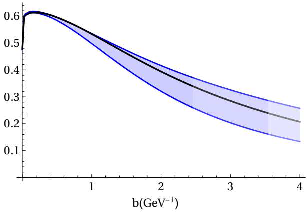
Kinematic ranges:

- Power corrections $q_T \sim Q$
- Resummation $\Lambda \gg q_T \gg Q$



$$F(x,b) = C(x, \ln(b)) \otimes f(x) + b^2 ...$$





Kinematic ranges:

- Power corrections $q_T \sim Q$
- Resummation $\Lambda \gg q_T \gg Q$
- Nonperturbative $q_T \lesssim \Lambda \sim 2 - 4\text{GeV}$

$$F(x, b) = C(x, \ln(b)) \otimes f(x) f_{\text{NP}}(b)$$

f_{NP} to fit

Low-energy measurements are
most interesting,
because they provide access
to NP structure.
Unfortunately,
all low-energy measurements
are inprecise.

End of part 1

Part 1

- Basics of TMD factorization
- TMD evolution and non-perturbative Collins-Soper kernel
- Determination of unpolarized distributions
- Kinematics of TMD processes

Part 2

- Zoo of TMD distributions
- Extraction of polarized distributions
- Nucleon tomography
- Problems and perspectives of TMD physics