Nucleon tomography in momentum space

Review on transverse momentum dependent (TMD) distributions and related topics

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SPD seminar

IPARCOS

Comunidad de Madrid

VNIVERSITAS COMPLVTENSIS MATRITENSIS

April 9, 2024

Plan

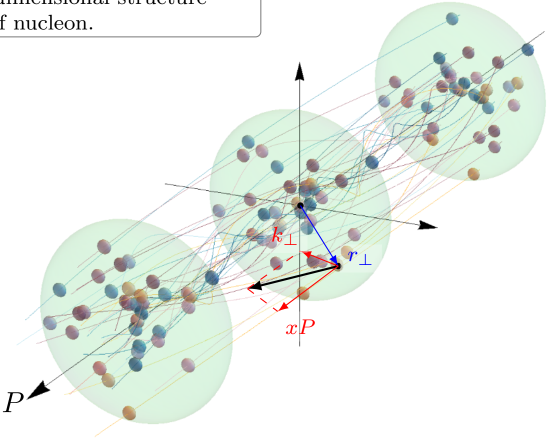
A general overview of TMD studies, including theory, interpretation, phenomenology. All topics are covered superficially.

For more details – interrupt and ask questions!

- Part 1
  - General ideology
  - TMD factorization in a nutshell
  - Evolution of TMD distributions
  - Unpolarized TMD distributions (properties)
  - Unpolarized TMD distributions (determination)
- Part 2
  - Zoo of TMD distributions
  - Polarized distributions (properties and determination)
  - Nucleon tomography
  - Problems and perspectives of TMD physics

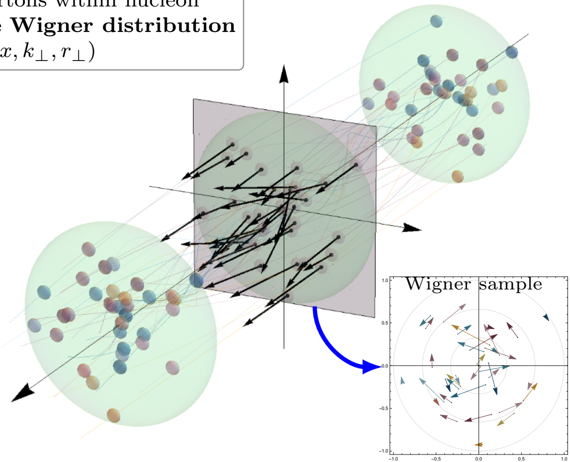
Hadron is a 3D object

Nucleon tomography aims to explore the multi-dimensional structure of nucleon.



Hadron is a 3D object

Complete information about motion of partons within nucleon is encoded in the **Wigner distribution** $W(x, k_\perp, r_\perp)$



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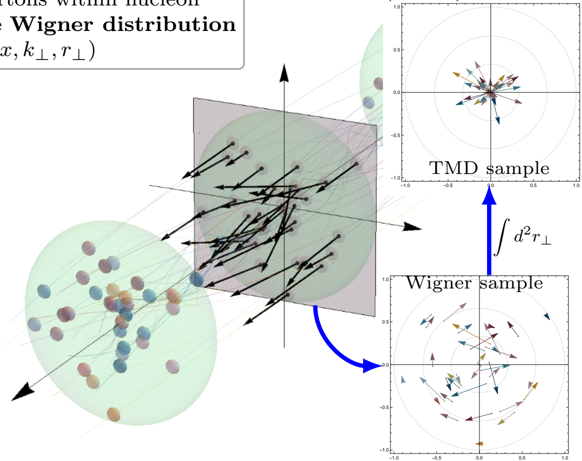
Complete information about motion of partons within nucleon is encoded in the **Wigner distribution** $W(x, k_\perp, r_\perp)$

Transverse Momentum Dependent (**TMD**) distribution

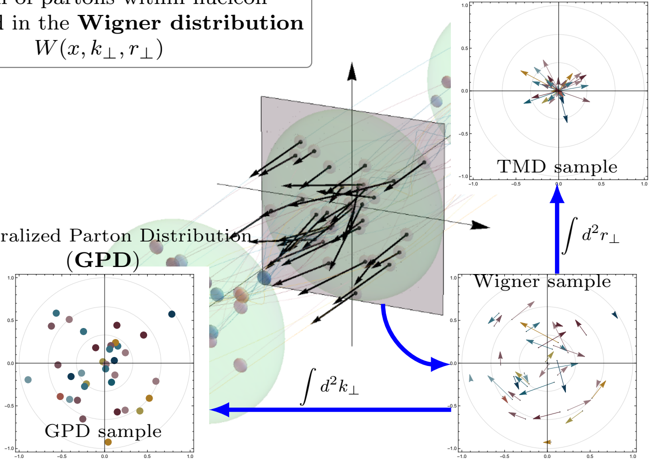


Hadron is a 3D object

Complete information about motion of partons within nucleon is encoded in the **Wigner distribution** $W(x, k_\perp, r_\perp)$

Transverse Momentum Dependent (**TMD**) distribution

Generalized Parton Distribution (**GPD**)

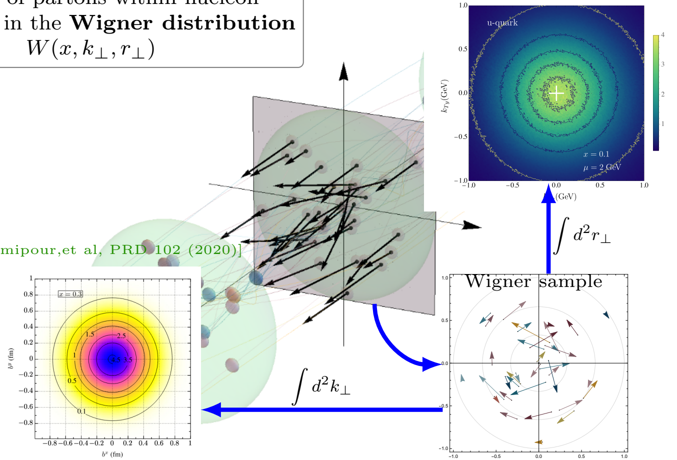


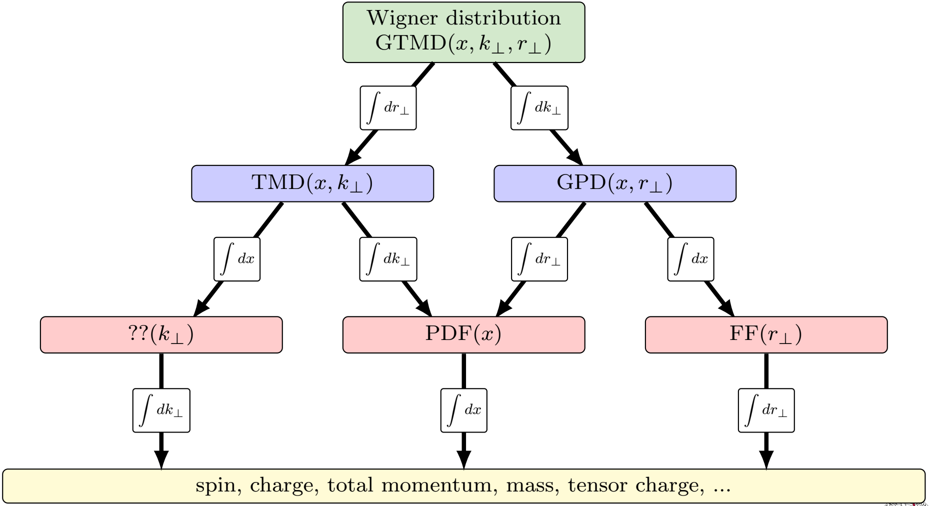
Hadron is a 3D object

Complete information about motion of partons within nucleon is encoded in the **Wigner distribution** $W(x, k_\perp, r_\perp)$

[Bury, Prokudin, AV, PRL 126 (2021)]

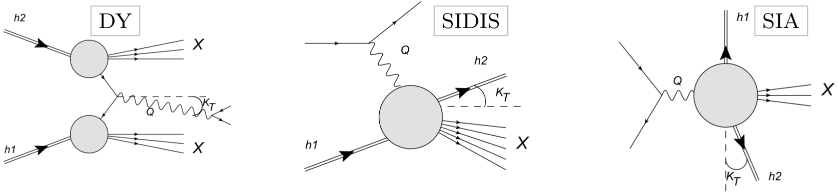
[Hashamipour,et al, PRD 102 (2020)]




Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
??$(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...

> Access 3D structure ⇒ process with 3D kinematic ⇒ at least 2 hadron states

Golden processes

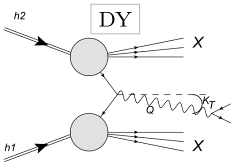


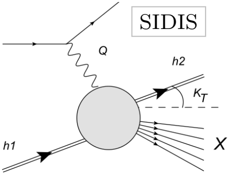
2 hadron states define the "scattering plane"

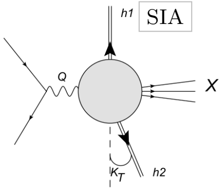
- Invariant mass of the photon $Q^2 \to \infty$
- Transverse momentum of the photon q_T

Access 3D structure ⇒ process with 3D kinematic ⇒ at least 2 hadron states

Golden processes







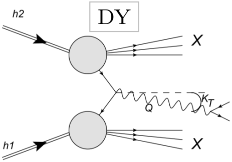
Sources of transverse momentum of photon

- **Perturbative**: from loops and multi-parton interaction $q_T \sim Q \gg \Lambda$
  collinear factorization
- **Non-Perturbative**: from non-colinearity of partons $q_T \sim \Lambda$
  TMD factorization

TMD factorization theorem

$s, Q^2 \to \infty,$ all other scales (x_1, x_2, q_T) are fixed

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta}) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$



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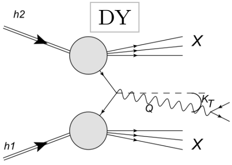

Hard coefficeint function

- Perturbative (known up to N^{4}LO)
- μ is hard-factorization scale ($\mu \sim Q$)

TMD factorization theorem

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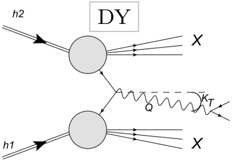


TMD distributions

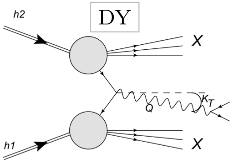
- Non-Perturbative functions
- One for each hadron (sum over quark-flavors is implied)
- Depend on two scales (μ, ζ)

Nucleon Polarization \ Quark Polarization	Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x,k_T^2)$ Unpolarized		$h_1^\perp(x,k_T^2)$ Boer-Mulders
L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^\perp(x,k_T^2)$ Kozinian-Mulders, "worm" gear
T	$f_{1T}^\perp(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Kozinian-Mulders, "worm" gear	$h_1(x,k_T^2)$ Transversity; $h_{1T}^\perp(x,k_T^2)$ Pretzelosity

TMD factorization theorem

$$s, Q^2 \to \infty, \qquad \text{all other scales } (x_1, x_2, q_T) \text{ are fixed}$$

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta}) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$



Fourier transform

- TMD factorization is "natural" in position space
- TMD distributions usually defined in position space
- In momentum space

$$\tilde{F}(x, k_T) \simeq \int d^2b e^{i(kb)_T} F(x, b)$$

$$d\sigma \sim \int d^2\mathbf{k}_{1,2} \delta(\mathbf{q}_T - \mathbf{k}_1 - \mathbf{k}_2) \tilde{F}(x_1, \mathbf{k}_1) \tilde{F}(x_2, \mathbf{k}_2)$$

TMD factorization theorem

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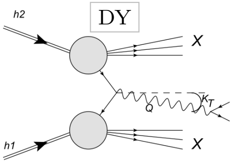


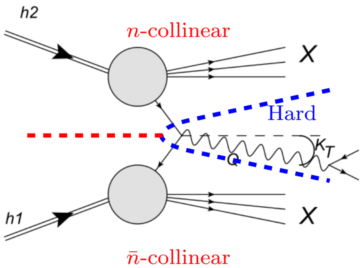
Power corrections

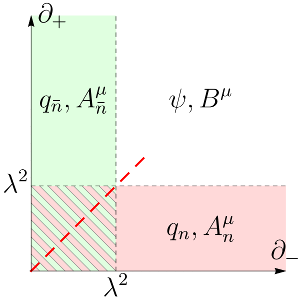
- So far, only theory (known at NLP!)
- Modern frontier..

The leading-power TMD factorization is **proven** at all orders of perturbation theory.
There are several approaches to prove it (each has pros. and cons.)

- Method of regios [Collins' textbook]
- SCET [Becher, Neubert, 2010, Scimemi, Echevarria, Idilbi 2011]
- OPE [AV, Moos, Scimemi, 2021]

Why there are two scales?





To derive TMD factorization one have to distinguish 3 regions

- Hard fields (well-localised interactions)
- $\bar{n}$-collinear fields (belongs to h_1)
- n-collinear fields (belongs to h_2)

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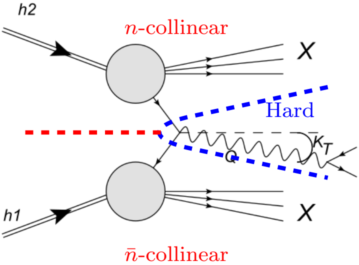

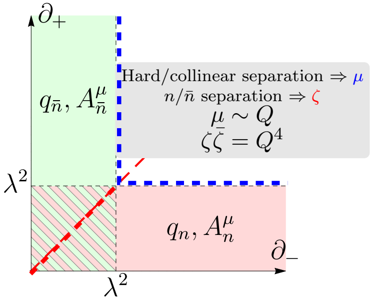

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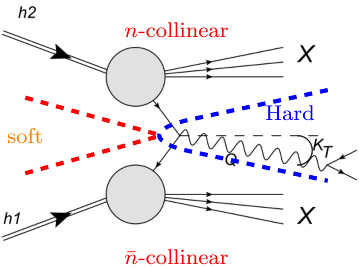

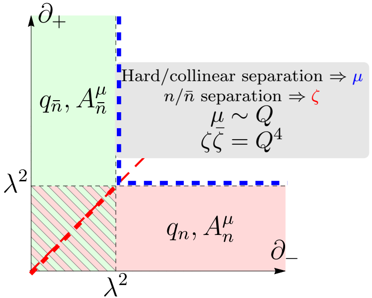

To derive TMD factorization one have to distinguish 3 regions

- Hard fields (well-localised interactions)
- $\bar{n}$-collinear fields (belongs to h_1)
- n-collinear fields (belongs to h_2)
- soft (not necessary)

TMD evolution
same for all TMD-distributions (polarized & unpolarized)

$$\mu^2 \frac{d}{d\mu^2} F(x, \mathbf{b}; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} F(x, \mathbf{b}; \mu, \zeta)$$
$$\zeta \frac{d}{d\zeta} F(x, \mathbf{b}; \mu, \zeta) = -\mathcal{D}(\mathbf{b}; \mu) F(x, \mathbf{b}; \mu, \zeta)$$

- γ_F anomalous dimension for hard/collinear separation
  - Usual UV anomalous dimension
  - Perturbative (known up to 4-loops)
- $\mathcal{D}$ Collins-Soper kernel (anomalous dimension for $n/\bar{n}$ separation)
  - also known as “rapidity anomalous dimension”
  - **Non-Perturbative function of b**
- Integrability condition

$$-\frac{d\mathcal{D}(\mathbf{b}; \mu)}{d \ln \mu^2} = \frac{1}{2} \frac{d\gamma_F(\mu, \zeta)}{d \ln \zeta} = \frac{\Gamma_{\text{cusp}}(\mu)}{2}$$

TMD evolution

$$\mu^2 \frac{d}{d\mu^2} F(x,\mathbf{b};\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2} F(x,\mathbf{b};\mu,\zeta)$$

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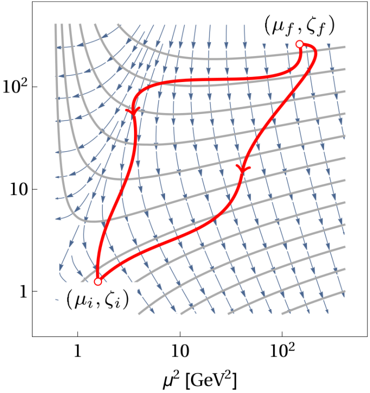


Solution

$$F(x,\mathbf{b};\mu,\zeta) = R[\mathbf{b};(\mu,\zeta)\to(\mu_i,\zeta_i)]F(x,\mathbf{b};\mu_i,\zeta_i)$$

$$R[\mathbf{b};(\mu,\zeta)\to(\mu_i,\zeta_i)] = \exp\left[\int_P \left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mathbf{b},\mu)\frac{d\zeta}{\zeta}\right)\right]$$

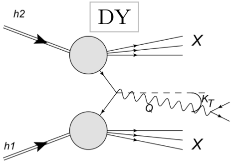
- Path independent

TMD factorization theorem
(practical form)

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta}) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$

Evolution

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) R^2[\mathcal{D}(\mathbf{b})] F(x_1, b) F(x_2, b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$



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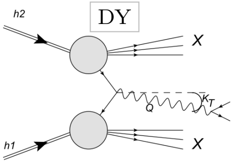
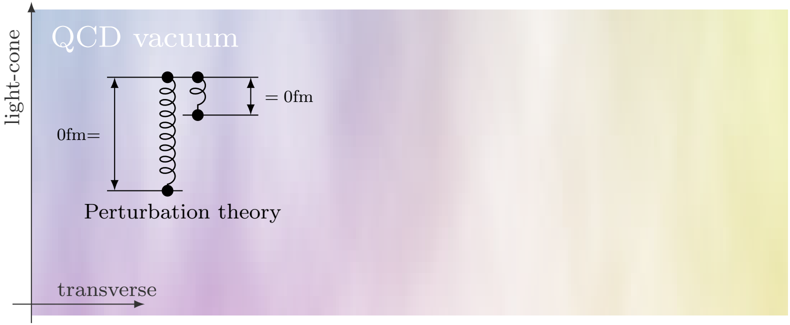


Collins-Soper kernel

- Evolution factor is function of CS kernel
- Universal for all processes
- Universal for all hadrons
- Can be computed with lattice methods

Collins-Soper kernel is about QCD vacuum



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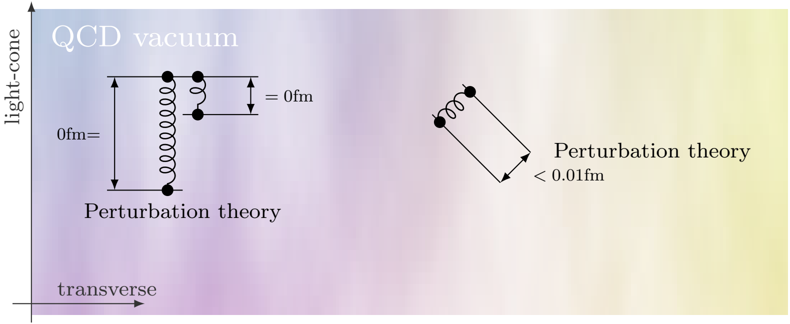

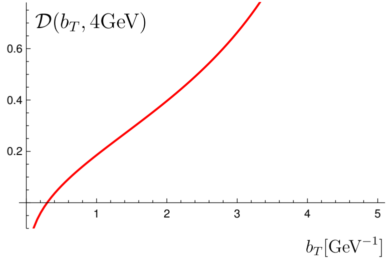

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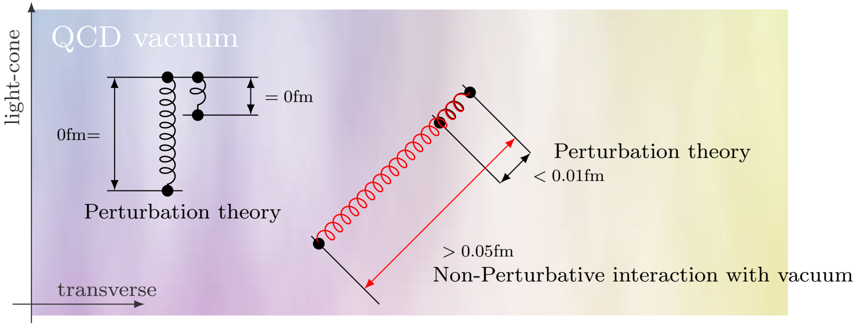
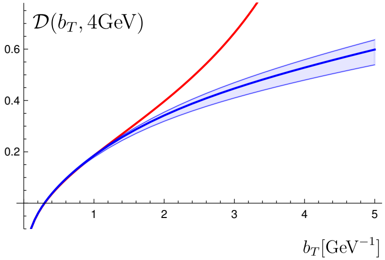
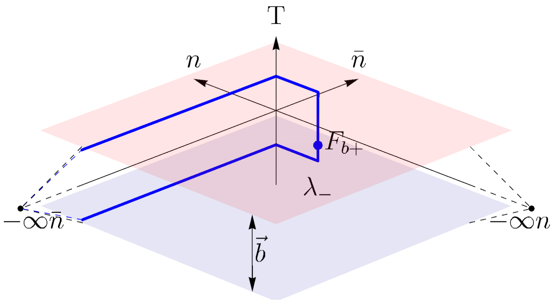




Collins-Soper kernel ∼ Wilson loop

[AV,PRL 125 (2020)]

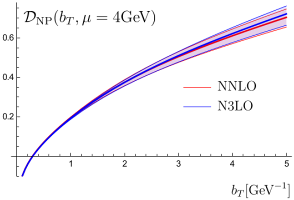


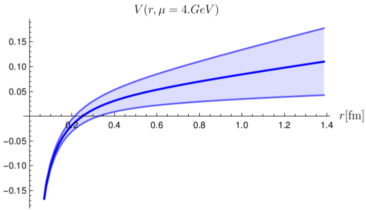
$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \frac{\text{Tr} \int_0^1 d\beta \langle 0|F_{b+}(-\lambda_- n + b\beta) W_{C'}|0\rangle}{\text{Tr}\langle 0|W_{C'}|0\rangle} + Z_{\mathcal{D}}(\mu)$$

Relation to the static potential

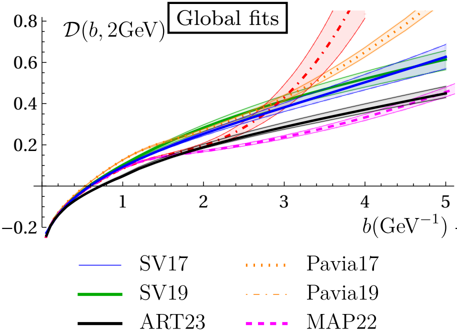
In SVM the potential between two quark sources (confining potential) is [Brambilla,Vairo,hep-ph/9606344]

$$V(\boldsymbol{b}) = 2\int_0^{\boldsymbol{b}} d\boldsymbol{y}(\boldsymbol{b}-\boldsymbol{y})\int_0^{\infty} d\boldsymbol{r}\Delta(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}) + \int_0^{\boldsymbol{b}} d\boldsymbol{y}\boldsymbol{y}\int_0^{\infty} d\boldsymbol{r}\Delta_1(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}).$$

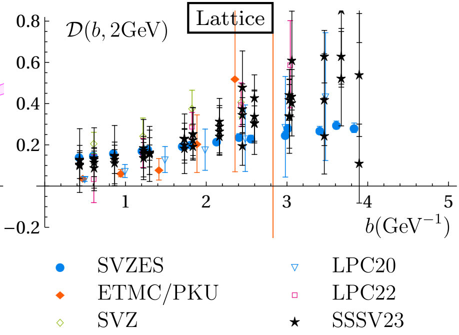




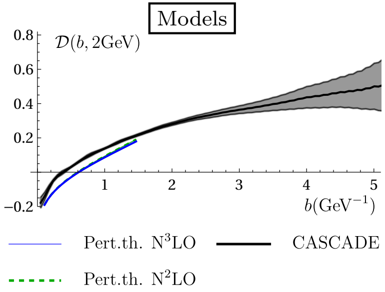
$$V(r) = r\frac{\pi}{4}\mathcal{D}''(0) + \frac{\mathcal{D}'(0)}{2} + \frac{r^2}{2}\int_r^{\infty}\frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}} + \ldots$$



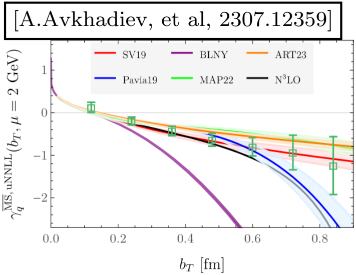
Global fits
$\mathcal{D}(b, 2\text{GeV})$
0.8
0.6
0.4
0.2
−0.2
1
2
3
4
5
$b(\text{GeV}^{-1})$
SV17
SV19
ART23
Pavia17
Pavia19
MAP22



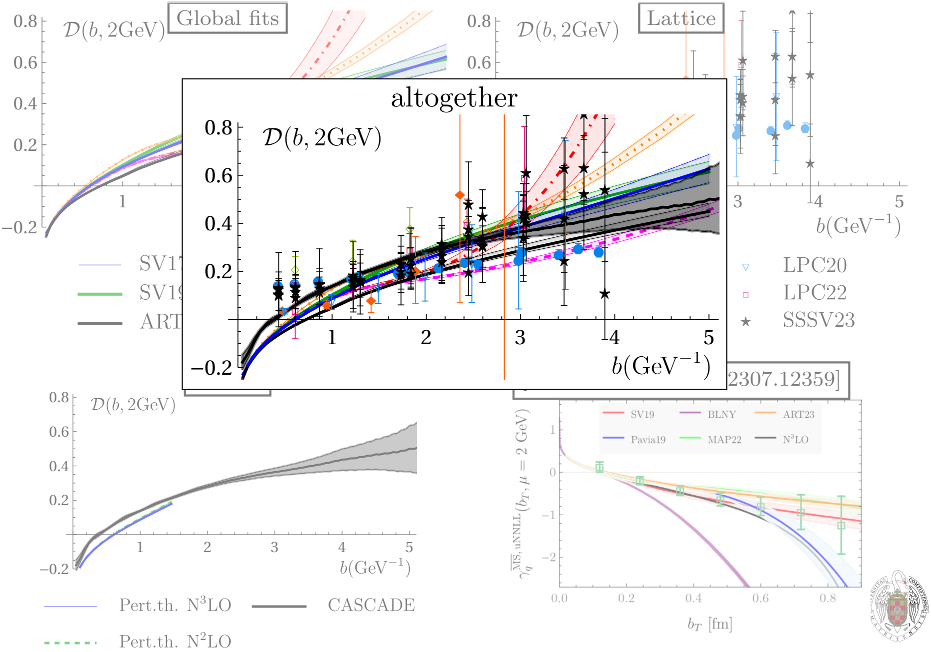
Lattice
$\mathcal{D}(b, 2\text{GeV})$
0.8
0.6
0.4
0.2
−0.2
1
2
3
4
5
$b(\text{GeV}^{-1})$
SVZES
ETMC/PKU
SVZ
LPC20
LPC22
SSSV23

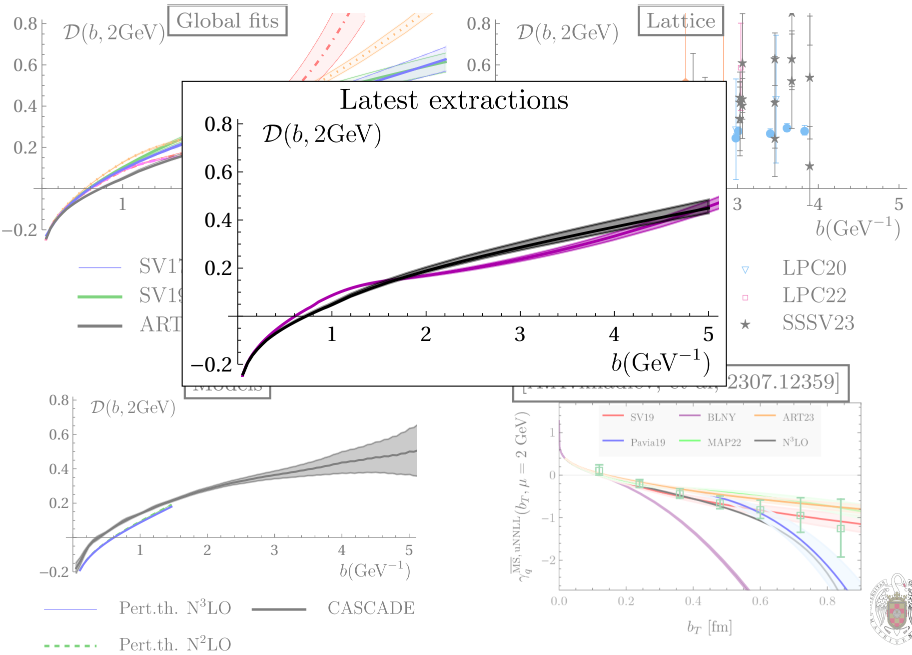


Models
$\mathcal{D}(b, 2\text{GeV})$
0.8
0.6
0.4
0.2
−0.2
1
2
3
4
5
$b(\text{GeV}^{-1})$
Pert.th. N^{3}LO
CASCADE
Pert.th. N^{2}LO



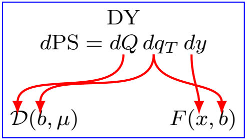
[A.Avkhadiev, et al, 2307.12359]
SV19
BLNY
ART23
Pavia19
MAP22
N^{3}LO
$\gamma_q^{\overline{\text{MS}},\text{uNNLL}}(b_T, \mu = 2\text{ GeV})$
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
b_T [fm]

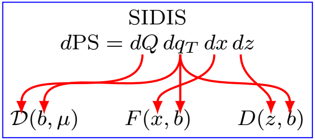

Global fits
$\mathcal{D}(b, 2\text{GeV})$
SV17
SV19
ART
Lattice
$\mathcal{D}(b, 2\text{GeV})$
LPC20
LPC22
SSSV23
altogether
$\mathcal{D}(b, 2\text{GeV})$
$b(\text{GeV}^{-1})$
2307.12359]
$\mathcal{D}(b, 2\text{GeV})$
$b(\text{GeV}^{-1})$
Pert.th. N^{3}LO
CASCADE
Pert.th. N^{2}LO
SV19
BLNY
ART23
Pavia19
MAP22
N^{3}LO
$\gamma_q^{\overline{\text{MS}},\text{uNNLL}}(b_T, \mu = 2\text{ GeV})$
b_T [fm]


Global fits
Lattice
Latest extractions
$\mathcal{D}(b, 2\text{GeV})$
$b(\text{GeV}^{-1})$
SV17
SV19
ART
LPC20
LPC22
SSSV23
Models
2307.12359]
SV19
BLNY
ART23
Pavia19
MAP22
N^{3}LO
$\gamma_q^{\overline{\text{MS}},\text{uNNLL}}(b_T, \mu = 2\text{ GeV})$
b_T [fm]
Pert.th. N^{3}LO
CASCADE
Pert.th. N^{2}LO

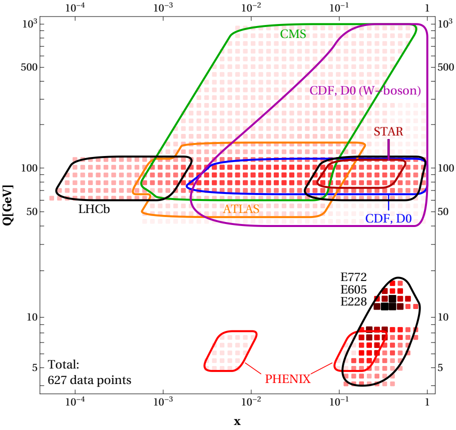
$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) R^2[\mathcal{D}(\mathbf{b})] F(x_1, b) F(x_2, b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$

- Each data-point is a convolution of **three independent nonperturbative** functions
- Each function is responsible for a separate kinematic variable
- Multi-dimensional bining is **essential**





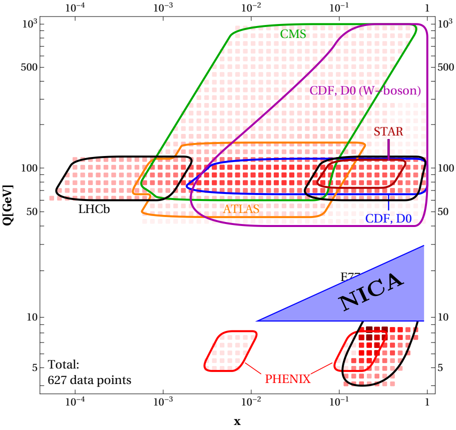
ART23=[Moos,Scimemi,AV,Zurita,2305.07473]
Global extraction of unpolarized TMD & CS-kernel from Drell-Yan data



- ATLAS
  - Z-boson at 8 (y-diff.)
  - Z-boson at 13 TeV **(0.1% prec.!)**
- CMS
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
  - Z/γ up to $Q = 1000$GeV
- LHCb
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
- **Further more:**
  - Z-boson at Tevatron
  - W-boson at Tevatron
  - Z-boson at RHIC
  - DY at PHENIX
  - DY at FERMILAB (fix target)

627 data points

ART23=[Moos,Scimemi,AV,Zurita,2305.07473]
Global extraction of unpolarized TMD & CS-kernel from Drell-Yan data



- ATLAS
  - Z-boson at 8 (y-diff.)
  - Z-boson at 13 TeV **(0.1% prec.!)**
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  - Z-boson at 7 and 8 TeV
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- **Further more:**
  - Z-boson at Tevatron
  - W-boson at Tevatron
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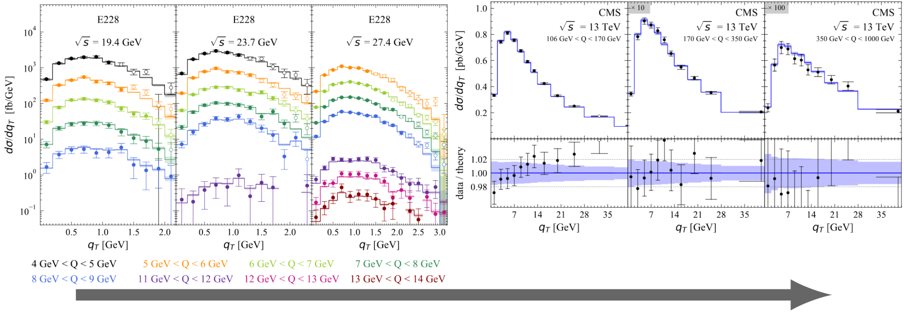
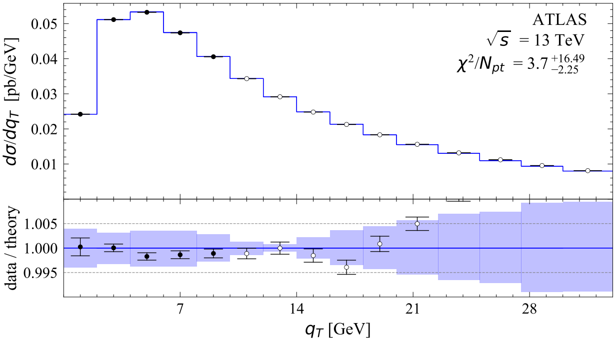
  - DY at PHENIX
  - DY at FERMILAB (fix target)

627 data points

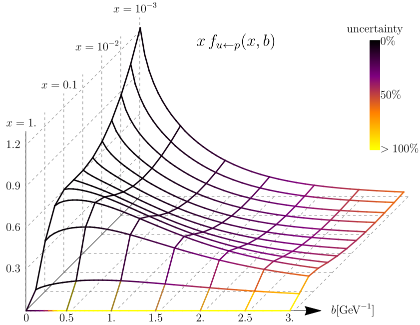


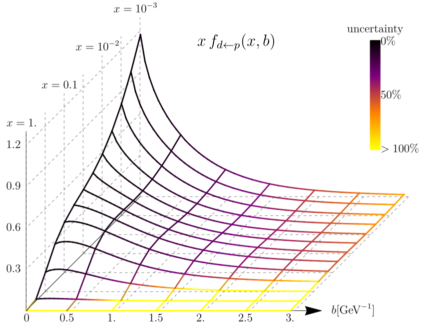
4GeV 1000GeV

Very presice test of TMD evolution



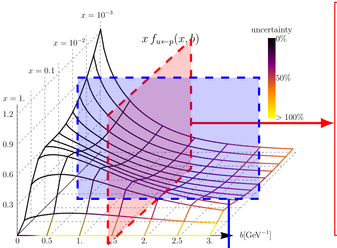
$$\text{TOTAL } (N_{\text{pt}} = 627): \quad \chi^2/N_{\text{pt}} = 0.96^{+0.09}_{-0.01}$$



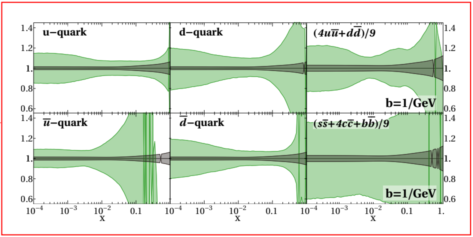


Some features of ART23:

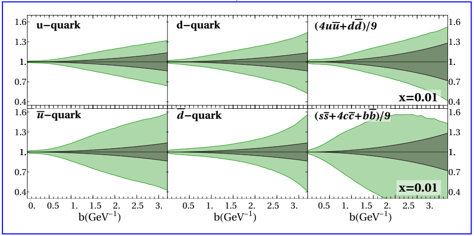
- Hard function and evolution at N^4LO
- Matching to PDF at N^3LO
- Flavor dependent NP-ansatz
- Consistent inclusion of the PDF uncertainty
- *artemide* (=ART23)



$x = 10^{-3}$
$x = 10^{-2}$
$x = 0.1$
$x = 1.$
$x\, f_{u\leftarrow p}(x, b)$
uncertainty
0%
50%
> 100%
1.2
0.9
0.6
0.3
0
0.5
1.
1.5
2.
2.5
3.
b[GeV^{-1}]



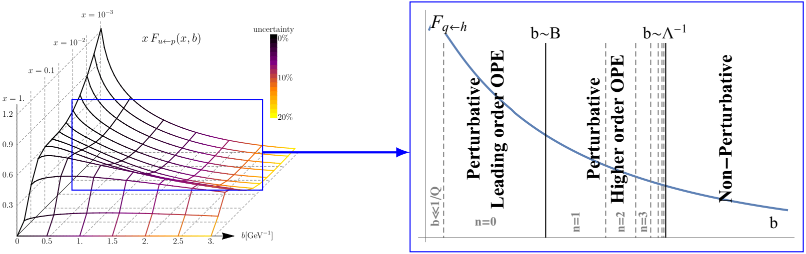
u−quark
d−quark
$(4u\bar{u}+d\bar{d})/9$
$\bar{u}$−quark
$\bar{d}$−quark
$(s\bar{s}+4c\bar{c}+b\bar{b})/9$
b=1/GeV
b=1/GeV
1.4
1.2
1.
0.8
0.6
10^{-4}
10^{-3}
10^{-2}
0.1
1.
x



u−quark
d−quark
$(4u\bar{u}+d\bar{d})/9$
$\bar{u}$−quark
$\bar{d}$−quark
$(s\bar{s}+4c\bar{c}+b\bar{b})/9$
x=0.01
x=0.01
1.6
1.3
1.
0.7
0.4
0.
0.5
1.
1.5
2.
2.5
3.
b(GeV^{-1})

TMD distributions are nonperturbative 3D functions

However, they match 1D PDFs at $b \to 0$ boundary



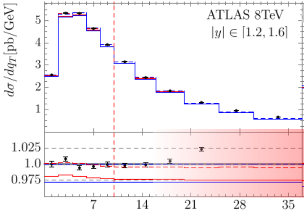
$$F(x,b) = \left[q(x) + \alpha_s\left(p(x)\ln(b^2\mu^2) + ...\right) + \alpha_s^2 ...\right] + b^2 ... + ...$$

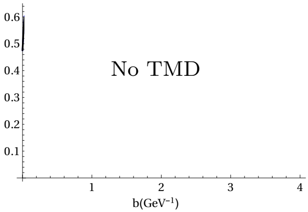
Lead.power OPE
up N^{3}LO

Higher power OPE

$$F(x,b) = C(x,b) \otimes q(x) f_{NP}(x,b)$$

Fitting ansatz



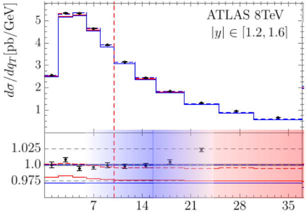


Kinematic ranges:

- Power corrections $q_T \sim Q$

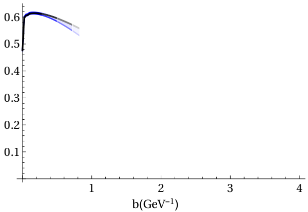
fixed order

$$F(x,b) \sim f(x) \to f(x)\delta(k_T)$$

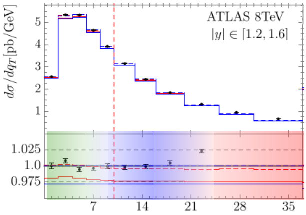


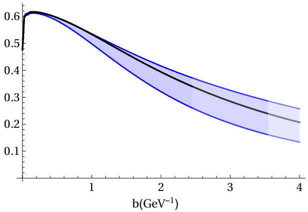
Kinematic ranges:

- Power corrections $q_T \sim Q$
- Resummation $\Lambda \gg q_T \gg Q$



$$F(x,b) = C(x, \ln(b)) \otimes f(x) + b^2 ...$$





Kinematic ranges:

- Power corrections $q_T \sim Q$
- Resummation $\Lambda \gg q_T \gg Q$
- Nonperturbative $q_T \lesssim \Lambda \sim 2 - 4$GeV

$$F(x, b) = C(x, \ln(b)) \otimes f(x) f_{\mathrm{NP}}(b)$$

f_{NP} to fit

Low-energy measurements are
most interesting,
because they provide access
to NP structure.
Unfortunately,
all low-energy measurements
are inprecise.

End of part 1

Part 1

- Basics of TMD factorization
- TMD evolution and non-perturbative Collins-Soper kernel
- Determination of unpolarized distributions
- Kinematics of TMD processes

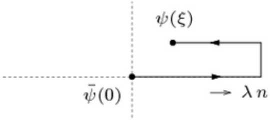
Part 2

- Zoo of TMD distributions
- Extraction of polarized distributions
- Nucleon tomography
- Problems and perspectives of TMD physics

Nucleon tomography in momentum space
Part 2

Polarized TMDs and outlook

- Zoo of TMD distributions
- Extraction of polarized distributions
- Nucleon tomography
- Problems and perspectives of TMD physics



PDF:

$$f^{[\Gamma]}(x) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda p^+} \langle p,s|\bar\psi(\lambda n)\frac{\Gamma}{2}[\lambda n,0]\psi(0)|p,s\rangle$$

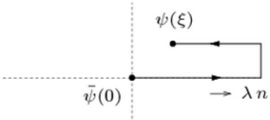
TMD:

$$\Phi^{[\Gamma]}(x,b) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda p^+} \langle p,s|\bar\psi(\lambda n+b)\frac{\Gamma}{2}[...]\psi(0)|p,s\rangle$$

$$\Gamma = \gamma^+, \gamma^+\gamma^5, i\sigma^{\alpha+}\gamma^5$$

There are 3 PDFs of twist-2

$$F^{[\gamma^+]}(x) = f_1(x),$$
$$F^{[\gamma^+\gamma^5]}(x) = \lambda g_1(x),$$
$$F^{[i\sigma^{\alpha+}\gamma^5]}(x) = s_T^\alpha h_1(x,b),$$



PDF:

$$f^{[\Gamma]}(x) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda p^+} \langle p,s|\bar{\psi}(\lambda n)\frac{\Gamma}{2}[\lambda n, 0]\psi(0)|p,s\rangle$$

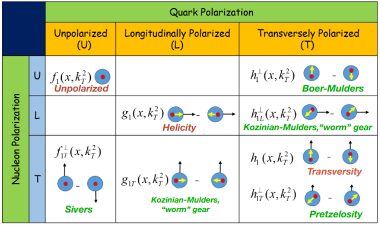
TMD:

$$\Phi^{[\Gamma]}(x,b) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda p^+} \langle p,s|\bar{\psi}(\lambda n + b)\frac{\Gamma}{2}[...]\psi(0)|p,s\rangle$$

$$\Gamma = \gamma^+, \gamma^+\gamma^5, i\sigma^{\alpha+}\gamma^5$$

There are 8 TMDs of twist-2

$$\begin{aligned}
\Phi^{[\gamma^+]}(x,b) &= f_1(x,b) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^\perp(x,b), \\
\Phi^{[\gamma^+\gamma^5]}(x,b) &= \lambda g_1(x,b) + i(b\cdot s_T) M g_{1T}^\perp(x,b), \\
\Phi^{[i\sigma^{\alpha+}\gamma^5]}(x,b) &= s_T^\alpha h_1(x,b) - i\lambda b^\alpha M h_{1L}^\perp(x,b) \\
&\quad + i\epsilon^{\alpha\mu} b_\mu M h_1^\perp(x,b) - \frac{M^2 b^2}{2}\left(\frac{g_T^{\alpha\mu}}{2} - \frac{b^\alpha b^\mu}{b^2}\right) s_{T\mu} h_{1T}^\perp(x,b),
\end{aligned}$$



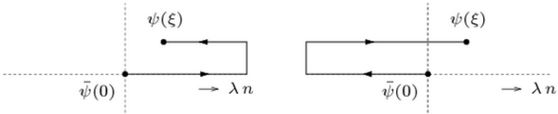
Nucleon Polarization	Quark Polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x,k_T^2)$ Unpolarized		$h_1^\perp(x,k_T^2)$ Boer-Mulders
L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^\perp(x,k_T^2)$ Kozinian-Mulders,“worm” gear
T	$f_{1T}^\perp(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Kozinian-Mulders, “worm” gear	$h_1(x,k_T^2)$ Transversity; $h_{1T}^\perp(x,k_T^2)$ Pretzelosity

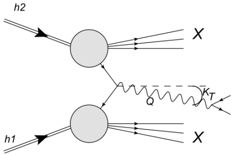
There are 8 TMDs of twist-2

$$\Phi^{[\gamma^+]}(x,b) = f_1(x,b) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^\perp(x,b),$$
$$\Phi^{[\gamma^+\gamma^5]}(x,b) = \lambda g_1(x,b) + i(b\cdot s_T) M g_{1T}^\perp(x,b),$$
$$\Phi^{[i\sigma^{\alpha+}\gamma^5]}(x,b) = s_T^\alpha h_1(x,b) - i\lambda b^\alpha M h_{1L}^\perp(x,b)$$
$$+ i\epsilon^{\alpha\mu} b_\mu M h_1^\perp(x,b) - \frac{M^2 b^2}{2}\left(\frac{g_T^{\alpha\mu}}{2} - \frac{b^\alpha b^\mu}{b^2}\right) s_{T\mu} h_{1T}^\perp(x,b),$$

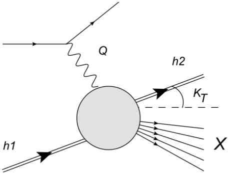
(Naive) Process-dependence

Shape of Wilson contour is dictated by the process

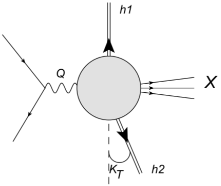




PDF($-\infty$)×PDF($-\infty$)



PDF($+\infty$)×FF($-\infty$)

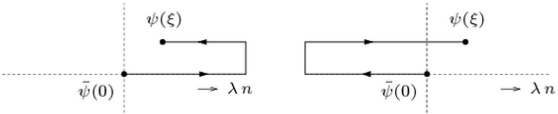


FF($+\infty$)×FF($+\infty$)

TMDPDF defined in Drell-Yan and SIDIS have different operators
(with Wilson lines pointing in different directions)

T-even & T-odd TMDs

The contour is reflected by T-conjugation



$$\text{Sivers function}: \quad f_{1T}^{\perp}(x,b)[\text{DY}] = -f_{1T}^{\perp}(x,b)[\text{SIDIS}]$$

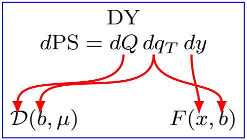
$$\text{Boer-Mulders function}: \quad h_{1}^{\perp}(x,b)[\text{DY}] = -h_{1}^{\perp}(x,b)[\text{SIDIS}]$$

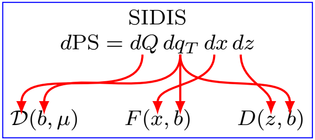
$$\text{rest TMDs}: \quad f(x,b)[\text{DY}] = f(x,b)[\text{SIDIS}]$$

Sign-change is one of conceptual predictions of QCD.
Can we check it experimentally?

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) R^2[\mathcal{D}(\mathbf{b})] F(x_1, b) F(x_2, b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda}{Q}\right)$$

- Each data-point is a convolution of **three independent nonperturbative** functions
- Each function is responsible for a separate kinematic variable
- Multi-dimensional bining is **essential**





We would like to extract Sivers function (for example)
Data in SIDIS $\sim f_{1T}^{\perp} \times D_1$
Data in DY $\sim f_{1T}^{\perp} \times f_1^{\pi}$

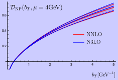
Low-energy
E228, E605,...
PHENIX

$R \times F_1 \times F_1$

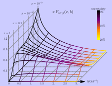
High-energy
ATLAS, LHCb,...
CDF, D0

Unpolarized DY

CS kernel

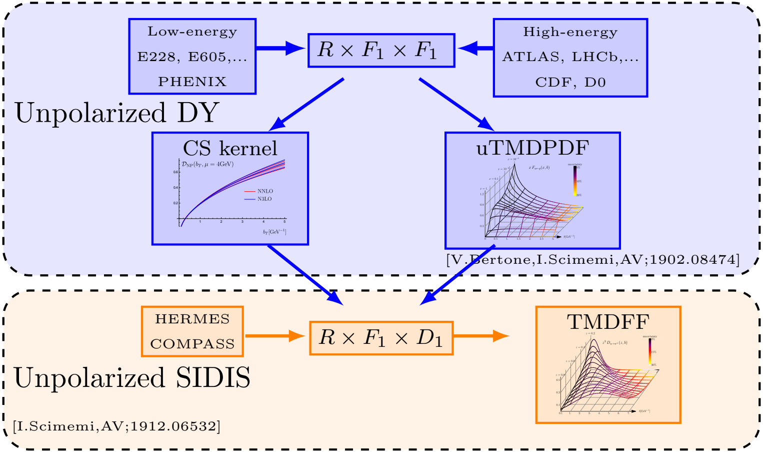


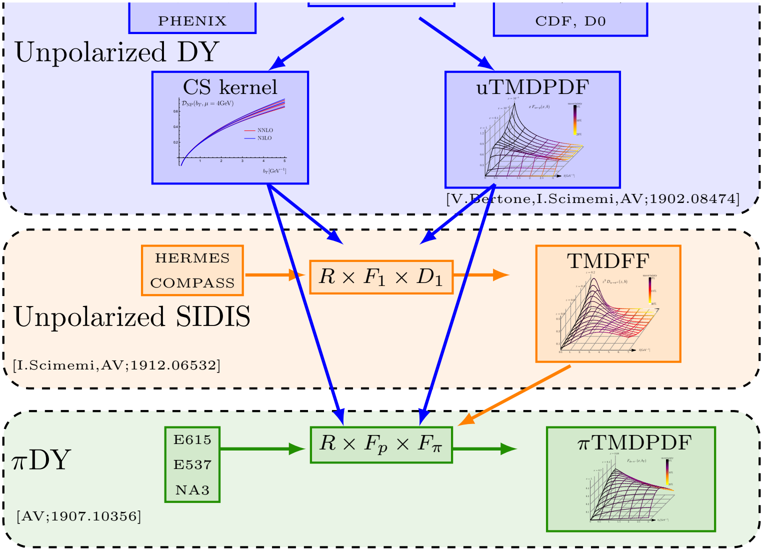
uTMDPDF



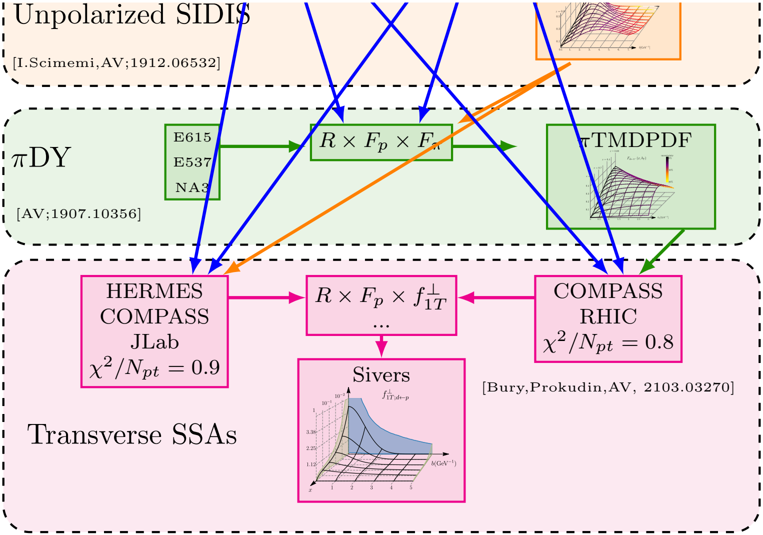
[V.Bertone,I.Scimemi,AV;1902.08474]

Universality & the chain of extractions

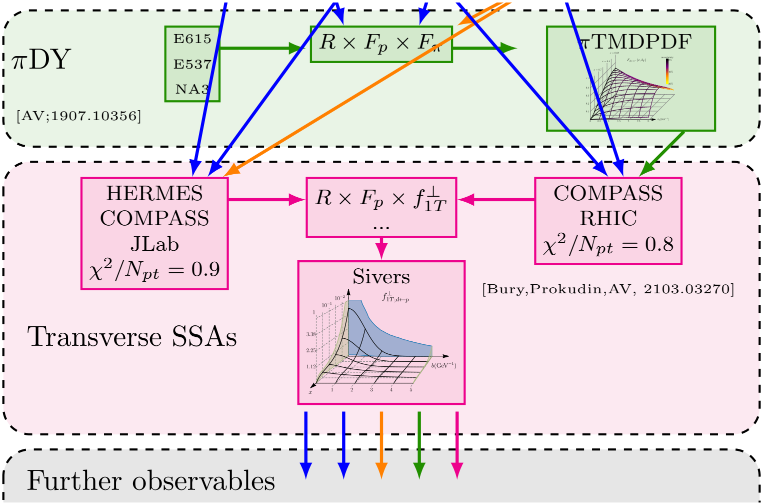



PHENIX
CDF, D0
Unpolarized DY
CS kernel
NNLO
N3LO
uTMDPDF
[V.Bertone,I.Scimemi,AV;1902.08474]
HERMES
COMPASS
$R \times F_1 \times D_1$
TMDFF
Unpolarized SIDIS
[I.Scimemi,AV;1912.06532]
πDY
E615
E537
NA3
$R \times F_p \times F_\pi$
πTMDPDF
[AV;1907.10356]

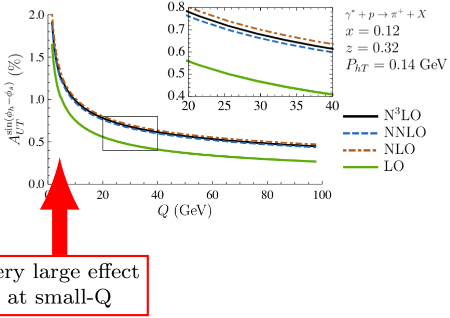
Universality & the chain of extractions



Universality & the chain of extractions



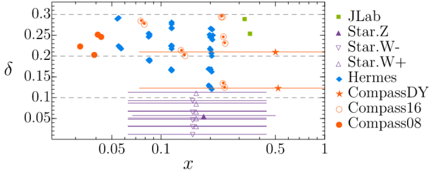
TMD evolution is very important



$$A_{UT}^{\sin(\phi_h-\phi_S)} = -M \frac{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} b\, J_1\left(\frac{b|P_{hT}|}{z}\right) R(b,Q) f_{1T,q\leftarrow h_1}^{\perp}(x,b) D_{1,q\to h_2}(z,b)}{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} J_0\left(\frac{b|P_{hT}|}{z}\right) R(b,Q) f_{1,q\leftarrow h_1}(x,b) D_{1,q\to h_2}(z,b)}$$

Quality of unpolarized input is also critical.

Data for the Sivers function



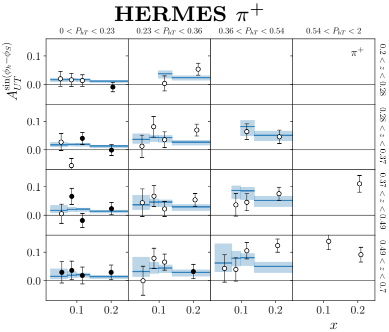
TMD factorization is valid
ONLY at
$q_T \ll Q, M \ll Q$
(it cuts a lot of data)

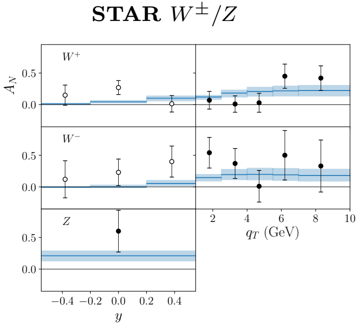
DY: $q_T = q_T$
SIDIS: $q_T = \frac{p_{h\perp}}{z}$

Each next measurement
in the chain of extractions has:
worse uncertainty
less data

Dataset name	Ref.	Reaction	# Points
Compass08	[36]	$d^\uparrow + \gamma^* \to \pi^+$	1 / 9
		$d^\uparrow + \gamma^* \to \pi^-$	1 / 9
		$d^\uparrow + \gamma^* \to K^+$	1 / 9
		$d^\uparrow + \gamma^* \to K^-$	1 / 9
Compass16	[39]	$p^\uparrow + \gamma^* \to h^+$	5 / 40
		$p^\uparrow + \gamma^* \to h^-$	5 / 40
Hermes	[35]	$p^\uparrow + \gamma^* \to \pi^+$	11 / 64
		$p^\uparrow + \gamma^* \to \pi^-$	11 / 64
		$p^\uparrow + \gamma^* \to K^+$	12 / 64
		$p^\uparrow + \gamma^* \to K^-$	12 / 64
JLab	[41, 42]	$p^\uparrow + \gamma^* \to \pi^+$	1 / 4
		$p^\uparrow + \gamma^* \to \pi^-$	1 / 4
		$p^\uparrow + \gamma^* \to K^+$	1 / 4
		$p^\uparrow + \gamma^* \to K^-$	0 / 4
SIDIS total			63
CompassDY	[40]	$\pi^- + d^\uparrow \to \gamma^*$	2 / 3
Star.W+	[43]	$p^\uparrow + p \to W^+$	5 / 5
Star.W-		$p^\uparrow + p \to W^-$	5 / 5
Star.Z		$p^\uparrow + p \to \gamma^*/Z$	1 / 1
DY total			13
Total			76

Example of data description



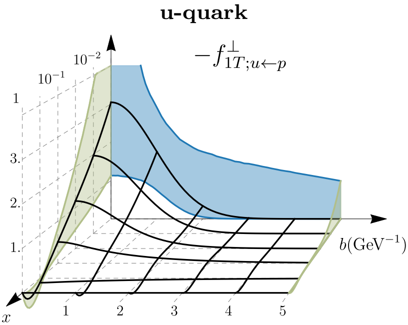


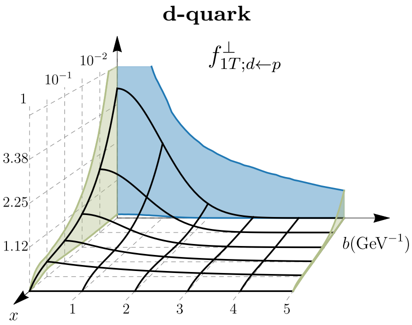
Filled points = in fit, Open point = prediction

Actually, we can explain more data (up to $q_T < 0.4Q$ in SIDIS)

Sivers function

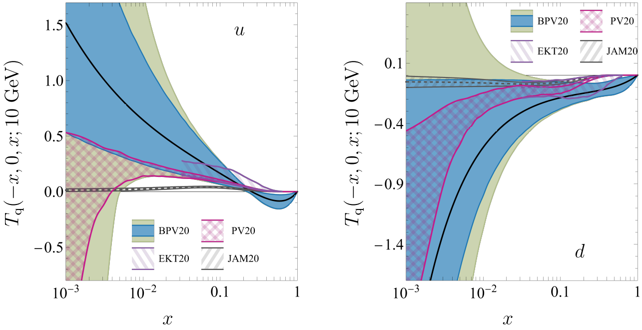
[M.Bury,A.Prokudin,AV,21]





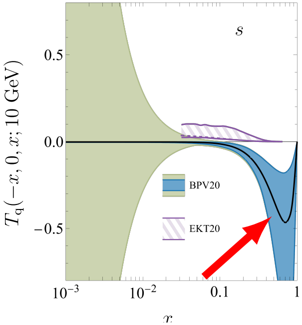
- Enormous uncertainties
- Even the sign is barely defined
- Still it is the most trustful extraction of Sivers function (included in PDG)

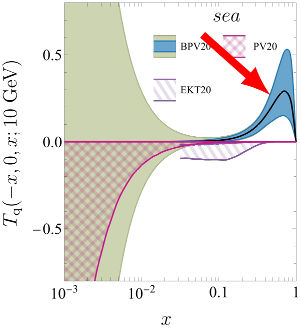
(Efremov-Teryaev-)Qiu-Sterman function
quark-gluon-quark correlator



$\lim_{b\to 0} f_1(x,b) \;\to\; f_1(x)$
$\lim_{b\to 0} f_{1T}^{\perp}(x,b) \;\to\; T(-x,0,x)$

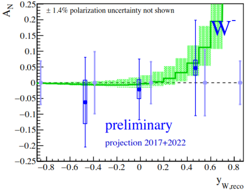
(Efremov-Teryaev-)Qiu-Sterman function
quark-gluon-quark correlator

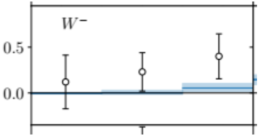




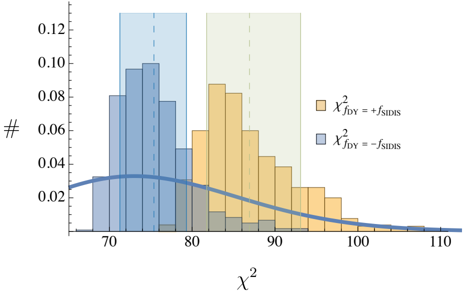
$$\lim_{b\to 0} f_1(x,b) \;\to\; f_1(x)$$
$$\lim_{b\to 0} f_{1T}^{\perp}(x,b) \;\to\; T(-x,0,x)$$

Large-x anomaly:
due to VERY large
A_N at STAR





$$f_{1T}^{\perp}(SIDIS) = -f_{1T}^{\perp}(DY)$$



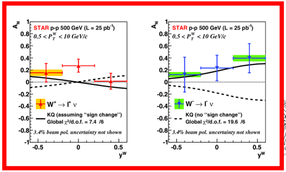
$$f_{1T}^{\perp}(sea) \to -f_{1T}^{\perp}(sea)$$

$\chi^2/N_{pt} = 0.88^{+0.16}_{-0.06}$ **vs.** $\chi^2/N_{pt} = 1.00^{+0.22}_{-0.08}$

Current data does not check sign-change!

Low-energy polarized Drell-Yan is needed!
NICA?

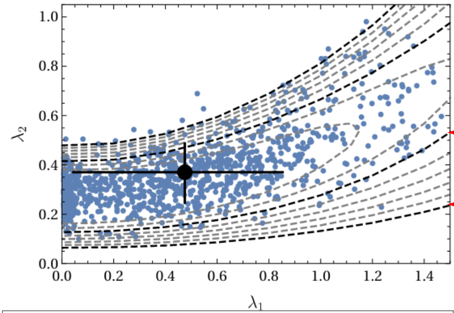
Naive picture



Situation with other TMDs is even worse
Data barely restricts them
Example of worm-gear-T function g_{1T} [Horstmann,et al, 2210.07268]

$\lambda_1 = 0.47^{+0.38}_{-0.43}$

$\lambda_2 = 0.37^{+0.12}_{-0.12}$



$\chi^2/N_{pt} = 0.95$

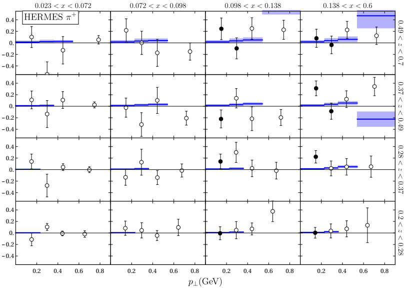
$\chi^2/N_{pt} = 1.0$

null-hypothesis: $\lambda_2 = 0 \quad \chi^2/N_{pt} = 1.06$

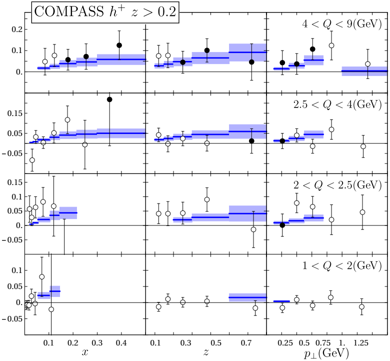
tw3-null-hypothesis: $\lambda_2 = 1 \quad \chi^2/N_{pt} = 0.95$

Example of data description

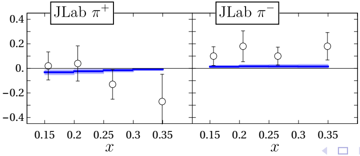
HERMES π^+



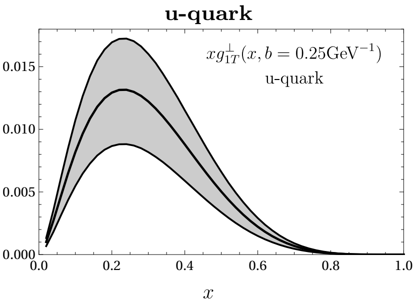
COMPASS π^+

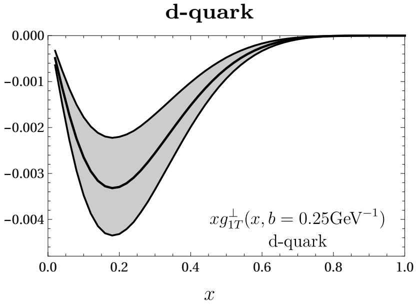


Filled points = in fit, Open point = prediction

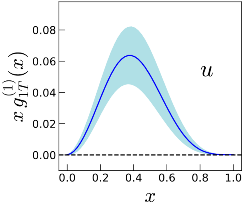


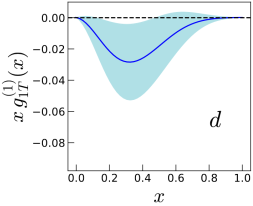
Worm-gear-T function (at $b = 0.25\text{GeV}^{-1}$)



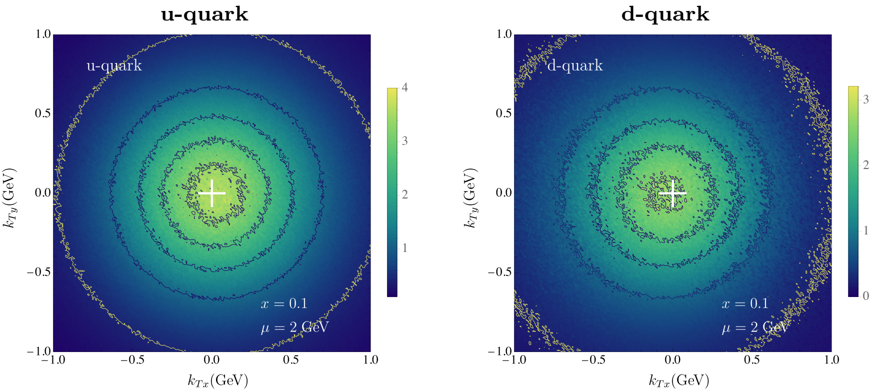


[Bhattacharya, Kang, Metz, Penn, Pitonyak: 2110.10253]





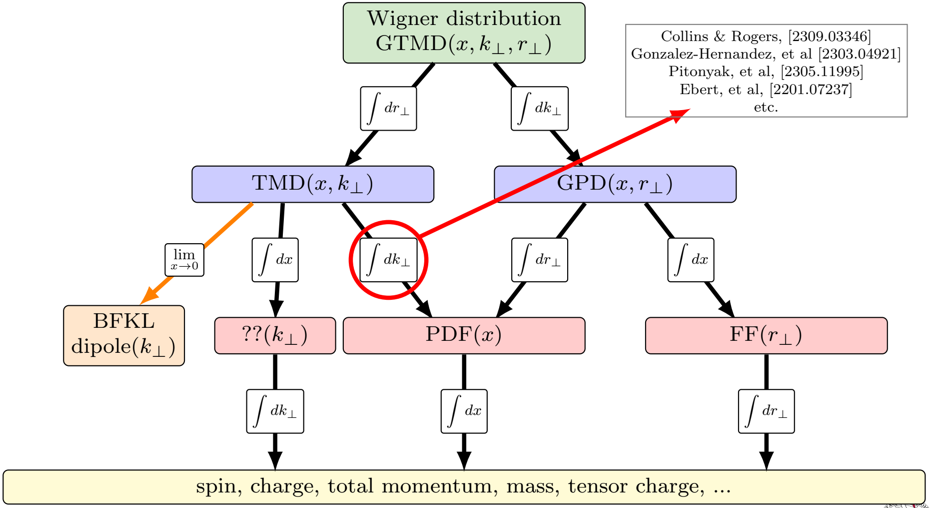
[M.Bury,A.Prokudin,AV,21]



Combination of unpolarized and Sivers function

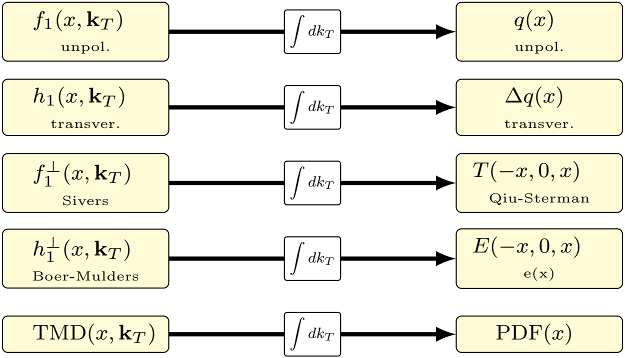
$$\rho_{1;q\leftarrow h^\uparrow}(x, \boldsymbol{k}_T, \boldsymbol{S}_T, \mu) = f_{1;q\leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f^\perp_{1T;q\leftarrow h}(x, k_T; \mu, \mu^2),$$

Interpreted as 3D momentum density of **unpolarized quark** in the nucleon


Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
Collins & Rogers, [2309.03346]
Gonzalez-Hernandez, et al [2303.04921]
Pitonyak, et al, [2305.11995]
Ebert, et al, [2201.07237]
etc.
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\lim_{x\to 0}$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
BFKL
dipole$(k_\perp)$
??$(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...

Collinear distribution from TMDs

Naively



Collinear distribution from TMDs
Properly

$f_1(x,\mathbf{k}_T,\mu,\zeta)$ unpol.	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ N^{3}LO	$q(x,\mu)$ unpol.
$h_1(x,\mathbf{k}_T,\mu,\zeta)$ transver.	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ N^{2}LO	$\Delta q(x,\mu)$ transver.
$f_1^\perp(x,\mathbf{k}_T,\mu,\zeta)$ Sivers	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ NLO	$T(-x,0,x,\mu)$ Qiu-Sterman
$h_1^\perp(x,\mathbf{k}_T,\mu,\zeta)$ Boer-Mulders	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ NLO	$E(-x,0,x,\mu)$ e(x)
$\mathrm{TMD}(x,\mathbf{k}_T,\mu,\zeta)$	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$	$\mathrm{PDF}(x,\mu)$

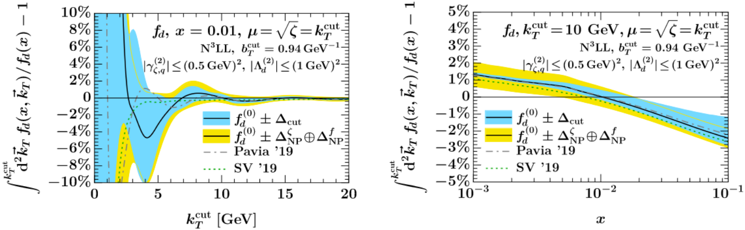
1. Coefficient function $\sim \ln^n(\mathbf{k}^2)/\mathbf{k}^2$
2. Three scales: (μ,ζ) in TMD, μ in collinear PDF

How determine collinear PDF from given TMDPDF?

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x, \mathbf{k}_T; \mu, \mu^2) \simeq q(x, \mu)$$

[Ebert, et al 2201.07237]
[Conzalez-Hernandez, et al, 2205.05750]

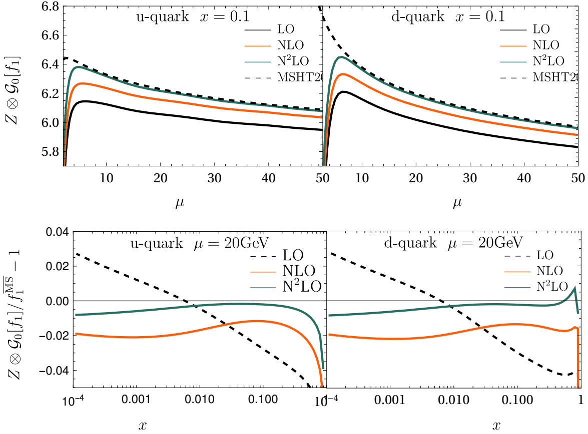


One can restore (tw2) collinear PDF up to few %. **Can we do better?**

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x, \mathbf{k}_T; \mu, \mu^2) = Z^{\mathrm{TMD}/\overline{\mathrm{MS}}}(\mu) \otimes q(x, \mu)$$

[O. del Rio, et al, 2402.01836]



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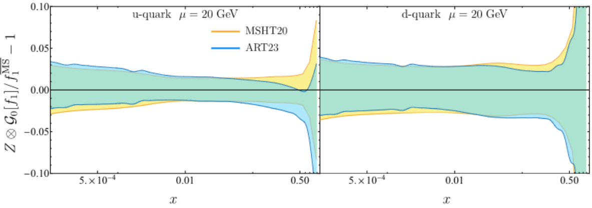
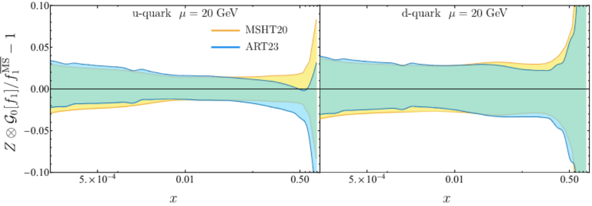
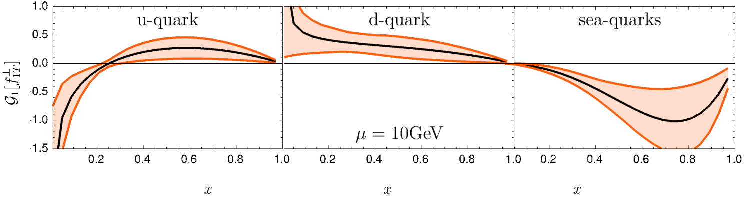


Collinear distribution from TMDs

[O. del Rio, et al, 2402.01836



$T(-x, 0, x)$ (from Sivers function [2103.03270])



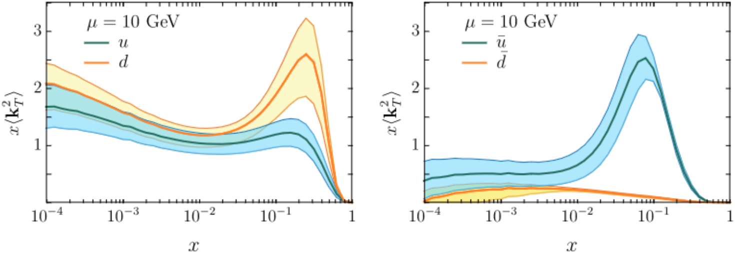
TMD-scheme (different from $\overline{\text{MS}}$ at NLO)

Second moment of TMD

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x, \mathbf{k}_T; \mu, \mu^2) - \text{subtraction} \simeq \langle \mathbf{k}_T^2 \rangle (x, \mu)$$

Equals to $\langle \bar{q} D^2 q \rangle$ in TMD-scheme ($\overline{\text{MS}}$-scheme up to NLO)

[O. del Rio, et al, 2402.01836



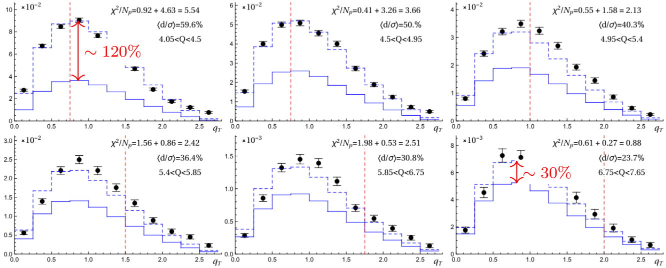
TMD factorization is very successful.
However, there are a lot of open problems.

List of problems

- **Normalization issue**
- Mismatch with high-q_T tale
- High cost of computation
- Mismatch with MC simulations
- Necessity of joint analysis (very expensive)
- Also many open theory questions
- ...
  - Power corrections
  - Interpretation of soft factor
  - Formal proof of evolution
  - Factorization for more involved processes
  - ...

Problem with normalization

πDY [1907.10356]



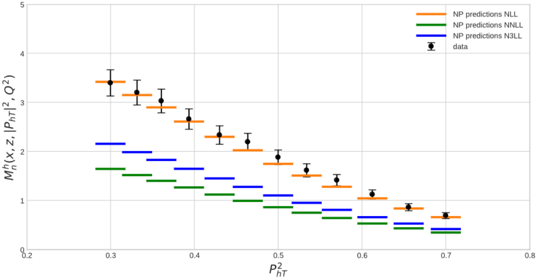
What is that?

- Problem with factorization?
- Problem with collinear PDF?
- Problem with data?

I think: this is evidence of power corrections

Problem with normalization

example of COMPASS-bin by MAP22 [2206.07598]



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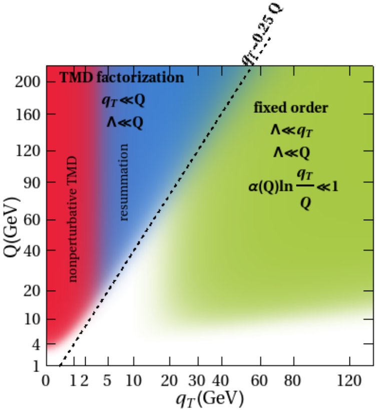


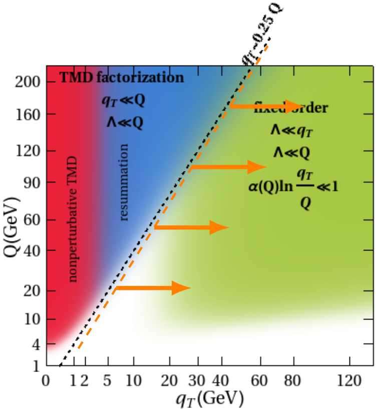
Power corrections:
(many works during last year)

- I.Stewart, A.Gao, et al,
- S.Rodini, AV, et al,
- I.Balitsky, et al,
- ...

NLP TMD factorization is done!
e.g. [2306.09495] for SIDIS
(it is much more complicated than one expected)

TMD factorization at NLP

- 4 TMDFFs, 16 TMDPDFs of twist-3
- NLP restoration of frame-invariance, gauge invariance, boost invariance
- NLO expression for coefficient functions
- LO evolution for twist-3 TMDs
- Qiu-Sterman-like terms in TMD factorization



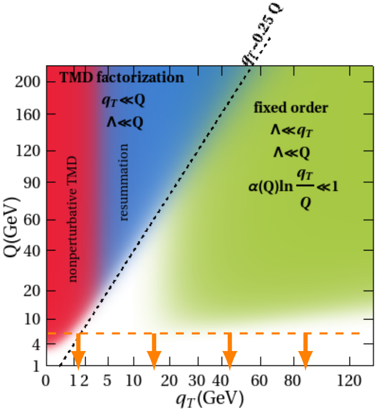
Power corrections:

1. q_T/Q-corrections
Y-term

2. Λ/Q & M/Q-corrections
higher-twist
target-mass

3. k_T/Q-corrections
kinematic

[AV,2307.13054]



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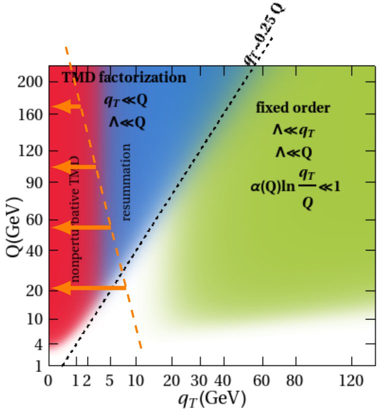

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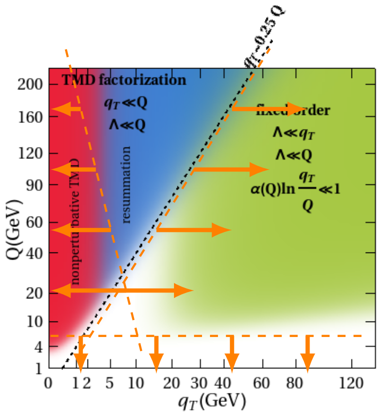

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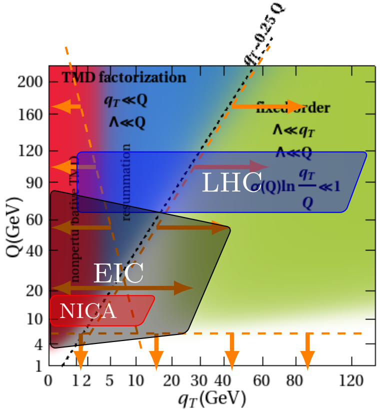

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Y-term

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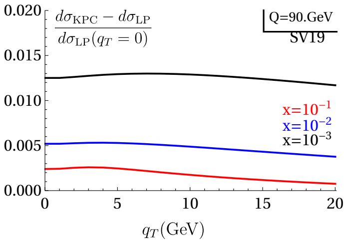
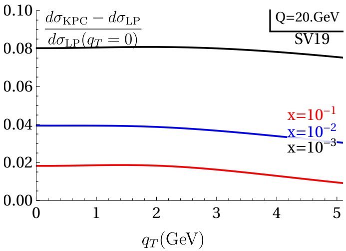
higher-twist
target-mass

3. k_T/Q-corrections
kinematic

[AV,2307.13054]

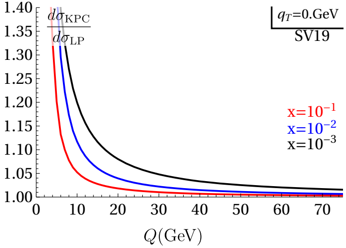
This explains why there are problems with low-k_T at $Q \sim 10$GeV
LHC is "pure" perturbation theory
EIC will be more interesting
NICA is very sensitive to these effects

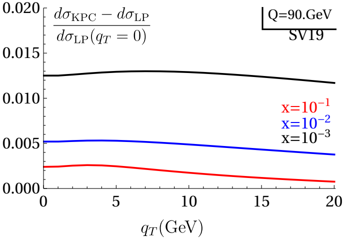


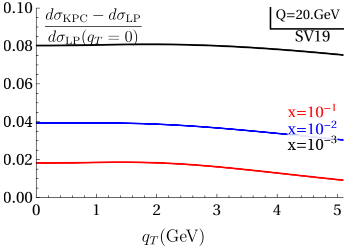


Kinematic power corrections

- Correction for the non-collinear parton momentum
- Restore EM-gauge-invariance (charge conservation)
- Restore frame-invariance
- Can be summed up at all powers [AV,2307.13054]
- Non-zero at $q_T = 0$







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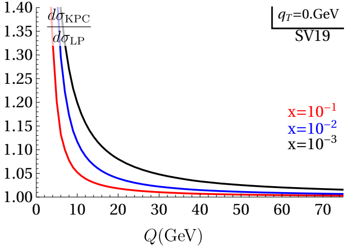
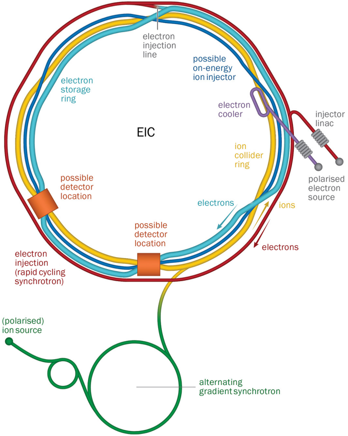


Requires further investigation

If true, all earlier phenomenology of TMDs is concerned.

EIC = Electron-Ion Collider

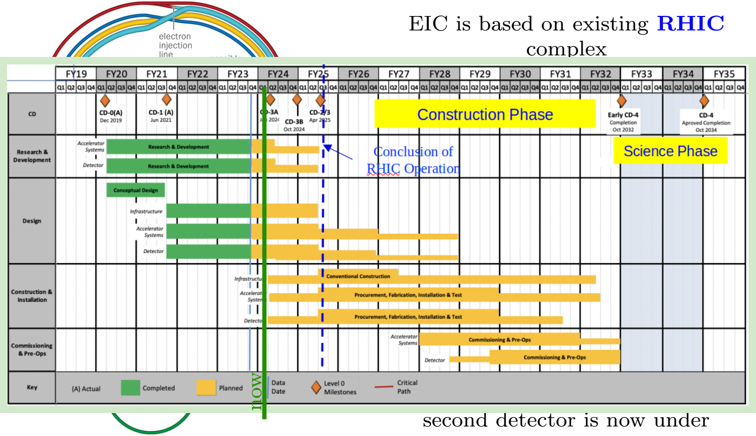


EIC is based on existing **RHIC** complex

- **High luminocity**: $\sim 10^{33} - 10^{34} \text{cm}^{-2}\text{s}^{-1}$ ($\sim$ 1000 higher than HERA)
- **Variable CM energy**: 20 – 100GeV (upg. to 140GeV)
- **Highly polarized:** 70% electron **and** nucleon beams
- **Ion beams:** proton → gold, lead, uranium
- **Two interaction regions**: second detector is now under discussion

EIC = Electron-Ion Collider

EIC is based on existing **RHIC** complex



second detector is now under discussion

Conclusion

Part 1 & 2

- Basics of TMD factorization
- TMD evolution and non-perturbative Collins-Soper kernel
- Determination of (un)polarized distributions
- Kinematics of TMD processes
- Problems and perspectives of TMD physics

Perspectives

- New experiments: EIC, AMBER, LHCspin, fixed target LHCb, JLab22, ...
- Ever improving theory
- New level of global fits

Can NICA/SPS contribute to it?

- Modern DY measurements (especially polarized) at ~ 10 GeV are very needed
- J/ψ-production in TMD is not that interesting: theory is not certain, description of J/ψ is worse that of TMDs
- Twist-three observables (asymmetries)

Conclusion

Thank you for attention!

Perspectives

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