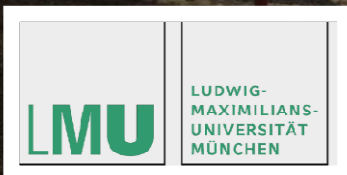


Transport Code Comparison under Controlled Conditions

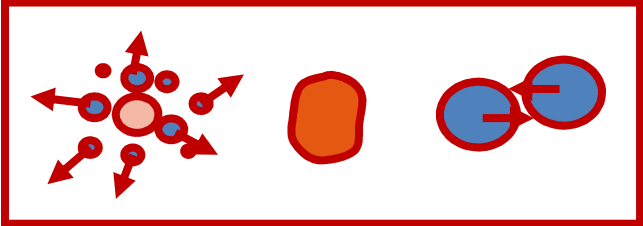
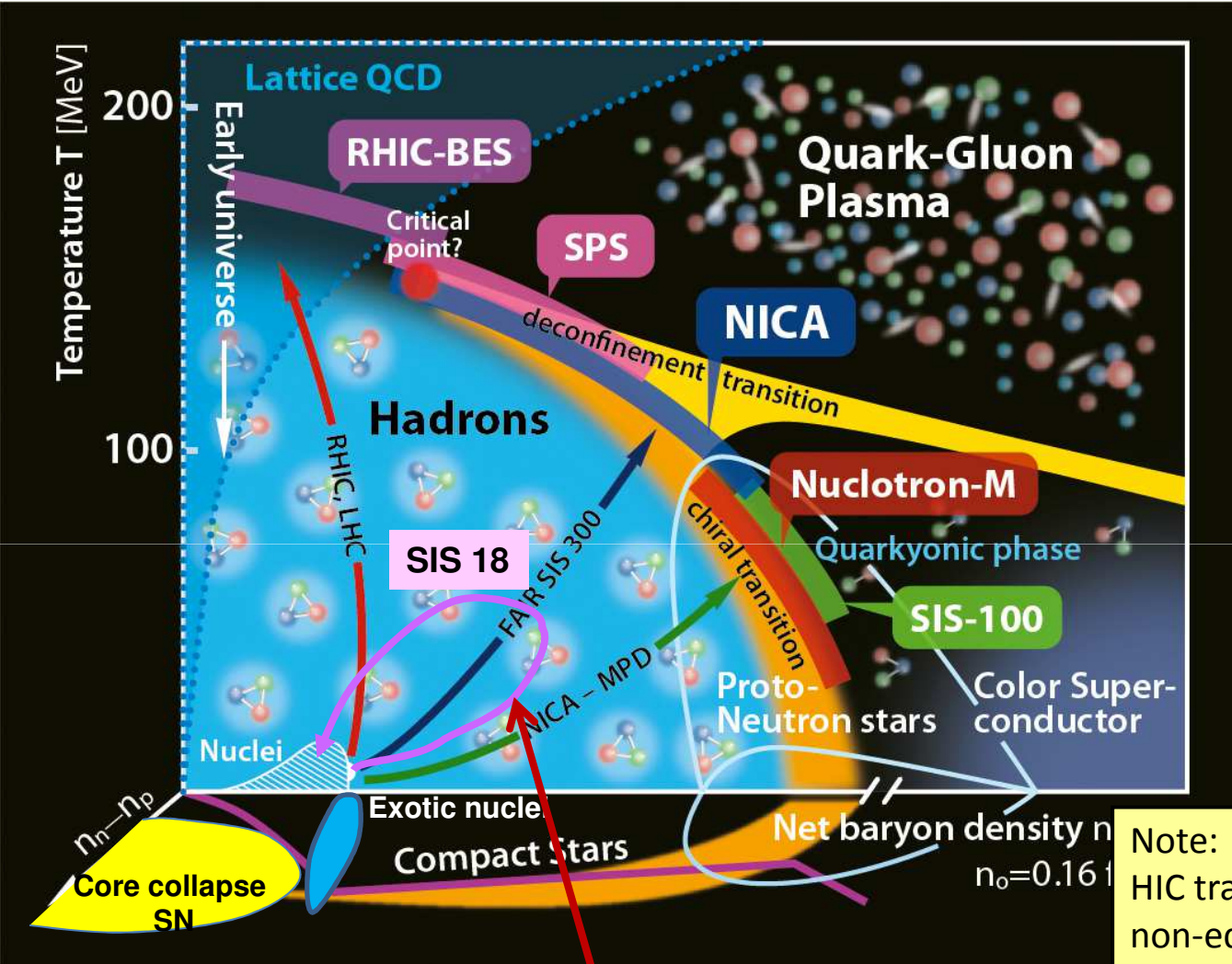
Hermann Wolter, University of Munich



II International Workshop on Simulations of HIC for NICA
Energies, Dubna, 16 - 18 April, 2018



Aim in Heavy Ion Reactions: The Phase Diagram of **Strongly Interacting Matter**

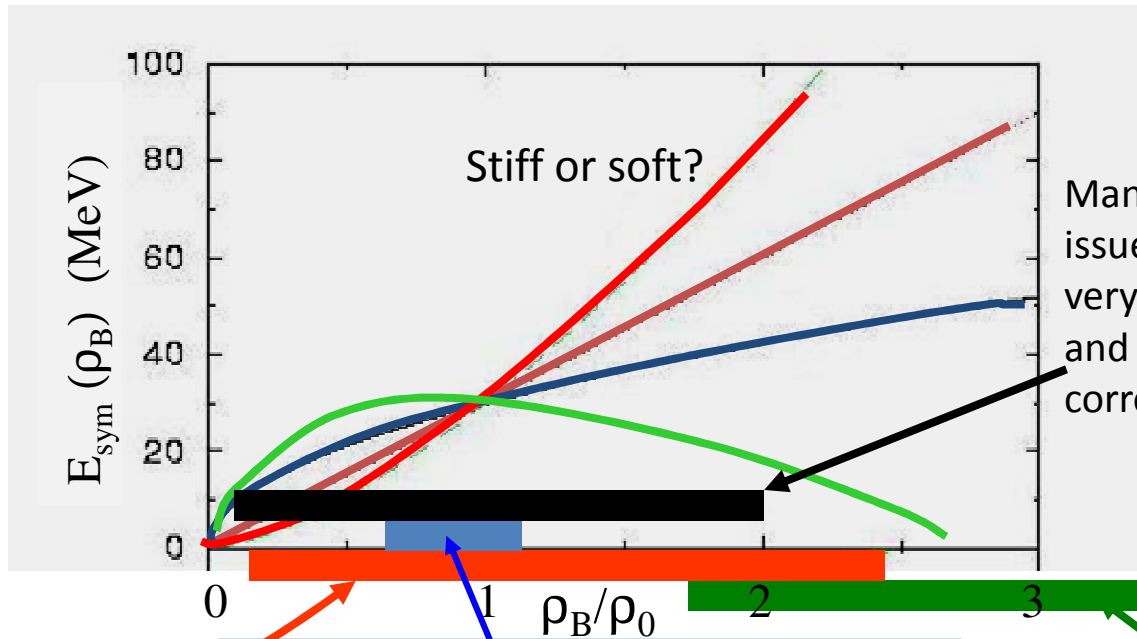


Note:
 HIC trajectories are non-equilibrium processes, and are not necessarily in this diagram
 → transport theory is necessary

The Search for the Nuclear Symmetry Energy

$$E(\rho_B, \delta)/A = E_{\text{nm}}(\rho_B) + E_{\text{sym}}(\rho_B)\delta^2 + O(\delta^4) + \dots$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$



Core collapse supernovae

heavy ion collisions

nuclear structure

neutron stars

$\rho \sim 2-4 \rho_0$

$E_{\text{sym}}(\rho_B)$

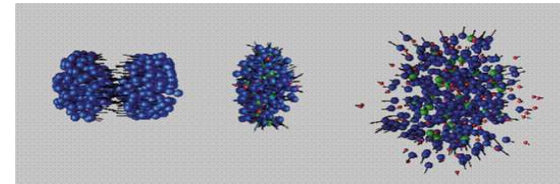
$$E_{\text{sym}}(\rho_B) = S_0 + \frac{L}{3} \left(\frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left(\frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \dots$$

→ Investigate using transport for HIC

HIC one way to obtain information on the EoS - but complex processes

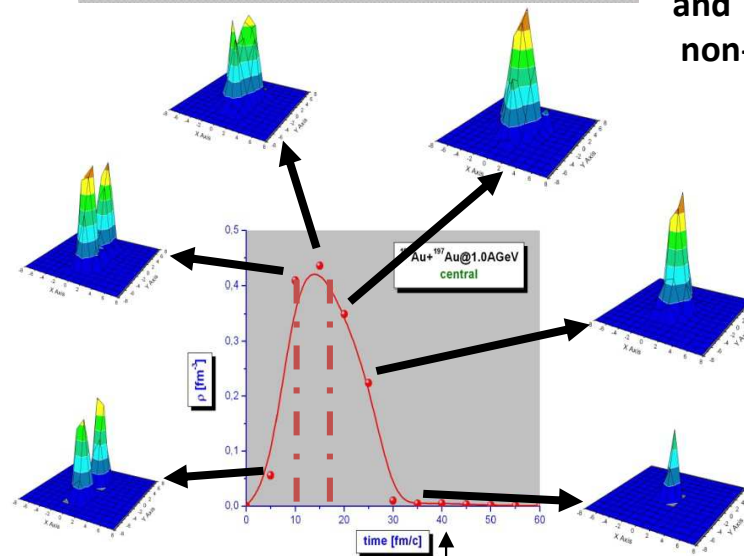
Fermi energies: (multi)-fragmentation in central collisions

Intermediate energies: several 100 MeV/A to several GeV/A
Vaporization, production of new particles, like pions, strangeness (kaons, hyperons), etc,

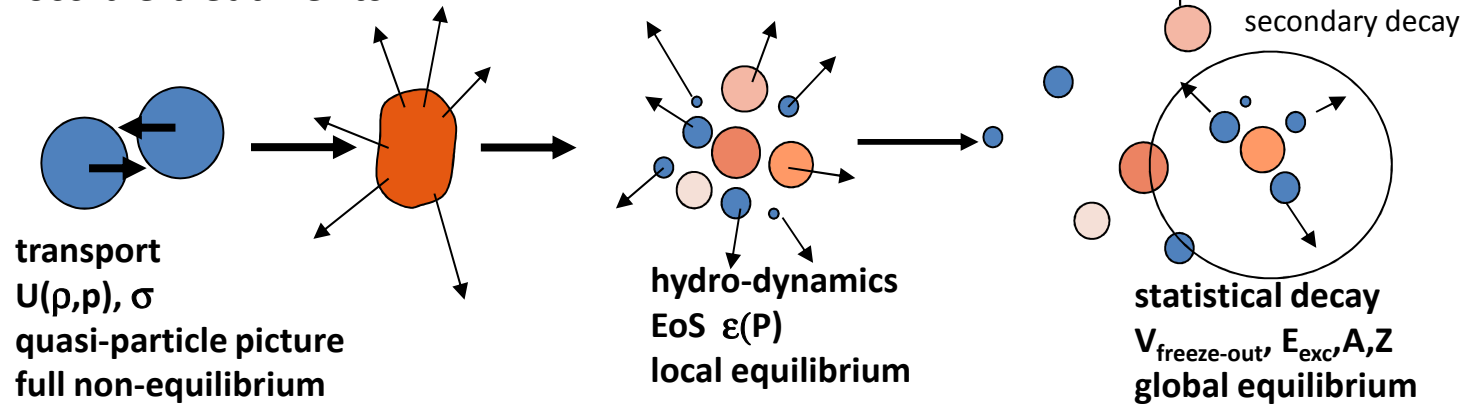


evolution in coordinate space

and momentum space: non-sphericity



Possible treatments:



Aim of this talk:

- **discussion of transport approaches to HIC**
- **not application to interpretation of data,
but rather to the accuracy of description of transport approaches**
- **comparison between different approaches among each other
for heavy ion collisions with identical physics input**
- **and with exact limits in nuclear matter (box calculations)**

- **limited to the hadronic regime, no phase transitions**
- **but hopefully an indication of what might be useful in other regimes**
- **also hybrid approaches use kinetic theory for initialization and hadronization**

- **highlight the role of fluctuations in the description of HIC**

On behalf of the Code Comparison Project

- **of the order of 30 participants**
- **core group: Akira Ono (Sendai), Yingxun Zhang (CIAE, Beijing), Jun Xu (SINAP, Shanghai), Jongjia Wang (Houzhou, China), Maria Colonna (Catania), Betty Tsang (MSU), Pawel Danielewicz (MSU), HW (Munich)**

Transport theory based on a chain of approximations

Martin-Schwinger real time formalism, irreversibility
 hierarchy in many-body Green functions, truncation,
 introduction of self energies (1-body quantities),

Quantum transport theory: Kadanoff-Baym theory

Semiclassical approximation :

Wigner transform, treat as phase space probabilities
 Gradient approximation (separation of short and long scales)

Quasi-particle approximation

Spectral function → delta function with effective momenta and masses
 neglect off-shell effects (or treat approximately)

→ kinetic equation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - (\vec{\nabla}^{(r)} U(r, p) \vec{\nabla}^{(p)} + \vec{\nabla}^{(p)} U(r, p) \vec{\nabla}^{(r)}) f(\vec{r}, \vec{p}; t) =$$

$$\int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') [f_1' f_2 \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_1' \bar{f}_2']$$

Pauli blocking factors,
main quantum ingredients

$\bar{f}_i := (1 - f_i)$

Mean field evolution (Vlasov) + uncorr. 2-body collisions (Boltzmann)
 + Pauli-blocking of final states (Uehling-Uhlenbeck)

physical input:

mf potential $U(r, p)$, momentum dependent
 σ^{in-med} in-medium cross sections

Obtainable
 e.g. from Bruecker theory microscopically
 or modeling (density functionals, cross sect.)

Two families of transport approaches

Boltzmann-Vlasov-like (BUU/BL/BLOB)

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} - \vec{\nabla} U(r) \vec{\nabla}^{(p)} \right) f(\vec{r}, \vec{p}; t) = I_{\text{coll}} [\sigma^{\text{in-med}}] + \delta I_{\text{fluct}}$$

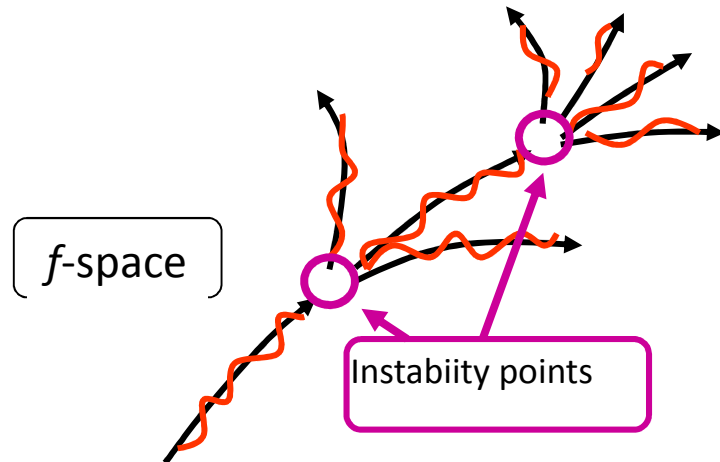
Dynamics of the 1-body phase space distribution function f with 2-body dissipation

fluctuations around diss. solution

$$f(\mathbf{r}, \mathbf{p}, t) = \bar{f}(\mathbf{r}, \mathbf{p}, t) + \delta f(\mathbf{r}, \mathbf{p}, t)$$

$$\frac{df}{dt} = I_{\text{coll}} + I_{\text{fluc}}$$

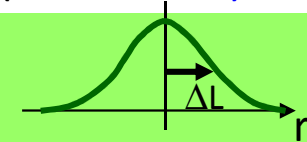
Boltzmann-Langevin eq.



Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = \mathcal{A} \prod_{i=1}^A \varphi(\mathbf{r}; \mathbf{r}_i, \mathbf{p}_i) |0\rangle$$

$$\dot{\mathbf{r}}_i = \{\mathbf{r}_i, H\}; \quad \dot{\mathbf{p}}_i = \{\mathbf{p}_i, H\}; \quad H = \sum_i t_i + \sum_{i,j} V(\mathbf{r}_i - \mathbf{r}_j)$$



TD-Hartree(-Fock)

(or classical molecular dynamics with extended particles)

plus stochastic NN collisions

No quantum fluctuations, but classical N-body fluctuations, damped by the smoothing.

More fluctuations than BUU, since degrees of freedom are nucleons:

→ amount controlled by width of single particle packet ΔL

Implementations of Transport Equation

1. BUU
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r) \vec{\nabla}^{(p)} f(\vec{r}, \vec{p}; t) = \int d\vec{v}_2 d\vec{v}_1 d\vec{v}_2' v_{21} \sigma_{12}(\Omega) (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \left[f_1' f_2' (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_1')(1 - f_2') \right]$$

non-linear integro-differential equation, no closed solution but deterministic !

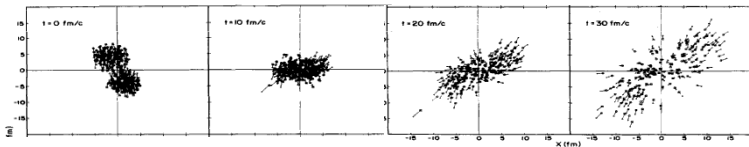
a) solution on a **lattice**: has been used for low-dimensional model systems, but too expensive for realistic cases

b) **test particle (TP) method (Wong 82)**

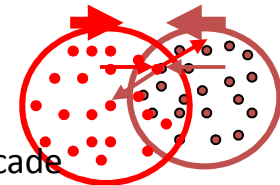
$$f(\vec{r}, \vec{p}; t) = \frac{1}{N_{TP}} \sum_{i=1}^{AN_{TP}} \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t))$$

where $\{\vec{r}_i(t), \vec{p}_i(t)\}$ are the positions and momenta of the TP as a funct. of time, and N_{TP} is the number of TP per nucleon (usually 50 – 200)

→ variant: Gaussian TP: smoother distribution with fewer TP



$$\frac{\partial \vec{r}_i}{\partial t} = \frac{\vec{p}_i}{m}; \quad \frac{\partial \vec{p}_i}{\partial t} = -\nabla U|_{\vec{r}_i}$$



c) the rhs (collision term) is simulated, stochastically, by collisions of test particles; like cascade

→ describes average effect of collisions (→dissipation), NO Fluctuations

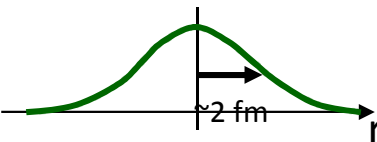
→ if $N_{TP} \rightarrow \infty$ exact solution of BUU eqn. !

$$b < b_{\max} = \frac{1}{\pi} \sqrt{\sigma^{\text{tot}}(\sqrt{s})}$$

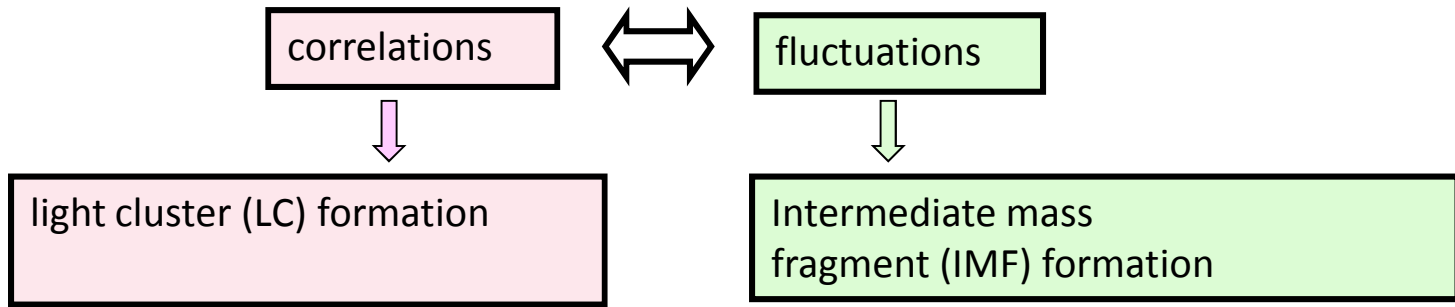
2. Molecular Dynamics (QMD)

Very similar equation of motions for centers of wave packets (nucleons, not test particles) and stochastic collisions of nucleons

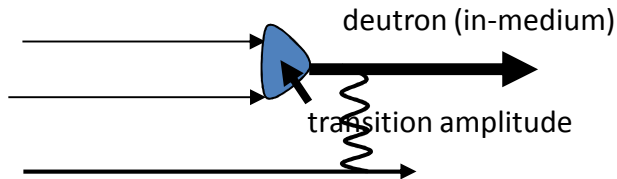
2b) Antisymmetrized MD (AMD,FMD) Gaussians are antisymmetrized wave packets collision term with stochastic features (wave packet splitting),



The issue in fragment and cluster formation in HIC collisions:

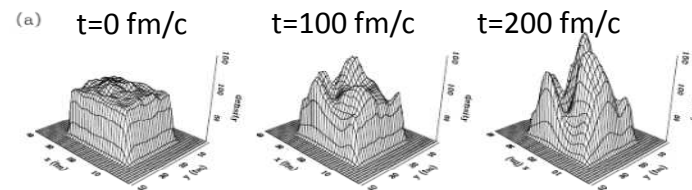


LC's are not stabilized by the mean field but by many body correlations. Introduce as explicit degrees of freedom, generated by the collision term



(P. Danielewicz and Q. Pan, PRC 46 (1992))
(d,t,3He, but no α !)

Fluctuation in the instable region are Amplified and stabilized by the mean field



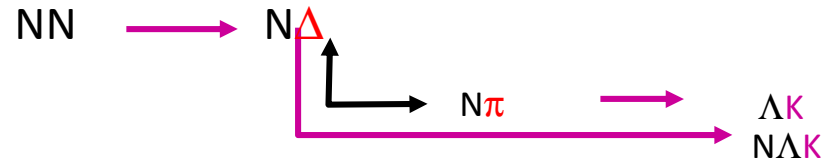
BUU calculation in a box with initial conditions inside the instability region: $\rho=\rho_0/3$, $T=5$ MeV, $\delta=0$

(V.Baran, et al., Phys.Rep.410,335(05))

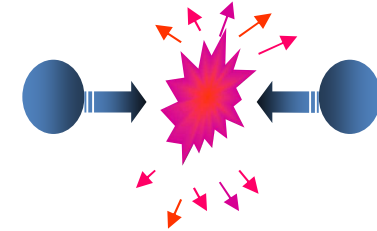
Perhaps the way to deal with the hadron-quark phase transition in transport approaches

Particle Production

Inelastic collisions: Production of particles and resonances



e.g. pion and kaon production;
coupling of Δ and strangeness channels.



$$\frac{d}{dt} f_N(x_\mu) = I_{coll}(\sigma_{NN \rightarrow NN'} f_N; \sigma_{NN \rightarrow N\Delta} f_\Delta; \dots)$$

$$\frac{d}{dt} f_\Delta(x_\mu) = I_{coll}(\sigma_{\Delta N \rightarrow NYK} f_Y f_K; \dots)$$

etc.

Coupled transport equations

Many new potentials, elastic and inelastic cross sections needed, Δ dynamics in medium

What can one learn from different species?

- **pions**: production at all stages of the evolution via the Δ -resonance
- **kaons** (strange mesons with high mass): subthreshold production, probe of high density phase
- **ratios** of π^+/π^- and K^0/K^+ :
→ **probe for symmetry energy**

Code Comparison Project

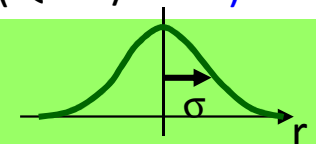
Boltzmann-Vlasov-like (BUU/BL/BLOB)

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} - \vec{\nabla} U(r) \vec{\nabla}^{(p)} \right) f(\vec{r}, \vec{p}; t) = I_{coll}[\sigma^{in-med}, f_i]$$

6-dim integro-differential, non-linear eq.

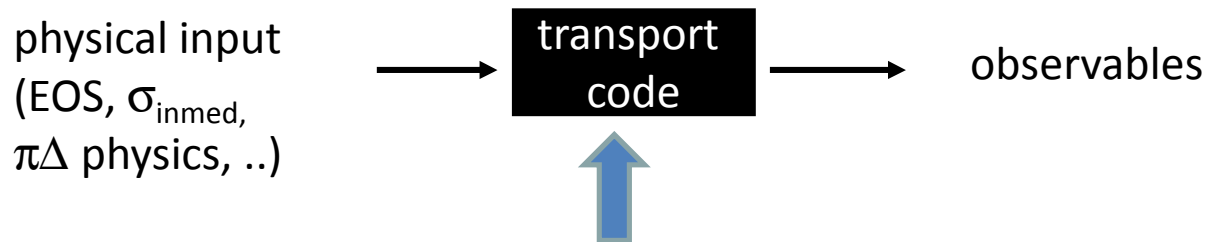
Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = \mathbf{A} \prod_{i=1}^A \varphi(r; r_i, p_i) |0\rangle$$

$$\dot{r}_i = \{r_i, H\}; \quad \dot{p}_i = \{p_i, H\}; \quad H = \sum_i t_i + \sum_{i,j} V(r_i - r_j)$$


6A-dim many body problem + stochastic coll.

→ very complex, simulate solutions
introduces many technical details

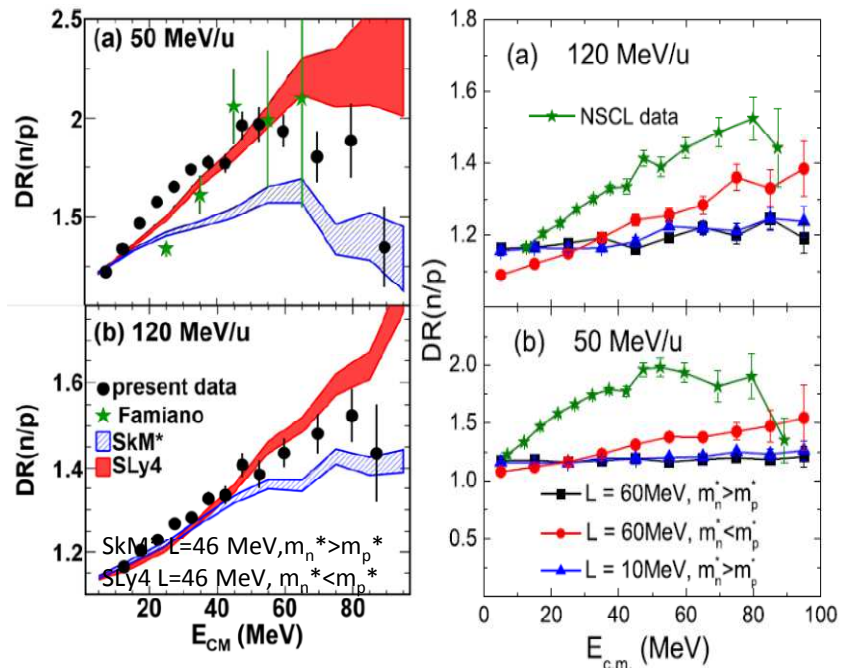


- unique?, e.g. like 2N transfer
- very complex, simulation of an equation rather than a solution
- results are sometimes not consistent
- establish a sort of systematical theoretical error

→ **Transport Code Evaluation (Comparison) Project**

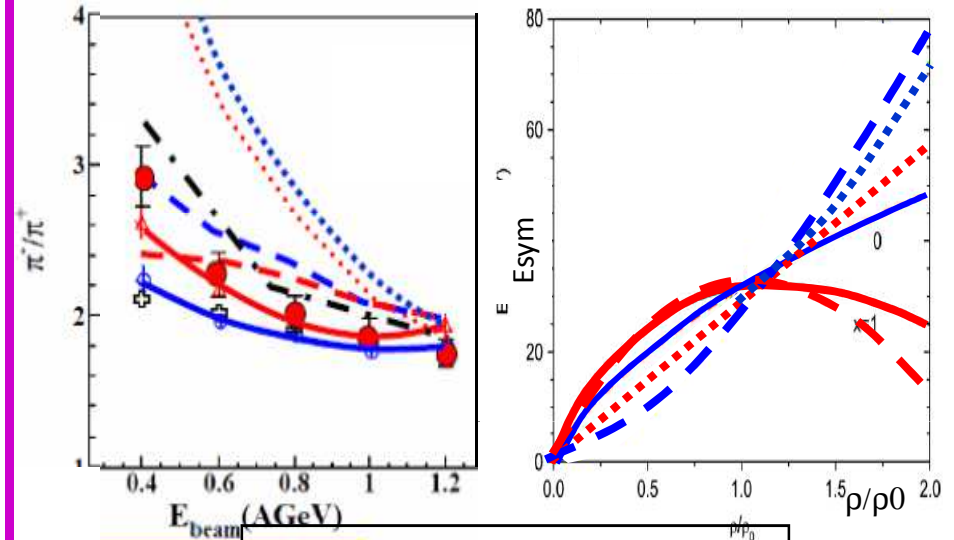
Code Comparison:
A need for more consistency in HI simulations: examples

double ratio of n/p pre-equilibrium emiss.



D.D.S.Coupland, et al., arXiv1406.4546
H.J.Kong, et al., PRC91,047601 (2015)

ratio of pion yields, Au+Au, 0.4-1.2 GeV/A



various models
blue: stiffer symm energy
red: softer symm energy
→ no consensus on ordering

Reasons for differences often not clear, since calculations slightly different in the physical parameters.
→ therefore comparison of calculations with same physical input, i.e. under controlled conditions

Code Comparison Project

Idea: Comparison of transport simulations

Determine a kind of - measure for the reliability

- i.e. a systematic theoretical error

History:

Workshop in Trento 2004 (1 AGeV regime, mainly particle production π, K)

E. Kolomeitsev, et al., J. Phys. G 31 (2005) S741)

Workshop in Trento 2009 (100, 400 A MeV)

Workshops in Shanghai and Lanzhou 2014, Shanghai 2015 (Au+Au collisions, 100, 400 A MeV)

J. Xu, et al., Phys. Rev. C 93, 044609 (2016)

Workshop ICNT and NuSYM 2017, MSU 2017 (Cascade box calculations)

Y.X.Zhang, et al., Phys. Rev. C 97, 034625 (2018)

to be continued : Zhuhai (China, 2018) and NuSYM 2018 (Busan, Korea)

Steps in Code Comparison of Transport Simulations

1. Full heavy ion collisions (Au+Au, 100, 400 AMeV)
comparison of initialization, collision rates and observables **done**
-> considerable discrepancies

2. Calculations of nuclear matter (box with periodic boundary conditions)
test separately ingredients in a transport approach:

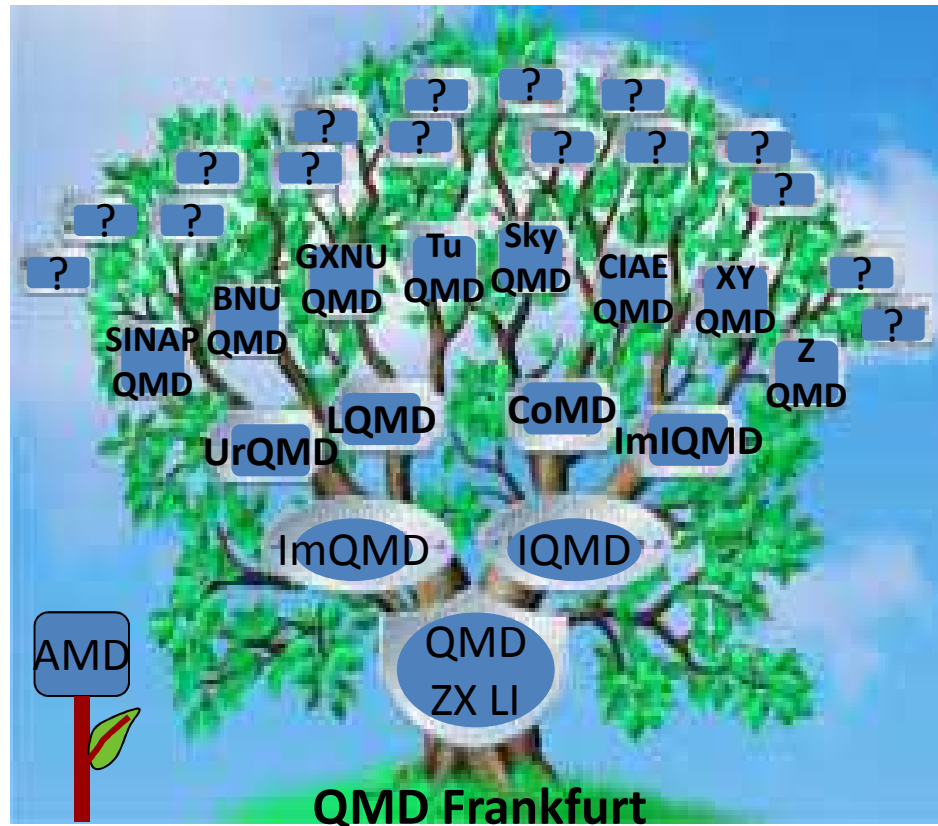
- a) collision term without and with blocking (Cascade) **done**
- b) mean field propagation (Vlasov) **in progress**
- c) pion , Δ production in cascade **in progress**
- d) instabilities , fragmentation **planned**
- e) momentum dependent fields **planned**

.....

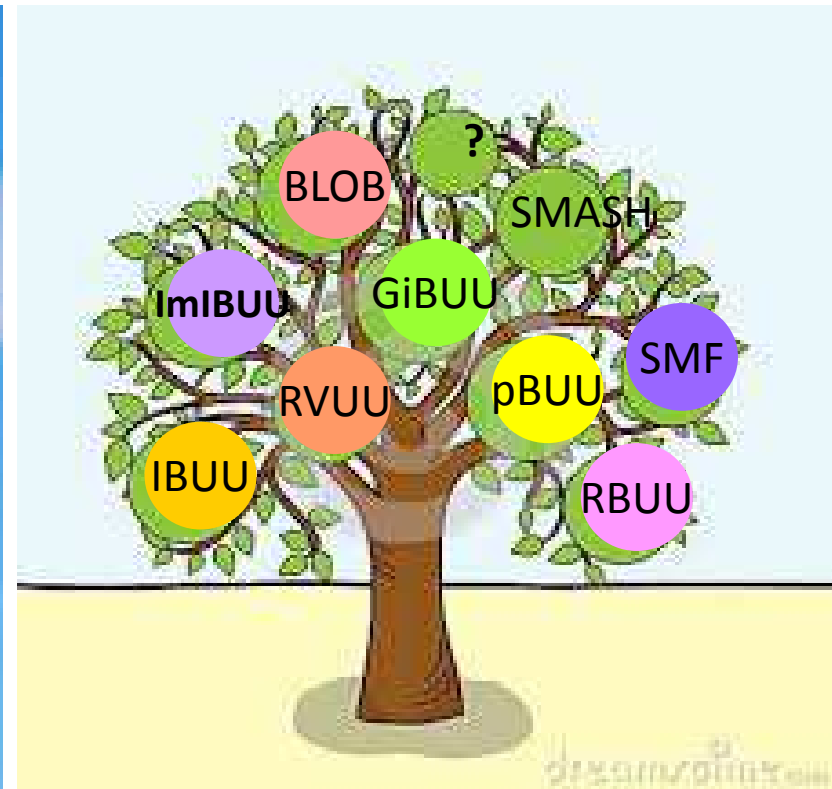
Codes participating in the code comparison

BUU type	Code correspondents	Energy range	Reference	QMD type	Code correspondents	Energy range	Reference
BLOB	P. Napolitani, M. Colonna	0.01–0.5	[19]	AMD	A. Ono	0.01–0.3	[28]
GIBUU-RMF	J. Weil	0.05–40	[20]	IQMD-BNU	J. Su, F. S. Zhang	0.05–2	[29]
GIBUU-Skyrme	J. Weil	0.05–40	[20]	IQMD	C. Hartnack, J. Aichelin	0.05–2	[30–32]
IBL	W. J. Xie, F. S. Zhang	0.05–2	[21]	CoMD	M. Papa	0.01–0.3	[33,34]
IBUU	J. Xu, L. W. Chen, B. A. Li	0.05–2	[11,22]	ImQMD-CIAE	Y. X. Zhang, Z. X. Li	0.02–0.4	[35]
pBUU	P. Danielewicz	0.01–12	[23,24]	IQMD-IMP	Z. Q. Feng	0.01–10	[36]
RBUU	K. Kim, Y. Kim, T. Gaitanos	0.05–2	[25]	IQMD-SINAP	G. Q. Zhang	0.05–2	[37]
RVUU	T. Song, G. Q. Li, C. M. Ko	0.05–2	[26]	TuQMD	D. Cozma	0.1–2	[38]
SMF	M. Colonna, P. Napolitani	0.01–0.5	[27]	UrQMD	Y. J. Wang, Q. F. Li	0.05–200	[39,40]

- BUU- and QMD-type
- non-rel. and relativistic codes
- antisymmetrized QMD code: AMD, CoMD
- BUU codes with explicit fluctuations: SMF, BLOB
- many new Chinese codes: (I)QMD-XXX: much new activity in China, often originally closely related



„... in full bloom...” often closely related



„...many individuals...”

Set-up of code comparison for Heavy Ion Collisions („homework“)

- typical reaction in low and intermediate energy: Au+Au, 100 and 400 A MeV
- impact parameter 20 fm (no collision, stability of initialization) and 7 fm (midcentral collision)
- simple physics case (not necessarily realistic)
 - standard Skyrme mean field, momentum independent, equivalent RMF
 - constant cross section, no inelastic collisions
- „close“ initialization of colliding nuclei
 - prescribed density profile, momentum in local Fermi sphere
- collision and blocking procedures as in standard use of code
- different „modes“: Vlasov (only mean field), Cascade (only collisions), „full“
- monitor: (test) particle motion, number, energy and time of collisions, Pauli-blocking, observables (rapidity, flow)

PHYSICAL REVIEW C 93, 044609 (2016)

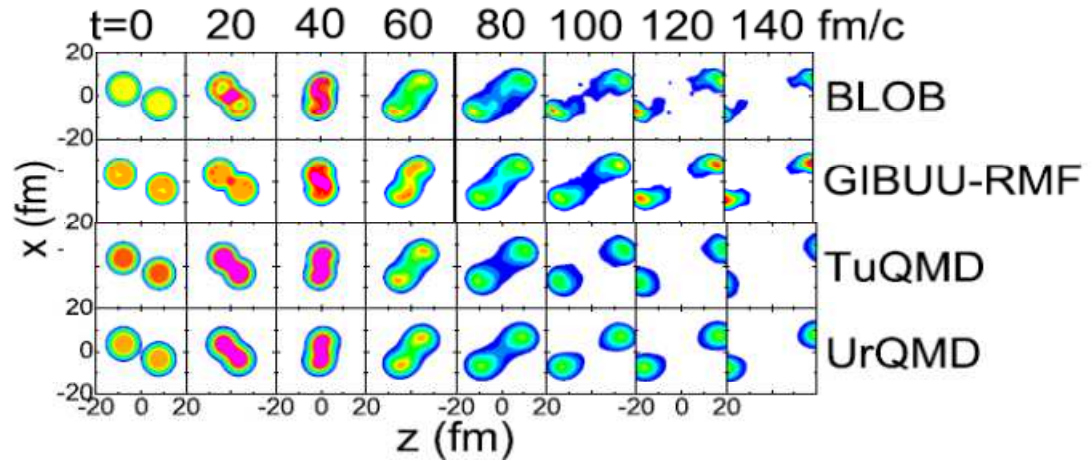
Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions

Jun Xu,^{1,*} Lie-Wen Chen,^{2,†} ManYee Betty Tsang,^{3,‡} Hermann Wolter,^{4,§} Ying-Xun Zhang,^{5,¶} Joerg Aichelin,⁶ Maria Colonna,⁷ Dan Cozma,⁸ Pawel Danielewicz,³ Zhao-Qing Feng,⁹ Arnaud Le Fèvre,¹⁰ Theodoros Gaitanos,¹¹ Christoph Hartnack,⁶ Kyungil Kim,¹² Youngman Kim,¹² Che-Ming Ko,¹³ Bao-An Li,¹⁴ Qing-Feng Li,¹⁵ Zhu-Xia Li,⁵ Paolo Napolitani,¹⁶ Akira Ono,¹⁷ Massimo Papa,¹⁸ Taesoo Song,¹⁹ Jun Su,²⁰ Jun-Long Tian,²¹ Ning Wang,²² Yong-Jia Wang,¹⁵ Janus Weil,¹⁹ Wen-Jie Xie,²³ Feng-Shou Zhang,²⁴ and Guo-Qiang Zhang¹

 editing group

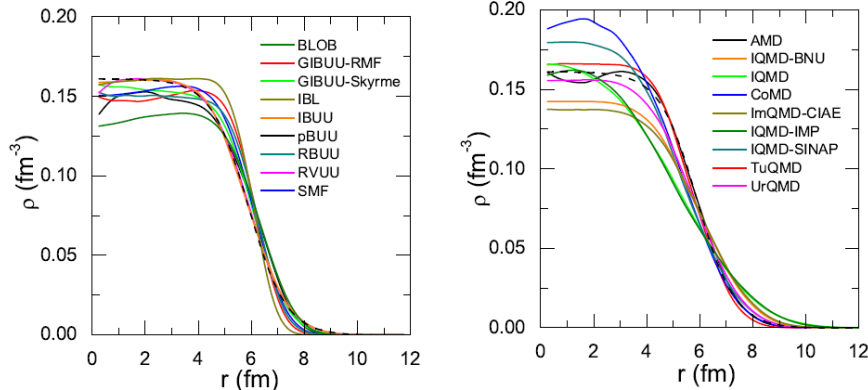
Code Comparison Project (1st stage):

HIC at $b=7m$ (midcentral)
 selected contour plots;
 different evolution apparent
 → compare collision numbers,
 blocking, and observables



Initialization and Stability

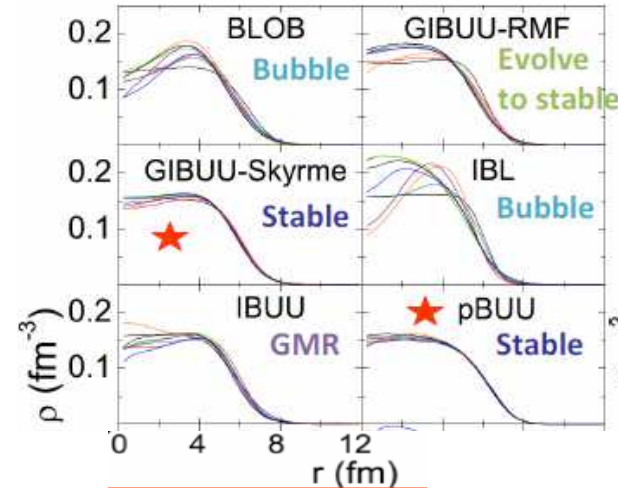
dashed curve \equiv prescribed density profile



„identical“ initialization difficult, since it depends also on representation of (test) particles

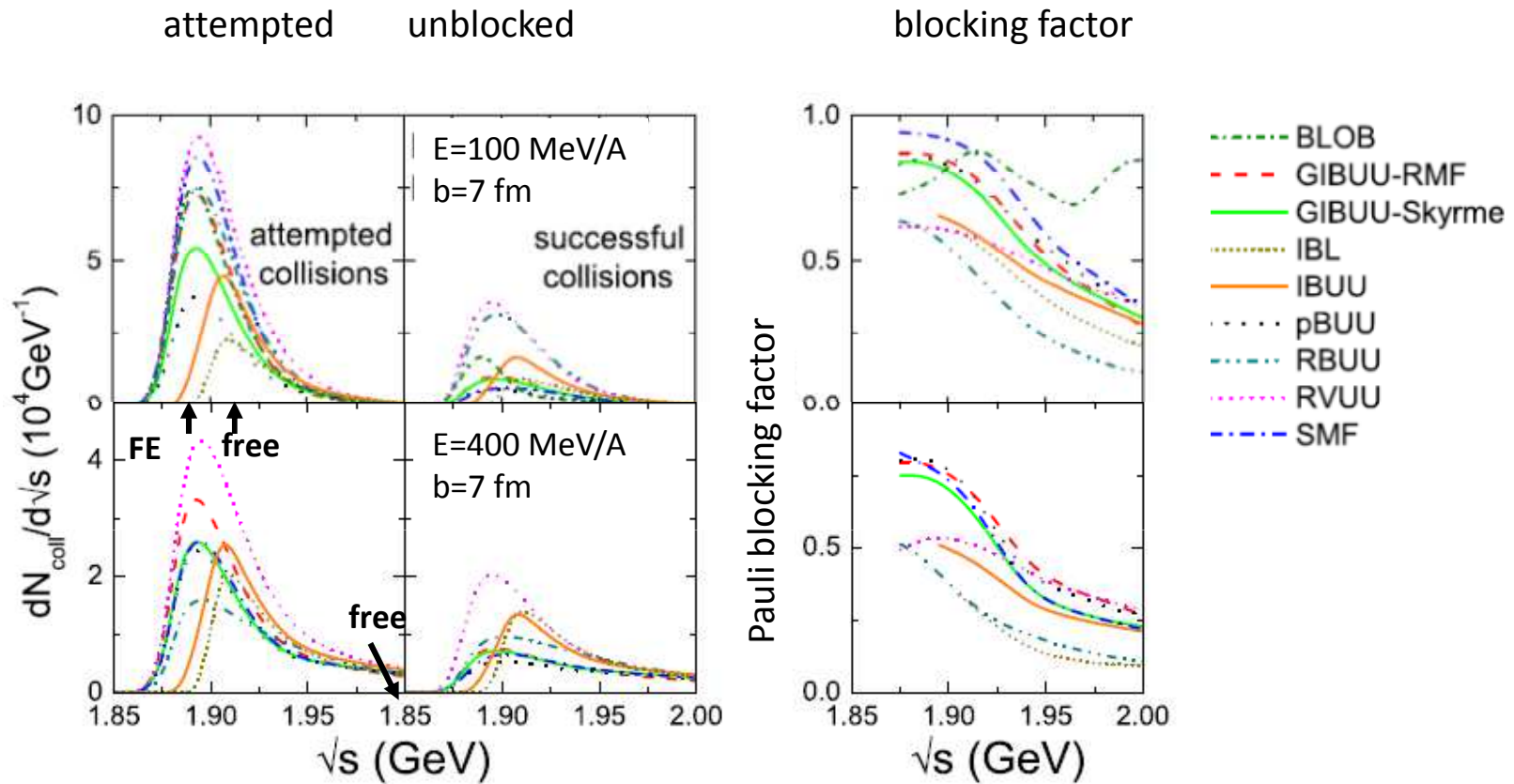
- prescribed density profile is not necessarily ground state and may be non-stationary
- diff. initializations affect evolution also in case of a collision

time evolution of isolated nucleus(examp)



★ Dynamical initialization (Thomas-Fermi)

NN Collision rates per energy bin



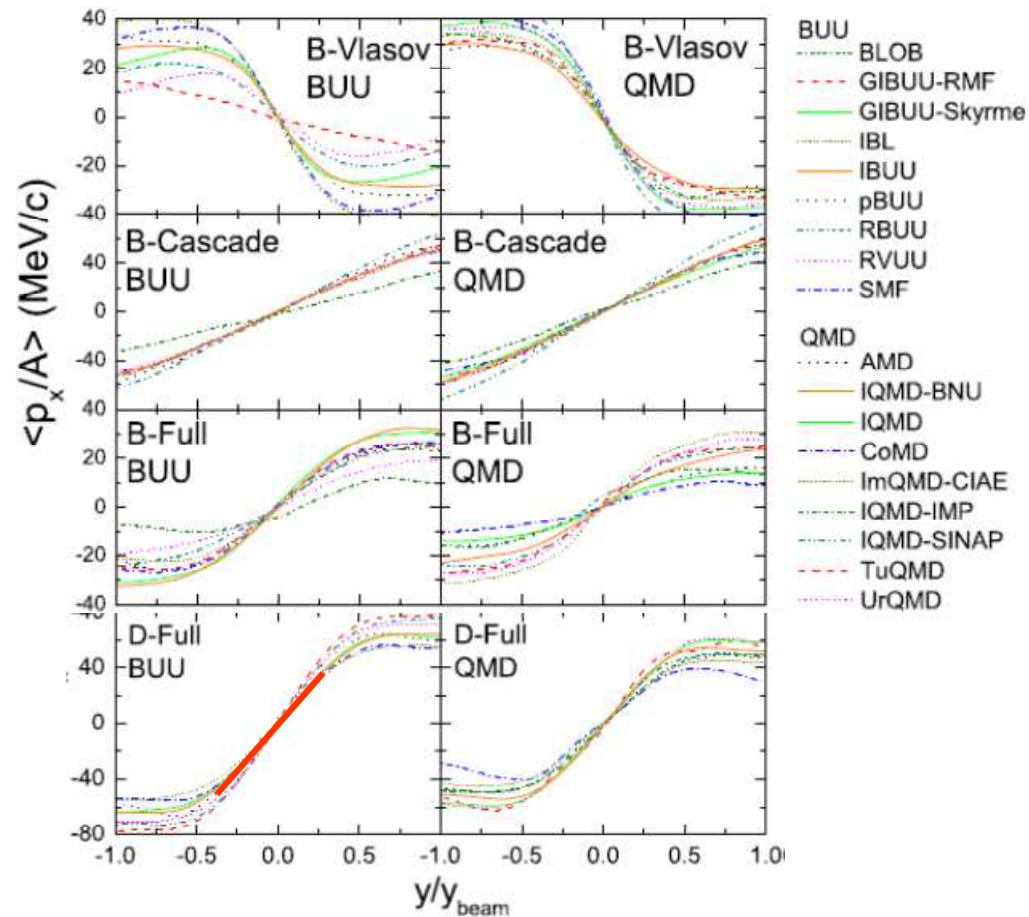
Considerable difference both for :

- attempted collisions, mostly low energy(!)
(depends on strategy for finding collision pairs)
- blocking factor (depends on occupation of final state)
- better consistency for higher energy
- not much difference for BUU and QMD

Observables: directed flow

Vlasov and Cascade
opposite slope:
~ balance energy at 100
MeV, sensitive region,
→ large discrepancies

at higher energy
more consistent



quantify spread of simulations by value of
„flow“=slope at midrapidity

Observables: directed flow

Vlasov and Cascade opposite slope:
 ~ balance energy at 100 MeV, sensitive region,
 → large discrepancies

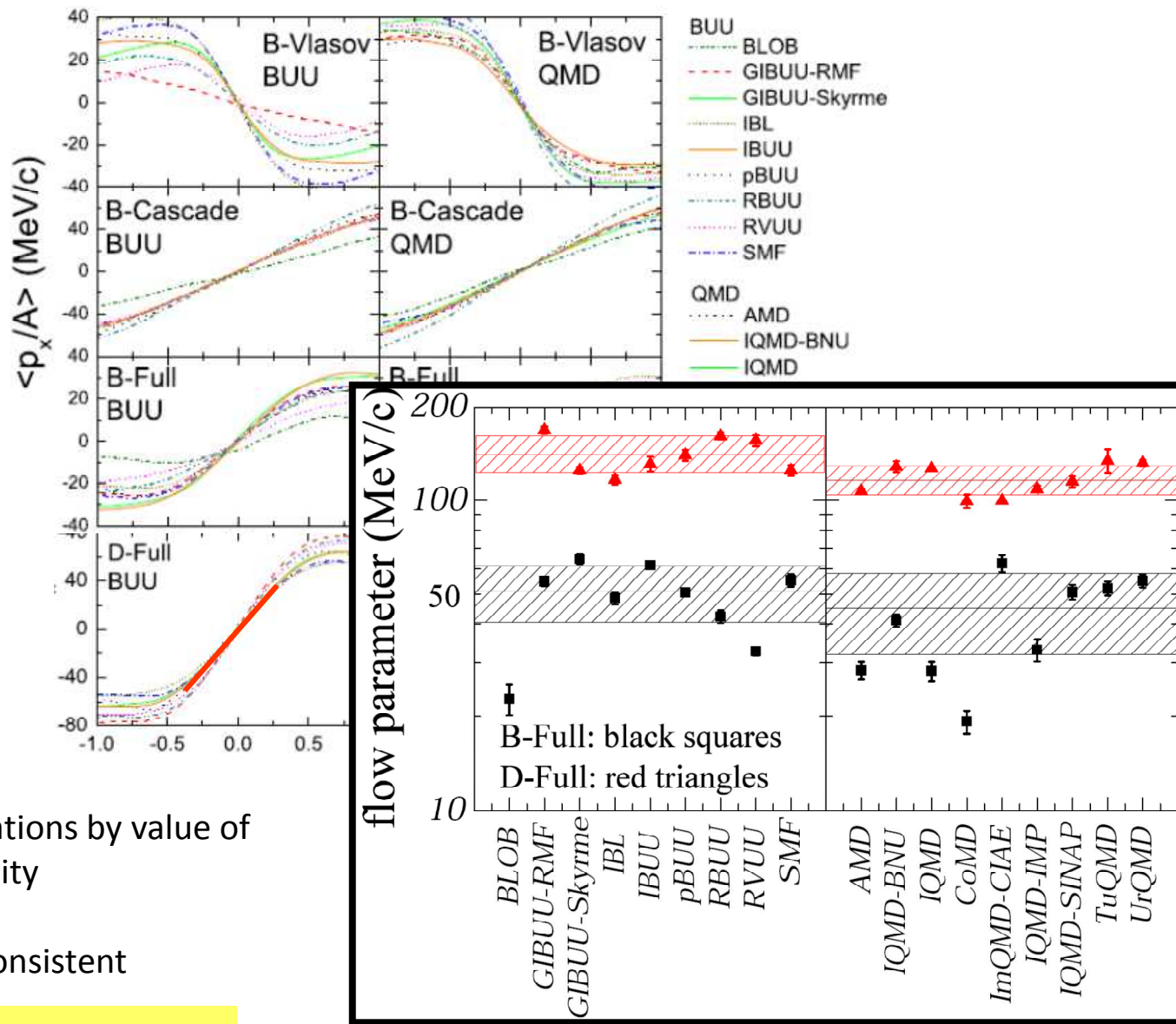
at higher energy more consistent

quantify spread of simulations by value of
 „flow“=slope at midrapidity

BUU and QMD approx. consistent

uncertainty

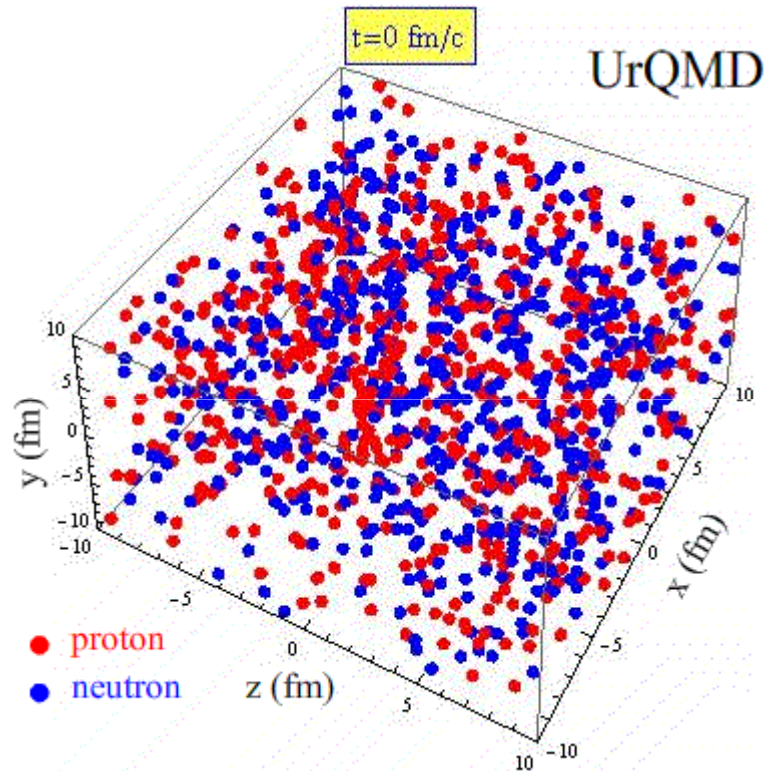
100 A MeV: ~30%
 400 A MeV: ~13%



Difficult to disentangle origin of discrepancies

2nd stage: Box calculation comparison

simulation of the static system of infinite nuclear matter,
→ solve transport equation in a periodic box



PHYSICAL REVIEW C 97, 034625 (2018)

Useful for many reasons:

- check consistency of calculation
e.g. EoS energy dens ϵ vs. pressure P
- check consistency of simulation:
collision numbers, blocking
(exact limits from kinetic theory)
- check aspects of simulation separately
Cascade: only collisions
without/with blocking
Vlasov: only mean field propagation
- check ingredients of particle production
e.g. pion production

Comparison of heavy-ion transport simulations: Collision integral in a box

Ying-Xun Zhang,^{1,2,*} Yong-Jia Wang,^{3,†} Maria Colonna,^{4,‡} Pawel Danielewicz,^{5,§} Akira Ono,^{6,||} Manyee Betty Tsang,^{5,¶}
Hermann Wolter,^{7,#} Jun Xu,^{8,**} Lie-Wen Chen,⁹ Dan Cozma,¹⁰ Zhao-Qing Feng,¹¹ Subal Das Gupta,¹² Natsumi Ikeno,¹³
Che-Ming Ko,¹⁴ Bao-An Li,¹⁵ Qing-Feng Li,^{3,11} Zhu-Xia Li,¹ Swagata Mallik,¹⁶ Yasushi Nara,¹⁷ Tatsuhiko Ogawa,¹⁸
Akira Ohnishi,¹⁹ Dmytro Oliinychenko,²⁰ Massimo Papa,⁴ Hannah Petersen,^{20,21,22} Jun Su,²³ Taesoo Song,^{20,21} Janus Weil,²⁰
Ning Wang,²⁴ Feng-Shou Zhang,^{25,26} and Zhen Zhang¹⁴

Collision term in box calculations

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f = \int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') \left[f_1' f_2' \cancel{f_1 f_2} - f_1 f_2 \cancel{f_1' f_2'} \right]$$

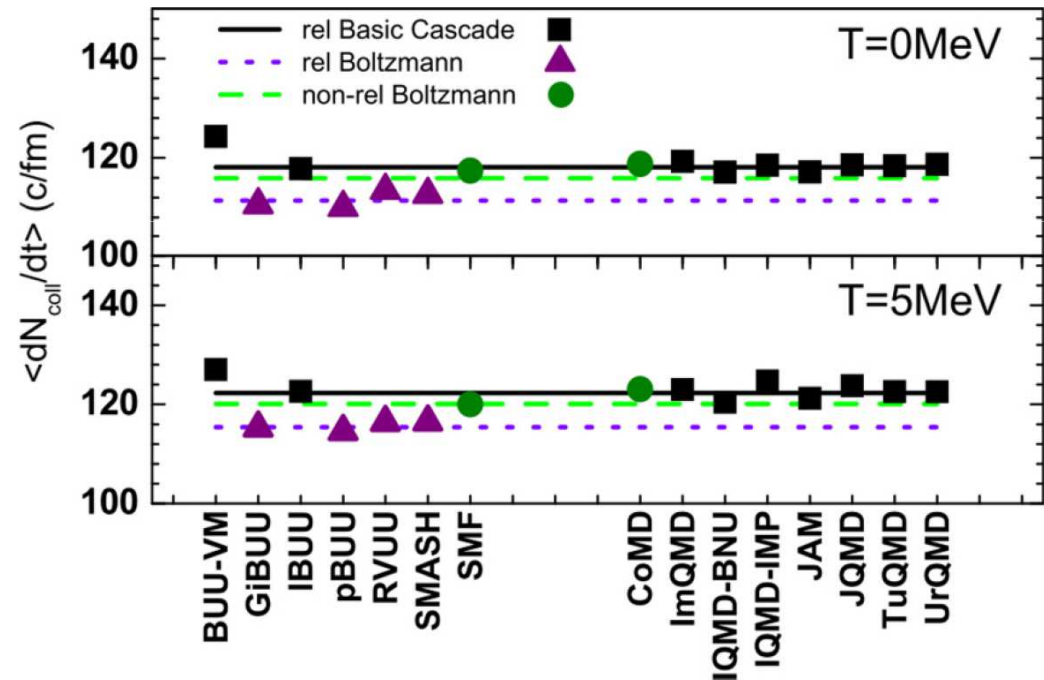
collision probability blocking

Collision rates in a cascade box calculation (w/o mean field, T=0 and 5 MeV)

**without blocking
Comparison to exact limit**

$$\begin{aligned} \frac{dN_{\text{coll}}}{dt} &= \frac{A}{2\rho} g^2 \int \frac{d^3 p d^3 p_1}{(2\pi \hbar)^6} v_{\text{rel}} \sigma^{\text{med}} f(p) f(p_1) \\ &= \frac{1}{2} A \rho \langle v_{\text{rel}} \sigma^{\text{med}} \rangle. \end{aligned}$$

(v_{rel} and average depend on treatment of relativity)

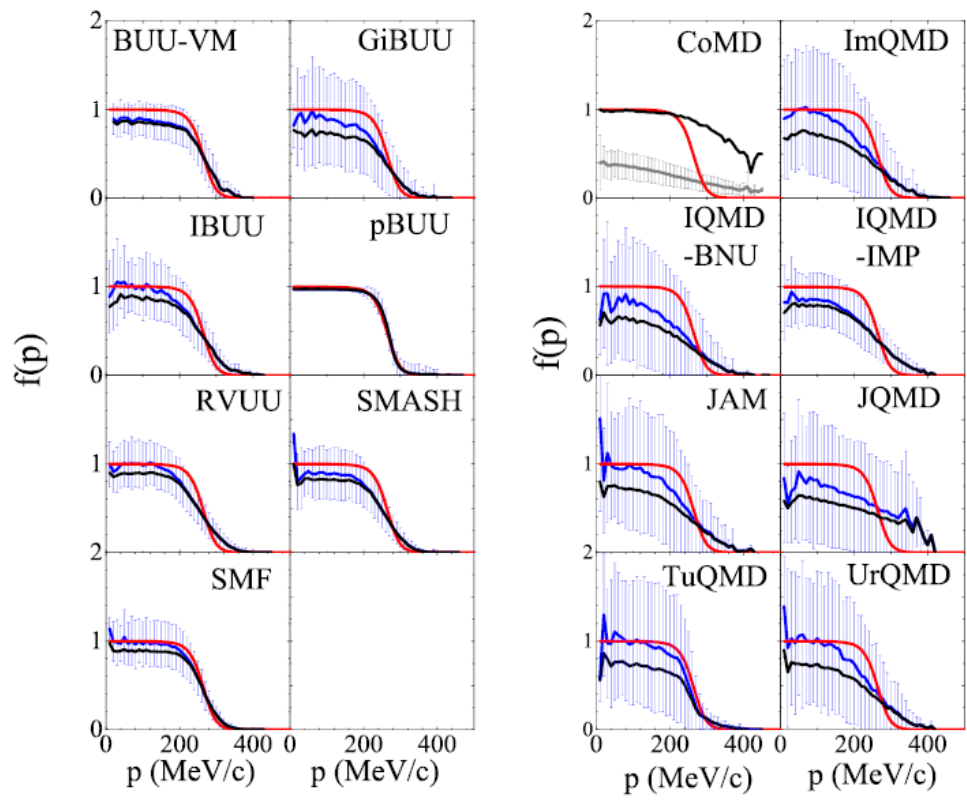
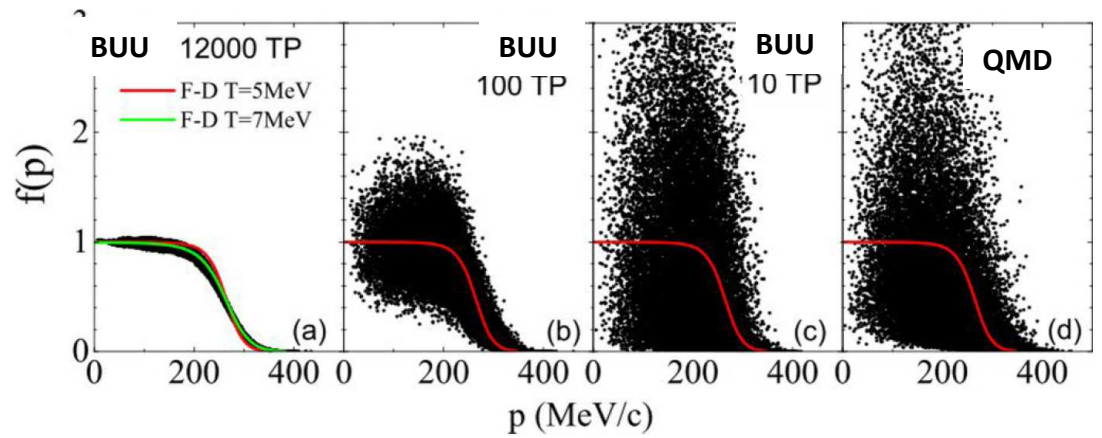


good agreement with corresponding exact result
collision probability ok

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') [f_1' f_2' \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_1' \bar{f}_2']$$

with blocking

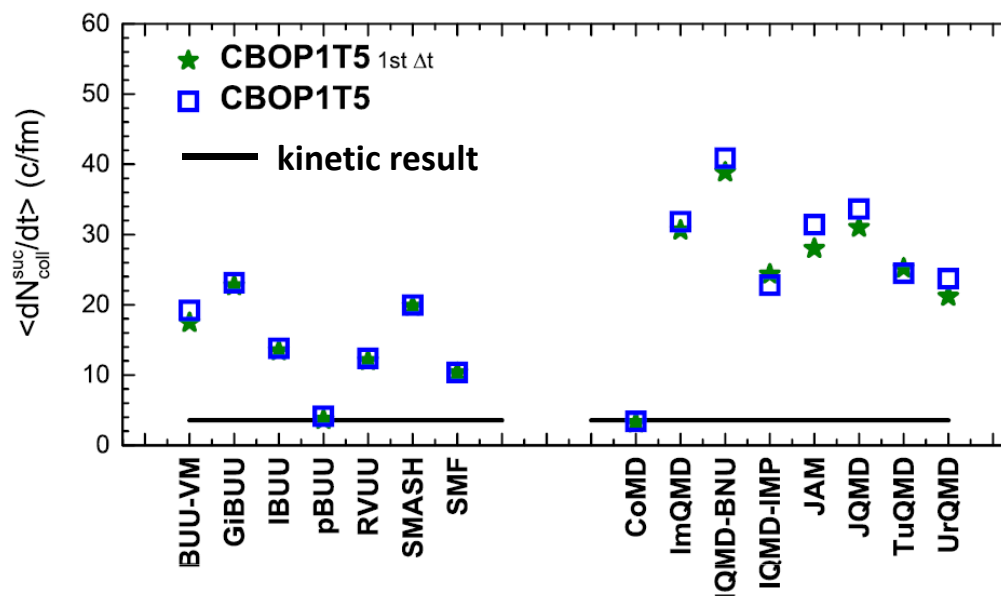
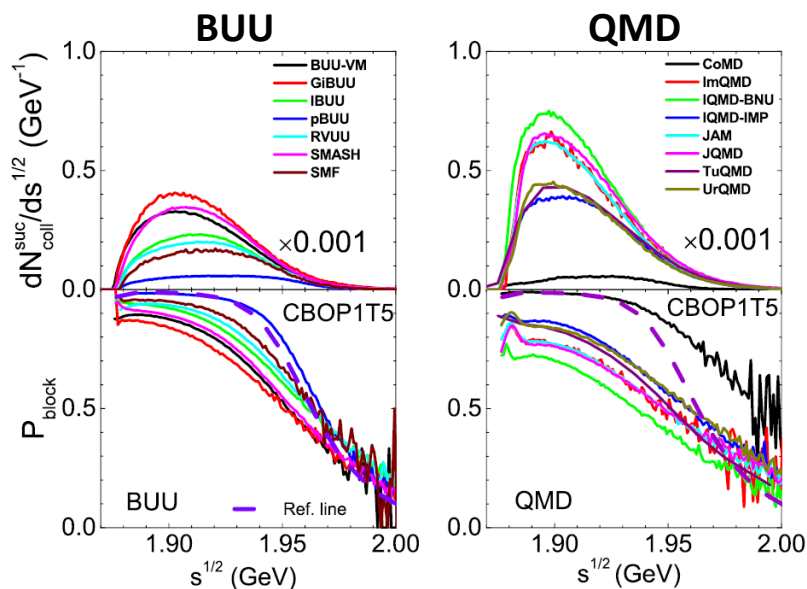
Sampling of occupation prob.
in comp. to prescribed FD distribution
(red)
- fluctuation in BUU controlled by TP
number, can be made arbitrarily small
- fluctuation in QMD given by width of +
wave packet



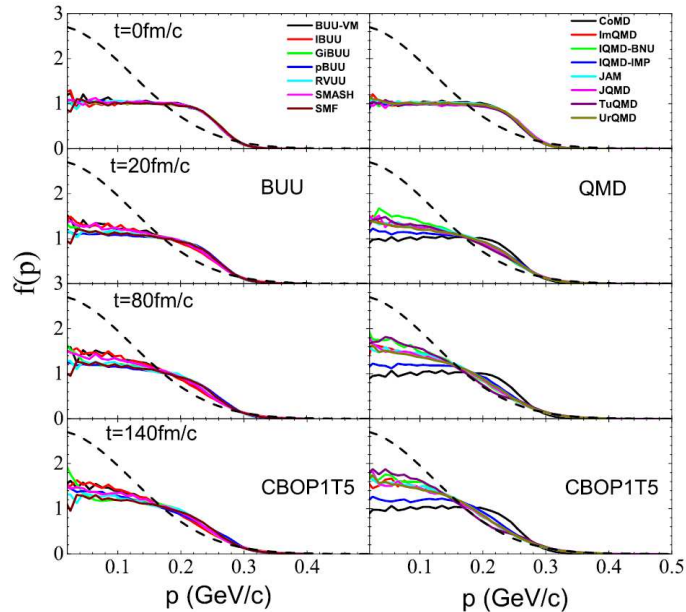
width and averages of calculated
occupation numbers in different codes

- prescribed occupation
- average calculated occupation
- average of $f < 1$ occupation
(used for the blocking)

Collision rates with blocking



Evolution of momentum distributions

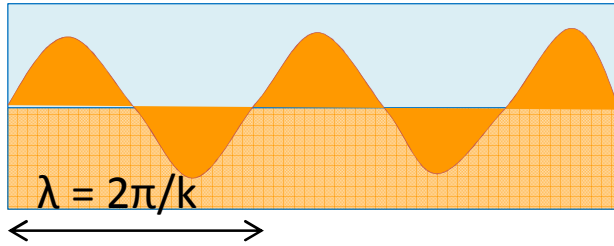


- almost all codes have too little blocking, i.e. allow too many collisions,
- QMD codes more, because of larger fluctuations
- the momentum distribution moves away from the stable Fermi-Dirac distribution towards the classical Maxwell-Boltzmann distribution (dotted line)

Fluctuations influence dynamics of transport calculations. However proper treatment open.

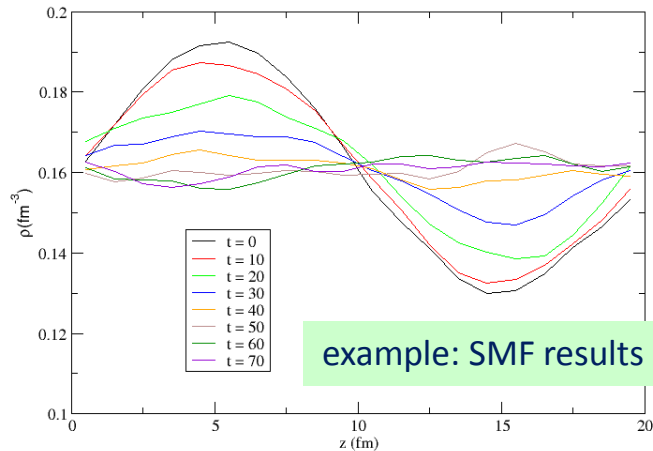
Box simulations: test of m.f. dynamics (in progress! preliminary)

- Study the time evolution of $\rho(z)$
 $L = 20$ fm



$$\rho(z, t=t_0) = \rho_0 + a_\rho \sin(k_i z)$$

$$k_i = n_i 2\pi/L, \quad a_\rho = 0.2 \rho$$

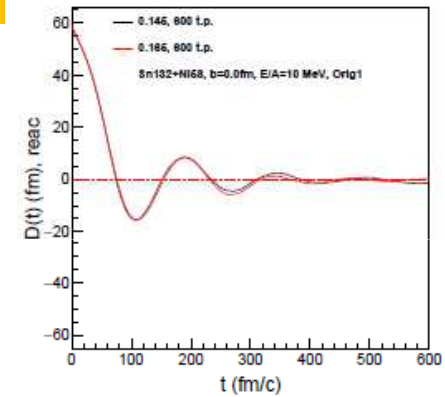


Maria Colonna

- Symmetric matter --
- Only mean-field potential
- No surface terms
- Compressibility $K = 240$ MeV

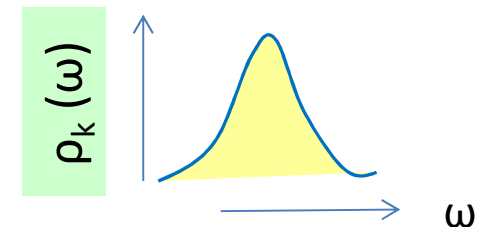
1. Extract the Fourier transform in space

$$\rho_k(t) = \int dz \sin(kz) \rho(z, t)$$

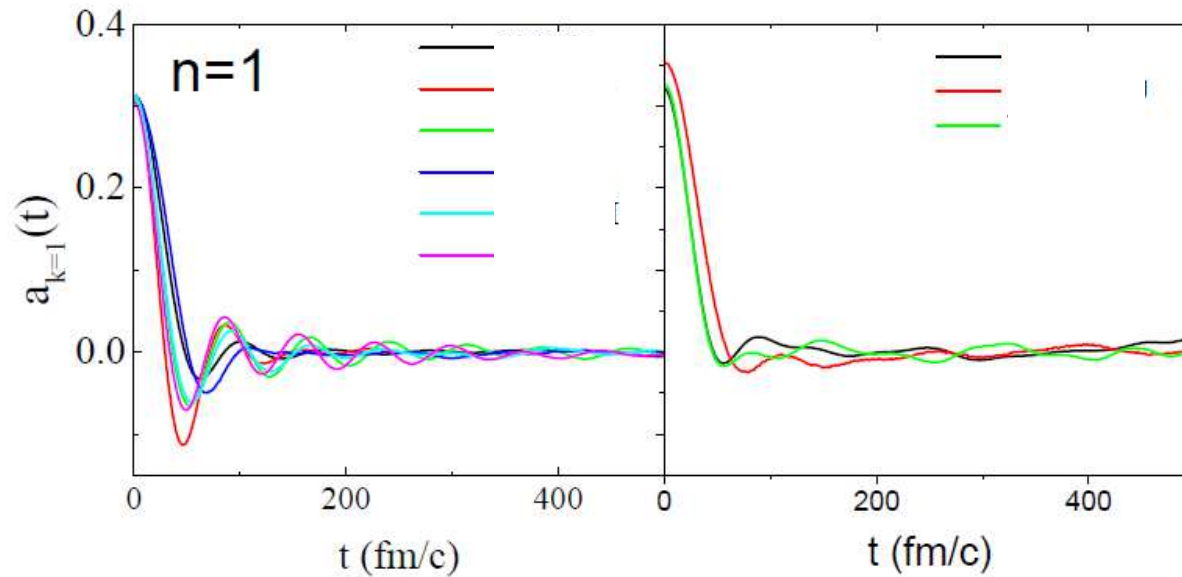


2. Fourier transform in time:
extract the oscillation frequency

$$\rho_k(\omega) = \int dt \cos(\omega t) \rho_k(t)$$



Time evolution of Fourier transform ρ_k

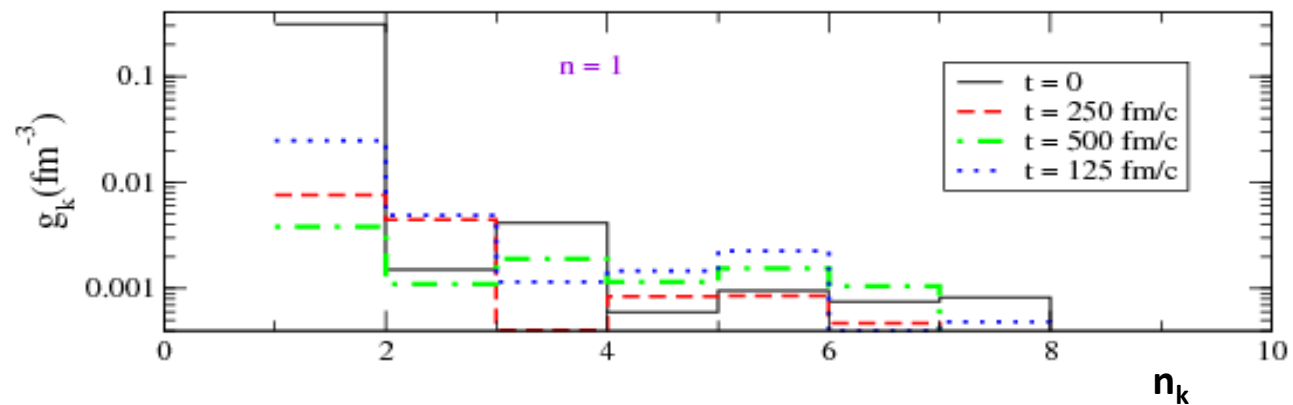


(code names removed,
results preliminary)

Different oscillation frequency in BUU-like

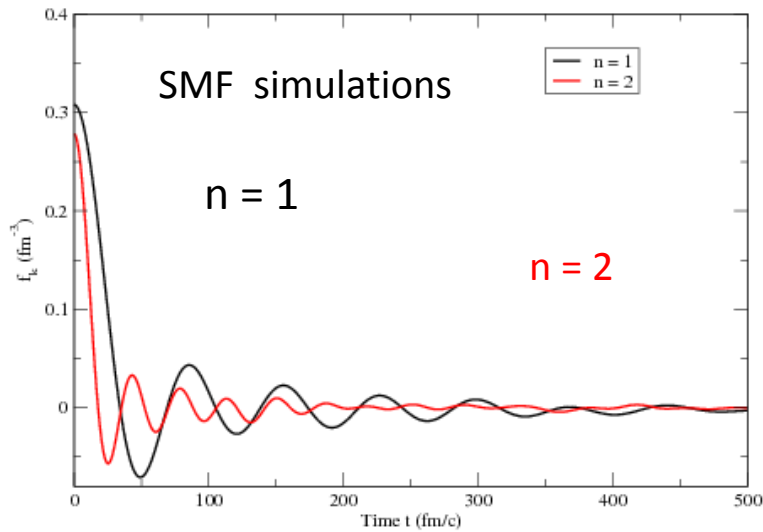
*Larger damping and
structureless fluctuations in QMD-like*

Coupling of modes (starting with $n=1$ mode) SMF



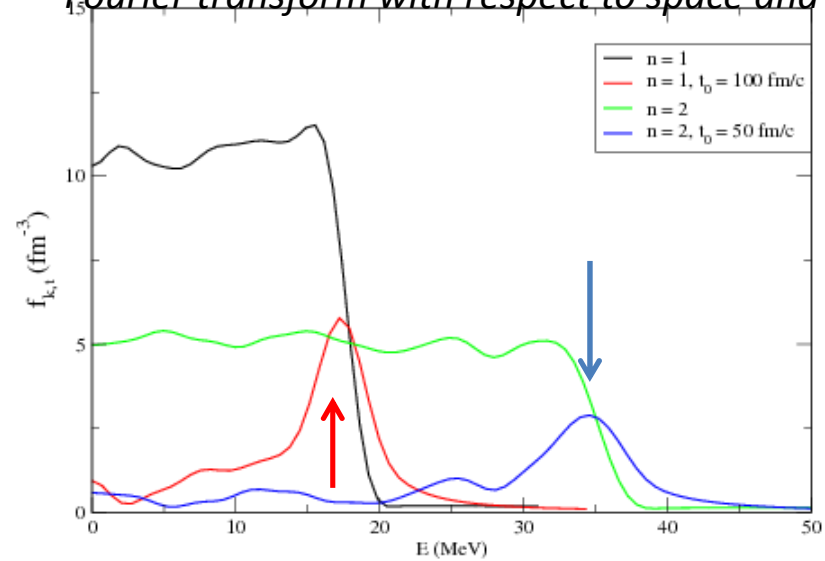
$$\rho_k(t) = \int dz \sin(kz) \rho(z,t)$$

Fourier transform with respect to space



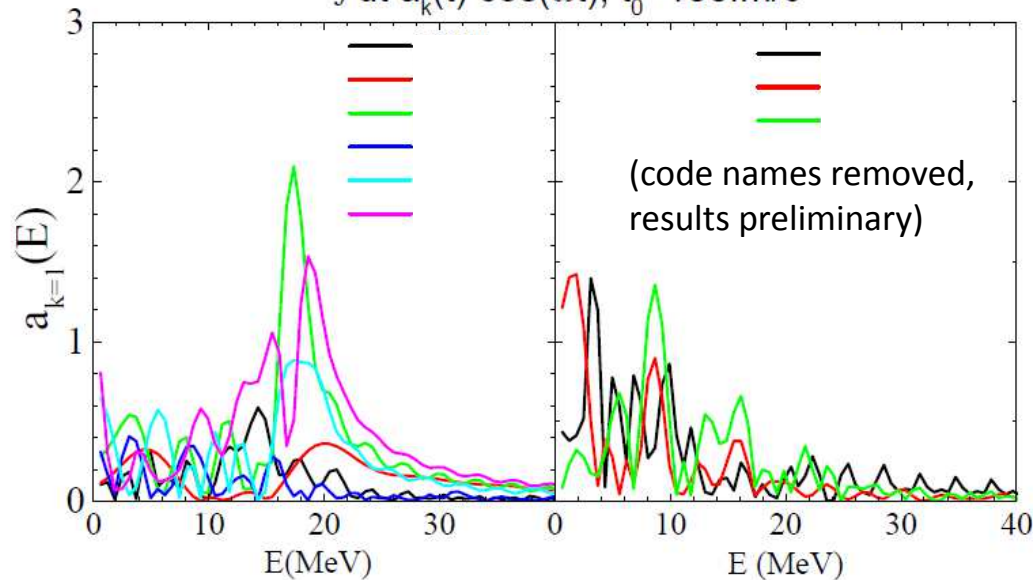
$$\rho_k(\omega) = \int dt \cos(\omega t) \rho_k(t)$$

Fourier transform with respect to space and time



$$\omega / (k v_F) \sim 1 \quad n = 1, E \sim 18 \text{ MeV}$$

$$\int dt a_k(t) \cos(\omega t), t_0 = 100 \text{ fm/c}$$

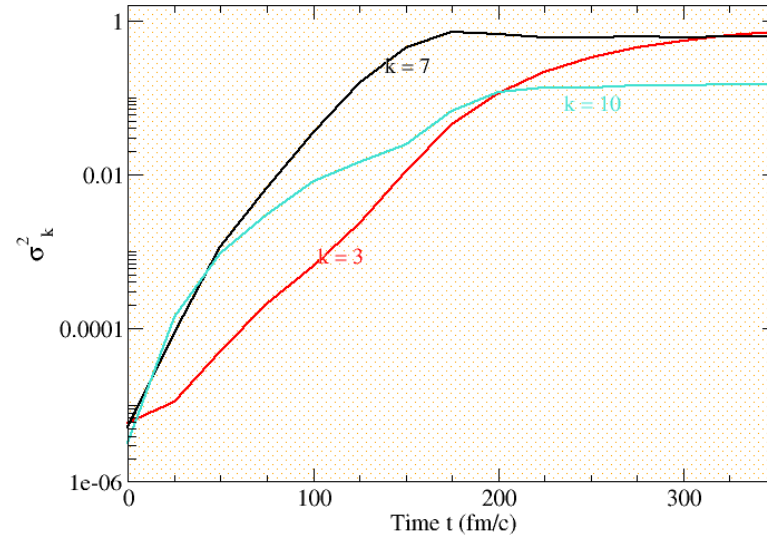
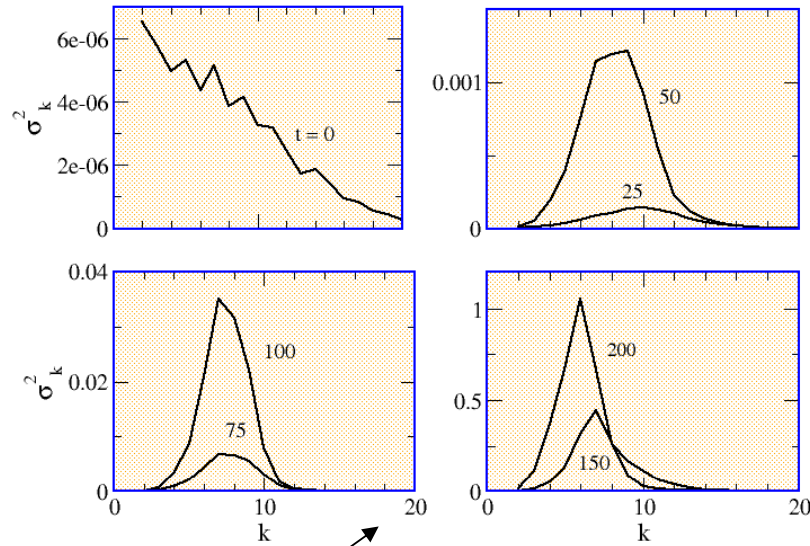


- QMD-like models: appear structureless, large damping
- BUU-like models: differences in frequency and damping

Fluctuations also influence mean field propagation

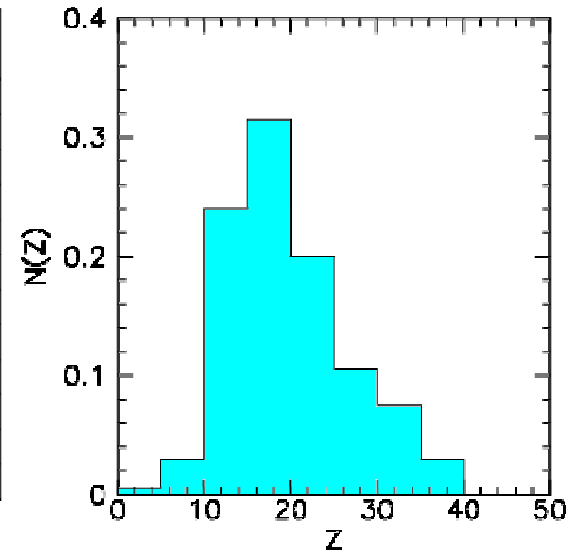
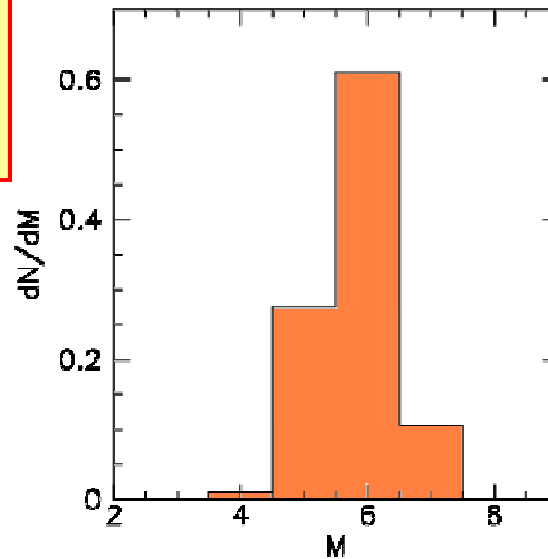
Propagation of fluctuations by the unstable mean-field (preliminary)

Box calculations : $\rho = 0.05 \text{ fm}^{-3}$, $T = 3 \text{ MeV}$



Fourier analysis of the density variance $\langle \delta\rho\delta\rho \rangle$: rapid growth of density fluctuations

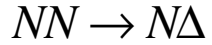
Fragment multiplicity and charge distributions (300 nucleons)



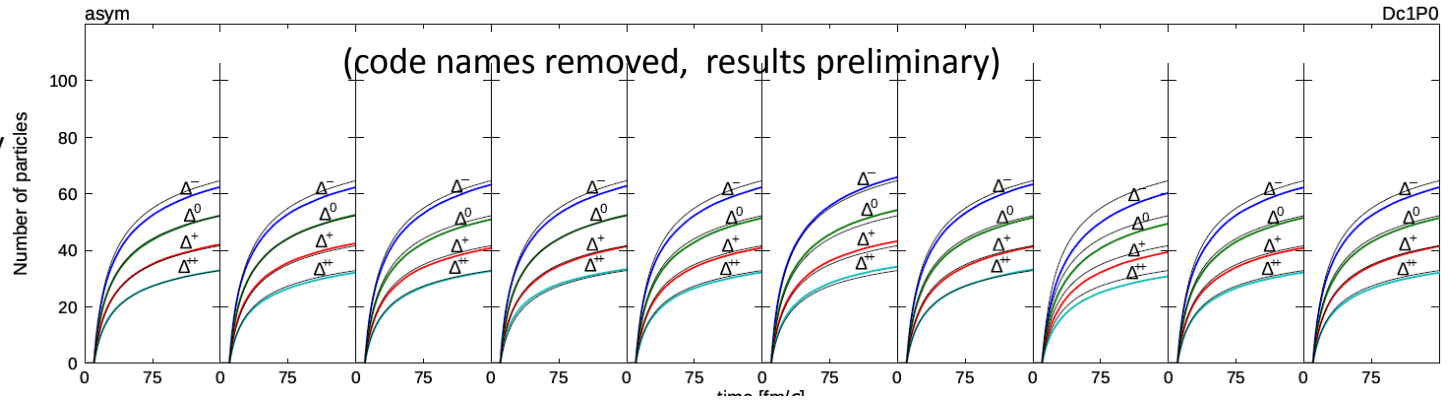
π, Δ production in box cascade calculation:
(in progress, preliminary!)

$NN \leftrightarrow N\Delta$ no pions
— kinetic solution

one-way only



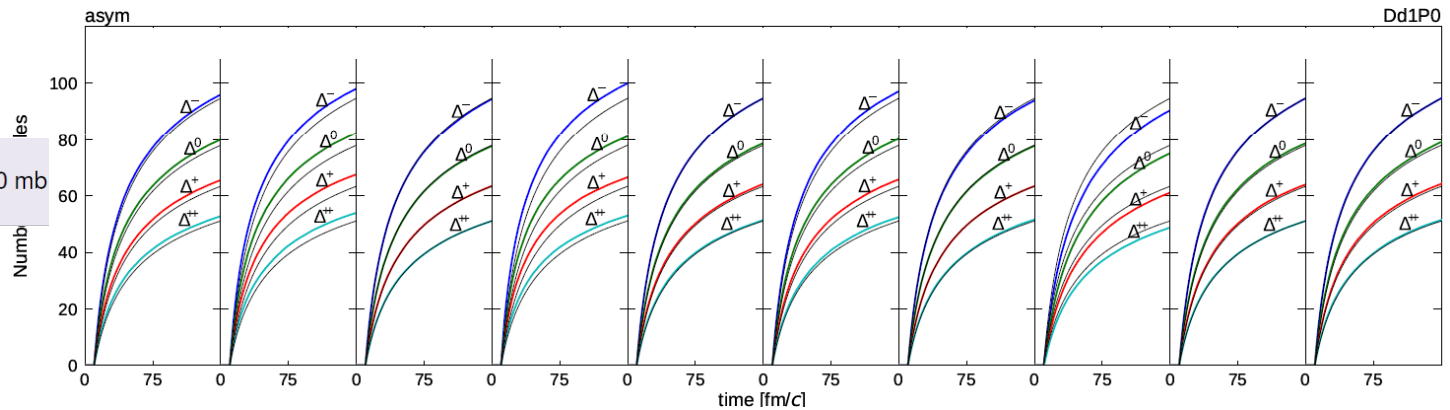
constant Δ mass $M_\Delta = 1.232$ MeV
constant $\sigma(NN \rightarrow N\Delta)$



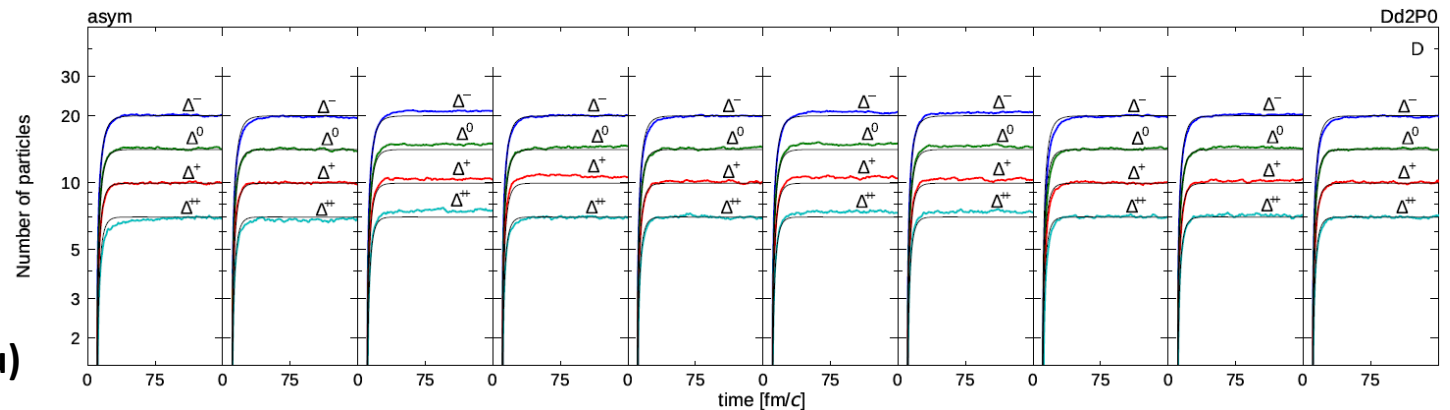
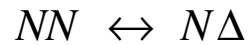
energy dep cross sect.
 Δ mass distribution

$$\sigma(NN \rightarrow N\Delta) = \frac{(\sqrt{s} - 2M_N - M_\pi)^2}{(0.015 \text{ GeV}^2) + (\sqrt{s} - 2M_N - M_\pi)^2} \times 20 \text{ mb}$$

$$A(m) = \frac{4M_\Delta^0 \Gamma_\Delta}{(m^2 - M_\Delta^0)^2 + M_\Delta^0 \Gamma_\Delta^2}$$



two-ways



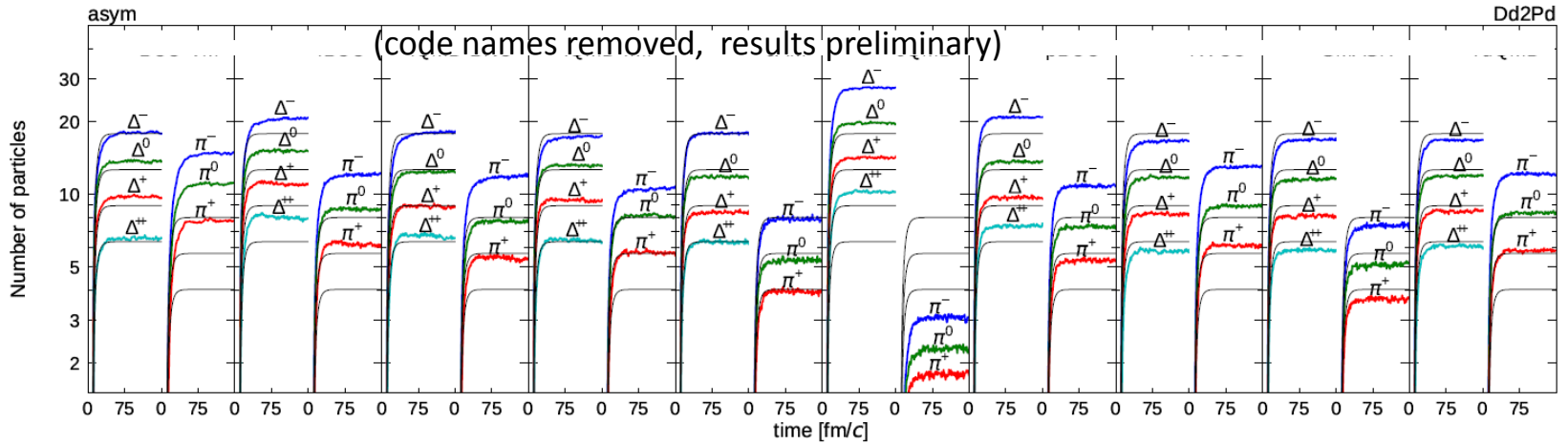
(Akira Ono and Jun Xu)

π, Δ production in box cascade calculation:
(in progress, preliminary!)

$$NN \leftrightarrow N\Delta, \quad \Delta \rightarrow N\pi$$

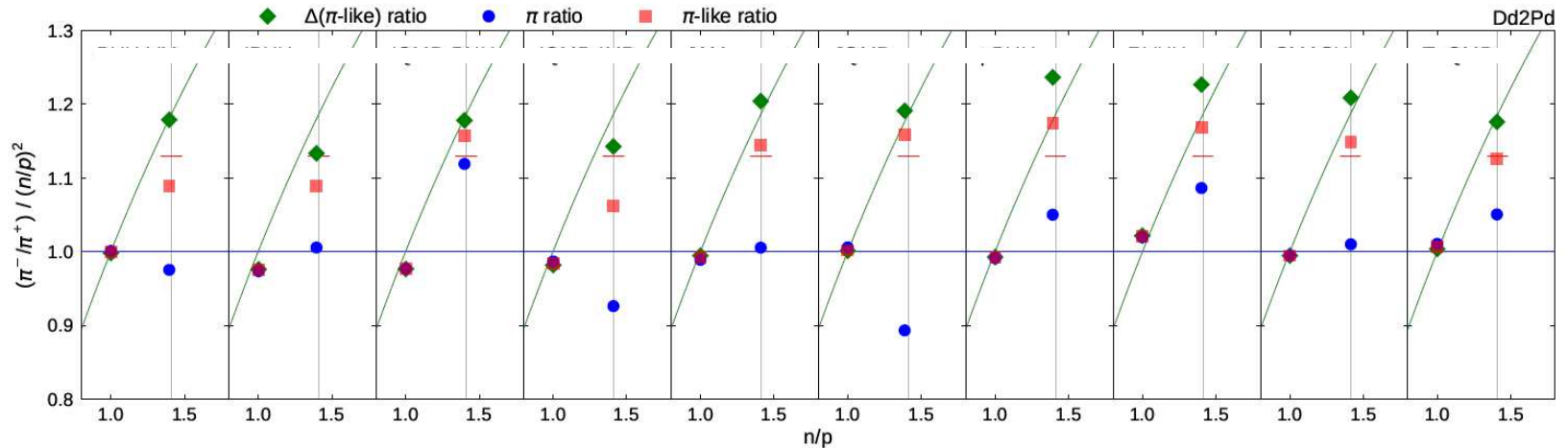
Δ mass distribution

large differences between models



pion ratio

$$\frac{\pi^-}{\pi^+} \left(\frac{n}{p} \right)^2$$



perhaps not surprising that there is now agreement on the interpretation of the pion ratios with respect to the symmetry energy. Differences may have to do with technical differences in the sequence of simulating creation and decay of Δ 's.

Summary and Conclusions

- Transport approaches are an important method to extract physics information from complex non-equilibrium processes, as e.g. heavy ion collisions.
- also in the NICA/FAIR energy range for the description of part (hybrid approaches) or all of the collision.

However, there are open problems in the application of transport theories:

- physical (which degrees of freedom, esp. for phase transitions, fluctuations, correlations, short range)
- questions of implementation: simulation, rather than solution of the transport equations
- involves strategies not strictly given by the equations, such as
 - representation of the phase space, coarse graining, criteria for collisions and Pauli blocking
- these may affect the deduction on physical properties from collisions and lead to a kind of systematical theoretical error
- here attempt to understand, quantify and hopefully reduce these uncertainties in a
Transport Code Comparison under Controlled Conditions

Results:

- Comparison of full HIC makes evident the discrepancies (initializations, collision term), but difficult to disentangle
- Box calculations to study the different ingredients of transport (collisions, blocking, mf evolution, particle production)
- Important finding is the importance of fluctuations on the simulations
- Fluctuations (and correlations) go beyond the one-body description. Implementations differ:
 - BUU --> explicit introduction in a fluctuation term (Boltzmann-Langevin eq.)
 - QMD --> smoothing by wave packet + classical correlations
- more investigations in the future, e.g. in fragmentation and near phase transitions

Thank you very much for your attention