Baryon-Antibaryon Annihilation and Reproduction in relativistic Heavy-Ion Collisions

Eduard Seifert

for the PHSD group

Institute for Theoretical Physics, Giessen

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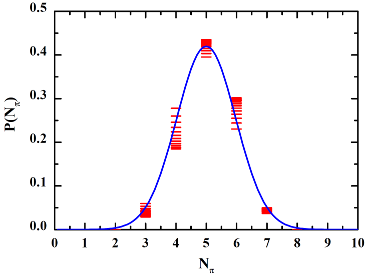
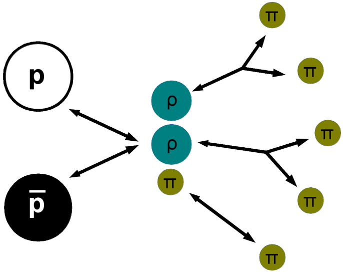
Motivation

- For a better understanding of data from HICs all hadronic channels have to be under control
- Most models neglect interactions of three or more particles
- With more particles the production thresholds are easier overcome
- We consider the annihilation of baryons with antibaryons to three mesons and the backward reaction $B\bar{B} \leftrightarrow 3M$ by the rearrangement of the quark content





[W. Cassing, NPA700(2002)618]

- $\langle N_\pi \rangle = 5$ realized through initial $\rho\rho\pi$ - with each ρ meson decaying to 2 pions
- For a physically correct description the backward reactions have to be implemented in transport
- Relative importance for different energy regions from FAIR/NICA to top SPS energies will be investigated

Overview

1. Theory for multi-particle interactions
2. Test of n-particle detailed balance
3. Parton-Hadron-String Dynamics (PHSD)
4. PHSD simulations with extended many-body reactions
5. Summary

Theory for multi-particle interactions

- Covariant on-shell reaction rate inside a volume element dV and time interval dt for general particle number changing process [W. Cassing, NPA700(2002)618]

$$\frac{\mathrm{d}N_{\mathrm{coll}}[n \to m]}{\mathrm{d}t\mathrm{d}V} = \sum_{\nu}\sum_{\lambda}\int \prod_{j=1}^{n}\left(\frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}\right)\prod_{k=1}^{m}\left(\frac{\mathrm{d}^3 p_k}{(2\pi)^3 2E_k}\right)$$
$$\times\, W_{n,m}(p_j;\nu|p_k;\lambda)(2\pi)^4\delta^4\left(\sum_{j=1}^{n} p_j^{\mu} - \sum_{k=1}^{m} p_k^{\mu}\right)\prod_{j=1}^{n}(f_j(x,p_j))\prod_{k=1}^{m}(\tilde{f}_k(x,p_k))$$

$W_{n,m}$=Transition matrix element squared
$f=$ phase-space distribution function
$\tilde{f} = 1 \pm f$ accounting for quantum statistics

- m-body phase-space, incorporates dynamics of the system in case of constant transition matrix element

$$R_m(P^{\mu}; M_1, \ldots, M_m) = \left(\frac{1}{(2\pi)^3}\right)^m \int \prod_{k=1}^{m} \frac{\mathrm{d}^3 p_k}{2E_k}\,(2\pi)^4\delta^4\left(P^{\mu} - \sum_{j=1}^{m} p_j^{\mu}\right)$$

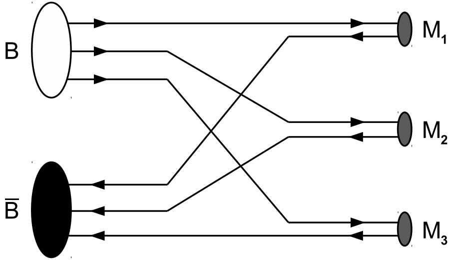
- Relation for a $2 \to m$ reaction to the cross section for a pair with quantum numbers i, j

$$\sum_{m}\sum_{\lambda_m} W_{2,m}(P^{\mu} = p_1^{\mu} + p_2^{\mu}; i, j; \lambda_m) R_m(P^{\mu}; M_3, \ldots, M_{m+1}) = 4E_1 E_2 v_{\mathrm{rel}}\sigma_{i,j}(\sqrt{s})$$

[E. Byckling, K. Kajantie, *Particle Kinematics*]

Quark Rearrangement Model

- $B, \bar{B}$: baryons and antibaryons from the baryon octet and decuplet plus N(1440) and N(1535) $M_1, M2, M3$: arbitrary mesons under conservation of the total quantum numbers (here 0^- and 1^- nonets)



- Reshuffle quark content from the baryon + antibaryon pair into 3 mesons or backwards
- Conserve quantum numbers in any combination of channels

Transition probability

$B\bar{B} \rightarrow 3$ mesons

- Assume W does not depend significantly on final momenta (only on $\sqrt{s}$)
- Assume dilute final phase space

$$\frac{\mathrm{d}N_{\mathrm{coll}}[B\bar{B} \rightarrow 3 \text{ mesons}]}{\mathrm{d}t\mathrm{d}V} =$$

$$\sum_{c}\sum_{c'} \frac{1}{(2\pi)^6} \int \frac{\mathrm{d}^3 p_1}{2E_1} \frac{\mathrm{d}^3 p_2}{2E_2} \; W_{2,3}\big(\sqrt{s}; c' = (M_1, M_2; i, j), c = (M_3, M_4, M_5; k, l, m)\big)$$

$$\times R_3(p_1^{\mu} + p_2^{\mu}; c) N_{\mathrm{fin}}^{c} f_i(x, p_1) f_j(x, p_2)$$

$$N_{\mathrm{fin}}^{c} = (2s_3 + 1)(2s_4 + 1)(2s_5 + 1) \frac{F_{\mathrm{iso}}}{N_{\mathrm{id}}!}$$

N_{fin}^{c}: Multiplicity of final state in channel c

F_{iso}: Number of isospin projections compatible with charge conservation

s: spin of respective meson

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Probability for $B\bar{B}$ pair c' to annihilate:

$$P^{c' \rightarrow c} = \frac{1}{4E_1E_2} W_{2,3}(\sqrt{s}, c, c') R_3(\sqrt{s}, c) N_{\mathrm{fin}}^c,$$

$$P_{\mathrm{tot}}^{c'}\frac{\mathrm{d}V}{\mathrm{d}t} = \sum_c P^{c' \rightarrow c}(\sqrt{s}) = v_{\mathrm{rel}}\sigma_{\mathrm{ann}}^{c'}(\sqrt{s})$$

Probability for final meson channel c

$$\tilde{P}^c(\sqrt{s}) = N_3(\sqrt{s}, c') R_3(\sqrt{s}, c) N_{\mathrm{fin}}^c,$$

$$N_3^{-1}(\sqrt{s}, c') = \sum_c R_3(\sqrt{s}, c) N_{\mathrm{fin}}^c$$

Transition probability

3 mesons $\rightarrow B\bar{B}$ (inverse channel)

$$P^{c\rightarrow c'}\frac{\mathrm{d}V^2}{\mathrm{d}t} = \frac{4E_1E_2}{8E_3E_4E_5}\sigma^{c'}_{\mathrm{ann}}(\sqrt{s})v_{\mathrm{rel}}N_3(\sqrt{s},c')R_2(\sqrt{s},c')N^{c'}_B$$

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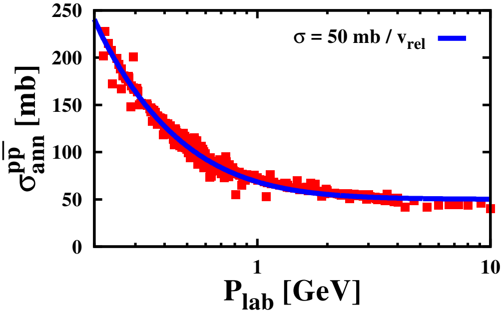
Annihilation cross section



- Good fit with $\sigma_{p\bar{p}} = 50\,\mathrm{mb}/v_{\mathrm{rel}}$ [http://pdg.lbl.gov/2015/hadronic-xsections/]
- Consistent with constant matrix element ($\sigma_{p\bar{p}}v_{\mathrm{rel}} = const.$)
- Particles with strange quark content: $\sigma_{\mathrm{ann}}^{c'} = \sigma_{p\bar{p}}\lambda^{s+\bar{s}}$ with $\lambda \in [0,1]$ and $s, \bar{s}$ signifying the number of strange and antistrange quarks respectively

Tests of n-particle detailed balance

Consider the light and strangeness sector:

- Baryon octet and decuplet: N,Δ(1232),N(1440),N(1535),Λ,Σ,Σ^*,Ξ,Ξ^*,Ω
- Meson 0^- and 1^- nonets: π,η,η',K,K^*,ρ,ω,Φ,a_1
- Hidden strangeness of η is taken into account as 50% $s\bar{s}$ content and the ϕ meson is assumed to have 83.1% $s\bar{s}$
  ⇒ 2800 possible mass channels

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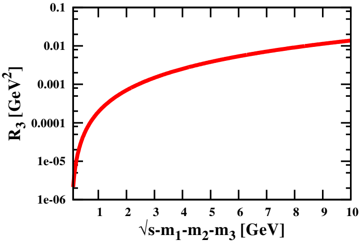
Strategy for calculations:

- Fit 3-body phase-space

$$R_3(t) = a_1\, t^{a_2} \left(1 - \frac{1}{a_3\, t + 1 + a_4}\right)$$

with $t = \sqrt{s} - m_1 - m_2 - m_3$ for 165 meson mass combinations



- Store multiplicities and possible final states for each combination of particles

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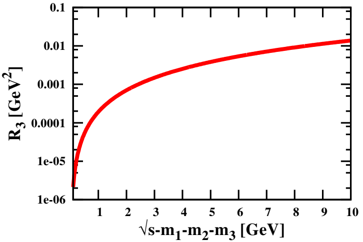


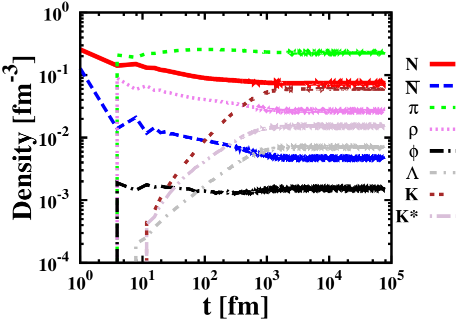
- Store multiplicities and possible final states for each combination of particles

Actual calculation:

- Divide space-time into 4-dimensional cells: $\Delta x, \Delta y, \Delta z, \Delta t$
- Particles inside the same cell may interact with each other
- Calculate transition probabilities and select via Monte Carlo the partners and final states

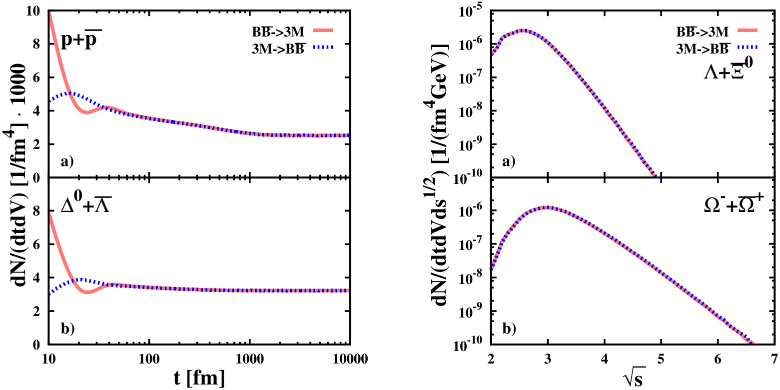
Test of n-particle detailed balance

- Box simulations with volumes around $V = 18000\,\text{fm}^3$
- Periodic boundary condition
- Initialization with only one type of baryon and antibaryon: N,Δ(1232),N(1440),N(1535),Λ,Σ,Σ^*,Ξ,Ξ^*,Ω
- Energy density $\epsilon = 0.4\,\text{GeV fm}^{-3}$, with 10% being kinetic energy
- Ratio baryon/antibaryon set to 2:1 $\longrightarrow$ baryon density ρ_B lies around $0.2\,\text{fm}^{-3}$
- Boltzmann-like initial momentum distribution
- No decays of resonances and no elastic scattering; only $B\bar{B} \leftrightarrow 3M$



Total Reaction Rates

Total reaction rates as a function of time t and invariant energy $\sqrt{s}$ in forward and backward-direction



- Detailed balance is fulfilled after $\approx 100\,\mathrm{fm/c}$
- Equilibrium is reached the latest after $2000\,\mathrm{fm/c}$
- $\Rightarrow$ Detailed balance is fulfilled also differentially for the total system in equilibrium

Deviation from Detailed Balance

Deviation from Detailed Balance on a channel by channel basis

$$\delta = \left| \frac{\frac{\mathrm{d}N}{\mathrm{d}t}(B\bar{B} \to 3M)}{\frac{\mathrm{d}N}{\mathrm{d}t}(3M \to B\bar{B})} - 1 \right|$$

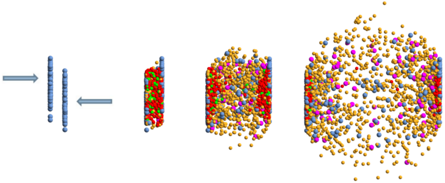
rank	$p + \bar{p}$		$\Delta^0 + \bar{\Lambda}$		$\Lambda + \bar{\Xi}^0$		$\langle\delta\rangle$ [%]
	channel	δ [%]	channel	δ [%]	channel	δ [%]	
1	$N\bar{N} \leftrightarrow \pi\pi\rho$	0.17	$N\bar{\Xi} \leftrightarrow \pi K K^*$	1.45	$N\bar{N} \leftrightarrow \pi\pi\rho$	0.13	1.24
2	$N\bar{N} \leftrightarrow \pi\rho\rho$	3.06	$N\bar{\Omega} \leftrightarrow K K^* K^*$	3.59	$N\bar{\Delta} \leftrightarrow \pi\rho\rho$	1.70	1.82
3	$N\bar{\Delta} \leftrightarrow \pi\pi\rho$	1.58	$\Delta\bar{\Xi} \leftrightarrow \pi K K^*$	1.32	$N\bar{\Delta} \leftrightarrow \pi\pi\rho$	2.04	1.70
4	$N\bar{\Delta} \leftrightarrow \pi\rho\rho$	0.84	$\Delta\bar{\Xi} \leftrightarrow K K^* \rho$	0.64	$N\bar{N} \leftrightarrow \pi\rho\rho$	3.31	1.54
5	$\Delta\bar{N} \leftrightarrow \pi\pi\rho$	2.43	$\Delta\bar{\Omega} \leftrightarrow K K^* K^*$	1.08	$\Delta\bar{N} \leftrightarrow \pi\rho\rho$	1.33	1.49
6	$\Delta\bar{N} \leftrightarrow \pi\rho\rho$	0.73	$N\bar{\Sigma} \leftrightarrow \pi K^* \rho$	3.58	$\Delta\bar{N} \leftrightarrow \pi\pi\rho$	2.71	1.97
7	$N\bar{N} \leftrightarrow \pi\pi a_1$	6.52	$\Delta\bar{\Sigma} \leftrightarrow \pi K^* \rho$	2.00	$\Delta\bar{\Delta} \leftrightarrow \pi\pi\rho$	2.69	2.04
8	$N\bar{N} \leftrightarrow \pi\pi\pi$	5.10	$N\bar{N} \leftrightarrow \pi\pi\rho$	0.23	$N\bar{\Sigma} \leftrightarrow \pi K^* \rho$	2.04	2.03
9	$N\bar{\Sigma} \leftrightarrow \pi K \rho$	0.31	$N\bar{\Sigma} \leftrightarrow \pi K \rho$	0.42	$\Delta\bar{\Delta} \leftrightarrow \pi\pi\rho$	2.12	2.11
10	$N\bar{\Sigma} \leftrightarrow \pi K^* \rho$	0.96	$N\bar{\Omega} \leftrightarrow KKK$	0.35	$N\bar{\Sigma} \leftrightarrow \pi K \rho$	0.35	2.11

Ranked by interaction rate and averaged over 100 system combinations
$\Rightarrow$ Detailed balance is fulfilled on a channel by channel basis in equilibrium better than 98%

E. Seifert, W. Cassing, Phys. Rev. C **97**, no. 2, 024913 (2018).

Parton-Hadron-String Dynamics (PHSD)

- Dynamical many-body transport approach.
- Consistently describes the full time evolution of a heavy-ion collision.
- Parton-parton interactions, explicit phase transition from hadronic to partonic degrees of freedom.



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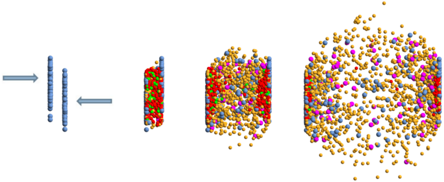
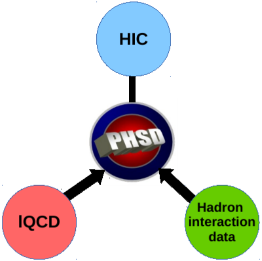




- Model applicable out-of equilibrium and in agreement with the lattice results in equilibrium as well as with the nuclear physics input.
- Transport theory: off-shell transport equations in phase-space representation based on Kadanoff-Baym equations for the partonic and hadronic phase.

W.Cassing, E.Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W.Cassing, EPJ ST 168 (2009) 3.

Solve generalized transport equations with extended test-particle ansatz

$$F_{XP} = iG^{<}(X,P) \sim \sum_{i=1}^{N} \delta^{(3)}(\mathbf{X} - \mathbf{X}_i(t))\delta^{(3)}(\mathbf{P} - \mathbf{P}_i(t))\delta(P_0 - \epsilon_i(t))$$

The equations of motion extracted from Kadanoff-Baym equations in first order gradient expansion in phase space read:

$$\frac{d\mathbf{X}_i}{dt} = \frac{1}{2\epsilon_i}\left[2\mathbf{P}_i + \nabla_{P_i}\mathrm{Re}\boldsymbol{\Sigma}^{ret}_{(i)} + \frac{\epsilon_i^2 - \mathbf{P}_i^2 - M_0^2 - \mathrm{Re}\boldsymbol{\Sigma}^{ret}_{(i)}}{\Gamma_{(i)}}\nabla_{P_i}\Gamma_{(i)}\right]$$

$$\frac{d\mathbf{P}_i}{dt} = -\frac{1}{2\epsilon_i}\left[\nabla_{X_i}\mathrm{Re}\boldsymbol{\Sigma}^{ret}_{(i)} + \frac{\epsilon_i^2 - \mathbf{P}_i^2 - M_0^2 - \mathrm{Re}\boldsymbol{\Sigma}^{ret}_{(i)}}{\Gamma_{(i)}}\nabla_{X_i}\Gamma_{(i)}\right]$$

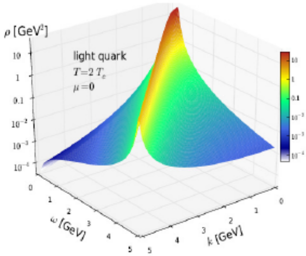
$$\frac{d\epsilon_i}{dt} = \frac{1}{2\epsilon_i}\left[\frac{\partial\mathrm{Re}\boldsymbol{\Sigma}^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \mathbf{P}_i^2 - M_0^2 - \mathrm{Re}\boldsymbol{\Sigma}^{ret}_{(i)}}{\Gamma_{(i)}}\frac{\partial\Gamma_{(i)}}{\partial t}\right]$$

Σ^{ret}: retarded self-energy
$\Gamma = \mathrm{Im}\Sigma^{ret}/2\epsilon$: effective width

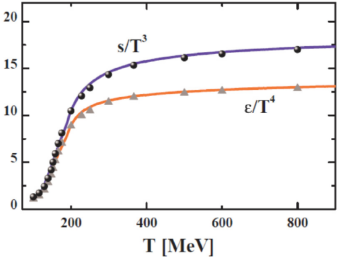
Dynamical Quasi-Particle Model (DQPM)

The QGP phase is described in terms of interacting quasi-particles with Lorentzian spectral functions:

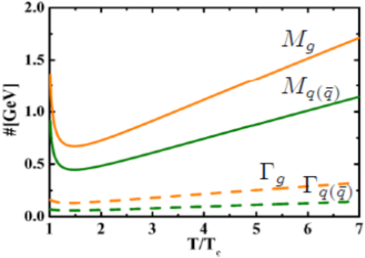
$$\rho_i(\omega, T) = \frac{4\omega\Gamma_i(T)}{(\omega^2 - \mathbf{p}^2 - M_i^2(T))^2 + 4\omega^2\Gamma_i^2(T)}, \quad (i = q, \bar{q}, g).$$



Properties of quasi-particles are fitted to the lattice QCD results:

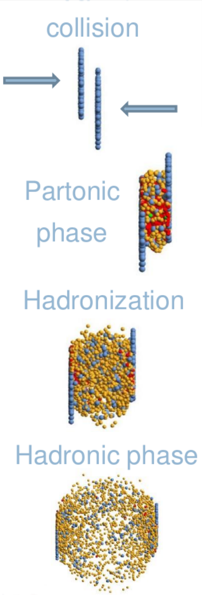


Masses and widths of partons depend on the temperature T and chemical potential μ_q of the medium:



Peshier, Casing, PRL 94 (2005) 172301; Casing, NPA 791 (2007) 365: NPA 793 (2007) .

Stages of a heavy-ion collision in PHSD



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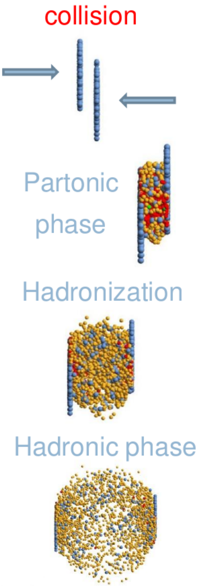
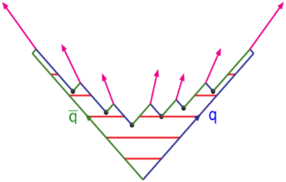


- **String formation** in primary NN Collisions.
- **String decays** to pre-hadrons(baryons and mesons).



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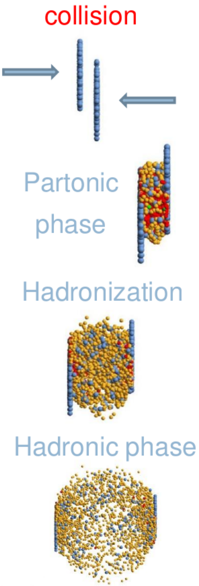

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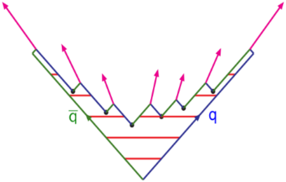


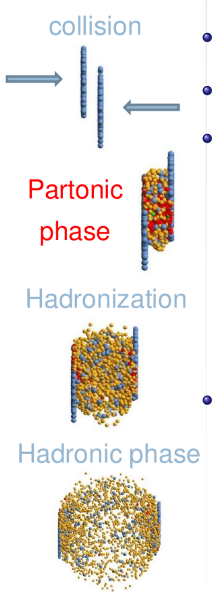
- Formation of a **QGP state** if the energy density $\epsilon > \epsilon_C \approx 0.5\,\text{GeV}\,\text{fm}^{-3}$.
- Dissolution of newly produced secondary hadrons into **massive colored quarks/antiquarks** and **mean-field energy** U_q:

$$B \to qqq\,(\bar{q}\bar{q}\bar{q}) \qquad M \to q\bar{q} \qquad + \quad U_q.$$

- **DQPM** defines the properties (masses and widths) of partons and mean-field potential at a given local energy density ϵ:

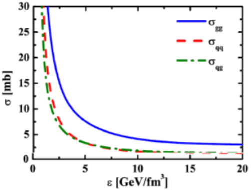
$$m_q(\epsilon) \qquad \Gamma_q(\epsilon) \qquad U_q(\epsilon).$$

Stages of a heavy-ion collision in PHSD



- Propagation of partons, considered as dynamical quasi-particles, in the self-generated mean-field potential from the DQPM.
- EoS of partonic phase: crossover from Lattice QCD fitted by DQPM.
- (Quasi-)elastic collisions:

$$q + q \Rightarrow q + q \qquad g + q \Rightarrow g + q$$
$$q + \bar{q} \Rightarrow q + \bar{q} \qquad g + \bar{q} \Rightarrow g + \bar{q}$$
$$\bar{q} + \bar{q} \Rightarrow \bar{q} + \bar{q} \qquad g + g \Rightarrow g + g$$



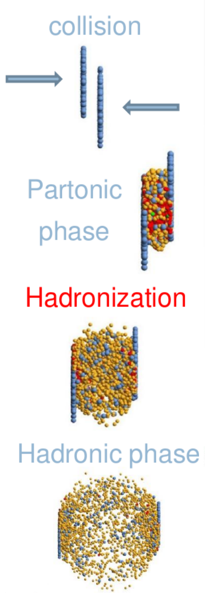
- Inelastic collisions:

$$q + \bar{q} \Rightarrow g$$
$$g \Rightarrow q + \bar{q}$$

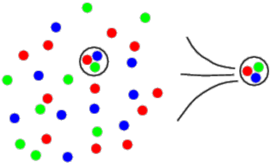
~~$q + \bar{q} \Rightarrow g + g$~~
~~$g \Rightarrow g + g$~~

Suppressed due to the large gluon mass.

Stages of a heavy-ion collision in PHSD

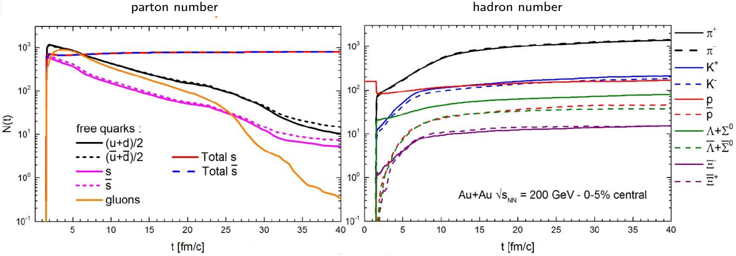


- Massive and off-shell (anti-)quarks hadronize to colorless off-shell mesons and baryons:

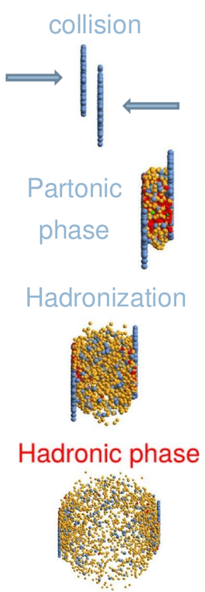


$$g \Rightarrow q + \bar{q}$$
$$q + \bar{q} \Rightarrow \text{meson ('string')}$$
$$q + q + q \Rightarrow \text{baryon ('string')}$$

- Local covariant off-shell transition rate.
- Strict 4-momentum and quantum number conservation.

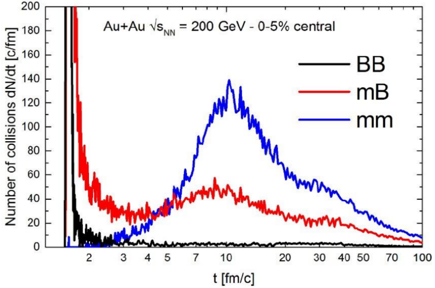


Stages of a heavy-ion collision in PHSD



- Hadron-string interactions – **off-shell HSD (Hadron String Dynamics)**.
- Elastic and inelastic collisions between baryons (B), mesons (m).

Distribution of hadron collisions as a function of time:



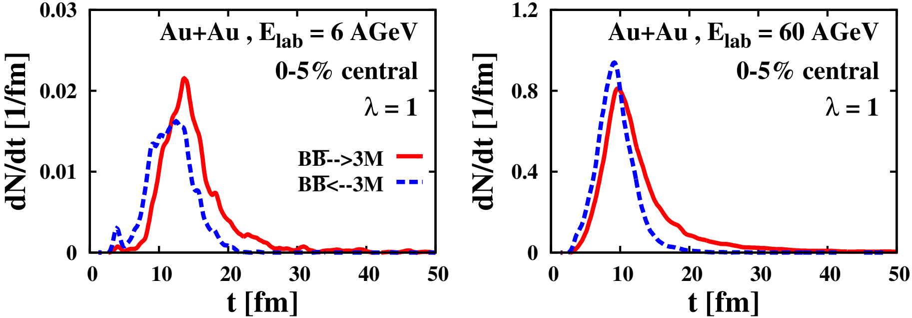
- Large amount of mm and mB collisions at times > 10 fm/c

PHSD simulations with extended many-body reactions

- The introduced many-body reactions have been implemented in PHSD in the light sector
- Now also the strangeness sector is implemented
- Check for sensitive observables in heavy-ion collisions

E. Seifert, W. Cassing, Phys. Rev. C **97**, 024913 (2018); Phys. Rev. C **97**, 044907 (2018)

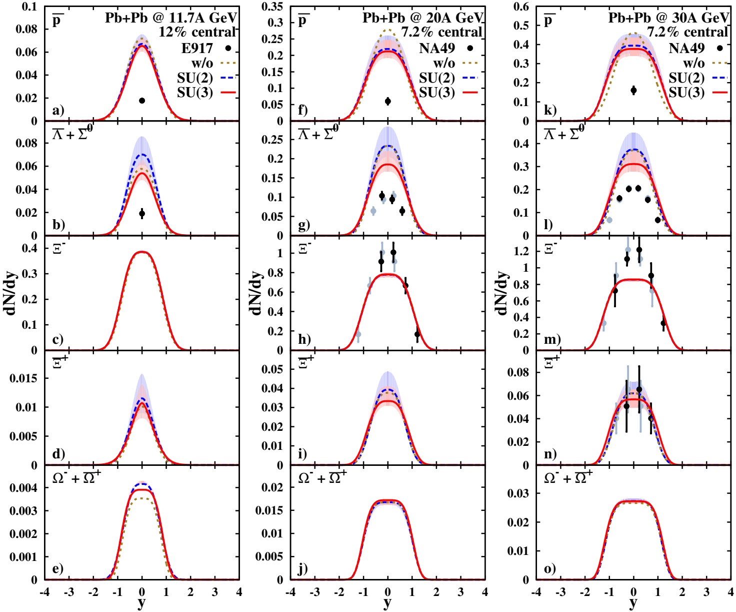
Reaction rates



Looking at the total $B\bar{B} \leftrightarrow 3M$ reaction rates we find:

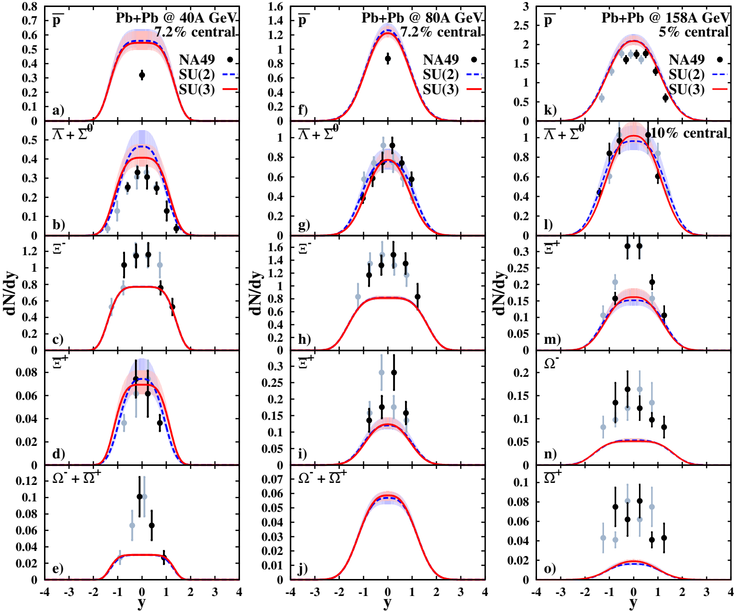
- The $B\bar{B} \leftrightarrow 3M$ impact a HIC only in the first 20 fm/c
- At these energies the annihilation and reproduction almost balance each other out
- A slight net-annihilation is found

AGS to SPS



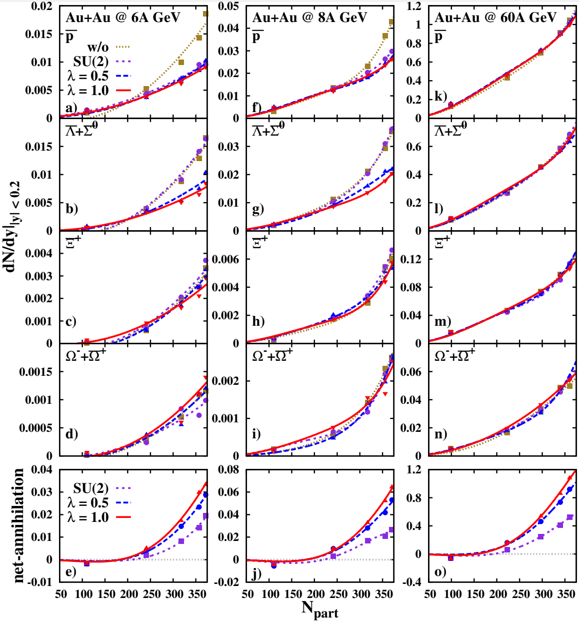
- Strangeness suppression of $\lambda = 0.5$ for SU(3) calculations
- No visible change in baryons and mesons
- Strangeness sector has weak influence on $\bar{p}$
- $B\bar{B} \leftrightarrow 3M$ reactions push PHSD results closer to experiment

AGS to SPS



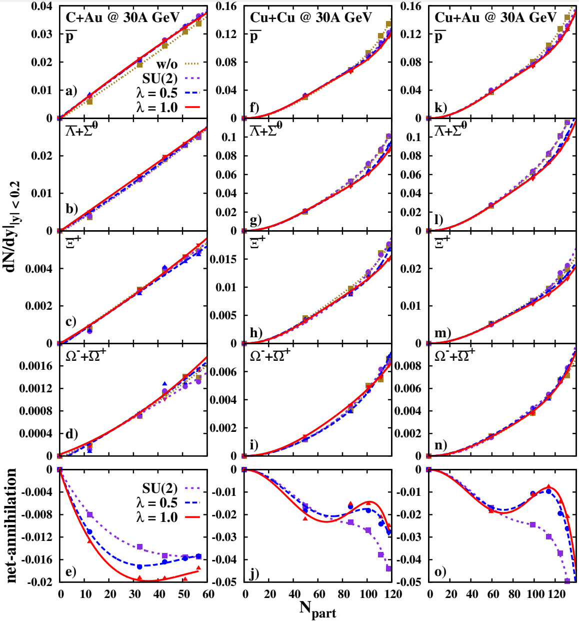
- With rising energy up to 158*A* GeV the sensitivity to the strangeness sector diminishes

FAIR and NICA Au+Au collisions



- At the lowest energies $\bar{p}$ has the highest yields for calculations without $B\bar{B} \leftrightarrow 3M$
- For other antibaryons calculations without $B\bar{B} \leftrightarrow 3M$ and with only the light sector lie on top of each other
- For lower strangeness suppression ($\lambda \rightarrow 1$) of the transition matrix element lower yields/ higher net-annihilations are found
- At 60A GeV the different calculations are not distinguishable from each other
- Net-annihilation as a function of $N_{\rm part}$ has almost quadratic behavior and is always positive

FAIR and NICA light systems



- No large differences between the calculations are found for the midrapidity yields as a function of N_{part}
- A negative net-annihilation is found with peculiar behavior as a function of N_{part} and system size

Summary

- We described $B\bar{B} \leftrightarrow 3M$ reactions as a rearrangement of the quark content
- The implementation is proven to fulfil the detailed balance relation using box simulations
- Strangeness sector affects mostly anti-hyperons and pushes results closer to data
- At low energies we find almost a balance between annihilation and recreation
- At FAIR and NICA energies simulations with and without $B\bar{B} \leftrightarrow 3M$ reactions and with and without strangeness suppression λ of the matrix element give different results at the lowest investigated energies of 6 and $8A$ GeV
- As soon as experimental data is available the relevance of the $B\bar{B} \leftrightarrow 3M$ can be verified or falsified

Thank you for your attention!