Nonequilibrium pion distribution in heavy-ion collisions from the Zubarev approach

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Colloquium Zubarev-100
Dubna 18./19.04.2018

Personal recollections – The academic family

N.N. Bogoliubov (1909 - 1992)

D.N. Zubarev (1917 – 1992)
PhD student of N.N.B.

G. Röpke (1941)
“Postdoc” of D.N.Z. (1968-70)

D. Blaschke (1959)

H. Reinholz (1959)

Recollections – The year 1959

Shirkov, Zubarev, Logunov, ?, Galasiewicz, Tavkhelidze
Visit to Mzcheta (Georgia), 1959

Recollections – The year 1959

Shirkov, Zubarev, Logunov, ?, Galasiewicz, Tavkhelidze
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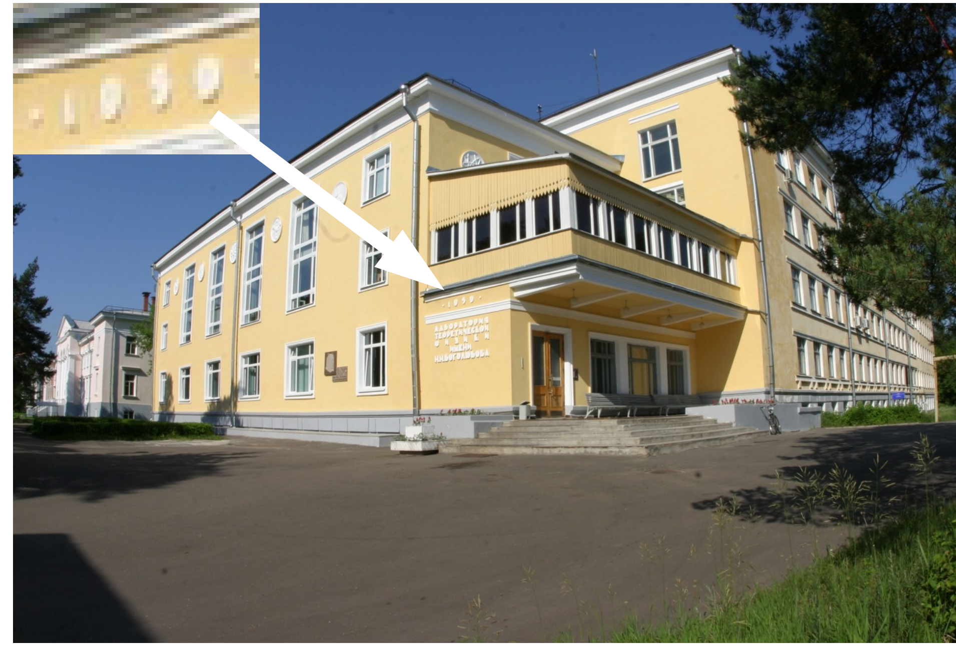
Recollections – The year 1959



Shirkov, Zubarev, Logunov, ?, Galasiewicz, Tavkhelidze
Visit to Mzcheta (Georgia), 1959

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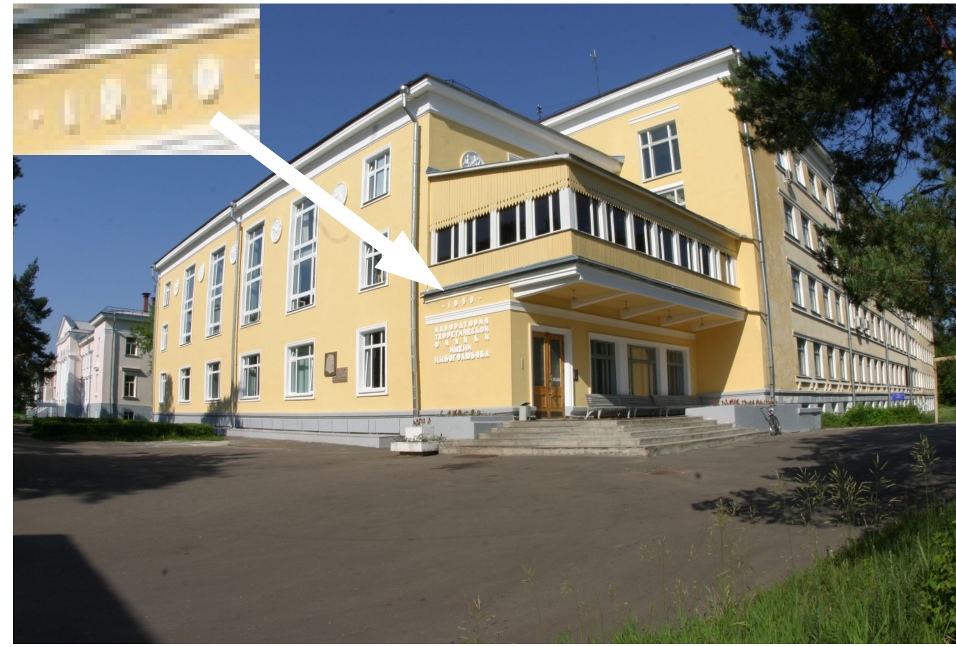

D.B. born 22.9.1959

Shirkov, Zubarev, Logunov, ?, Galasiewicz, Tavkhelidze
Visit to Mzcheta (Georgia), 1959

Recollections – The year 1959

Shirkov with former vice Directors of BLTP (2009)

Shirkov, Zubarev, Logunov, ?, Galasiewicz, Tavkhelidze
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Zubarev & Tokarchuk visit Rostock (1987)

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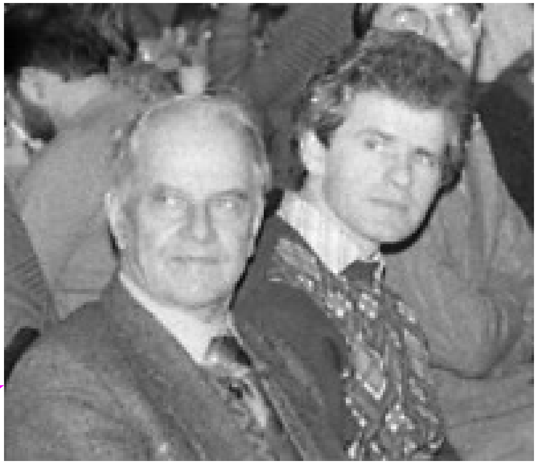

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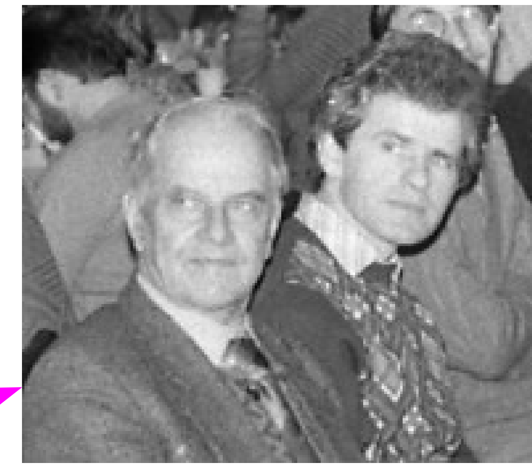

Zubarev & Tokarchuk visit Rostock (1987)

Galasiewicz at Wroclaw University (2008)

Shirkov, Zubarev, Logun
Visit to Mzcheta (Georgia


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ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ
Ф И З И К И
ИМЕНИ
Н.Н.БОГОЛЮБОВА

May 1984 – Inauguration of the Blokhintsev plate

Low-momentum pion enhancement – early stage

First heavy-ion collision experiments at CERN SPS with O-Au and S-S at 60 and 200 GeV/n
→ low momentum enhancement of negative particle (pions) yields relative to Bose-distribution

Possible solution of the puzzle →

$$f(p,x)=\frac{1}{e^{(E-\mu)/T}\pm 1}\,,\quad E=\sqrt{p_T^2+p_L^2+m^2}$$

$$\frac{dN}{p_T\,dp_T\,dy}=(\pi R^2\tau_{\rm dec})\,\frac{m_T}{(2\pi)^2}$$

$$\times\sum_{n=1}^{\infty}(\pm 1)^{n+1}e^{n\mu/T}K_1\left(n\frac{m_T}{T}\right),$$

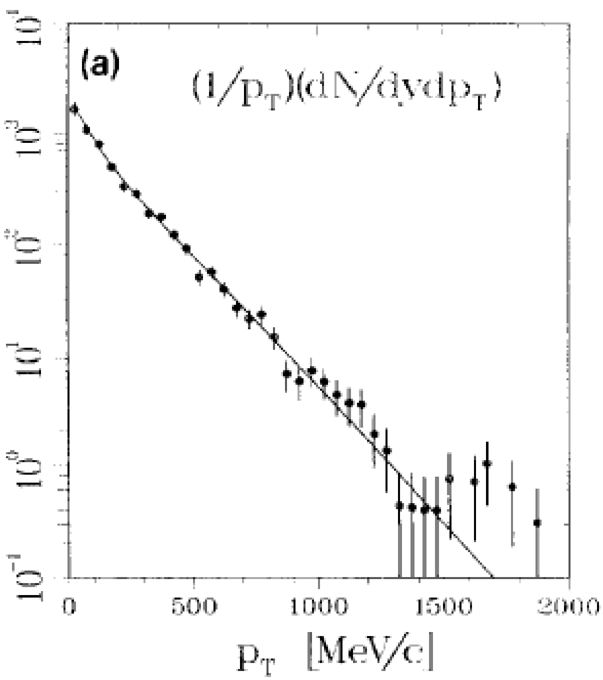
Transverse mass $m_T=\sqrt{p_T^2+m^2}$

Nonequilibrium chemical potential
for pions μ = 126 MeV; T=167 MeV

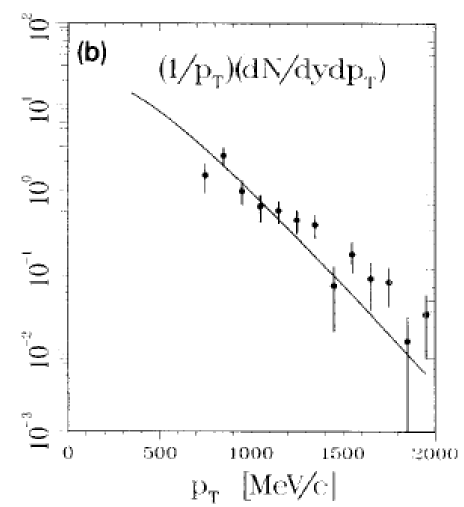
Non-zero chemical potential
and the shape of the p_T-distribution of hadrons
in heavy-ion collisions

PLB 243 (1990) 181

M. Kataja and P.V. Ruuskanen
Department of Physics, University of Jyväskylä, Seminaarinkatu 15, SF-40100 Jyväskylä, Finland



← Pions and Kaons

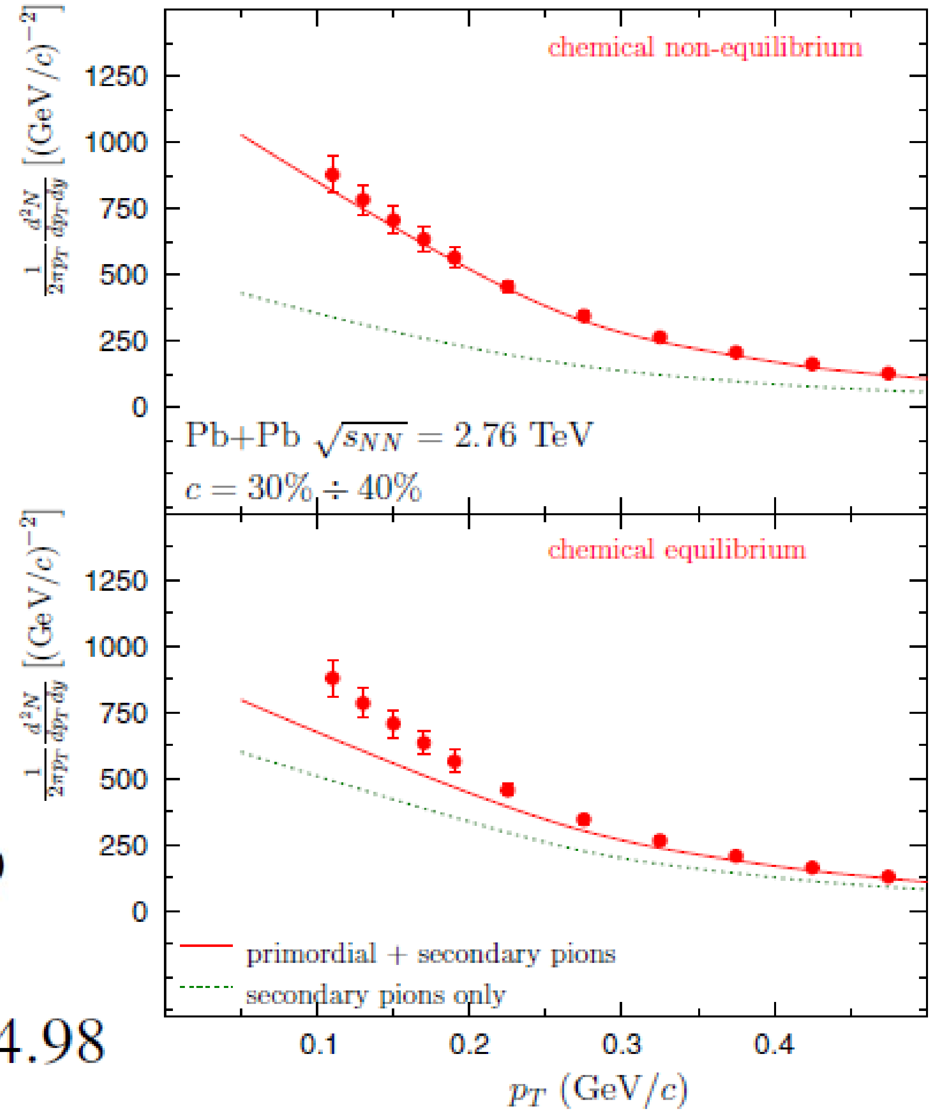


See also:
S. Gavin, "Pions in and out of equilibrium"
NPA 544 (1992) 459; NPB 351 (1991) 561

Low-momentum pion enhancement – LHC revival

Explanation of hadron transverse-momentum spectra in heavy-ion collisions at $\sqrt{s_{NN}} = 2.76$ TeV within a chemical nonequilibrium statistical hadronization model

V. Begun, W. Florkowski, M. Rybczynski,
PRC 90, 014906 (2014)



Cooper-Frye formula

$$\frac{dN}{dyd^2p_T} = \int d\Sigma_\mu p^\mu f(p\cdot u),$$

Primordial distribution

$$f_i = \frac{g_i}{\Upsilon_i^{-1}\exp\left(\sqrt{p^2+m_i^2}/T\right)\mp 1}.$$

Fugacity factor

$$\Upsilon_i = (\lambda_q\gamma_q)^{N_q^i}(\lambda_s\gamma_s)^{N_s^i}(\lambda_{\bar q}\gamma_{\bar q})^{N_{\bar q}^i}(\lambda_{\bar s}\gamma_{\bar s})^{N_{\bar s}^i}$$
$$= \gamma_q^{N_q^i+N_{\bar q}^i}\gamma_s^{N_s^i+N_{\bar s}^i}\exp\left(\frac{\mu_B B_i+\mu_S S_i}{T}\right)$$

Fit for semiperipheral collisions, c=30%-40%

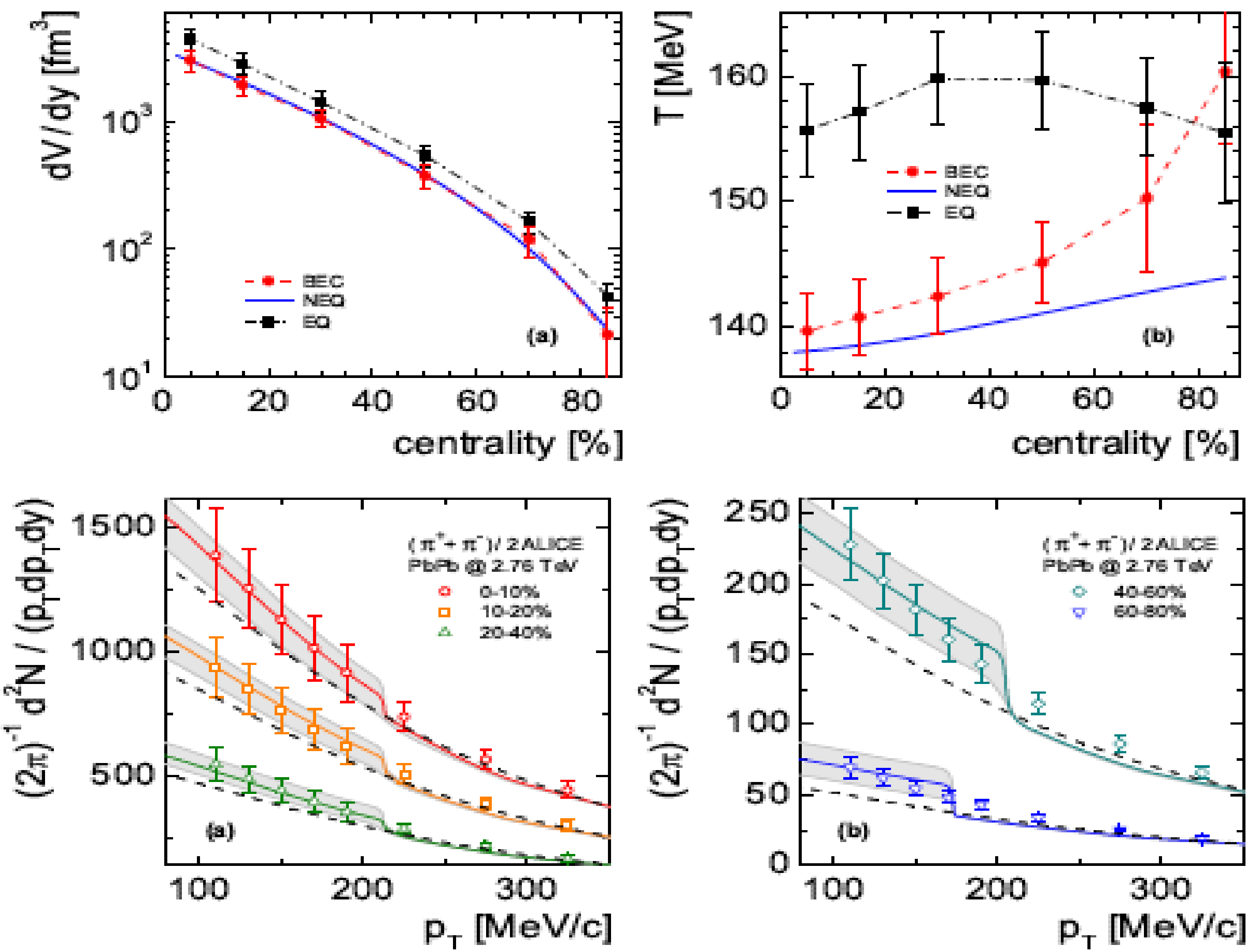
$$T = 139.85\ \mathrm{MeV},\quad \gamma_q = 1.62,\quad \gamma_s = 2.0$$

Low-pT Bose enhancement of pions

$$\mu_\pi = 2T\ln\gamma_q \simeq 134.9\ \mathrm{MeV} \leftrightarrow m_{\pi^0} \simeq 134.98$$

Low-momentum pion enhancement at LHC - Onset of Bose-Einstein Condensation of pions ?

$$n = \int d^3p \, \frac{1}{(2\pi)^3} \frac{g}{\exp\left(\frac{\sqrt{p^2+m^2}-\mu}{T}\right) - 1} \left[1 + \frac{(2\pi)^3}{\mathcal{V}} \, \delta(p_x) \, \delta(p_y) \, \delta(p_z) \right]$$



V. Begun, W. Florkowski, PRD (2015); arxiv:1503.04040

Nonequilibrium statistical approach

nonequilibrium statistical operator $\rho(t)$

$$\rho(t) = \lim_{\varepsilon\to 0} \varepsilon \int_{-\infty}^{t} dt' e^{\varepsilon(t-t')} U(t,t') \rho_{\rm rel}(t') U^\dagger(t,t') \qquad U(t,t') = \exp[(i/\hbar)H(t-t')]$$

limit $\varepsilon \to 0$ has to be taken after the thermodynamic limit

relevant statistical operator $\rho_{\rm rel}(t)$ according to Zubarev

The missing irrelevant correlations $\rho_{\rm irrel}(t) = \rho(t) - \rho_{\rm rel}(t)$

are assumed to be formed dynamically by the action of the time evolution operator $U(t,t')$

The relevant information is given by the averages of the relevant observables B_n

$$\langle B_n \rangle^t = {\rm Tr}\{\rho(t) B_n\}$$

The maximum of the information entropy at given averages is the generalized Gibbs distribution

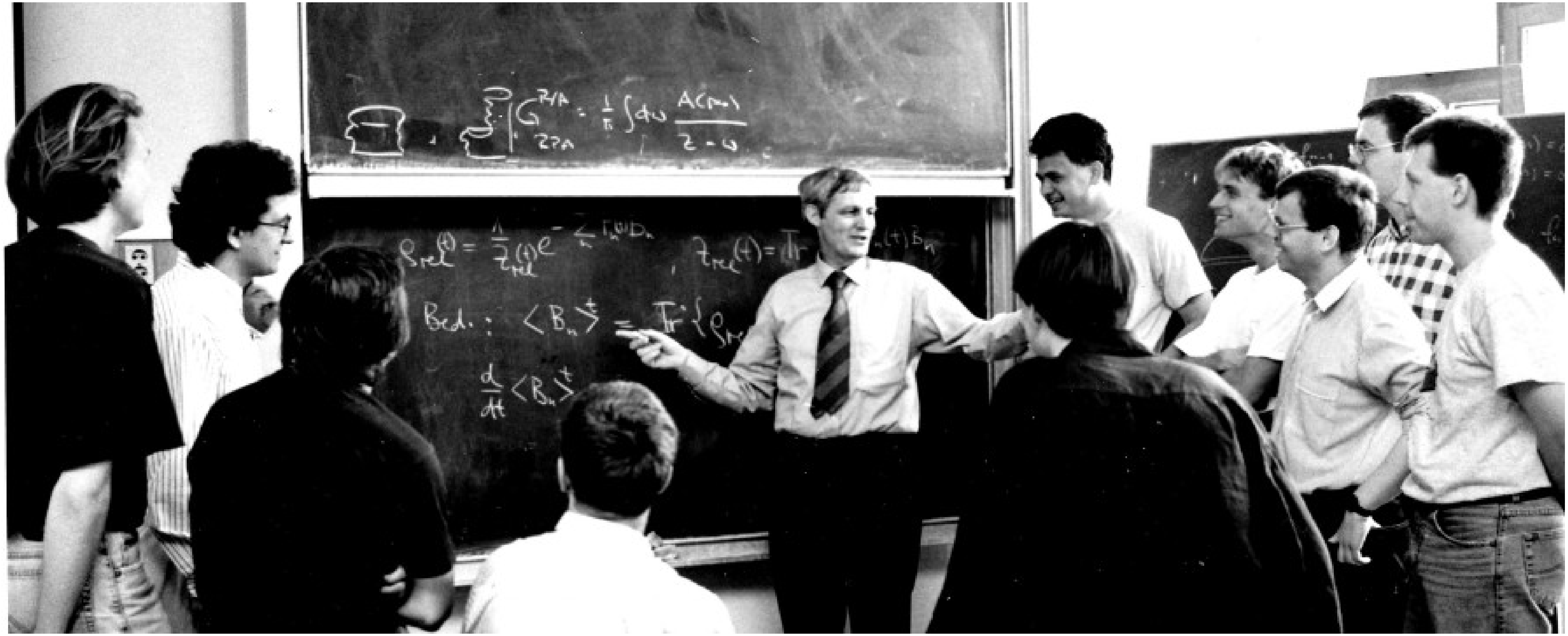
$$\rho_{\rm rel}(t) = \frac{1}{Z_{\rm rel}(t)} e^{-\sum_n F_n(t) B_n}; \qquad Z_{\rm rel}(t) = {\rm Tr} e^{-\sum_n F_n(t) B_n}$$

where the Lagrange multipliers $F_n(t)$ are determined by the self-consistency conditions

$$\langle B_n \rangle^t_{\rm rel} = \langle B_n \rangle^t$$

with $\langle B_n \rangle^t_{\rm rel} \equiv {\rm Tr}\{\rho_{\rm rel}(t) B_n\}$.

Nonequilibrium statistical operator formalism



Gerd Roepke explains the Zubarev approach to nonequilibrium in the Max-Planck group at Rostock University (1995)

Nonequilibrium approach to pion enhancement effect

Hamiltonian with instantaneous and retarded part

$$\mathcal{H} = \mathcal{H}^{(0)}\delta(t-t') + \mathcal{H}^{(r)}(t-t')$$

Occupation numbers $N_c = \sum_{\mathbf{p}} n_{\mathbf{p},c}$

Hamiltonian $H = \sum_c H_c + \frac{1}{2}\sum_{c,c'} V_{c,c'}^{\rm interaction}$

Relevant statistical operator

$$\rho_{\rm rel}(t) = \frac{1}{Z_{\rm rel}(t)} e^{-\sum_c \beta_c(t)(H_c - \mu_c(t) N_c)}$$

Occupation numbers

$$\langle n_{\mathbf{p},c}\rangle_{\rm rel}^t = \frac{1}{e^{\beta_c(t)[E_c^{\rm mf}(\mathbf{p}) - \mu_c(t)]} \pm 1}$$

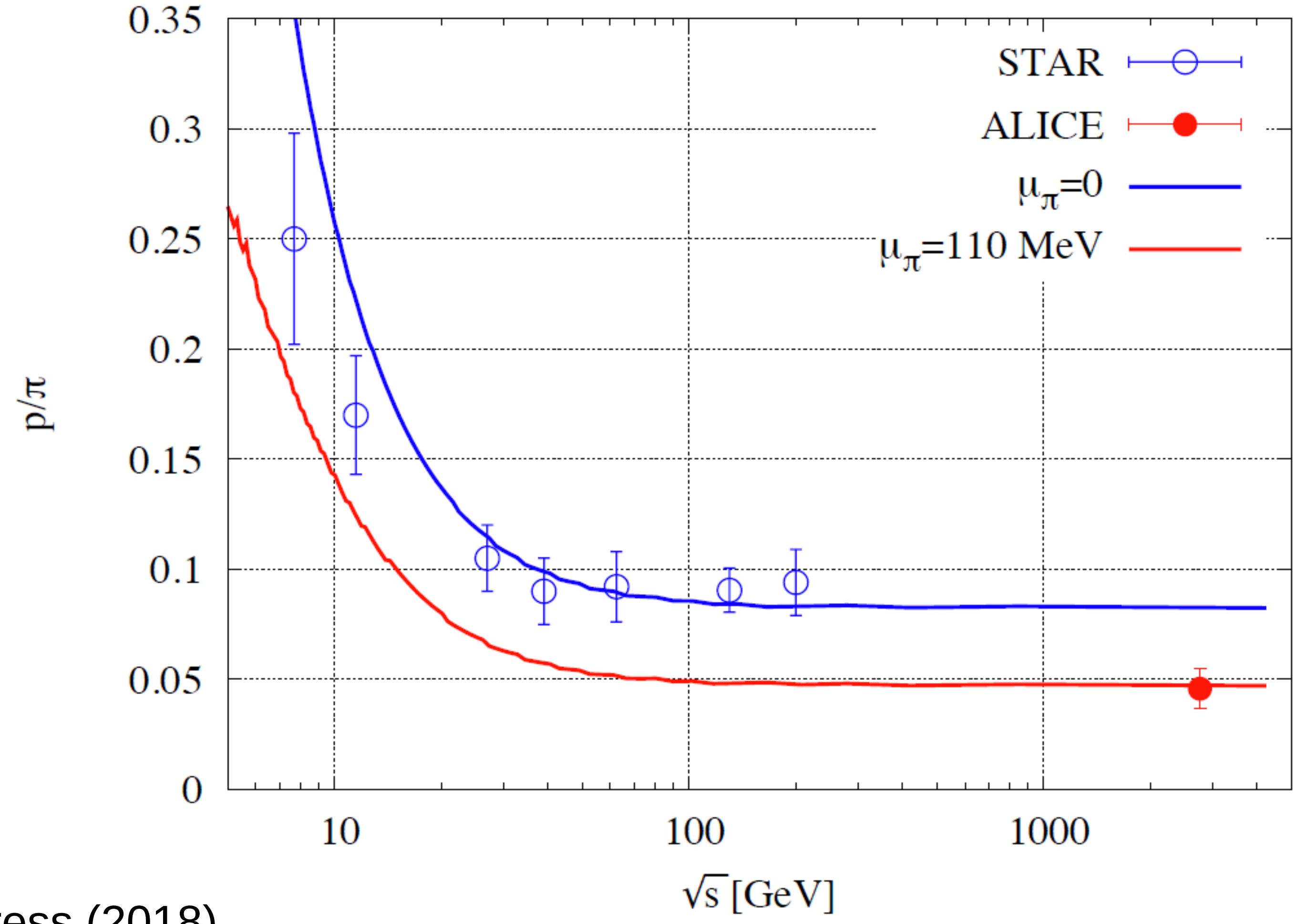
Equations of motion

$$\frac{d}{dt}\langle N_c\rangle^t = \frac{i}{\hbar}\langle [H, N_c]\rangle_{\rm rel}^t - \frac{1}{\hbar^2}\int_{-\infty}^{0} d\tau e^{\epsilon\tau} {\rm Tr}\{\rho_{\rm rel}(t)[H(\tau),[H,N_c]]\}$$

$$\frac{d}{dt}\langle H_c^{\rm mf}\rangle^t = \frac{i}{\hbar}\langle [H, H_c^{\rm mf}]\rangle_{\rm rel}^t - \frac{1}{\hbar^2}\int_{-\infty}^{0} d\tau e^{\epsilon\tau} {\rm Tr}\{\rho_{\rm rel}(t)[H(\tau),[H,H_c^{\rm mf}]]\}$$

Nonequilibrium approach to pion enhancement effect

LHC pion and proton puzzle related ….



G. Roepke et al., in progress (2018)

Conclusions

- Zubarev approach to nonequilibrium gives a sound conceptual basis to describe deviations from equilibrium distributions from particles produced In heavy-ion collisions, under nonequilibrium conditions

- perfect example for applications is the low-pT pion enhancement as described with a pion chemical potential that appears as a Lagrange multiplier of a generalized nonequilibrium statistical operator

- Extension to chemical nonequilibrium situations (e.g., strangeness production) shall be straightforward

- A microscopic process for the overpopulation of the pion phase space need to be worked out, e.g., a gluon-to-pion conversion process