Bose-Einstein Condensation from a Gluon Transport Equation

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Acknowledgements

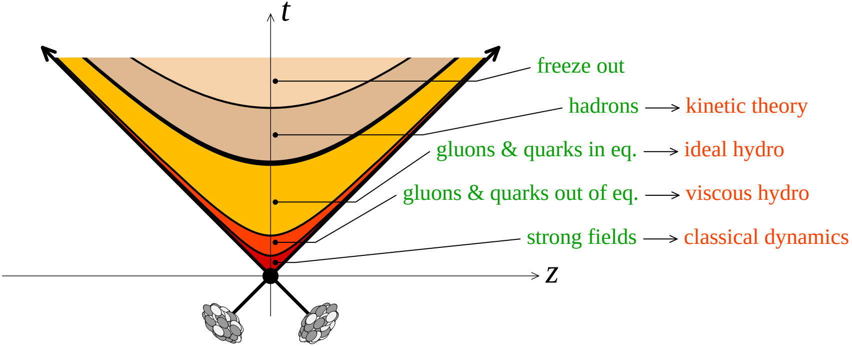
- J. P. Blaizot, J. Liao, and L. McLerran. Gluon transport equation in the small angle approximation and the onset of Bose-Einstein condensation. Nuclear Physics A, 920:58, 2013.

- Spatially homogeneous system of gluons, isotropic in momentum space. Evolve using QCD Boltzmann equation.

- For certain initial conditions a Bose-Einstein condensate forms in finite time

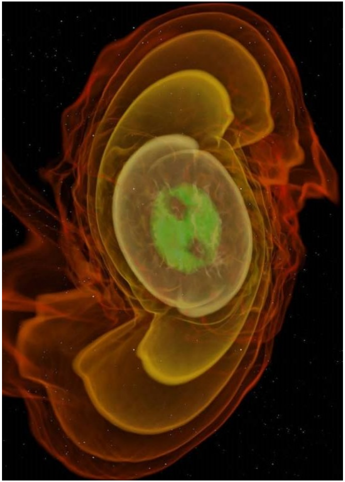
- We extend this beyond the onset of condensation, and introduce anisotropy

The Anatomy of a Collision



- Equilibriation is *fast* - $\mathcal{O}(1\ \text{fm})$
- So why the Boltzmann equation?

Relativistic Hydrodynamics



- Ideal Hydro
  - Works well
  - “Equilibriation” is instantaneous
- Dissipative Hydro
  - Can consider off-equilibrium systems
  - No relativistic equivalent to Navier-Stokes equations

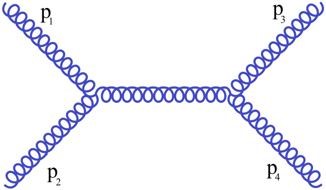
A more microscopic approach

- Can derive hydrodynamics from the Boltzmann equation
- Perhaps worth investigating the problem using this directly?
- Boltzmann equation treats the QGP as a dilute particle gas
- Hydro only assumes energy-momentum conservation - is the particle approach appropriate at high energies?
- Boltzmann equation is computationally difficult - but let's do it!

Boltzmann equation

- Gluon plasma subject to elastic, number-conserving two-body collisions. $D_t f = C[f]$, where

$$C[f] = \frac{1}{2} \int_{2,3,4} |\mathcal{M}_{12\to 34}|^2 (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)(f_3 f_4 \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_3 \bar{f}_4).$$



- This is a nonlinear integro-differential equation. Solving it is... non-trivial.

The H-theorem

- Can, however, describe some properties without solving it. By the H-theorem,

$$f_{eq}(x,p) = \left[Exp\left(\frac{p^{\nu} u_{\nu}(x) - \mu(x)}{T(x)} \right) - 1 \right]^{-1}$$

- Here u^{ν} is the particle 4-current, μ is a chemical potential and T is the temperature.

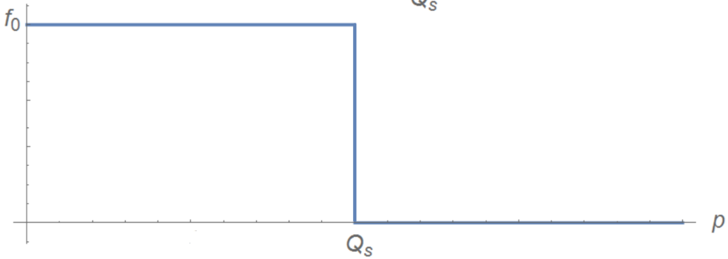
- There is one caveat.

Bose-Einstein condensation

- There exist certain initial distributions of gluons that are "overpopulated" with respect to equilibrium.

- Consider the CGC-inspired family of spherically symmetric distribution functions

$$f(p) = f_0 \Theta\left(1 - \frac{p}{Q_s}\right)$$



- For $f(p) = f_0\theta(1 - p/Q_s)$

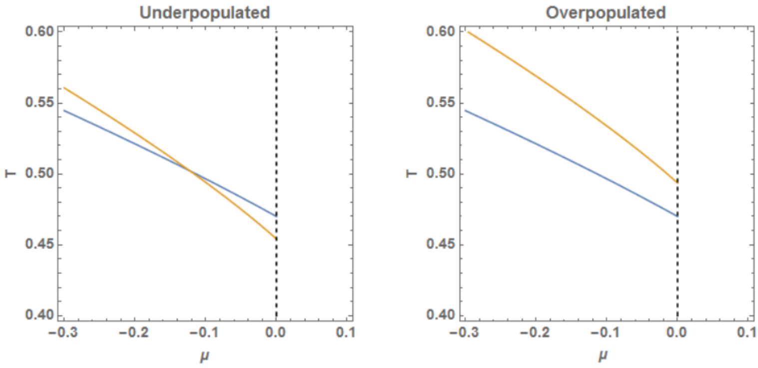
$$\epsilon_0 = f_0 \frac{1}{2\pi^2} \int_0^{Q_s} dp\, p^3 = f_0 \frac{Q_s^4}{8\pi^2},$$

$$n_0 = f_0 \frac{1}{2\pi^2} \int_0^{Q_s} dp\, p^2 = f_0 \frac{Q_s^3}{6\pi^2}.$$

- Now consider the number and energy densities for the equilibrium Bose distribution:

$$\epsilon_{eq}(T, \mu) = \frac{1}{2\pi^2} \int_0^{\infty} dp \frac{p^3}{e^{(p-\mu)/T} - 1} = \frac{3T^4}{\pi^2} \mathrm{Li}_4(e^{\mu/T})$$

$$n_{eq}(T, \mu) = \frac{1}{2\pi^2} \int_0^{\infty} dp \frac{p^2}{e^{(p-\mu)/T} - 1} = \frac{T^3}{\pi^2} \mathrm{Li}_3(e^{\mu/T})$$

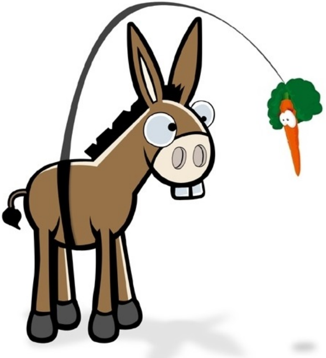


- Consider contours of constant n and ϵ

$$\color{blue}{n_0 = n_{eq}}; \quad \color{orange}{\epsilon_0 = \epsilon_{eq}}$$

$$r \equiv n\epsilon^{-3/4}; \quad r_{crit} = 0.28$$

Relaxation Time Approximation



- From the initial condition we always know the final distribution function - can make an ansatz.

$$\partial_t f = \frac{p^\mu u_\mu}{p_0} \frac{f_\infty - f}{\tau}$$

- In a sense we have "averaged over" the collision term. An analytic solution exists, viz.

$$f(t) = f_\infty + (f_0 - f_\infty) e^{-\left(\frac{p^\mu u_\mu}{p_0} \frac{t}{\tau}\right)}.$$

How good is the RTA?

- Approaches equilibrium exponentially
- Can model the growth of the condensate
- Relaxation time parameter τ has to be set by hand
- No QCD features though
- Ultimately we would like to use something closer to the truth

The Fokker-Planck Equation

- Under the assumption that small scattering angle collisions dominate, it is possible to recast the collision term as the divergence of a current,

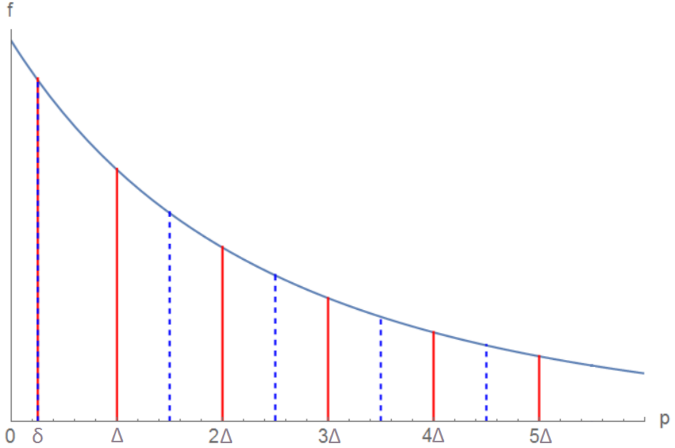
$$D_t f = C[f] \xrightarrow[\text{scattering}]{\text{soft}} \nabla \cdot \mathcal{J}$$

  where

$$\mathcal{J}(p) = I_a \nabla f + I_b f \bar{f} \hat{p} + (\nabla f \cdot \hat{p})\mathcal{I} + (\nabla f \times \hat{p}) \times \mathcal{I}.$$

- $I_a = \int f\bar{f}$, $I_b = \int \frac{2f}{p}$ and $\mathcal{I} \equiv (\mathcal{I}_x, \mathcal{I}_y, \mathcal{I}_z) = \int \frac{\vec{p}}{p} f\bar{f}$ are functionals of the distribution function.

The scheme for isotropic initial conditions



The basic idea of the implementation

- First discretize the phase space. Next interpolate over the arbitrary initial distribution function.
- Numerically integrate to obtain the particle number in each cell.
- Calculate the particle flux at the boundaries between cells and update the particle number using the forward Euler method.
- From analytical expressions for the integrals of your interpolating functions, use rootfinding to obtain the new distribution function
- As my supervisor is fond of saying, the devil is in the details.

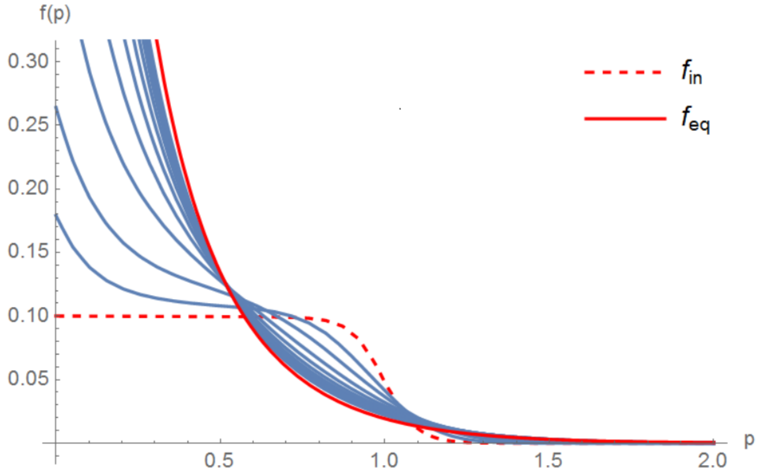
An interpolation ansatz

- For overpopulated initial conditions, the equilibrium distribution is singular at the origin

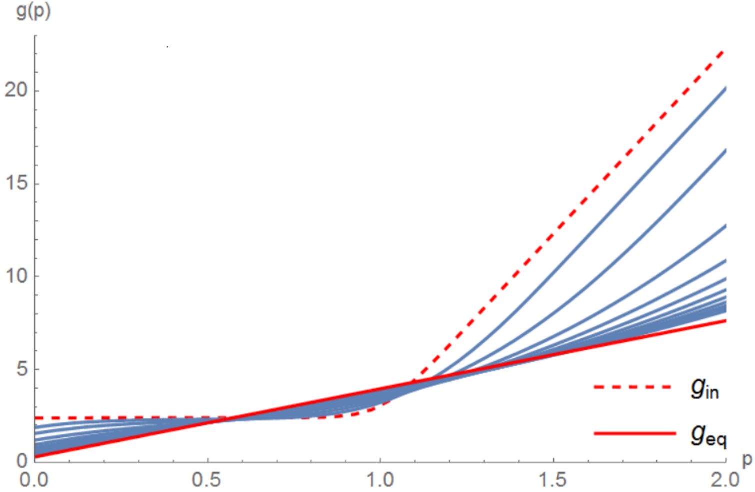
$$f_{eq} = \frac{1}{e^{p/T} - 1}$$

- A linear interpolation would fail
- Instead we interpolate with piecewise Bose distributions
- Many nice properties, including an exact interpolation of the equilibrium distribution

Some results



$$f_{eq} = 1/(e^{g(p)} - 1); \quad \ln\left(\frac{1+f}{f}\right) = g(p)$$



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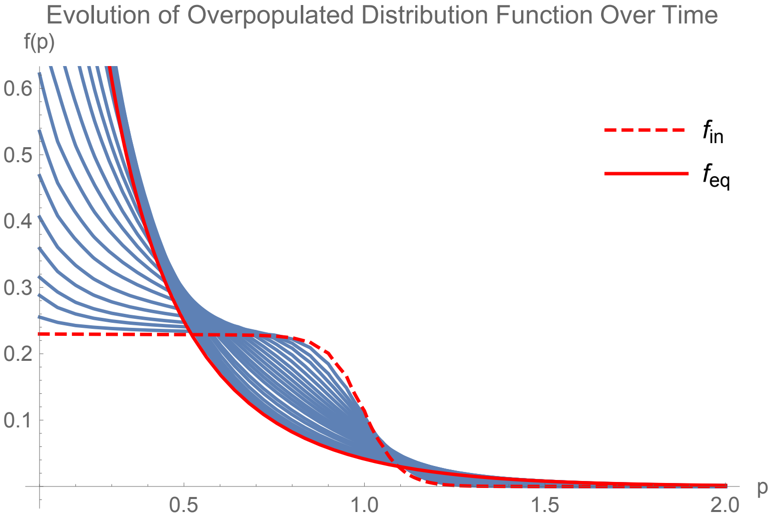

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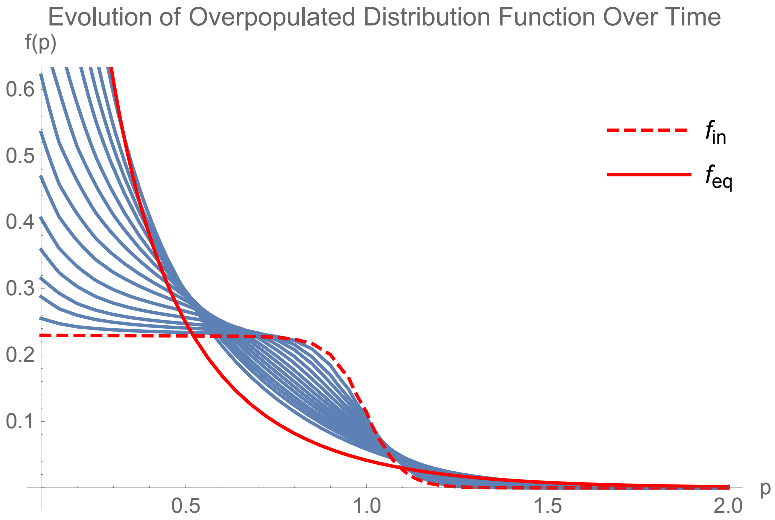

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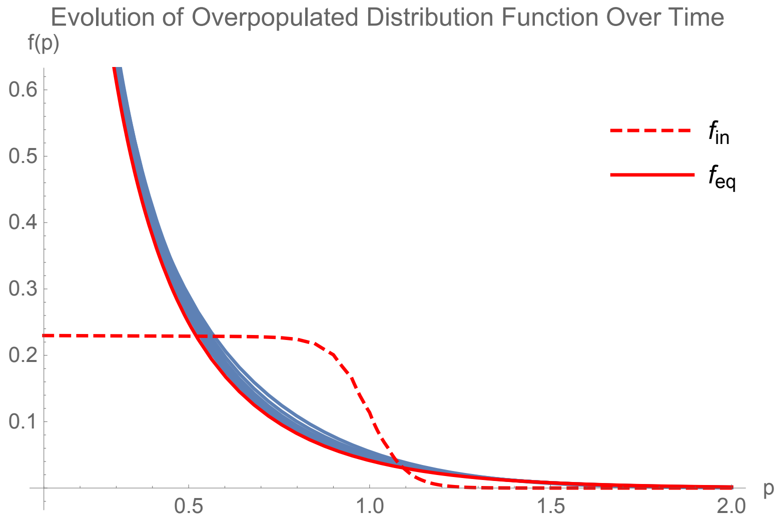
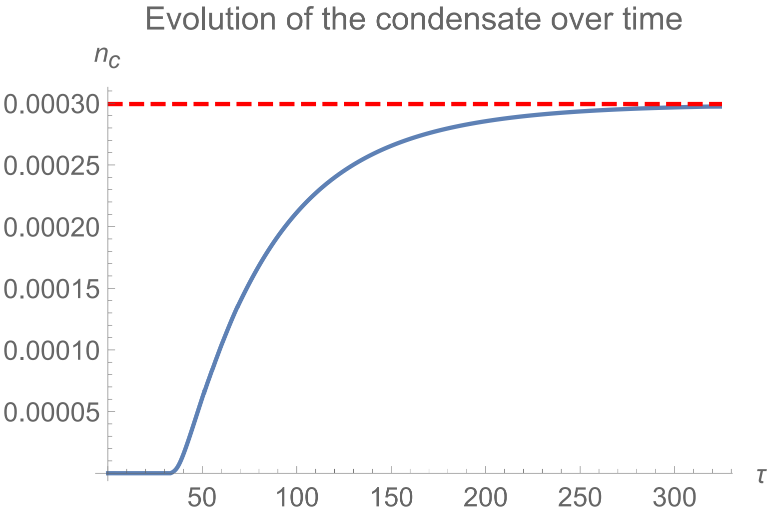
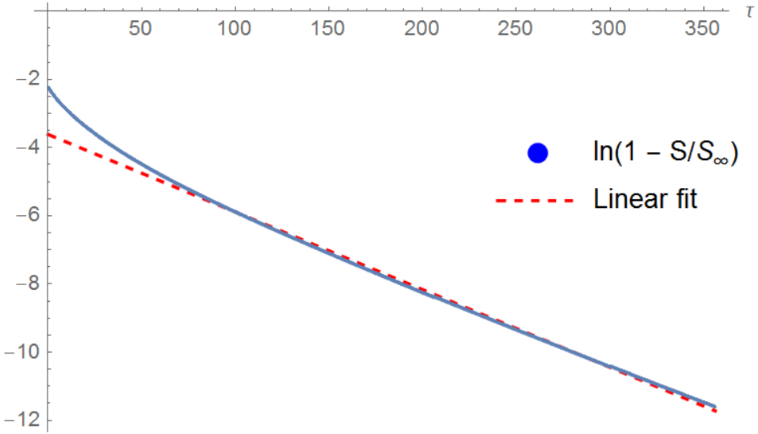




- Onset of condensation: $50\tau \approx 2$ fm$/c$.

Some results



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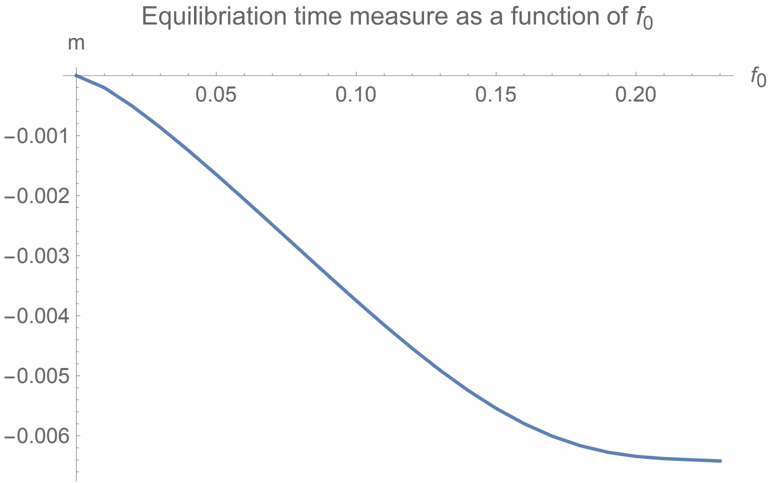
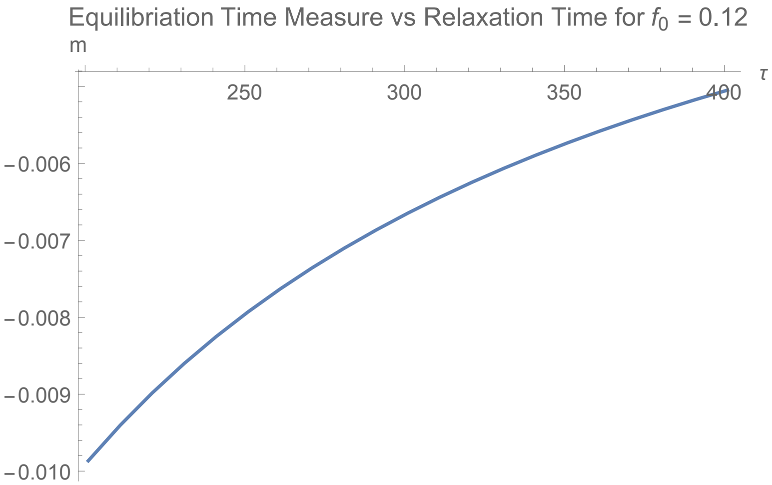
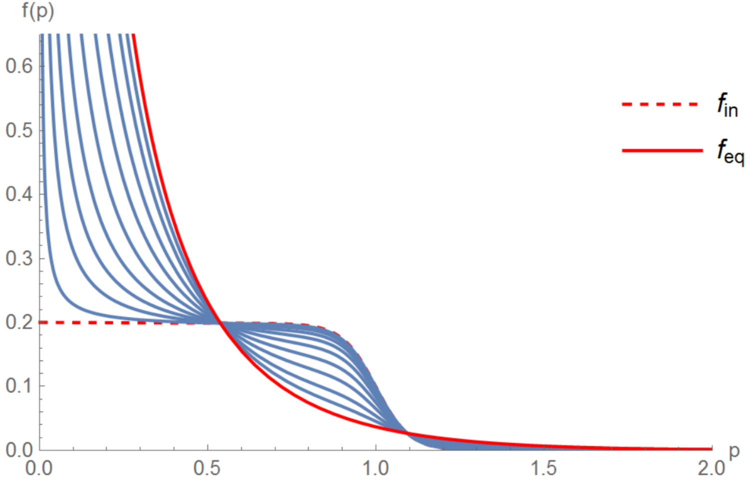


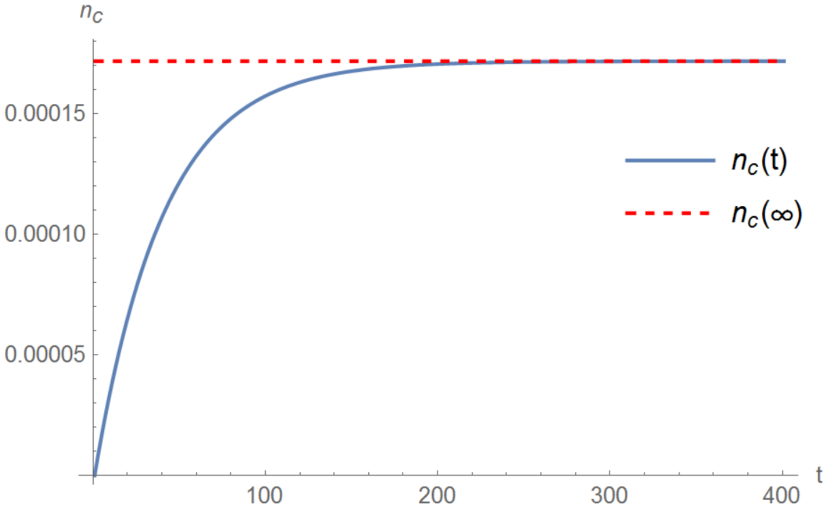
Estimating the relaxation time



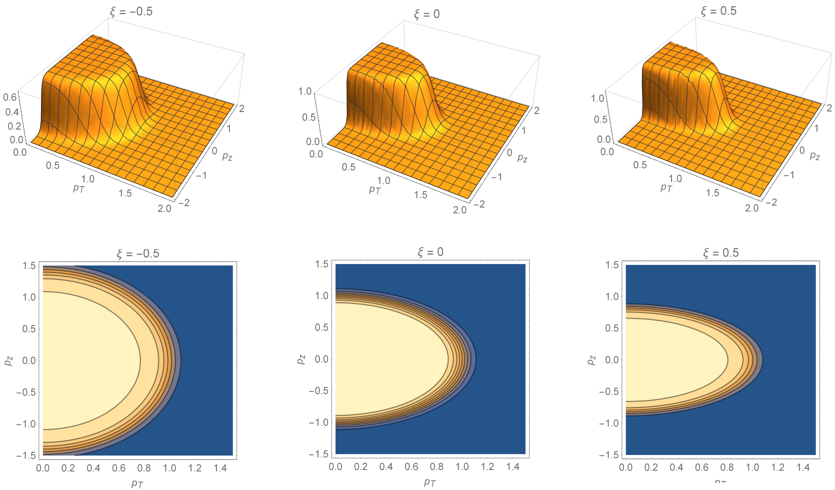
RTA vs Fokker-Planck: Overpopulated f_i



RTA vs. Fokker-Planck: Condensate Formation



Cylindrical Symmetry & Anisotropy



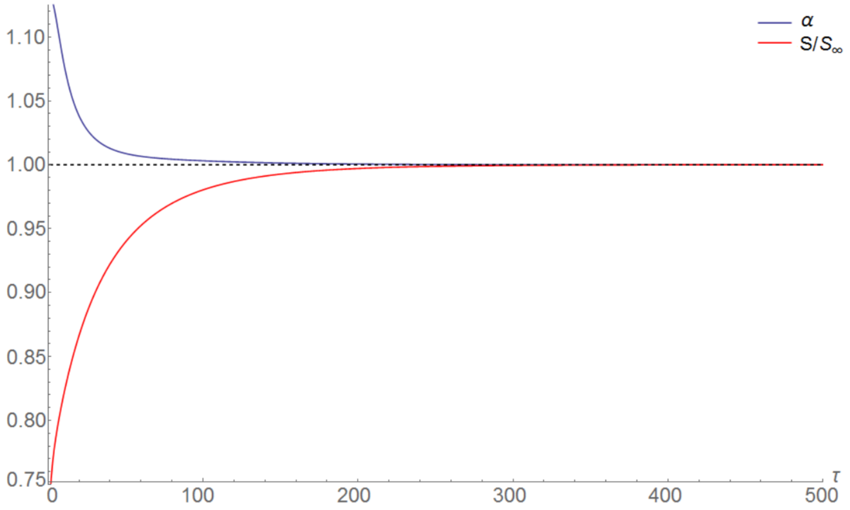
A Measure of Anisotropy

- We define the "anisotropy parameter",

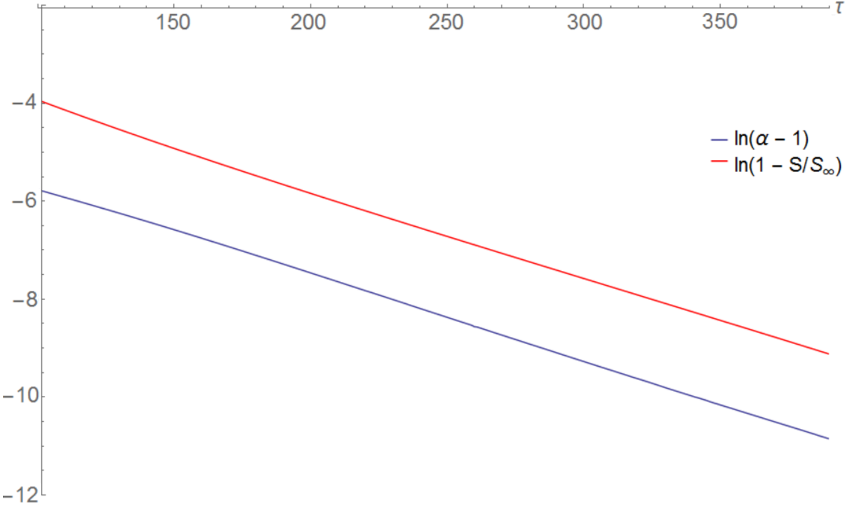
$$\alpha \equiv \frac{T^{22}_{LRF}}{T^{33}_{LRF}}$$

- $T^{\mu\nu}_{LRF}$ is the energy-momentum tensor in the local rest frame
- For cylindrically symmetric $f(p)$, $T^{11} = T^{22} = P_\perp$ is the transverse pressure
- $T^{33} = P_z$ is the longitudinal pressure.

Equilibriation vs Isotropization



Equilibriation vs Isotropization



Conclusion

- In summary, we have developed an efficient numerical scheme to solve the QCD Boltzmann equation in the small scattering angle approximation.
- Our work extends the results of Blaizot et al. to systems with cylindrically symmetric momentum distributions
- We also handle the dynamics of the formation of the Bose-Einstein condensate.
- Thank you!