Challenges for hydrodynamic modeling at [NICA] energies

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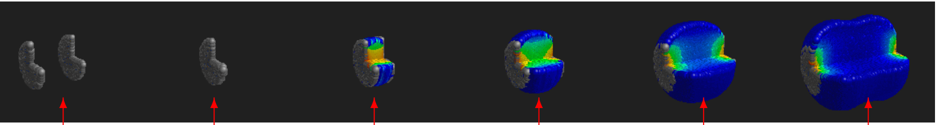
CNRS - SUBATECH Nantes

IK, Huovinen, Petersen, Bleicher, Phys. Rev. C 91, 064901 (2015)
Auvinen, IK, Bernhard, Bass, Phys. Rev. C 97, 044905 (2018)
P. Batyuk et al., Phys. Rev. C 96, 024911 (2017)

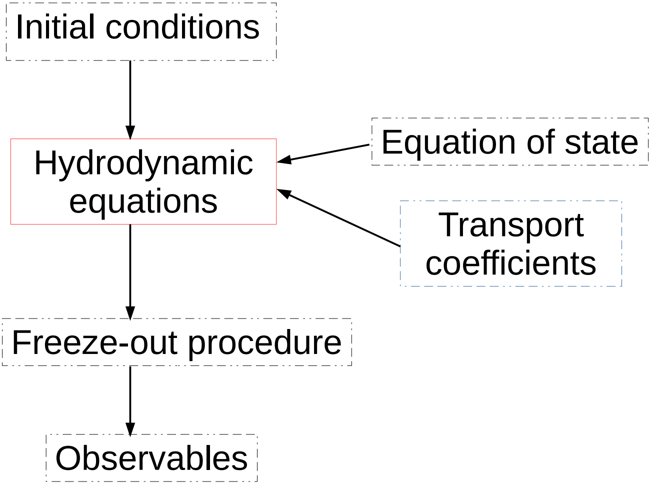
Introduction: heavy ion collision in pictures

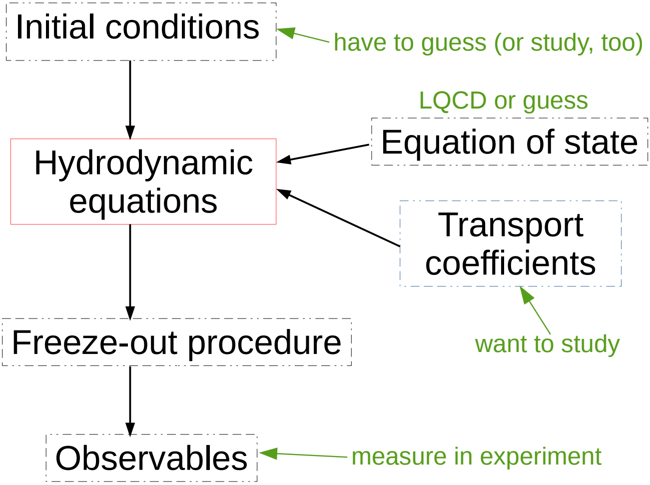
https://www.jyu.fi/fysiikka/tutkimus/suurenergia/urhic/anim1.gif/image_view_fullscreen



Hybrid model: initial state + hydrodynamic phase + hadronic cascade

thermalization — particlization


Initial conditions
Hydrodynamic equations
Equation of state
Transport coefficients
Freeze-out procedure
Observables

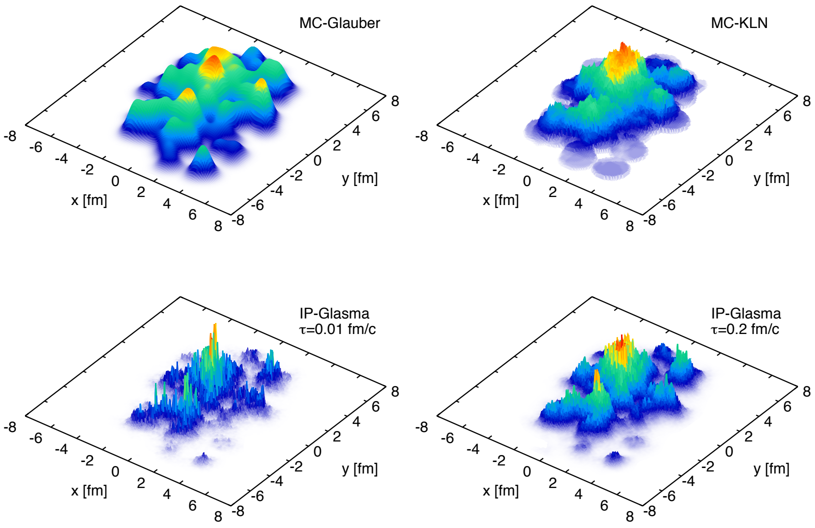

Initial conditions
have to guess (or study, too)
LQCD or guess
Hydrodynamic equations
Equation of state
Transport coefficients
Freeze-out procedure
want to study
Observables
measure in experiment

Initial stage

Models for initial conditions at high energies (full RHIC, LHC)

- **Glauber:** geometric model determining wounded nucleons based on the inelastic nucleon-nucleon cross section (whole family of variants)
  - Wounded quark model
- **Color Glass Condensate (CGC)**
  - **MC-KLN:** CGC-based model using kT-factorization
  - **IP-Glasma:** CGC-based model using classical Yang-Mills evolution of early-time gluon fields
- **EKRT:** perturbative QCD + saturation
- **EPOS:** pomeron picture
- **AMPT, HIJING:** minijets + strings
- **UrQMD** (hadron/string), **BAMPS** (partons)

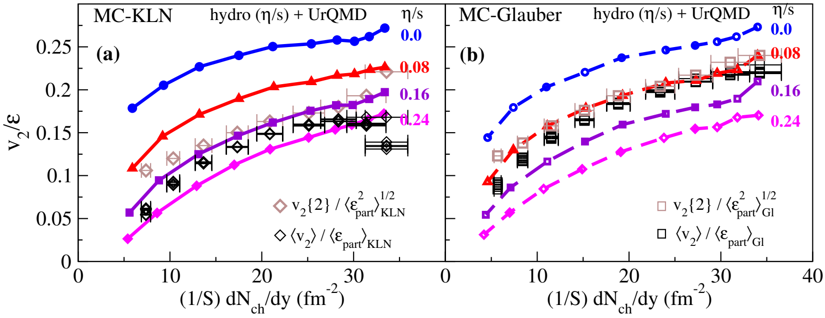
Different models provide different shapes of initial state



C. Gale et al, Int. J. of Mod. Phys. A, Vol. 28, 1340011 (2013)

...and it affects observables (from the model)

C. Shen et al., J. Phys. G38, 124045 (2011)



Different initial eccentricity in from Glauber and KLN models

$\Downarrow$

Different resulting elliptic flow

$\Downarrow$

Factor 2 difference in ‘extracted’ η/s from comparison to experimental data!

None of those models lead to thermalization!

What we need: energy-momentum tensor of ideal fluid in the local rest frame

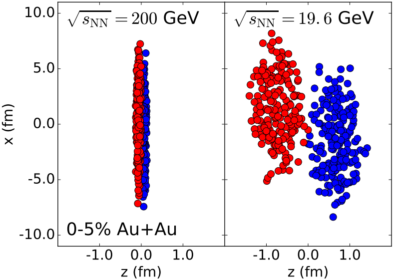
$$T^{\mu\nu}_{\mathsf{LRF}} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

How we can match initial state to hydrodynamics:

- Take only $T^{00} = \varepsilon$ from initial state, assume zero initial velocities (and ignore the rest of $T^{\mu\nu}$ from the initial state)
- Take T^{00}, T^{0i} and recalculate initial energy density ε and initial velocities u^{μ}_{ini} assuming hydrodynamical EoS $p = p(\varepsilon)$ (and ignore T^{ij} from the initial state)
- Take $T^{\mu\nu}$, find Landau frame and extract $\varepsilon = T$ and u^{μ} (and ignore that T^{ii} differ from equilibrium pressure)

Additional challenges at [NICA] energies

- Initial state: thick pancakes
  - boost ivariance is not a good approximation
    $\rightarrow$ need for 3 dimensional initial state
  - CGC picture does not work as well anymore
- Nonzero baryon and electric charge densities



C. Shen, B. Schenke,
Phys. Rev. C 97, 024907 (2018)

Which of the models above qualify for NICA?

- **Glauber:** geometric model determining wounded nucleons based on the inelastic nucleon-nucleon cross section (whole family of variants)
  - Wounded quark model
- **Color Glass Condensate (CGC)**
  - **MC-KLN:** CGC-based model using kT-factorization
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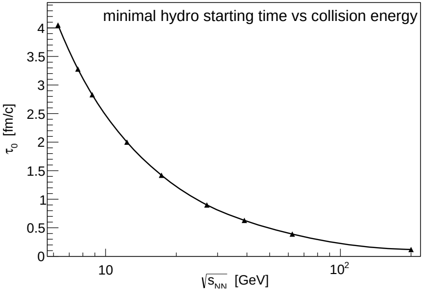
Initial stage in vHLLE+UrQMD

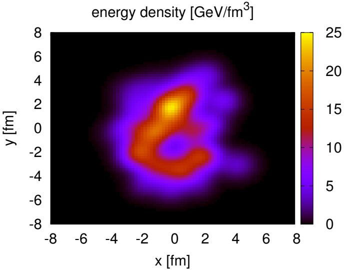
- pre-thermal evolution: UrQMD cascade
- scatterings allowed until $\sqrt{t^2 - z^2} = \tau_0$
- minimal starting time is $\tau_0 = \frac{2R}{\gamma v_z}$



"Thermalization"

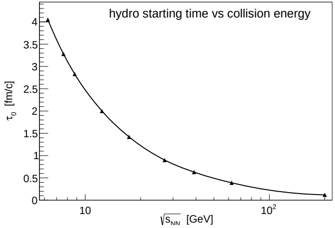
At $\tau = \tau_0$ the energy/momentum P^α, baryon and electric charges N^0 of every particle are deposited into fluid cells according to:

$$\Delta P^\alpha_{ijk} = P^\alpha \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2 \gamma_\eta^2 \tau_0^2/R_\eta^2\right)$$
$$\Delta N^0_{ijk} = N^0 \cdot C \cdot \exp\left(-(\Delta x_i^2 + \Delta y_j^2)/R_\perp^2 - \Delta\eta_k^2 \gamma_\eta^2 \tau_0^2/R_\eta^2\right)$$



Next challenge:

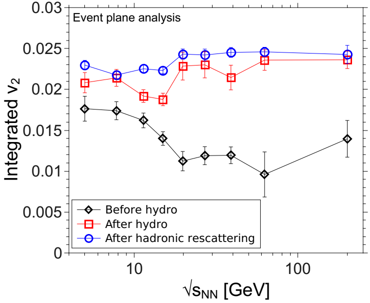
At [NICA], pre-hydro stage in a "sandwich" approach is too long:



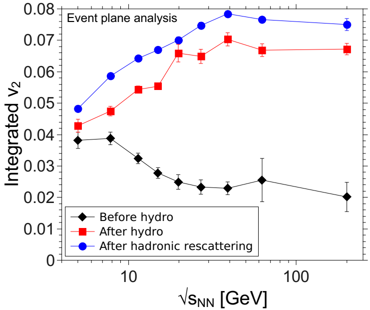
UrQMD 3.4 (UrQMD IC + ideal hydro + UrQMD afterburner)

J. Auvinen, H. Petersen, Phys.Rev.C 88:064908,2013

a) Charged hadrons, b = 0 - 3.4 fm



b) Charged hadrons, b = 8.2 - 9.4 fm



One must start hydro description early!

Multi-fluid dynamics

Hydrodynamic description starts from the very beginning of the collision.

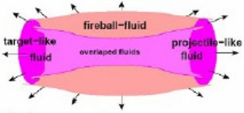
Difficulty: reasonability of fluid description at the very start of heavy ion collision?

Dynamical fluidization (1 fluid)

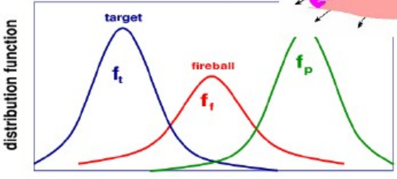
Regions of fluid phase are created dynamically, where (and when) the density is large enough.

Difficulty: how to treat non-fluid and fluid phase together (in the intial state)?

3-Fluid Dynamics



Produced particles populate mid-rapidity
$\Rightarrow$ fireball fluid



Target-like fluid: $\partial_\mu J_t^\mu = 0$ $\qquad \partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$

Leading particles carry bar. charge $\qquad$ exchange/emission

Projectile-like fluid: $\partial_\mu J_p^\mu = 0,$ $\qquad \partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$

Fireball fluid: $J_f^\mu = 0,$ $\qquad \partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$

Baryon-free fluid $\qquad$ Source term $\qquad$ Exchange

The source term is delayed due to a formation time $\tau \sim 1$ fm/c

Total energy-momentum conservation:

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

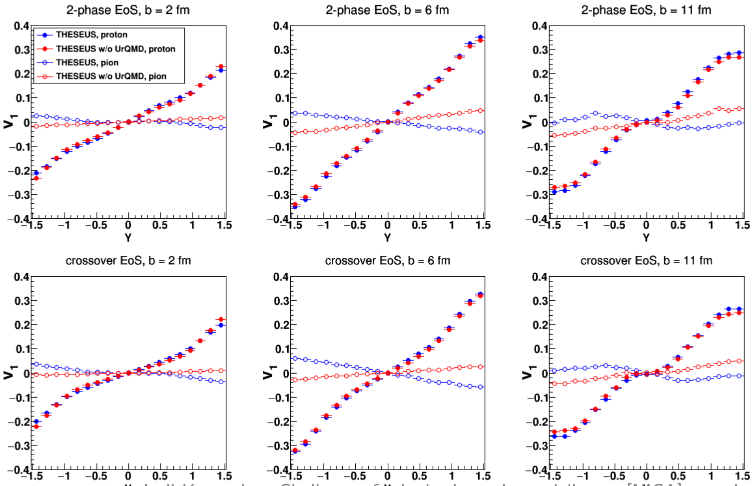
http://theory.gsi.de/~ivanov/mfd/

Yu.B. Ivanov, V.N. Russkikh and V.D. Toneev, Phys. Rev. C73, 044904 (2006)

[advertisement] THESEUS: 3-Fluid Dynamics + UrQMD

P. Batyuk, D. Blaschke, M. Bleicher, Yu. B. Ivanov, Iu. Karpenko, S. Merts, M. Nahrgang, H. Petersen, and O. Rogachevsky, Phys. Rev. C 94, 044917 (2016)

The scheme: 3-fluid hydro + particlization + hadronic cascade



Hydrodynamic stage

Hydrodynamic stage

Energy-momentum/charge conservation:

$$\partial_{;\nu} T^{\mu\nu} = 0, \quad \partial_{;\nu} N^{\nu} = 0$$

Evolution equations for shear/bulk, coming from Israel-Stewart formalism:

$$< u^{\gamma}\partial_{;\gamma}\pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\text{NS}}}{\tau_{\pi}} - \frac{4}{3}\pi^{\mu\nu}\partial_{;\gamma}u^{\gamma}$$

$$u^{\gamma}\partial_{;\gamma}\Pi = -\frac{\Pi - \Pi^{\mu\nu}_{\text{NS}}}{\tau_{\pi}} - \frac{4}{3}\Pi\partial_{;\gamma}u^{\gamma}$$

Couple public 3+1D hydro codes available, one of them is:

vHLLE code: free and open source. Comput. Phys. Commun. 185 (2014), 3016
https://github.com/yukarpenko/vhlle

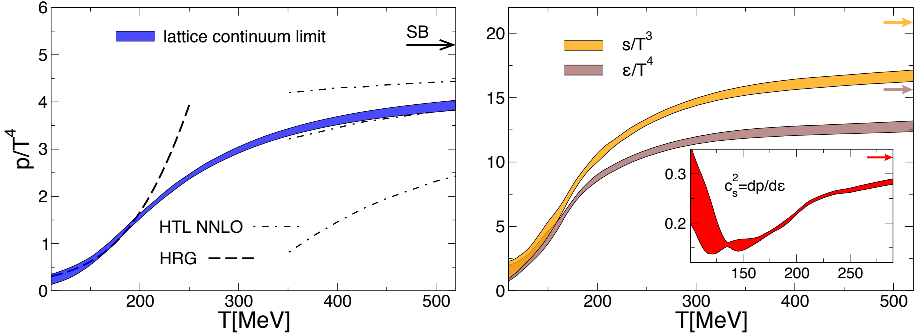
Inputs:

1. equation of state (EoS)
2. transport coefficients: τ_{π}, η/s

1) EoS at high energies: kingdom of Lattice QCD

Lattice QCD calculations are constantly improving, and as for now

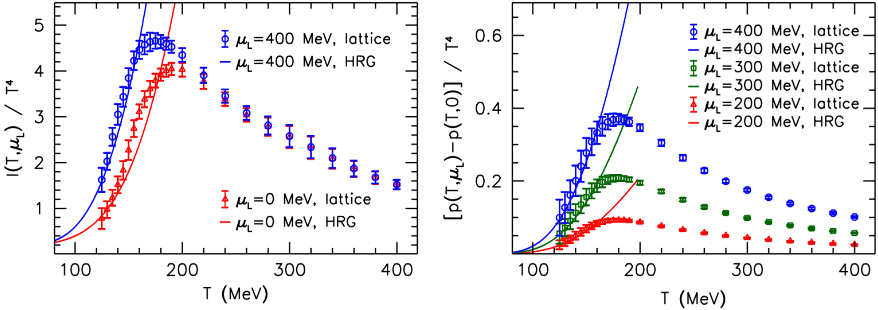
- there is a good agreement with hadron resonance gas at low temperatures



Borsányi et al, Phys. Lett. B 370 (2014) 99-104

Lattice QCD calculations are constantly improving, and as for now

- thermodynamic quantities can be evaluated at finite baryon chemical potential, up to $\mu_B/T = 2.5 \ldots 3$



Borsányi et al, JHEP 1208, 053 (2012) $\mu_L = 3 \cdot \mu_B$

This is not enough for hydro modeling at lower energies!

⇒ EoS models (used in vHLLE+UrQMD)

Chiral model

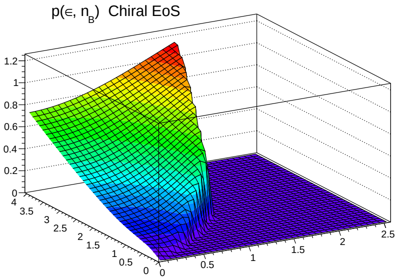
J. Steinheimer, et al, J. Phys. G 38, 035001 (2011)

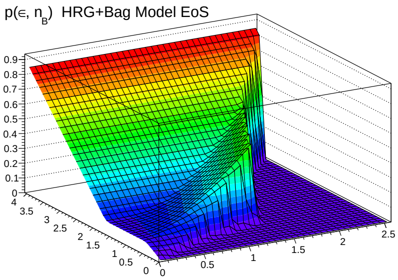
- good agreement with lattice QCD at $\mu_B = 0$
- crossover type PT between confined and deconfined phases at all μ_B

Hadron resonance gas + Bag Model

P.F. Kolb, et al, Phys.Rev. C 62, 054909 (2000)
(a.k.a. `EoS Q`)

- hadron resonance gas made of u, d quarks including repulsive meanfield
- Maxwell construction resulting in 1^{st} order PT





2) Transport coefficients:

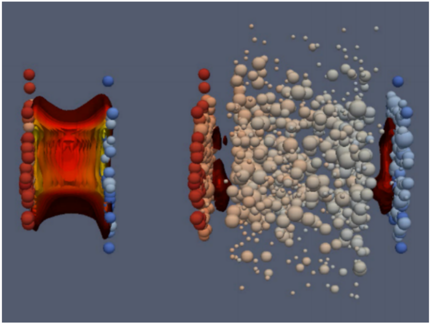
- η/s may grow with μ_B, and potentially hit the limit for applicability of I-S viscous hydro.

Evolution near T_C:

- Bulk viscosity ζ/s may be strongly enhanced near T_c (F. Karsch et al, 2008; Y. Aoki et al, 2008)
- which may be a manifestation of a more general phenomenon: timescale separation $\tau_{\rm micro} << \tau_{\rm macro}$ is broken.
  M. Stephanov, Y. Yin, arXiv:1712.10305 (Hydro+)

Post-hydro stage

Fluid→particle transition and hadronic phase



Credit: MADAI.us

- Cooper-Frye prescription at $\varepsilon = \varepsilon_{\text{sw}}$:

$$p^0 \frac{d^3 n_i}{d^3 p} = \sum f(x,p) p^\mu \Delta\sigma_\mu$$

$$f(x,p) = f_{\text{eq}} \cdot \left(1 + (1 \mp f_{\text{eq}}) \frac{p_\mu p_\nu \pi^{\mu\nu}}{2T^2(\varepsilon + p)}\right)$$

- $\Delta\sigma_i$ using Cornelius subroutine*
- Hadron gas phase: back to UrQMD cascade

*Huovinen and Petersen, *Eur.Phys.J.* A **48** (2012), 171

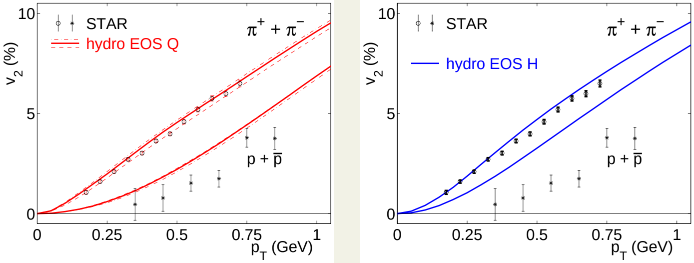
Modeling

And (hopefully) making physics conclusions from it

An example from (relatively) early days

Success of ideal hydrodynamics

Kolb, Heinz, Huovinen et al ('01) **minbias Au+Au at RHIC**



not perfect agreement but plasma EoS favored

ideal fluid? — so how ideal is plasma actually. . . ?

P. Huovinen @ QM Student lecture, May 18, 2014 30

As hydro models evolved, this became more complicated...

Although all models based on hydrodynamic approach are called 'hydro models', they have freedom in:
1) initial state
2) EoS
3) transport coefficients (η/s, ζ/s, τ_π, ...)
4) freezeout procedure.

• All the freedom (parameters) cannot be fixed by comparison to just one observable (i.e. elliptic flow)
• The parameter dependence is non-factorizable

Fitting experimental data with a model is not possible, because evaluating model output for single point in the parameter space requires many CPU hours.

A challenge?

Eyeball fitting with `vHLLE+UrQMD`

Free parameters of the model:

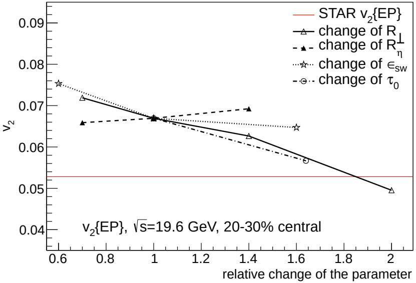
- hydro starting time (=duration of pre-hydro phase) τ_0
- transverse smearing of initial state $R_\perp$
- longitudinal smearing of the initial state R_z
- shear viscosity η/s
- fluid-to-particle transition (particlization) $\varepsilon = 0.5$ GeV/fm^3

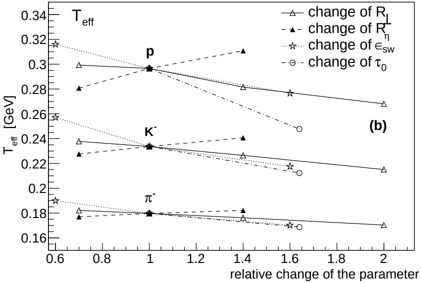
Learning parameter dependence

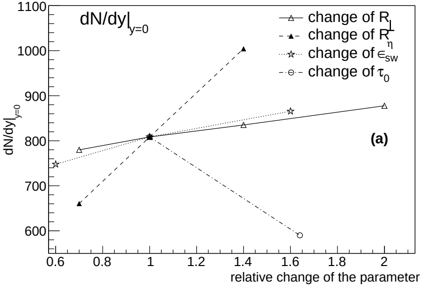
Response of the observables:

- T_{eff}, inverse slope of p_T spectrum
- dN/dy at midrapidity
- p_T integrated $v_2\{\mathrm{EP}\}$

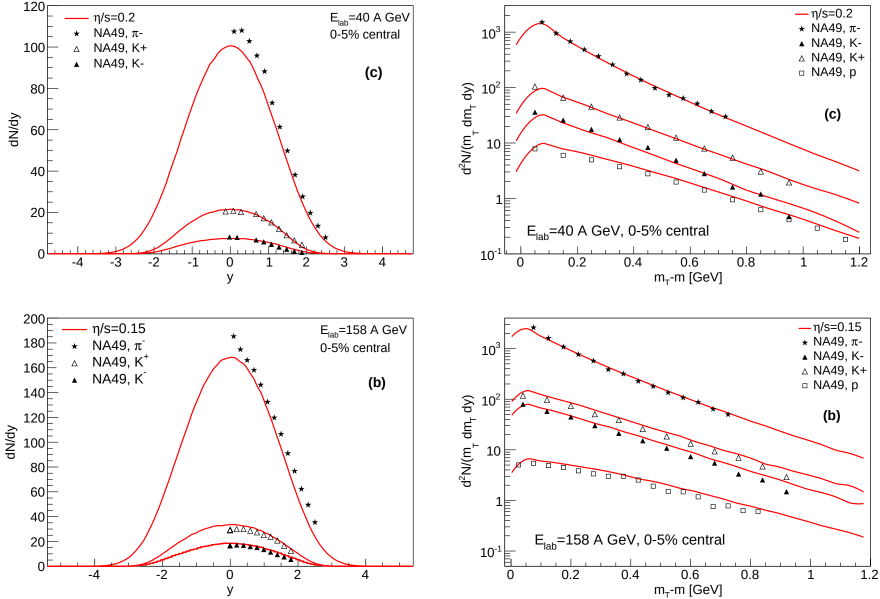
to the change of every parameter with respect to its default value.



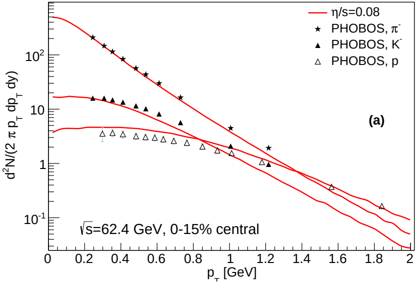


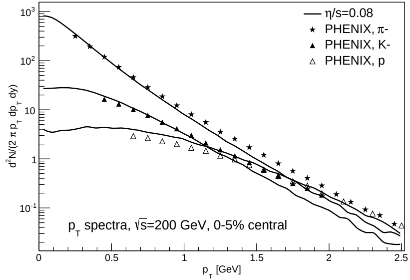


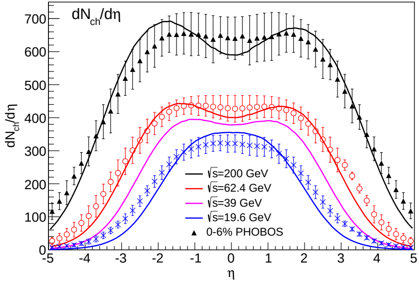
40 + 158 A GeV PbPb SPS ($\sqrt{s} = 8.8$ and 17.3 GeV)



RHIC BES + top RHIC



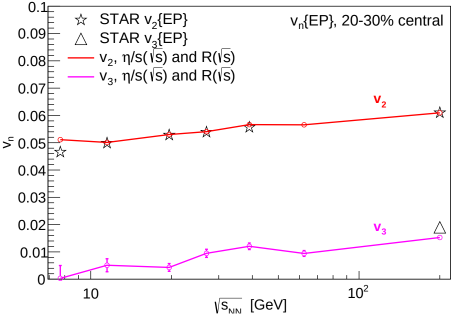




The rapidity/pseudorapidity and p_T distributions from SPS/NA49 together with RHIC are reasonably reproduced.

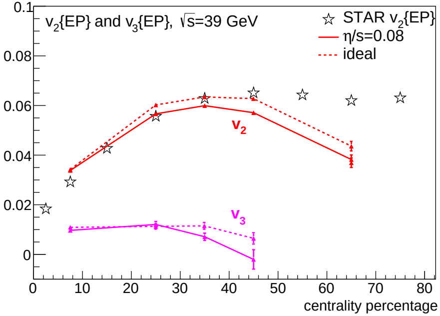
Elliptic and triangular flows at RHIC BES + top RHIC

v_2, v_3 vs collision energy



v_3: prediction!

v_2, v_3 vs centrality



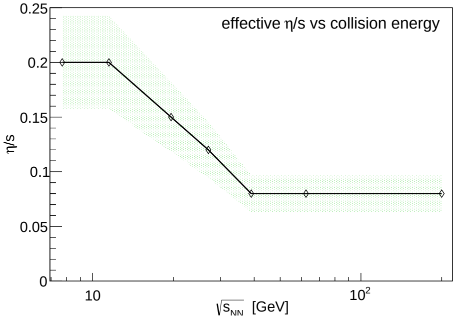
Peripheral events: too strong smearing for smaller system

Parameter values used to approach the data

EoS: Chiral model, $\varepsilon_{\rm sw} = 0.5$ GeV/fm^3.

$\sqrt{s}$ [GeV]	τ_0 [fm/c]	$R_\perp$ [fm]	R_z [fm]	η/s
7.7	3.2	1.4	0.5	0.2
8.8	2.83	1.4	0.5	0.2
11.5	2.1	1.4	0.5	0.2
17.3	1.42	1.4	0.5	0.15
19.6	1.22	1.4	0.5	0.15
27	1.0	1.2	0.5	0.12
39	0.9*	1.0	0.7	0.08
62.4	0.7*	1.0	0.7	0.08
200	0.4*	1.0	1.0	0.08

*here we increase τ_0 as compared to $\tau_0 = \frac{2R}{\gamma v_z}$.



Green band:
same v_2 and $\pm 5\%$ change in $T_{\rm eff}$.

* The green band is not an actual error bar.

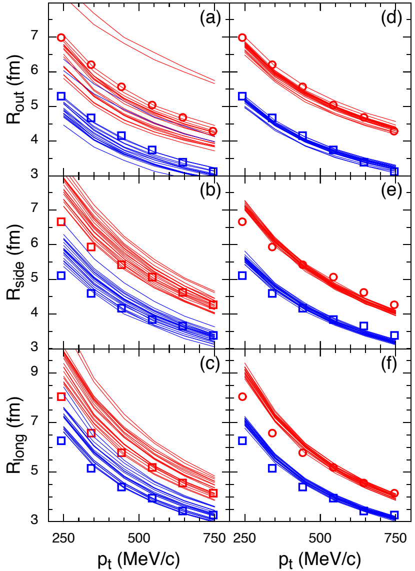
Can we do better?

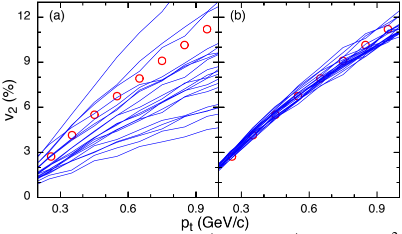
Emulator + MCMC technique:

(Hydrodynamic) model	- slow, but ‘exact’
⇓	
Gaussian process emulator	- a fast but inexact proxy for the model
⇓	
Markov Chain Monte Carlo (MCMC)	- walking in the parameter space

One example: constraining the EoS of QGP matter

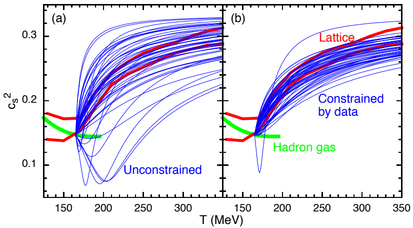
S. Pratt, E. Sangaline, P. Sorensen, H. Wang, Phys. Rev. Lett. 114, 202301 (2015)



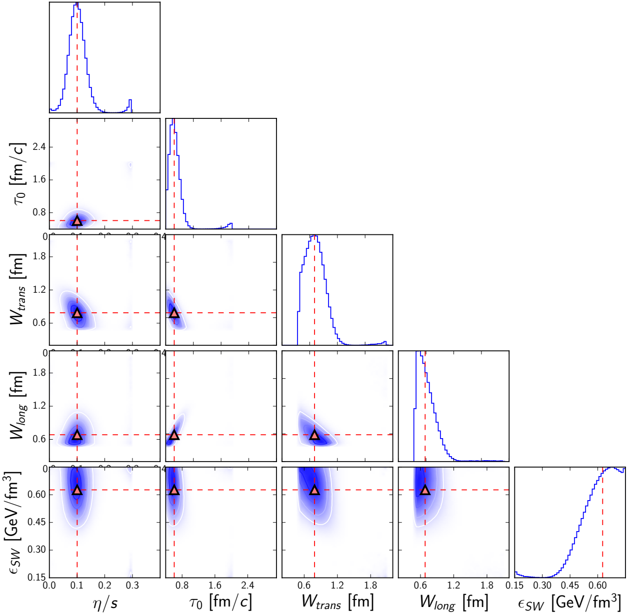


$$c_s^2(\varepsilon) = c_s^2(\varepsilon_h) + \left(\frac{1}{3} - c_s^2(\varepsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2}, \quad (2)$$

$$X_0 = X' R c_s(\varepsilon)\sqrt{12}, \quad x \equiv \ln \varepsilon / \varepsilon_h,$$



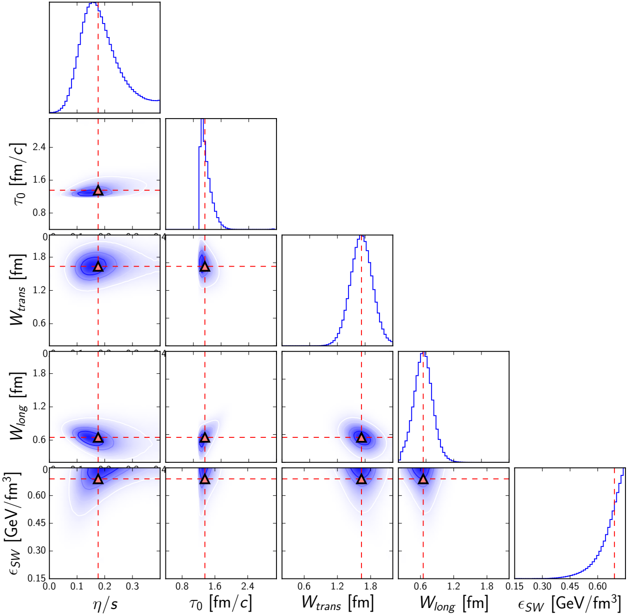
What about vHLLE+UrQMD? J. Auvinen, analysis at $\sqrt{s_{NN}} = 62.4$ GeV



This hybrid model
+
Gaussian processes
(emulator)
+
Markov chain Monte Carlo

Auvinen, Bernhard,
Bass, IK
Phys. Rev. C 97, 044905
(2018)

What about vHLLE+UrQMD? J. Auvinen, analysis at $\sqrt{s_{NN}} = 19.6$ GeV

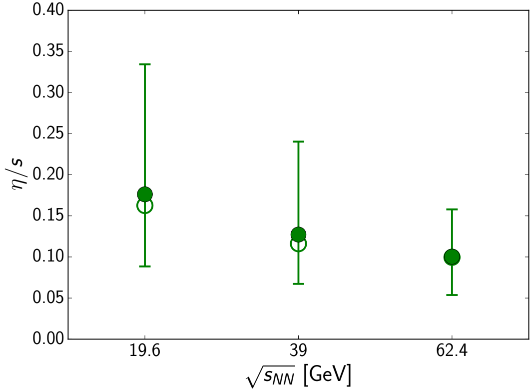


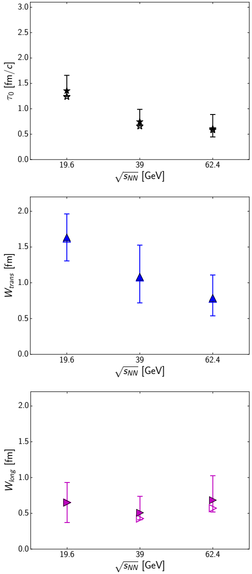
This hybrid model
+
Gaussian processes
(emulator)
+
Markov chain Monte Carlo

Auvinen, Bernhard, Bass, IK
Phys. Rev. C 97, 044905 (2018)

J. Auvinen, Bayesian analysis with vHLLE+UrQMD

Auvinen, Bernhard, Bass, IK, Phys. Rev. C 97, 044905 (2018)



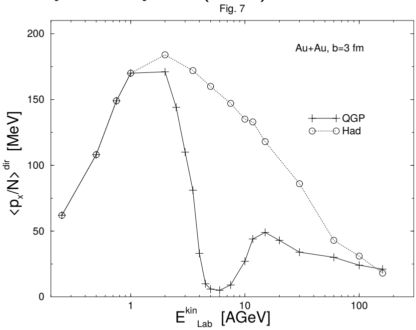


Error bars represent 90% confidence range around the median value.

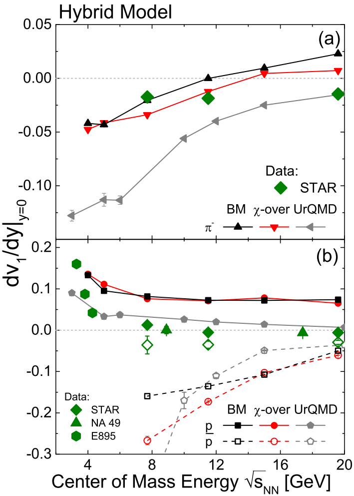
Few more thoughts

Finding signals of mixed phase is not as easy as it seemed

D. Rischke et al,
Heavy Ion Phys. 1 (1995) 309



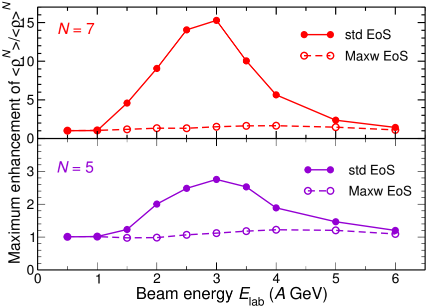
J. Steinheimer et al,
Phys. Rev. C 89 (2014) 054913



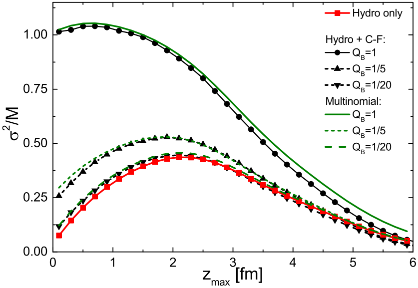
- non-monotonic directed flow of net protons predicted as a signal of 1st order PT
- in up-to-date hybrid model the signal vanishes

There are challenges to properly include some effects

Steinheimer, Randrup, PRL 109 (2012), 212301



Steinheimer, Koch, arXiv:1705.08538



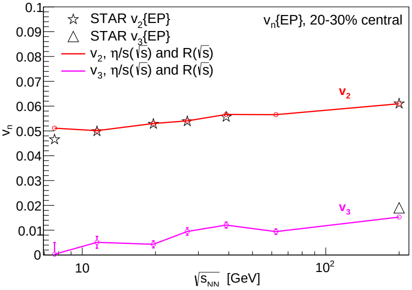
Basic experimental observables from BES are important

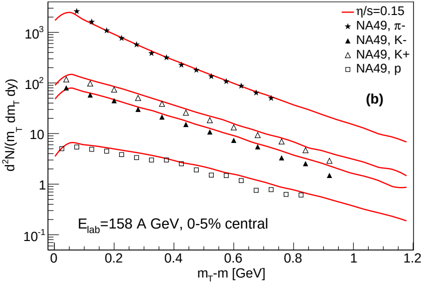
They are as important as 'exciting' ones!

To extract the η/s we used:

- v_2: new STAR results
  Phys. Rev. C 86 (2012) 54908
  Au-Au system

- p_T distributions: old NA49 data
  (in limited p_T range)
  Pb-Pb system





...2017: p_T spectra for BES energies from STAR, arXiv:1701.07065

List of challenges as conclusions

- Initial state: rapidity dependence
- Initial state: baryon, electric, strange densities
- Complex geometry of initial state
- EoS: baryon density dependence
- Break-up of hydro near T_C
- Shorter duration of hydro, so more dependence on the initial state
- Non-factorizable dependence on the model parameters

The end (so far).

Thank you for your attention!