Anomalous axial current from covariant Wigner function and thermodynamic equilibrium density operator

G.U. Prokhorov [1]

O.V. Teryaev [1]

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[1]Joint Institute for Nuclear Research, BLTP, Dubna

Literature

- F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].
- M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].
- Prokhorov, G. and Teryaev O. arXiv:1707.02491 [hep-th], accepted in PhysRevD.

Thermal vorticity tensor

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

$$\beta_\mu = \frac{u_\mu}{T}$$

4-vector of inverse temperature

Global thermodynamic equilibrium

$$b_\mu = \text{const} \qquad \varpi_{\mu\nu} = \text{const}$$

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x_\nu$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

Acceleration

$$\alpha_\mu = \varpi_{\mu\nu} u^\nu$$

$$\alpha_\mu = \frac{a_\mu}{T}$$

Vorticity

$$w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} u^\nu \varpi^{\alpha\beta}$$

$$w_\mu = \frac{\omega_\mu}{T}$$

Temperature gradient

Ansatz of the Wigner function, taking into account the thermal vorticity tensor

Generators of Lorentz transformations of spinors

$$\Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$$

Ansatz of the Wigner function

$$X(x,p) = \left(\exp[\beta \cdot p - \xi(x)] \exp\left[-\frac{1}{2}\varpi(x) : \Sigma \right] + I \right)^{-1}$$

- Limit of zero thermal vorticity tensor

$$f(x,p) = \frac{1}{e^{\beta(x)\cdot p - \xi(x)} + 1}$$

- Nonrelativistic limit

F. Becattini and L. Tinti, Annals Phys. 325 (2010) 1566 doi:10.1016/j.aop.2010.03.007 [arXiv:0911.0864 [gr-qc]].

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

Axial current

The axial current is expressed in terms of an integral with the Wigner function

$$\langle : j_\mu^5 : \rangle = -\frac{1}{16\pi^3}\epsilon_{\mu\alpha\nu\beta}\int \frac{d^3p}{\varepsilon} p^\alpha \Big\{ \mathrm{tr}\big(X\Sigma^{\nu\beta}\big) - \mathrm{tr}\big(\bar{X}\Sigma^{\nu\beta}\big) \Big\}$$

Can be calculated analytically outside perturbation theory

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

The existence of CVE is shown

$$j_5^\mu(x) = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega^\mu$$

It is convenient to introduce scalar combinations constructed from the thermal vorticity tensor

$$g_1 = \frac{1}{4}\left(\sqrt{(\varpi : \varpi)^2 + (\varpi : \widetilde{\varpi})^2} + \varpi : \varpi\right)^{1/2}$$

$$g_2 = \frac{1}{4}\left(\sqrt{(\varpi : \varpi)^2 + (\varpi : \widetilde{\varpi})^2} - \varpi : \varpi\right)^{1/2}$$

Prokhorov, G. and Teryaev O. arXiv:1707.02491 [hep-th], accepted in PhysRevD.

Trace calculation

Particles	$\xi \to -\xi$	Antiparticles
$\mathrm{tr}\big(X\Sigma^{\nu\beta}\big)$	⇐ Common sign	$\mathrm{tr}\big(\bar{X}\Sigma^{\nu\beta}\big)$

- **Expand in Taylor's series**

$$X = \sum_{n=0}^{\infty}(-1)^n \exp\left[t(n+l)(\beta\cdot p - \xi - \frac{1}{2}\varpi : \Sigma)\right] =$$

$$\sum_{n=0}^{\infty}(-1)^n \exp\left[t(n+l)(\beta\cdot p - \xi)\right]\sum_{m=0}^{\infty}\frac{1}{m!}\Big(t(n+l)(-\frac{1}{2}\varpi : \Sigma)\Big)^m$$

- Use the properties of gamma matrices

$$(\varpi : \Sigma)^{2k} = \eta^k \frac{1+\gamma^5}{2} + \theta^k \frac{1-\gamma^5}{2}\,, \quad k = 0, 1, 2\dots.$$

- One can find a trace in each term of the series

$$\mathrm{tr}\Big((\varpi : \Sigma)^{2k+1}\Sigma^{\nu\beta}\Big) = (\varpi^{\nu\beta} + i\tilde{\varpi}^{\nu\beta})\eta^k + (\varpi^{\nu\beta} - i\tilde{\varpi}^{\nu\beta})\theta^k$$

$$\mathrm{tr}\Big((\varpi : \Sigma)^{2k}\Sigma^{\nu\beta}\Big) = 0\,, \quad k = 0, 1, 2\dots.$$

⇒

- Sum up the series back

Prokhorov, G. and Teryaev O. arXiv:1707.02491 [hep-th], accepted in PhysRevD.

Axial current, mass is not zero

$$\langle :j^5_\mu: \rangle = C_1 w_\mu + \mathrm{sgn}(\varpi : \widetilde{\varpi})\, C_2 \alpha_\mu \,,$$

$$C_1 = \frac{1}{4\pi^2} \frac{g_2 \cosh(g_1)\sin(g_2) + g_1 \sinh(g_1)\cos(g_2)}{g_1^2 + g_2^2}\big(I_1(\xi) + I_1(-\xi)\big) +$$

$$\frac{1}{8\pi^2} \frac{g_1 \sinh(2g_1) + g_2 \sin(2g_2)}{g_1^2 + g_2^2}\big(I_2(\xi) + I_2(-\xi)\big)\,,$$

$$C_2 = \frac{1}{4\pi^2} \frac{g_2 \sinh(g_1)\cos(g_2) - g_1 \cosh(g_1)\sin(g_2)}{g_1^2 + g_2^2}\big(I_1(\xi) + I_1(-\xi)\big) +$$

$$\frac{1}{8\pi^2} \frac{g_2 \sinh(2g_1) - g_1 \sin(2g_2)}{g_1^2 + g_2^2}\big(I_2(\xi) + I_2(-\xi)\big)\,,$$

$$I_1(\xi) = \int \frac{dp\, \mathbf{p}^{\,2} \cosh(\frac{\varepsilon}{T} - \xi)}{\Big(\cosh(\frac{\varepsilon}{T} - \xi - g_1) + \cos(g_2)\Big)\Big(\cosh(\frac{\varepsilon}{T} - \xi + g_1) + \cos(g_2)\Big)}\,,$$

$$\frac{dp\, \mathbf{p}^{\,2}}{\mathrm{h}(\frac{\varepsilon}{T} - \xi - g_1) + \cos(g_2)\Big)\Big(\cosh(\frac{\varepsilon}{T} - \xi + g_1) + \cos(g_2)\Big)}\,.\; I_2(\xi) = \int \frac{}{\Big(\cos}$$

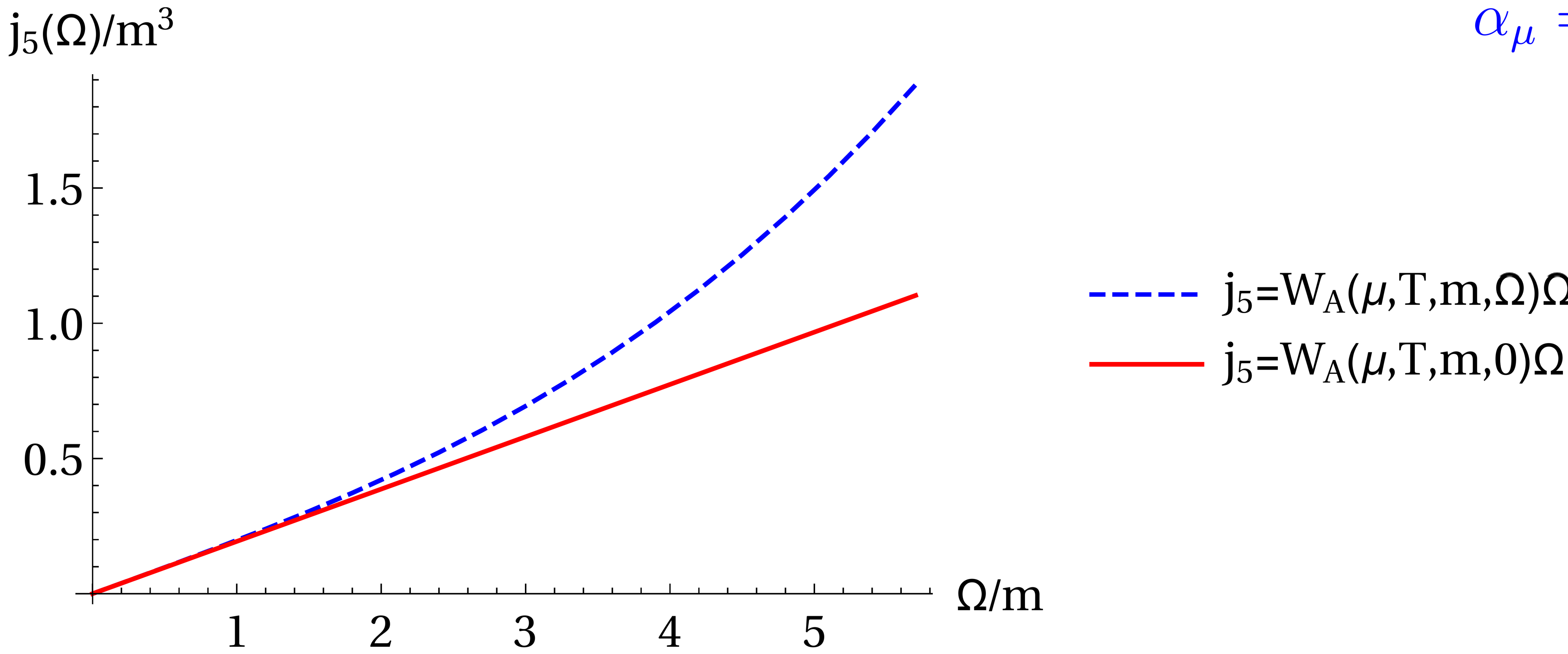
2 terms: along 4-acceleration (1-st order) and vorticity (3-rd order of magnitude by $\varpi_{\mu\nu}$)

The convergent 1-dimensional integral from an elementary function

Prokhorov, G. and Teryaev O. arXiv:1707.02491 [hep-th], accepted in PhysRevD.

Axial current, mass is not zero

$\alpha_\mu = 0$

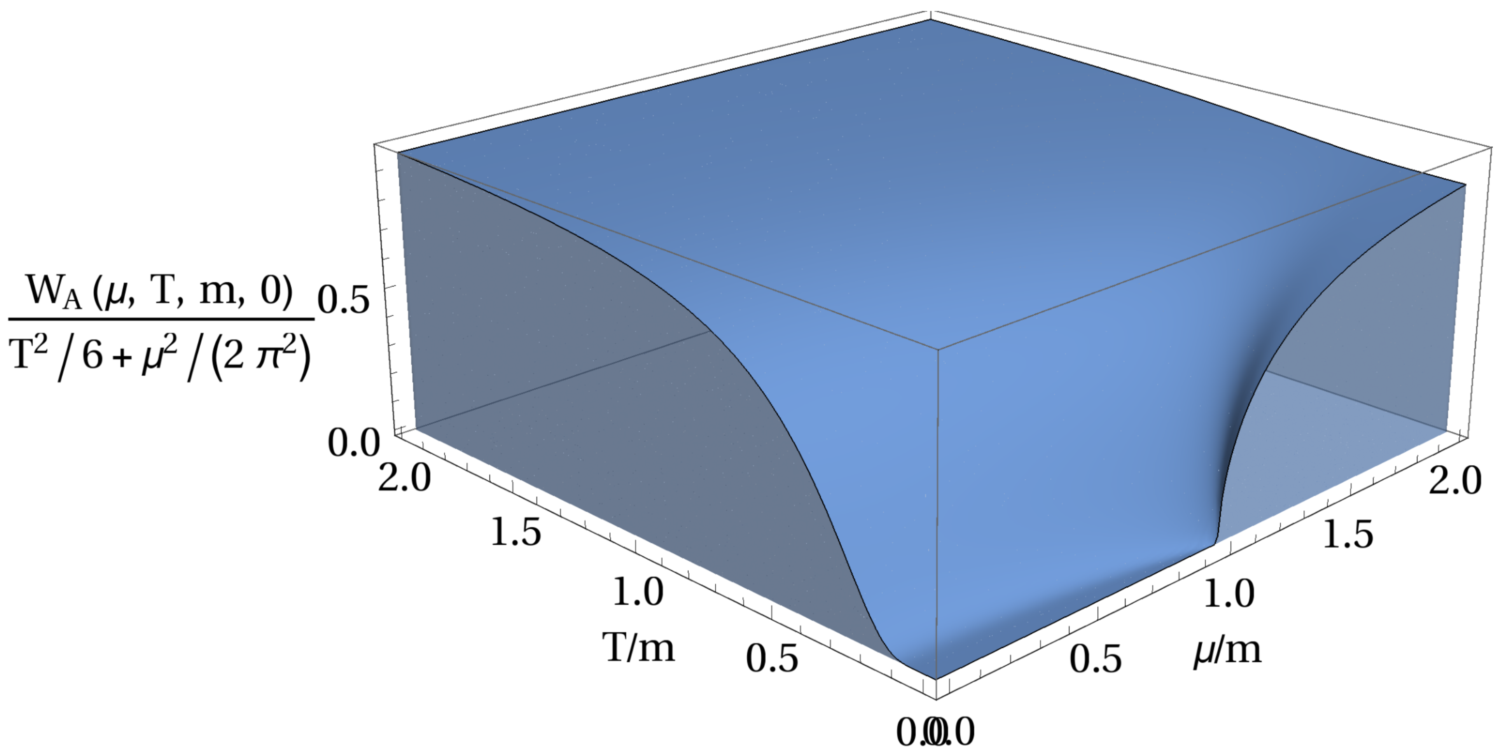


Typical behavior of the axial current as a function of temperature for arbitrary parameter values in comparison with conventional CVE

- The axial current increases due to the growth of W^A with an increase of the rotational speed

Axial current, mass is not zero

$\alpha_\mu = 0$



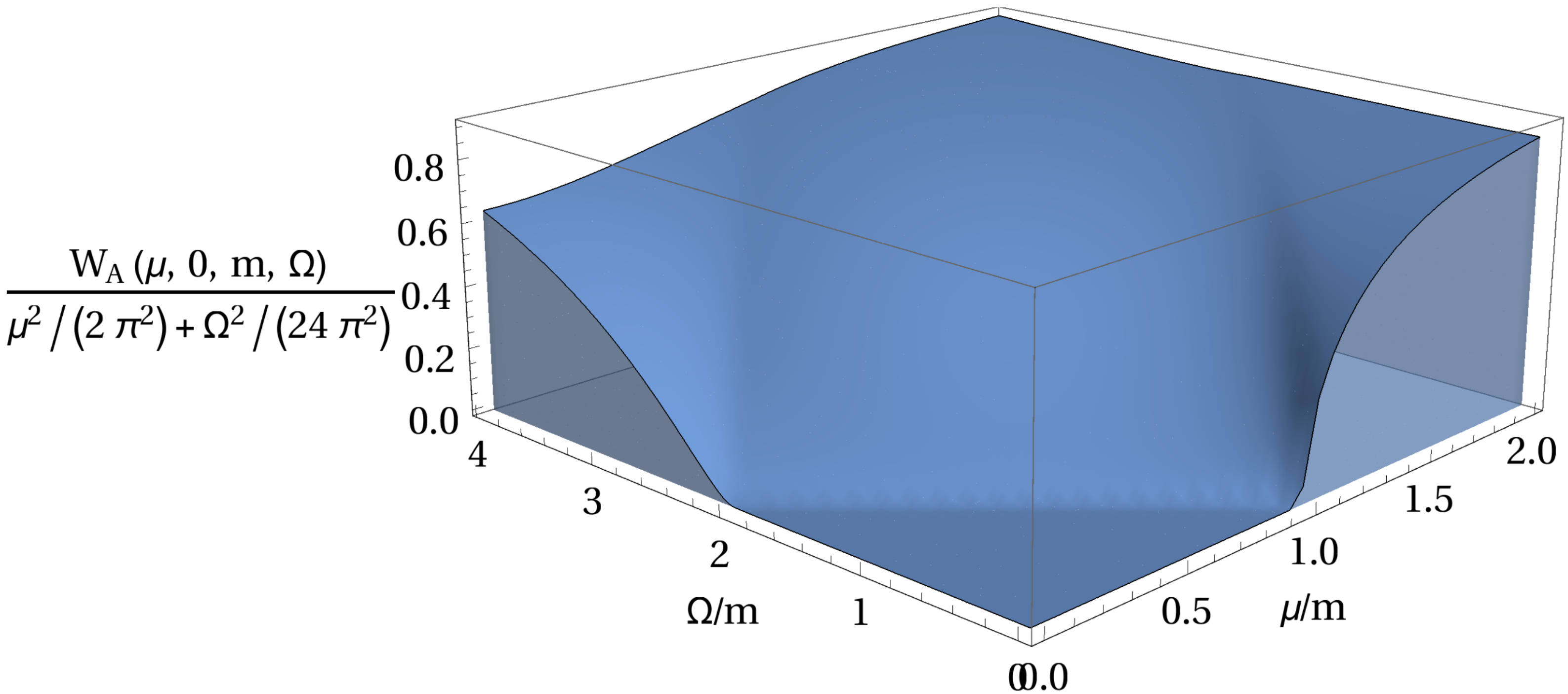
Coefficient in axial current as a function of temperature and chemical potential, normalized to normal CVE at zero speed of rotation

W^A coincides with the result

M. Buzzegoli, E. Grossi and F. Becattini,
JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

- ***The step at a chemical potential equal to the mass***
- ***The step smoothes with increasing temperature***
- ***Asymptotically tends to the normal CVE at zero mass at a temperature and chemical potential much larger than the mass***

Axial current, mass is not zero



$\alpha_\mu = 0$

The coefficient in the axial current, as a function of the rotation speed and chemical potential, normalized to the formula at zero temperature and mass

- ***Plateau, where the axial current is zero, bounded by the straight line*** $\Omega = 2(m-\mu)$
- ***Asymptotically tends to the result at zero mass and at a temperature and speed of rotation much larger than the mass***

The limit of massless fermions

The momentum integrals are analytically expressed in terms of polylogarithms

$$\langle : j^5_\mu : \rangle = -\frac{T^3}{4\pi^2(g_1^2+g_2^2)}\Big((g_1+ig_2)\mathrm{Li}_3(-e^{g_1-ig_2-\xi}) - (g_1-ig_2)\mathrm{Li}_3(-e^{-g_1-ig_2-\xi}) + \ldots$$

A remarkable property of polylogarithms

$$\mathrm{Li}_3(-e^{-x}) - \mathrm{Li}_3(-e^{x}) = \frac{\pi^2}{6}x + \frac{1}{6}x^3$$

$$\langle : j^5_\mu : \rangle = \Big(\frac{1}{6}\big[T^2 + \frac{a^2-\omega^2}{4\pi^2}\big] + \frac{\mu^2}{2\pi^2}\Big)\omega_\mu + \frac{1}{12\pi^2}(\omega\cdot a)\,a_\mu$$

- **A simple analytic formula for an axial current, derived outside the perturbation theory.**
- Members of higher orders are reduced in terms of chemical potential, temperature, and thermal vorticity.
- Maximal order of acceleration and rotation speed: 3.

Prokhorov, G. and Teryaev O. arXiv:1707.02491 [hep-th], accepted in PhysRevD.

The limit of massless fermions comparison with other approaches

A normal CVE, a term along the vorticity of the first order

The third-order term in vorticity corresponds to

The term of order 3 along the 4-acceleration, leads to a nonzero divergence

$$\langle : j_\mu^5 : \rangle = \langle : j_\mu^5 : \rangle_{\mathrm{Tvort}} + \langle : j_\mu^5 : \rangle_{\mathrm{vort}} + \langle : j_\mu^5 : \rangle_{\mathrm{acc}} \, ,$$

$$\langle : j_\mu^5 : \rangle_{\mathrm{Tvort}} = \Big(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \Big) \omega_\mu \, , \quad \langle : j_\mu^5 : \rangle_{\mathrm{vort}} = \frac{a^2 - \omega^2}{24\pi^2} \omega_\mu \, ,$$

$$\langle : j_\mu^5 : \rangle_{\mathrm{acc}} = \frac{1}{12\pi^2} (\omega \cdot a) \, a_\mu \, .$$

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

A. Vilenkin, Phys. Rev. D 20 (1979) 1807. doi:10.1103/PhysRevD.20.1807

A. Vilenkin, Phys. Rev. D 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260

Non-conservation of axial charge if $(\omega \cdot a) \neq 0$

$$\partial^\mu \langle : j_\mu^5 : \rangle = \partial^\mu \langle : j_\mu^5 : \rangle_{\mathrm{acc}} = \frac{1}{6\pi^2} (\omega \cdot a)(a^2 + \omega^2)$$

Prokhorov, G. and Teryaev O. arXiv:1707.02491 [hep-th], accepted in PhysRevD.

The covariant Wigner function conclusion

- **The result obtained is in full accordance with all known theoretical calculations of the CVE** ***(the same conclusion in M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].).***
- **The simplicity of the formula obtained.**
- **The cubic term for the vorticity exactly coincides with the other two conclusions for the axial current made by A. Vilenkin.**
- **Symmetry between vorticity and acceleration.**
- **Non-conservation of the axial charge in the case when the acceleration and the rotation speed are not equal to zero and not perpendicular.**
- **An ansatz for the Wigner function is used.**
- **An approximation is used for the Wigner function.**

AN ADDITIONAL INDEPENDENT APPROACH FOR THE INSPECTION OF THE OBTAINED RESULT SHOULD BE USED

The second approach: the equilibrium density operator

- Thermodynamics can be derived from field theory, constructed in terms of the path integral in imaginary time
- The central role is played by the density operator
- **General covariant form of the density operator for a medium in local thermodynamic equilibrium**

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}(x)\beta_\nu(x) - \zeta(x)\widehat{j}^\mu(x) \right) \right] \Rightarrow$$

Maximum $S = -\mathrm{tr}(\widehat{\rho} \log \widehat{\rho})$

D.N. Zubarev, A.V. Prozorkevich and S.A. Smolyanskii, Derivation of nonlinear generalized equations of quantum relativistic hydrodynamics, Theor. Math. Phys. 40 (1979) 821.

⇓

Provided that $\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$ ⇒

$$\beta_\mu(x) = b_\mu + \varpi_{\mu\nu} x^\nu \qquad \zeta = \mathrm{const.}$$

$$b_\mu = \mathrm{const} \qquad \varpi_{\mu\nu} = \mathrm{const}$$

⇓

$\widehat{\rho}$ does not depend on the choice of a hypersurface $\mathrm{d}\Sigma_\mu$

⇓

global thermodynamic equilibrium

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

The density operator for a medium with a thermal vorticity tensor

Global equilibrium conditions

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

The form of the density operator for a medium with rotation and acceleration

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

Thermal vorticity tensor

4-momentum operator

Charge operator

Generators of Lorentz transformations

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right)$$

- *F. Becattini and E. Grossi, Phys. Rev. D 92 (2015) 045037 [arXiv:1505.07760] [INSPIRE].*
- *F. Becattini, arXiv:1712.08031 [gr-qc], to appear in Phys. Rev. D.*

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

Mean value of the physical quantity operator

Mean in terms of the path integral

$$\langle \widehat{O}(x)\rangle = \mathrm{tr}(\hat{\rho}\,\hat{O}(x))_{\mathrm{ren}}$$

Statistical sum: reduction of disconnected correlators

Perturbation theory in the <u>third order</u>

$$\langle \widehat{O}(x)\rangle = \langle \widehat{O}(x)\rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2|\beta|}\int_0^{|\beta|} d\tau \langle T_\tau J^{\mu\nu}_{-i\tau u}\widehat{O}(0)\rangle_{\beta(x),c}$$

$$+\frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}}{8|\beta|^2}\int_0^{|\beta|} d\tau_x d\tau_y \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u}\widehat{O}(0)\rangle_{\beta(x),c}$$

$$+\frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}\varpi_{\alpha\beta}}{48|\beta|^3}\int_0^{|\beta|} d\tau_x d\tau_y d\tau_z \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} J^{\alpha\beta}_{-i\tau_z u}\widehat{O}(0)\rangle_{\beta(x),c} + \dots$$

Ordering by imaginary time

Connected correlators

$$\langle \widehat{J}\widehat{O}\rangle_c = \langle \widehat{J}\widehat{O}\rangle - \langle \widehat{J}\rangle\langle \widehat{O}\rangle$$

- Reduction of the members by vorticity, starting with order 4 in the approach with the Wigner function and in

A. Vilenkin, Phys. Rev. D 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

Axial current in the third order of perturbation theory

Axial current in the third order of perturbation theory

$$\langle \widehat{j}_5^\lambda(x) \rangle_3 = A_1 w^2 w^\lambda + A_2 \alpha^2 w^\lambda + \sout{A_3 (w\alpha)} \alpha^\lambda$$

The results of calculating the coefficients

$$A_1 = -\frac{1}{24|\beta|^3\pi^2}$$

$$A_2 = -\frac{1}{8|\beta|^3\pi^2}$$

$$A_3 = 0$$

⇒ **The axial charge is conserved**

Comparison with the result obtained using the Wigner function

Calculation based on the density operator in the framework of a QFT at a finite temperature

$$j_5^\lambda(x) = (\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2})\omega^\lambda$$

Calculation based on the Wigner function

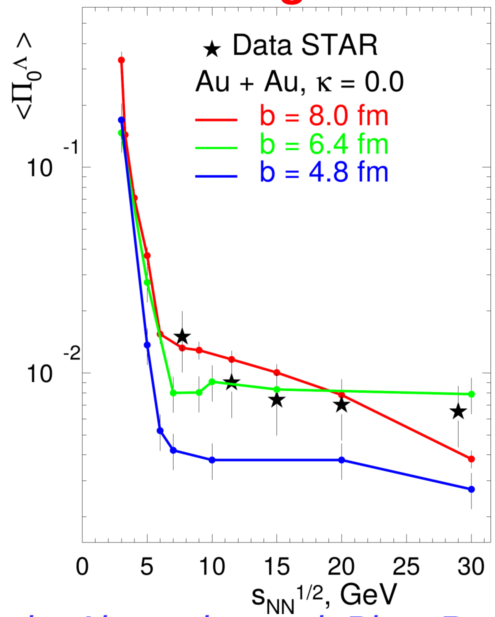
$$\langle : j_\mu^5 : \rangle = \left(\frac{1}{6}\left[T^2 + \frac{a^2 - \omega^2}{4\pi^2}\right] + \frac{\mu^2}{2\pi^2}\right)\omega_\mu + \frac{1}{12\pi^2}(\omega \cdot a)\, a_\mu$$

- Cubic terms coincide.
- Coincide linear terms corresponding to CVE.
- The second current has a term along the acceleration vector, which violates the conservation of the axial charge.
- The coefficients that stand before $a^2\omega^\lambda$ are different.

2 approaches to the calculation of the polarization of hyperons

1-st approach

The fall of polarization of hyperons with increasing collision energy.



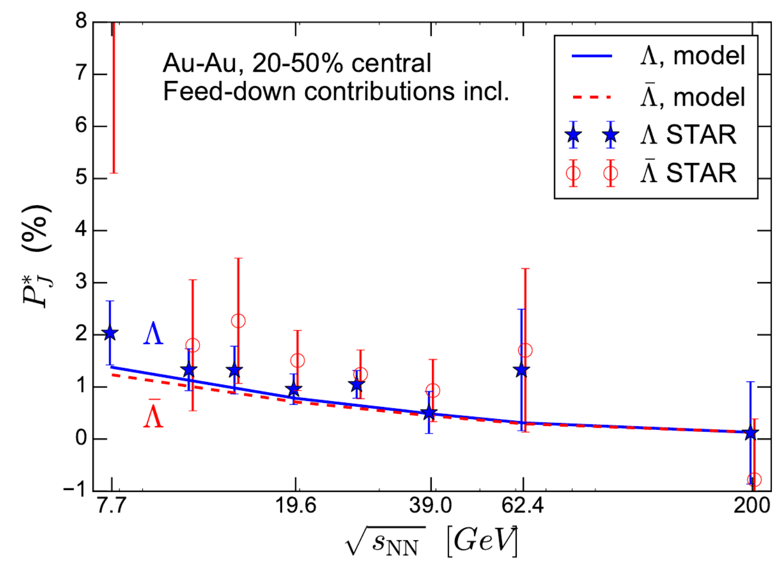
- *Sorin, Alexander et al. Phys.Rev. C95 (2017) no.1, 011902 arXiv:1606.08398 [nucl-th]*
- *M. Baznat, K. Gudima, A. Sorin and O. Teryaev, EPJ Web Conf. 138 (2017) 01008.doi:10.1051/ epjconf/201713801008; arXiv:1701.00923*

2-nd approach

- *Karpenko, Iu. et al. Nucl.Phys. A967 (2017) 764-767 arXiv:1704.02142 [nucl-th]*

Based on the Wigner function

$$S^\mu(x,p) = -\frac{1}{8m}(1-f(x,p))\epsilon^{\mu\nu\rho\sigma}p_\sigma\varpi_{\nu\rho}$$



The polarization is determined by the axial charge of strange quarks

according to CVE

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6},$$

$$Q_5^s = N_c \int d^3x\, C(r)\, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

The fall of polarization is caused by the decrease of the strange chemical potential

2 approaches to the calculation of the polarization of hyperons

- **It was proved that the Wigner function used in the second approach leads to CVE.**
- **In this case, the CVE underlies the first approach.**
- **Thus, CVE is essential for both approaches, which can explain the same polarization behavior in them.**

There is a connection between the two approaches to the calculation of the polarization of hyperons.

Main results

- **CVE calculation based on the covariant Wigner function proposed by F. Becattini, V. Chandra, L. Del Zanna, E. Grossi** (a similar conclusion is given previously in *M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].*).
- **On the basis of the Wigner function, a formula is derived for the axial current in a rotating and accelerated medium outside the limits of perturbation theory.**
- **The correspondence of two approaches to the calculation of the polarization of Λ-hyperons is shown: on the basis of calculating the axial charge of strange quarks and on the basis of the Wigner function.**
- **Using the quantum density operator F. Becattini, the axial current is calculated in the third order of perturbation theory.**

Thank you for attention!

Chiral phenomena in relativistic hydrodynamics

Investigation of Chiral Magnetic Effect (CME) and Chiral Vortical Effect (CVE)

- *A. Vilenkin, Phys. Rev. D 21 (1980) 2260. doi:10.1103/PhysRevD.21.2260*
- *A. Vilenkin, Phys. Rev. D 22 (1980) 3080. doi:10.1103/PhysRevD.22.3080*
- *D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601 doi:10.1103/PhysRevLett.103.191601 [arXiv:0906.5044 [hep-th]].*
- *A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025 doi:10.1103/PhysRevD.83.105025 [arXiv:1012.1958 [hep-th]].*
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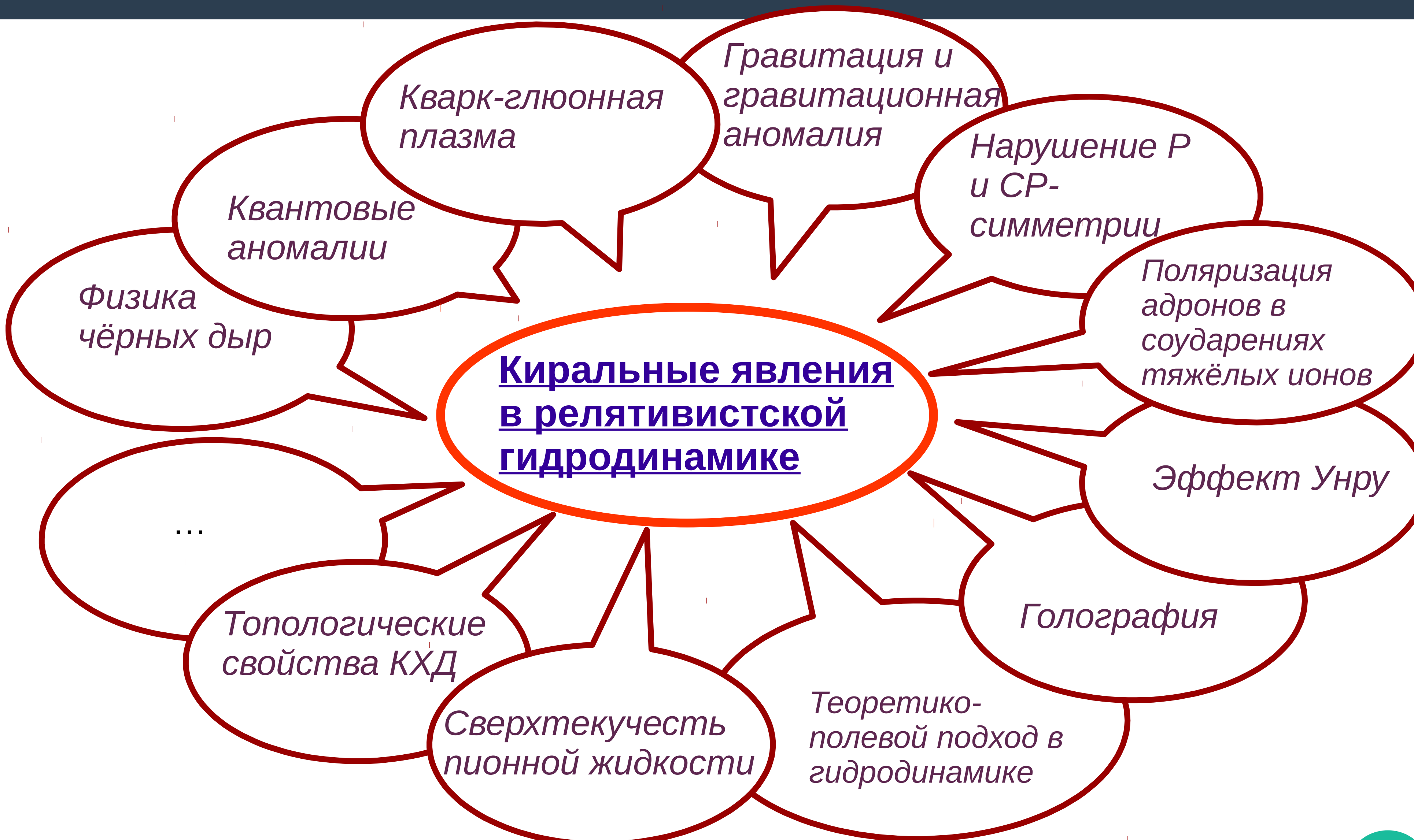
- ***Prokhorov, G. et al. arXiv:1707.02491 [hep-th], accepted in PhysRevD.***

Some applications of the theory of chiral liquids

- **Chiral phenomena detection = check of the physics that lies behind them.**
- **Hadron polarisation in heavy ion collisions.**
- **Chiral batteries.**

Области физики, проявляющиеся в киральных явлениях



Достижение Сона и Суровки

D. T. Son and P. Surowka, Phys. Rev. Lett. 103 (2009) 191601 doi:10.1103/PhysRevLett.103.191601 [arXiv:0906.5044 [hep-th]]. (формулы на слайде)

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York (1959).

Закон сохранения энергии/импульса во внешнем электромагнитном поле

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

+

Квантовая аномалия в дивергенции аксиального тока

$$\partial_\mu j^\mu = C E^\mu B_\mu$$

Второй закон термодинамики $\partial_\mu s^\mu \geq 0$

Идеальная жидкость

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu},$$

$$j^\mu = n u^\mu + \nu^\mu,$$

Вязкость+электропроводность **+аксиальная аномалия**

Киральный вихревой эффект CVE

Киральный магнитный эффект CME

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu,$$

$$s^\mu = s u^\mu - \frac{\mu}{T} \nu^\mu + D\omega^\mu + D_B B^\mu,$$

$$\xi = C \left(\mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right)$$

Киральные явления как проявления аксиальной аномалии в эффективной теории поля

A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025 doi:10.1103/PhysRevD.83.105025 [arXiv:1012.1958 [hep-th]]. (формулы на слайде)

Химический потенциал как эффективное внешнее электромагнитное поле $qA^\mu \rightarrow \mu u^\mu$

Гидродинамика как эффективная теория поля в реальном времени

$$S_{eff} = \int dx \left(i\bar{\psi}\gamma^\rho D_\rho \psi + \mu u_\mu \bar{\psi}\gamma^\mu \psi + \mu_5 u_\mu \bar{\psi}\gamma^\mu \gamma_5 \psi \right)$$

Аксиальная аномалия в гидродинамике

$$\partial_\mu j_5^\mu = -\frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}\left(\partial^\mu (A^\nu + \mu u^\nu)\partial^\alpha (A^\beta + \mu u^\beta) + \partial^\mu \mu_5 u^\nu \partial^\alpha \mu_5 u^\beta\right)$$

$$\partial_\mu j^\mu = -\frac{1}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\mu (A^\nu + \mu u^\nu)\partial^\alpha \mu_5 u^\beta$$

Киральные эффекты в векторном токе

$$j^\mu = n u^\mu + \frac{\mu\mu_5}{\pi^2}\omega^\mu + \frac{\mu_5}{2\pi^2}B^\mu$$

Киральные эффекты в аксиальном токе

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2 + \mu_5^2}{2\pi^2}\omega^\mu + \frac{\mu}{2\pi^2}B^\mu$$

Исследованные вопросы и методы

- Эффекты вращения и ускорения в аксиальном токе.
- Сохранение аксиального заряда во вращающихся и ускоренно движущихся системах отсчёта.
- Поляризация Λ-гиперонов в соударениях тяжёлых ионов.

Методы

- Ковариантная функция Вигнера для систем, характеризующихся тензором термальной завихрённости.
- Метод равновесного квантового оператора плотности, учитывающего эффекты, связанные с тензором термальной завихрённости.

Теория возмущений в приложении к теории поля при конечных температурах.

Учёт аксиального химического потенциала в первом порядке теории возмущений

Термальный пропагатор с учётом векторного и аксиального химического потенциалов: случай массы равной нулю

$$\langle T_\tau \Psi_a(X)\bar{\Psi}_b(Y)\rangle_{\beta(x)} = \sum\!\!\!\!\!\!\int_{\{P\}} \left[e^{iP^{+-}(X-Y)}\mathcal{P}_+ \frac{\widetilde{\gamma}_\alpha(-iP^{+-}_\alpha)}{(P^{+-})^2} + e^{iP^{++}(X-Y)}\mathcal{P}_- \frac{\widetilde{\gamma}_\alpha(-iP^{++}_\alpha)}{(P^{++})^2}\right]$$

$$\mu_5 \neq 0$$
$$m = 0$$

$$\mathcal{P}_\pm = \frac{1 \pm \gamma_5}{2}$$

$$P^{+-\lambda} = (p_n + i\mu - i\mu_5, \mathbf{p})$$

Аксиальный ток в первом порядке

$$j_5^\lambda(x) = n_5 u^\lambda + (\frac{T^2}{6} + \frac{\mu^2+\mu_5^2}{2\pi^2})\omega^\lambda$$

Векторный ток в первом порядке

$$j^\lambda = n u^\lambda + \frac{\mu\mu_5}{\pi^2}\omega^\lambda$$

Соответствует другим подходам: эффективная теория поля с аксиальными аномалиями и кинетический

- *A. V. Sadofyev, V. I. Shevchenko and V. I. Zakharov, Phys. Rev. D 83 (2011) 105025 doi:10.1103/PhysRevD.83.105025 [arXiv:1012.1958 [hep-th]].*
- *J. H. Gao, Z. T. Liang, S. Pu, Q. Wang and X. N. Wang, Phys. Rev. Lett. 109 (2012) 232301 doi:10.1103/PhysRevLett.109.232301 [arXiv:1203.0725 [hep-ph]].*

Учёт гравитационной аномалии

K. Landsteiner, E. Megias, F. Pena-Benitez: Gravitational Anomaly and Transport. Phys. Rev. Lett. 107, 021601 (2011), arXiv:1103.5006 [hep-ph].

Киральные эффекты в аксиальном токе

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2 + \mu_5^2}{2\pi^2}\omega^\mu + \frac{\mu}{2\pi^2}B^\mu$$

Киральный вихревой эффект (CVE)

$$j_5^\lambda = \left(\frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2}\right)\omega^\lambda$$

Covariant Wigner function: a method of describing a system in quantum kinetic theory

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

Wigner function for Dirac fields

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y}\langle : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) :\rangle$$

⇒ Contains thermodynamic information about the system

⇓

The interaction is weak, W has inhomogeneities only on macroscopic scales

⇓

$$W(x,k) = \frac{1}{2}\int \frac{d^3p}{\varepsilon}\Big(\delta^4(k-p)U(p)f(x,p)\bar{U}(p) - \delta^4(k+p)V(p)\bar{f}^{\,T}(x,p)\bar{V}(p)\Big)$$

⇓

Mean values of operators

$$\langle : \overline{\Psi}(x)A\Psi(x) :\rangle = \int \mathrm{d}^4k\ \mathrm{tr}(AW(x,k))$$

⇓

$$f(x,p) = \frac{1}{\mathrm{e}^{\beta(x)\cdot p - \xi(x)} + 1}$$

Fermi-Dirac distribution under local thermodynamic equilibrium

Ansatz of the Wigner function, taking into account the thermal vorticity tensor

Examples of calculation of corrections for the thermal vorticity tensor

- *Vector current*

$$j^0(x) = 2\int \mathrm{d}^3\mathrm{p}\ (n_F - \bar{n}_F) + \varpi(x) : \varpi(x)\frac{1}{4}\int \mathrm{d}^3\mathrm{p}\ \left[n_F(1-n_F)(1-2n_F) - \bar{n}_F(1-\bar{n}_F)(1-2\bar{n}_F)\right]$$

- *Axial current*

$$j_A^\mu(x) = w^\mu W^A$$

$$W^A = \frac{1}{2\pi^2|\beta|}\int_0^\infty \frac{\mathrm{d}p}{E_p}\left(n_F(E_p-\mu) + n_F(E_p+\mu)\right)(2p^2+m^2) \longrightarrow \frac{T^3}{6} + \frac{T\mu^2}{2\pi^2}$$

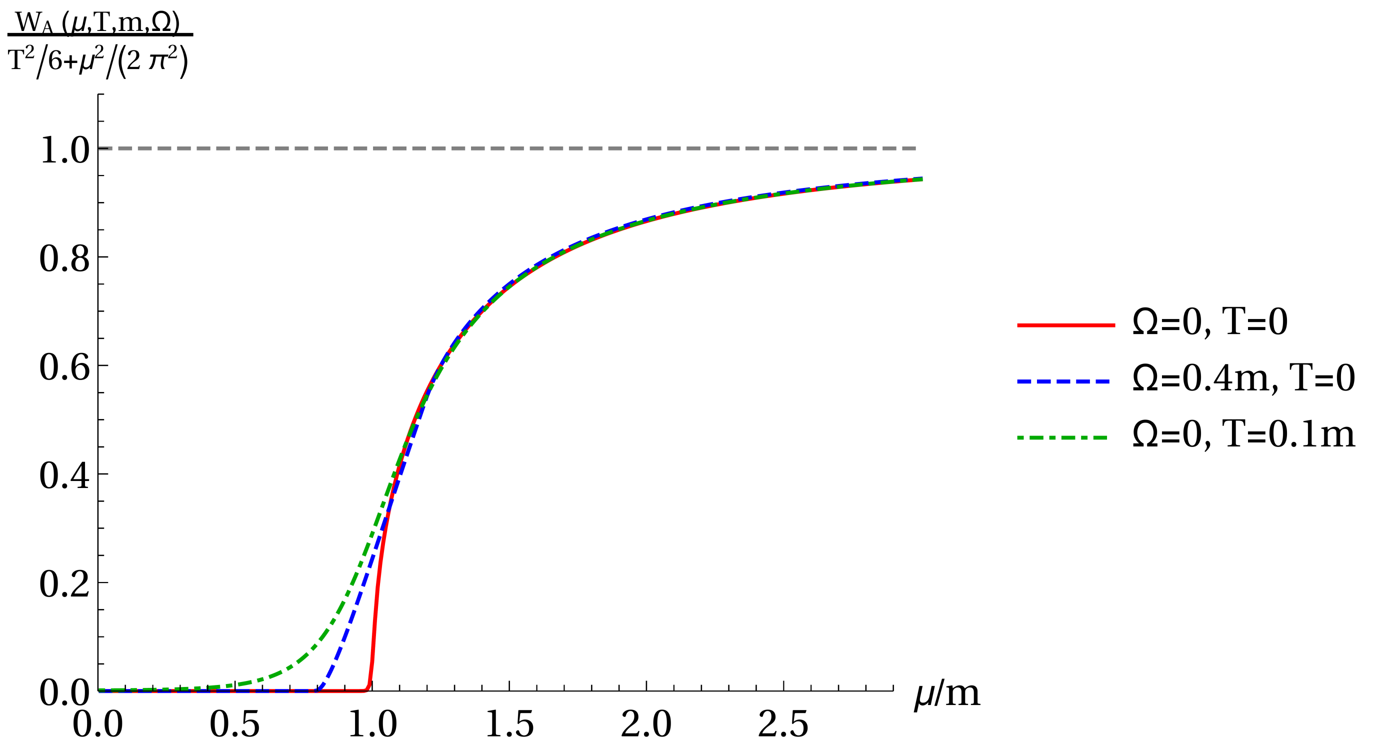
CVE

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32 doi:10.1016/j.aop.2013.07.004 [arXiv:1303.3431 [nucl-th]].

Axial current, mass is not zero

$\alpha_\mu = 0$



Axial current, as a function of the chemical potential at different values of the rotational speed and temperature

- ***The presence of a step at a chemical potential equal to the mass***

- *The step is smoothed with increasing temperature or speed of rotation*

The limit of massless fermions
Diagonalization

$$\varphi_\mu = \frac{a_\mu}{2\pi} + \frac{i\omega_\mu}{2\pi}$$

Complex superposition of acceleration and vorticity vectors

⇩

$$\langle : j^5_\mu : \rangle = 2\pi \, \mathrm{Im} \Big[\big(\frac{1}{6}(T^2 + \varphi^2) + \frac{\mu^2}{2\pi^2} \big) \varphi_\mu \Big]$$

- **The symmetry between vorticity and acceleration is an analog of the symmetry between the magnetic and electric fields in electrodynamics**

Предельные случаи

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\widehat{Q}\right]$$

$\varpi = 0$ **Предельные случаи** $\alpha_\mu = 0$

$$\widehat{\rho} = \frac{1}{Z} \exp[-b \cdot \widehat{P} + \zeta\widehat{Q}]$$

Великое каноническое распределение

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\widehat{H}/T_0 + \omega_0 \widehat{J}_z/T_0\right]$$

V.E. Ambrus and E. Winstanley, Rotating fermions inside a cylindrical boundary, Phys. Rev. D 93 (2016) 104014 [arXiv:1512.05239] [INSPIRE].

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

Axial current in the third order of perturbation theory

Three types of admissible parity:

$$\langle \hat{j}_5^\lambda(x) \rangle_3 = A_1 w^2 w^\lambda + A_2 \alpha^2 w^\lambda + A_3 (w\alpha)\alpha^\lambda$$

Violates the conservation of axial charge

The main points of the technique of computing of hydrodynamic coefficients

- Representation of composite operators in a split form *(point splitting)*

$$\widehat{T}_{\mu\nu}(X) = \lim_{X_1, X_2 \to X} \mathcal{D}_{\mu\nu}(\partial_{X_1}, \partial_{X_2}) \bar{\Psi}(X_1) \Psi(X_2)$$

$$\mathcal{D}_{\mu\nu}(\partial_{X_1}, \partial_{X_2}) = \frac{\mathrm{i}^{\delta_{0\mu}+\delta_{0\mu}}}{4} \left[\tilde{\gamma}_\mu (\partial_{X_2} - \partial_{X_1})_\nu + \tilde{\gamma}_\nu (\partial_{X_2} - \partial_{X_1})_\mu \right]$$

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

- Summation over the Matsubara frequencies

$$\frac{1}{\beta} \sum_{\{\omega_n\}} \frac{(\omega_n \pm \mathrm{i}\mu)^k \mathrm{e}^{\mathrm{i}(\omega_n \pm \mathrm{i}\mu)\tau}}{(\omega_n \pm \mathrm{i}\mu)^2 + E^2}$$

$$= \frac{1}{2E} \left[(-\mathrm{i}E)^k \mathrm{e}^{\tau E} (\theta(-\tau) - n_F(E \pm \mu)) + (\mathrm{i}E)^k \mathrm{e}^{-E\tau} (\theta(\tau) - n_F(E \mp \mu)) \right]$$

$$= \frac{1}{2E} \sum_{s=\pm 1} (-\mathrm{i}sE)^k \mathrm{e}^{\tau s E} [\theta(-s\tau) - n_F(E \pm s\mu)]$$

Fermi distribution

Детали вычислений

Термальный пропагатор фермионов:

M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091 doi:10.1007/JHEP10(2017)091 [arXiv:1704.02808 [hep-th]].

$$\langle \mathrm{T}_\tau \Psi_a(X)\bar{\Psi}_b(Y)\rangle = \sum\!\!\!\!\!\!\int_{\{P\}} \mathrm{e}^{\mathrm{i}P^+\cdot(X-Y)} \frac{(-\mathrm{i}\not{P}^+ + m)_{ab}}{(P^+)^2+m^2} = \sum\!\!\!\!\!\!\int_{\{P\}} \mathrm{e}^{\mathrm{i}P^+\cdot(X-Y)}(-\mathrm{i}\not{P}^+ + m)_{ab}\Delta(P^+)$$

$$\Delta(P^\pm) = \frac{1}{P^{\pm 2}+m^2}$$

Интегрирование по фазовому пространству и суммирование по мацубаровским частотам

Явная зависимость от координат переписывается в виде производных по импульсу:

$$\int d^3x d^3y d^3z\, f(\mathbf{p},\mathbf{q},\mathbf{k},\mathbf{r}) e^{-i\mathbf{p}(\mathbf{x}-\mathbf{y})-i\mathbf{q}(\mathbf{x}-\mathbf{z})-i\mathbf{k}\mathbf{y}-i\mathbf{r}\mathbf{z}} x^i y^j z^k =$$

$$i\left(\frac{\partial^3}{\partial r^k \partial k^j \partial p^i} + \frac{\partial^3}{\partial r^k \partial k^j \partial k^i}\right) f(\mathbf{p},\mathbf{q},\mathbf{k},\mathbf{r})\Big|_{\substack{\mathbf{q}=-\mathbf{p}\\ \mathbf{k}=\mathbf{p}\\ \mathbf{r}=-\mathbf{p}}}$$

Запланированные задачи

- Сравнение двух подходов к расчёту поляризации Λ-гиперонов: вывод связи поляризации с аксиальным током.
- Исследование вопроса о неперенормируемости киральных эффектов в теории поля при конечных температурах.
- Исследование влияния эффектов, связанных с гравитаций и, в частности, с гравитационной аномалией, на физику киральных жидкостей.
- Дополнительная проверка сохранения аксиального заряда.

$$\mathrm{tr}\big(X\Sigma^{\nu\beta}\big) = \Big\{\big(\exp\left[(\beta\cdot p - \xi - g_1 + ig_2)\right] + 1\big)^{-1} - \big(\exp\left[(\beta\cdot p - \xi + g_1 - ig_2)\right] + 1\big)^{-1}\Big\}$$
$$\frac{1}{4(g_1 - ig_2)}[\varpi^{\nu\beta} - i\,\mathrm{sgn}(\varpi : \widetilde{\varpi})\widetilde{\varpi}^{\nu\beta}] +$$
$$\Big\{\big(\exp\left[(\beta\cdot p - \xi - g_1 - ig_2)\right] + 1\big)^{-1} - \big(\exp\left[(\beta\cdot p - \xi + g_1 + ig_2)\right] + 1\big)^{-1}\Big\}$$

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{1}{|\beta|^3}\int d\tau_x d\tau_y d\tau_z d^3x d^3y d^3z \langle T_\tau \widehat{T}^{\alpha_1\alpha_2}(\tau_x, \mathbf{x})$$
$$\widehat{T}^{\alpha_3\alpha_4}(\tau_y, \mathbf{y})\widehat{T}^{\alpha_5\alpha_6}(\tau_z, \mathbf{z})\widehat{j}_5^{\lambda}(0)\rangle_{\beta(x),c} x^i y^j z^k$$

$$A_1 = -\frac{1}{6}(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} +$$
$$C^{01|01|02|3|221} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112} - C^{01|01|01|3|222})$$

$$\langle T_\tau \widehat{T}^{\alpha_1\alpha_2}(\tau_x,\mathbf{x})\widehat{T}^{\alpha_3\alpha_4}(\tau_y,\mathbf{y})\widehat{T}^{\alpha_5\alpha_6}(\tau_z,\mathbf{z})\widehat{j}_5^{\lambda}(0)\rangle_{\beta(x),c} = \lim_{\substack{X_1,X_2\to X\\ Y_1,Y_2\to Y\\ Z_1,Z_2\to Z\\ F_1,F_2\to F=0}} \mathcal{D}^{\alpha_1\alpha_2}_{a_1a_2}(\partial_{X_1},\partial_{X_2})$$

$$\mathcal{D}^{\alpha_3\alpha_4}_{a_3a_4}(\partial_{Y_1},\partial_{Y_2})\mathcal{D}^{\alpha_5\alpha_6}_{a_5a_6}(\partial_{Z_1},\partial_{Z_2})\mathcal{J}^{\lambda}_{A\,a_7a_8}\langle\bar{\Psi}_{a_1}(X_1)\Psi_{a_2}(X_2)\bar{\Psi}_{a_3}(Y_1)\Psi_{a_4}(Y_2)$$
$$\bar{\Psi}_{a_5}(Z_1)\Psi_{a_6}(Z_2)\bar{\Psi}_{a_7}(F_1)\Psi_{a_8}(F_2)\rangle_{\beta(x),c}$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

$\Updownarrow$

Разложение на генераторы углового момента и сдвига

$$\widehat{J}^{\mu\nu} = u^\mu \widehat{K}^\nu - u^\nu \widehat{K}^\mu - u_\rho \epsilon^{\rho\mu\nu\sigma}\widehat{J}_\sigma$$

Аксиальный ток в третьем порядке теории возмущений

Каждый из гидродинамических коэффициентов представляется как сумма величин C

$$A_1 = -\frac{1}{6}(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} + C^{01|01|02|3|221} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112} - C^{01|01|01|3|222})$$

Каждый из этих коэффициентов имеет вид:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{i}{|\beta|^3(2\pi)^3}\int [d\tau]d^3p\left(\frac{\partial^3 D_1}{\partial r^k \partial k^j \partial p^i} + \frac{\partial^3 D_1}{\partial r^k \partial k^j \partial k^i}\right)\Bigg|_{\substack{\mathbf{q}=-\mathbf{p}\\ \mathbf{k}=\mathbf{p}\\ \mathbf{r}=-\mathbf{p}}} + \dots$$

$$D_1 = \frac{-1}{16E_pE_qE_kE_r}\sum_{\substack{s_1,s_2,s_3,\\ s_4=\pm 1}} e^{(\tau_x-\tau_y)s_1E_p+(\tau_x-\tau_z)s_2E_q+\tau_y s_3 E_k+\tau_z s_4 E_r} A^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6\lambda}(\widetilde{P},\widetilde{K},\widetilde{Q},\widetilde{R})$$

$$\Big(\Theta(-s_1[\tau_x-\tau_y]) - n_F(E_p - s_1\mu)\Big)\Big(\Theta(-s_2[\tau_x-\tau_z]) - n_F(E_q + s_2\mu)\Big)$$

$$\Big(\Theta(-s_3) - n_F(E_k - s_3\mu)\Big)\Big(\Theta(-s_4) - n_F(E_r + s_4\mu)\Big)$$

Многочлен от аргументов

$$C^{02|02|02|3|111} = -C^{01|01|01|3|222} = \frac{29}{80|\beta|^3\pi^2}$$

$$C^{01|02|02|3|211} = C^{02|01|02|3|121} = C^{02|02|01|3|112} = -C^{02|01|01|3|122} =$$

$$-C^{01|02|01|3|212} = -C^{01|01|02|3|221} = \frac{19}{240|\beta|^3\pi^2}$$