Nonperturbative kinetic approach for describing the pre-equilibrium dynamics of HIC

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1. The processes in the collision of heavy ions that precede the appearance of QGP.
2. Schwinger mechanism is a source of soft particles.
3. Kinetic theory in the quasiparticle representation for the abelian projection of QCD.
4. Model parameters.
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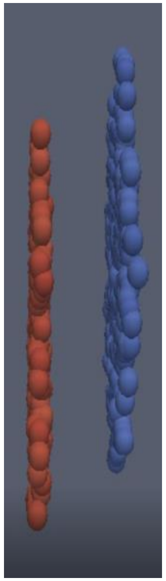
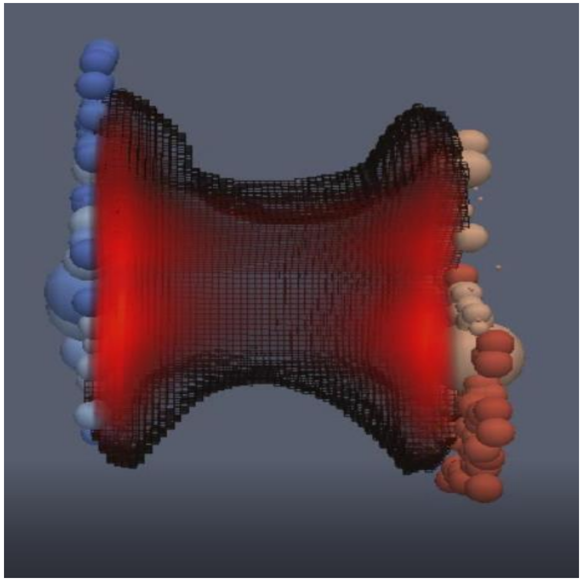
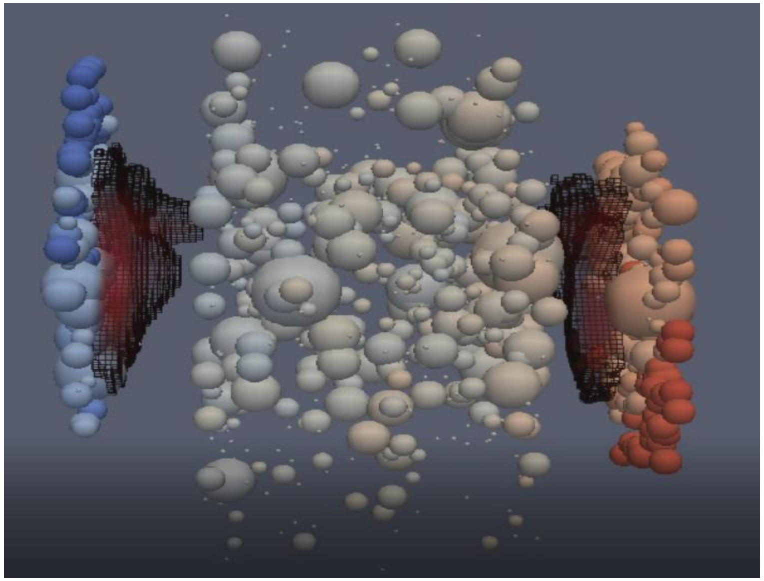
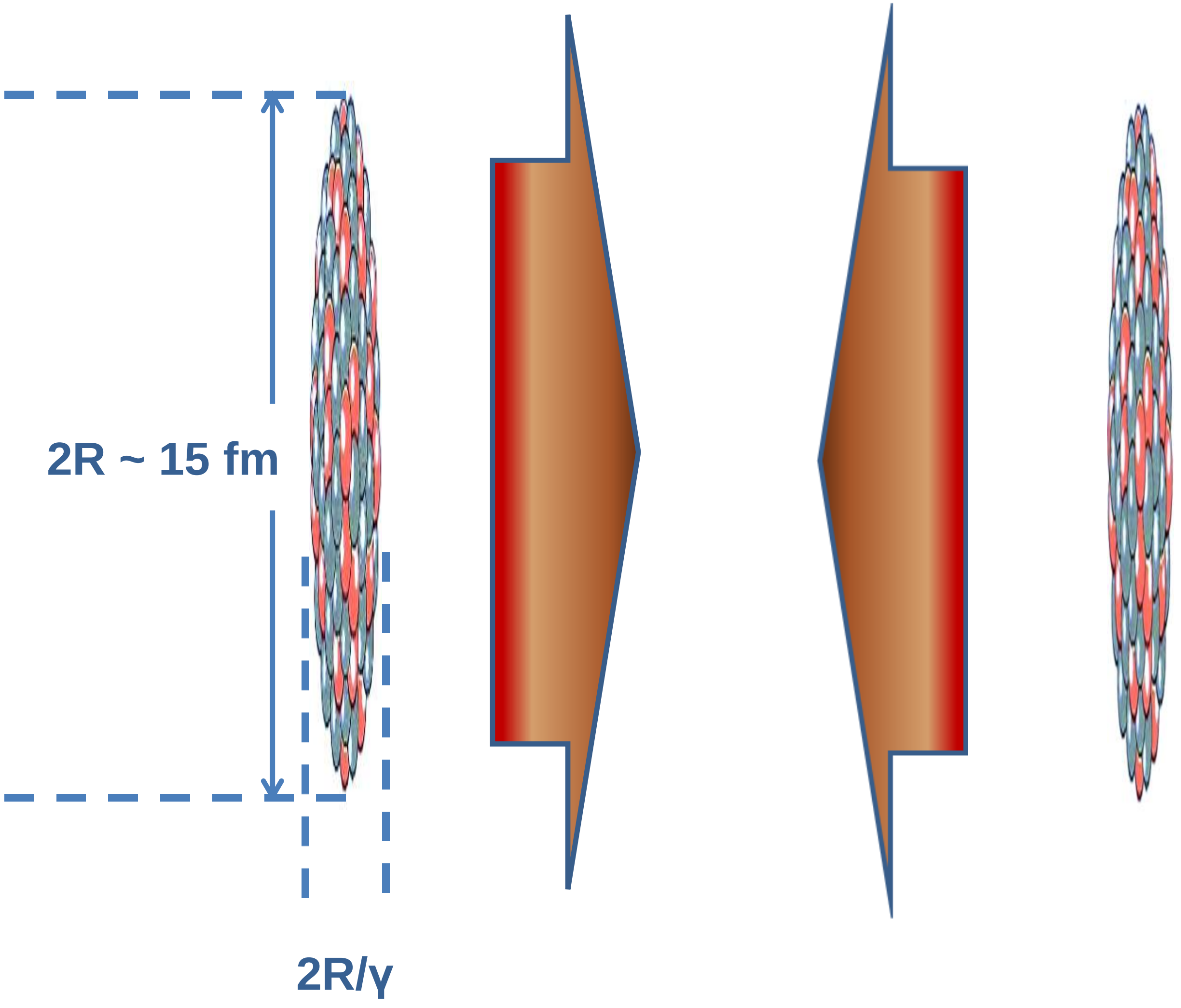


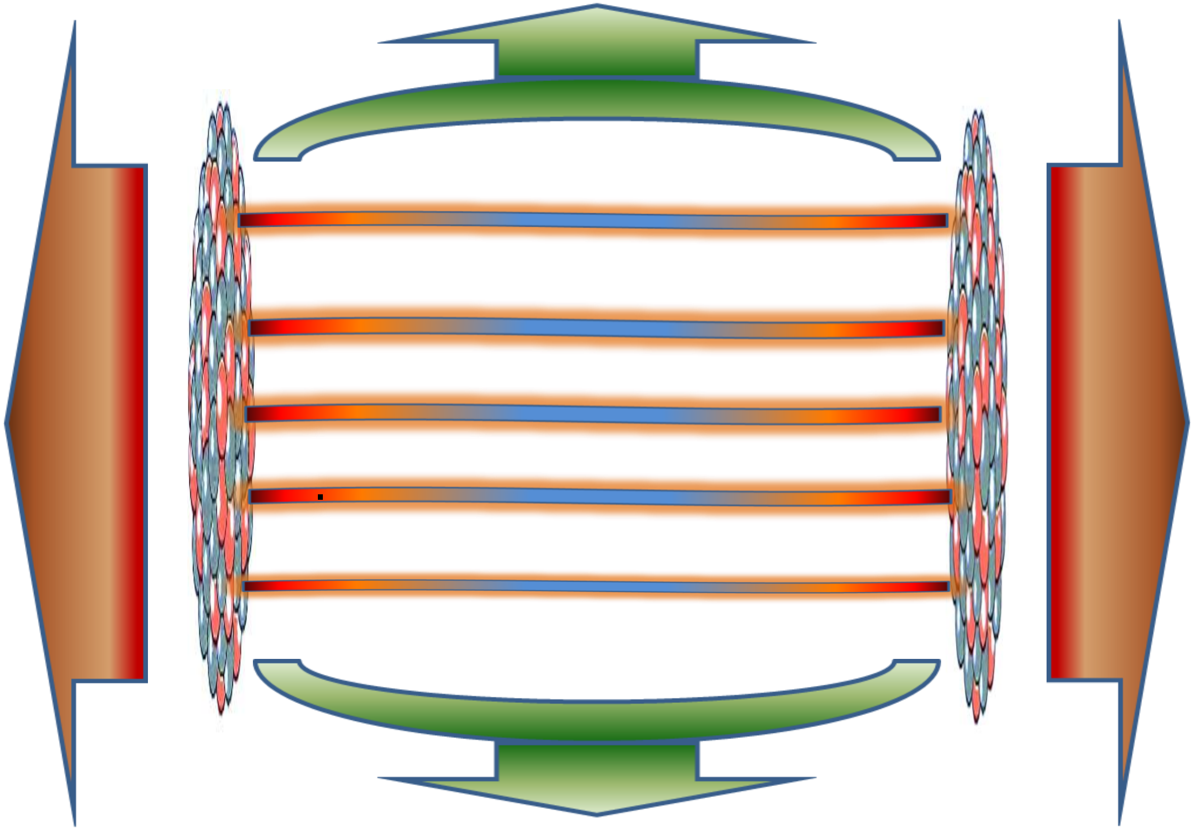


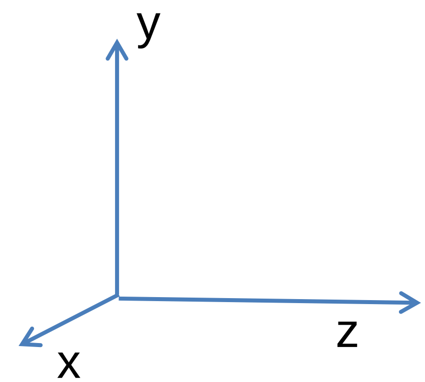


1. What do we know about the processes in the collision of heavy ions that precede the appearance of QGP?



2. Schwinger mechanism is a source of soft particles.





$$W \sim exp\left(-\frac{m_f^2}{K}\right)$$

J. Schwinger, Phys. Rev. 82 (1951) 664.
A. Casher, H. Neuberger, A. Nussinov, Phys. Rev. D 20 (1979) 179;
A. Casher, H. Neuberger, A. Nussinov, Phys. Rev. D 21 (1980) 1966.

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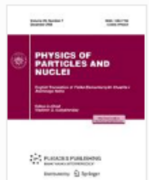
Fluctuations of the string tension and transverse mass distribution

A. Bialasab



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Low momentum π-meson production from evolvable quark condensate

Authors Authors and affiliations

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Qualitative development of the model by using the kinetic approach:

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Vacuum creation of quarks at the time scale of QGP thermalization and strangeness enhancement in heavy-ion collisions

A.V. Prozorkevich [a], S.A. Smolyansky [a], V.V. Skokov [b], E.E. Zabrodin [c,d]

3. Kinetic theory in the quasiparticle representation for the abelian projection of QCD.

Next, we will use the coordinate system shown on the previous slide. We can not say that the formed color strings are strictly parallel to the axis Z. But as a simplifying assumption, such a choice will be fully justified in the model under consideration. This allows us to determine the vector potential of the color field in the form:

$$B_a^{\mu}(t) = (0,0,0,B_a^3(t) = B_a(t)) \qquad (1)$$

We call attention to the Lagrangian of a free color field:

$$\mathcal{L}_{JM} = -\frac{1}{4}\sum G_a^{\mu\nu} G_{\mu\nu}^a \qquad G_a^{\mu\nu} = B_a^{\mu,\nu} - B_a^{\nu,\mu} + g\sum f_{abc} B_b^{\mu} B_c^{\nu} \qquad (2)$$

Due to the complete antisymmetry of the structural constants f_{abc} of the group SU(3) in a color field of the form (1), there is no non-Abelian contribution to the Lagrangian. For completeness of the picture, we can include into consideration also the electromagnetic field

$$A^{\mu}(t) = (0,0,0,A^3(t) = A(t))$$

$$\mathcal{L}_M = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \qquad F^{\mu\nu} = A^{\nu,\mu} - A^{\mu,\nu} \qquad (3)$$

To construct a kinetic theory for the quark sector of QCD define the Lagrangian function of the quark in an external classical gluon (B) and electromagnetic (A) fields ($\hat{m}$ is the mass matrix):

$$\mathcal{L} = i\bar{q}\gamma^{\mu} D_{\mu} q - \bar{q}\hat{m}q, \qquad \textbf{(4)}$$

equation of motion

$$(iD_{\mu}\gamma^{\mu} - \hat{m})\, q = 0, \qquad \textbf{(5)}$$

energy-momentum tensor

$$T_{\mu\nu} = \frac{i}{2}\left[\bar{q}\gamma_{\mu} D_{\mu} q - (D_{\nu}^{+}\bar{q})\gamma_{\mu} q\right] \qquad \textbf{(6)}$$

The electromagnetic A^{μ} and gluon fields B_a^{μ} bring in theory by standard way of lengthening the derivatives $(e > 0, g > 0)$

$$D^{\mu} q^{j} = \sum_{j'} \left[\left(\partial^{\mu} + i e_f A^{\mu}\right) \delta_{jj'} - \frac{i}{2} g \sum_{a} B_a^{\mu} \lambda_{jj'}^{a} \right] q^{j'}. \qquad \textbf{(7)}$$

Here $f, f', \ldots$ are flavour and $j, k, \ldots$ are color index; e_f is the electric charge of the quark with flavour f, g is the color charge.

The corresponding Hamiltonian function after exception of the time derivative of the quark field with help of equation of motion is equal

$$\mathcal{H} = T_{00} = \frac{i}{2}\left[\bar{q}\vec{\gamma}\vec{D}q - \left(\vec{D}^{+}\bar{q}\right)\vec{\gamma}q\right]. \quad (8)$$

In general case transition in the quasiparticle representation implies diagonalization of the Hamiltonian in the Fock representation relative to the quark and antiquark creation and annihilation operators. Bogolubov's canonical transformation method is used for this aim. In QED it is possible for the space homogeneous time dependent classical electromagnetic fields, which provides locality in the momentum space of the Hamiltonian and does not violate of homogeneity of the quadratic form of the creation and annihilation operators. However the last feature of the QED interaction is violated in QCD, where the no diagonal color terms in the quark quadratic form are presented. Therefore separate the terms from the Hamiltonian which do not change color of quarks in act interaction with the gluon field. Thus, the Hamiltonian function can be represented in the form:

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_{int}, \quad (9)$$

where $\mathcal{H}_Q$ **is the homogeneous flavour and color quadratic form**

$$\mathcal{H}_Q = \frac{i}{2}[\bar{q}\vec{\gamma}\vec{\mathfrak{D}}q - (\vec{\mathfrak{D}}\bar{q})\vec{\gamma}q] \quad (10)$$

and $\mathfrak{D}^{\mu}_{fj}$ contains interaction of the electromagnetic and gluon fields with the quark of a flavour f and a color j only

$$\mathfrak{D}^{\mu}_{fj} = \partial^{\mu} + ie_f A^{\mu} - \frac{i}{2} g \sum B^{\mu}_{a} \lambda^{a}_{jj} = \partial^{\mu} + ie_f A^{\mu} - \frac{i}{2} g \mathcal{A}^{\mu}_{j} \qquad (11)$$

Used in this expression the gluon fields $\mathcal{A}^{\mu}_{j}$ do not change the quark color

$$\mathcal{A}^{\mu}_{j} = \begin{cases} B^{\mu}_{3} + \frac{1}{\sqrt{3}} B^{\mu}_{8}, & j = 1; \\ -B^{\mu}_{3} + \frac{1}{\sqrt{3}} B^{\mu}_{8}, & j = 2; \\ -\frac{2}{\sqrt{3}} B^{\mu}_{8}, & j = 3. \end{cases} \qquad (12)$$

Just the Hamiltonian function (10) is the basis for transition in quasiparticle representation in the quark Fock space.
The $\mathcal{H}_{int}$ describes the residual part of the Hamiltonian function (9) which connect with the color changing interaction of a quark with the gluon field and should be taken into account in the presence of spatial heterogeneity. Thus, the need to ensure the locality of the Hamiltonian in the representation of the second quantization limits us to considering only spatially homogeneous systems.
Or, in general, systems that can satisfy this requirement at some level of consideration.

The further procedure of derivation of the KE's in the quark sector QCD does not differ from the well known one in QED. The resulting system of KE's relative to the distribution functions of quarks

$$f_{fj}(\vec{p},t) = < t|q^{+}_{fj}(\vec{p},t)q^{-}_{fj}(\vec{p},t)|t >$$

and antiquarks

$$\tilde{f}_{fj}(\vec{p},t) = < t|\tilde{q}^{+}_{fj}(\vec{p},t)\tilde{q}^{-}_{fj}(\vec{p},t)|t > \qquad (13)$$

has the following form

$$\begin{aligned} \dot{f}_{fj}(\vec{p},t) &= \dot{\tilde{f}}_{fj}(\vec{p},t) = \frac{1}{2}\lambda_{fj}(\vec{p},t)\int_{t_0}^{t} dt'\lambda_{fj}(\vec{p},t') \times \\ &\times \left[1 - f_{fj}(\vec{p},t) - \tilde{f}_{fj}(-\vec{p},t)\right]\cos\theta_{fj}(\vec{p};t,t'), \end{aligned} \qquad (14)$$

where

$$\lambda_{fj}(\vec{p},t) = \frac{F_{fj}(\vec{p},t)}{\varepsilon^{2}_{fj}(\vec{p},t)} \qquad (15)$$

is the amplitude of vacuum transitions.

The generalized force acting on the quark with the quantum numbers $f,\ j,\ \vec{p};$ **:**

$$F_{fj}(\vec{p},t) = -e_f\dot{A}(t) + \frac{1}{2}g\dot{\mathcal{A}}_j(t) \qquad (16)$$

The phase of the high frequency oscillations with the gap energy $2\varepsilon_{fj}(\vec{p},t)$ **in the presence of the classical electric and chromo-electric fields.**

$$\theta_{fj}(\vec{p};t,t') = 2\int_{t'}^{t} d\tau\varepsilon_{fj}(\vec{p},\tau) \qquad (17)$$

KE's for quarks and antiquarks are the same, but the functions $f_{fj}(\vec{p},t)$ and $\tilde{f}_{fj}(-\vec{p},t)$ can differ if difference of the initial conditions, which can take place in the general case. Strict equality will take place if we use zero initial conditions.
This is justified for quark flavors, which are certainly not in the original baryonic matter. This is also true if there are grounds for believing that the states occupied by the pair are free. Even in the presence of quarks of this flavor. Further, we use this version of determining the initial conditions for modeling the production of soft quarks.
In QED the equality $f=\tilde{f}$ is using, as a rule, that corresponds to the electroneutrality condition.

The system KE's contains 2 x 3N integro-differential KE's of the non-Markovian type for the three combinations of the color fields (11). For numerical investigations it is convenient to rewrite each KE in the form of system of the three ordinary differential equations every:

$$\begin{aligned} \dot{f} &= \dot{\tilde{f}} = \frac{1}{2}\lambda u, \\ \dot{u} &= \lambda(1-f-\tilde{f}) - 2\varepsilon v, \\ \dot{v} &= 2\varepsilon u. \end{aligned} \qquad \textbf{(18)}$$

4. Model parameters

In the previously presented formalism, the electric and color fields are represented equally. However, the color (strong) interaction plays an important role in the processes under consideration. The ratio of their roles can be estimated by determining the ratio of the Coulomb interaction of the quark-antiquark pair to the tension force of the color string binding them. In addition, in the phase of the expansion of the remains of two nuclei, the electric fields between us are directed in opposite and are mutually suppressed. Based on the above estimates, at this stage of the model study we exclude the electric field from consideration.

To estimate the strength of the action of the color field on virtual quarks, we use the results of calculations of the tension force of a single string 0.18 GeV^2. We will treat this value as the upper bound. It will be valid with a relatively small number of color strings. When they are not yet merged and melted. The violation of this condition can be expressed by a corresponding decrease in this parameter in the simulation.

Itself string tension force defines only the multiplication $g\dot{\mathcal{A}}_j^\mu$**. Since the color charge** g **is not a constant, we must define it independently or consider it as an additional degree of freedom for our model.**

Apparently, attempts to determine the value of the effective coupling constant from the parameters of the partons of the colliding nuclei are not justified. The energy density before the thermalization of the system also can not serve as a guide for the choice of this value.

In the kinetic equations (18), the energy of quasiparticle excitations of quark fields is present as a parameter. It is determined through the mass for the quarks of the corresponding flavor and the kinematic moment, taking into account the presence of color fields:

$$\varepsilon_{fj}(\vec{p},t) = \sqrt{m_f^2 + p_1^2 + p_2^2 + P_j^2}\ , \qquad (19)$$

$$P_j^3 = p^3 + \frac{1}{2} g \mathcal{A}_j(t)\ .$$

This value can be a natural energy scale for determining the coupling constant in the kinetic equations. But due to the presence in the definition of the kinematic momentum term

$$\frac{1}{2} g \mathcal{A}_j(t),$$

such a definition will be unambiguous only in the absence of an external color field. However, even in the presence of a color field, approximate estimates can be obtained.

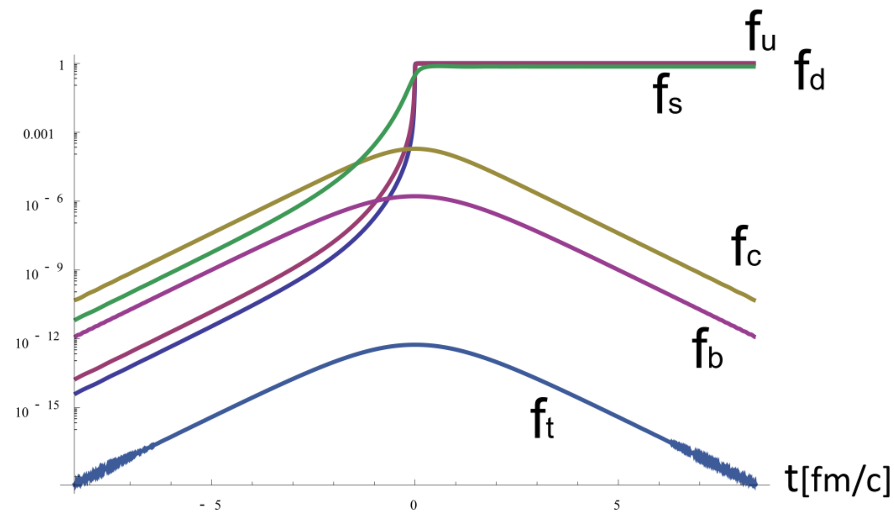
The next question requiring detailed discussion is the quark masses m_f of various flavors present in the kinetic equations. In the Lagrangian (4), there are current («naked») masses. Quarks with such masses can be produced only in conditions of deconfinement. At the initial stage, when there is no quark-gluon plasma, the energy costs for the production of quarks will be determined by their constituent mass. In the framework of modeling the transition process, we can consider effective intermediate values.

5. Examples of numerical simulations.

To begin with, we present the simplest example illustrating the behavior of the distribution functions in the single pulse of a color field of a soliton type:

$$\mathcal{A}_j^3(t) = \mathcal{G}_0 \tau \tanh\left(\frac{t}{\tau}\right)$$

Parameter τ determine haracteristic time scale of the process. The value is used for it 2 fm/c.

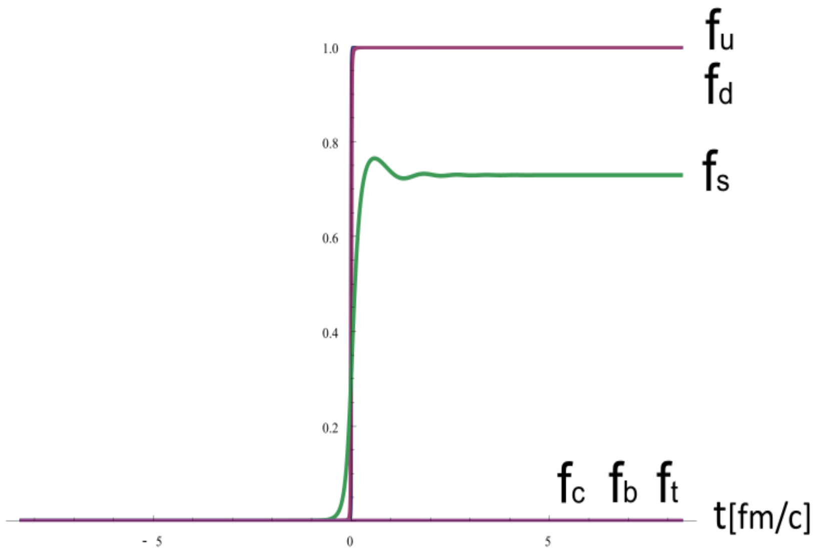


The maximum value of the intensity of the color field $\mathcal{G}_0$ at the time $t = 0$ is determined from the condition that the action force of such a color field is equal to the quark tension force of the gluon string. The masses of the quarks participating in the process are determined by their current values. The values used are shown in the table:

The figure shows the simulation results for zero momentum $p_1 = p_2 = p_3 = 0$. It can be stated that in the formalism used, the quark fields of all the flavors respond to the external field. The triplet of their heaviest ones returns to the initial vacuum state after the field is turned off. And three light quarks with a probability close to the one become real.

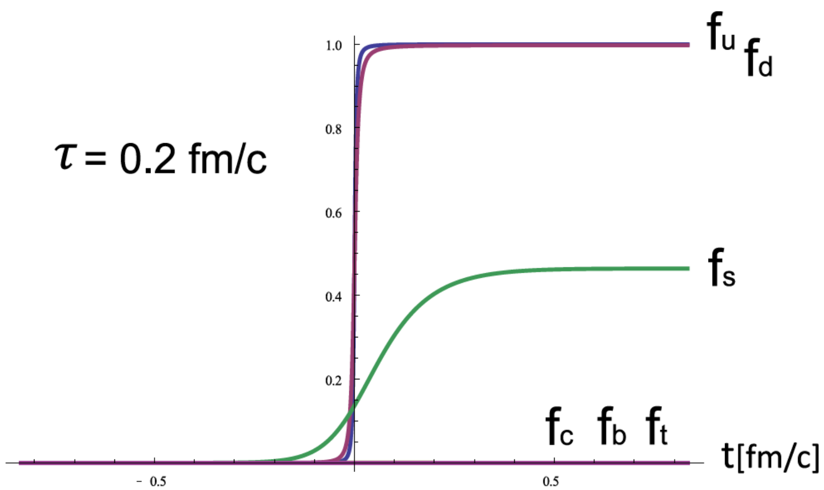
m_u = 2.3 MeV/c^2
m_d = 4.8 MeV/c^2
m_c = 1275 MeV/c^2
m_s = 95 MeV/c^2
m_t = 173210 MeV/c^2
m_b = 4180 MeV/c^2

On a linear scale, it is seen that for the lightest u end d quarks the probability of creation is almost exactly one. For a s quark it is somewhat less. Its exact value is 0.729 . This means that in the processes with the parameters used, quite a few s quarks are produced. Their number can be comparable with the number of u and d quarks.

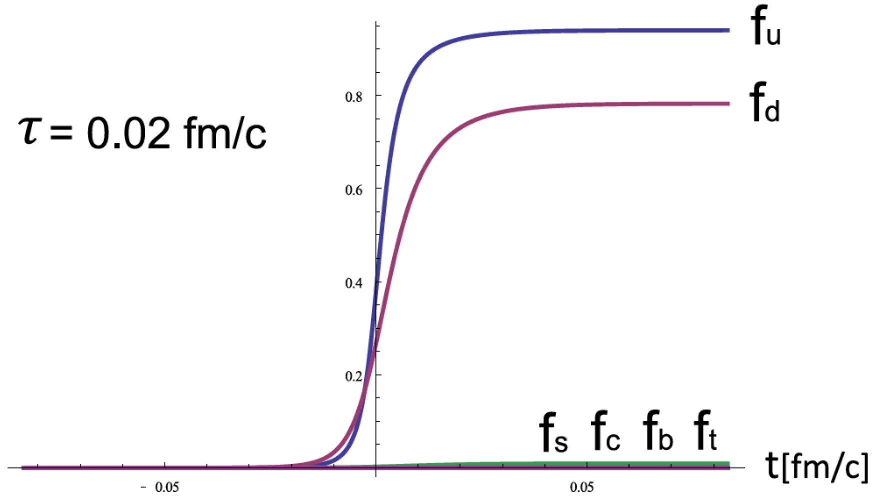


Already in this example, one can demonstrate the influence of the color field parameters on the characteristics of the generated quarks. Depending on the value of the Lorentz factor γ in the laboratory reference frame, the characteristic time τ of the parton interaction can vary within wide limits.

The value of 2 fm/c used for the top figure is rather an upper bound. The result for 0.2 fm/c is shown in the lower figure. There are no qualitative changes, but the relative number of strange quarks has decreased.



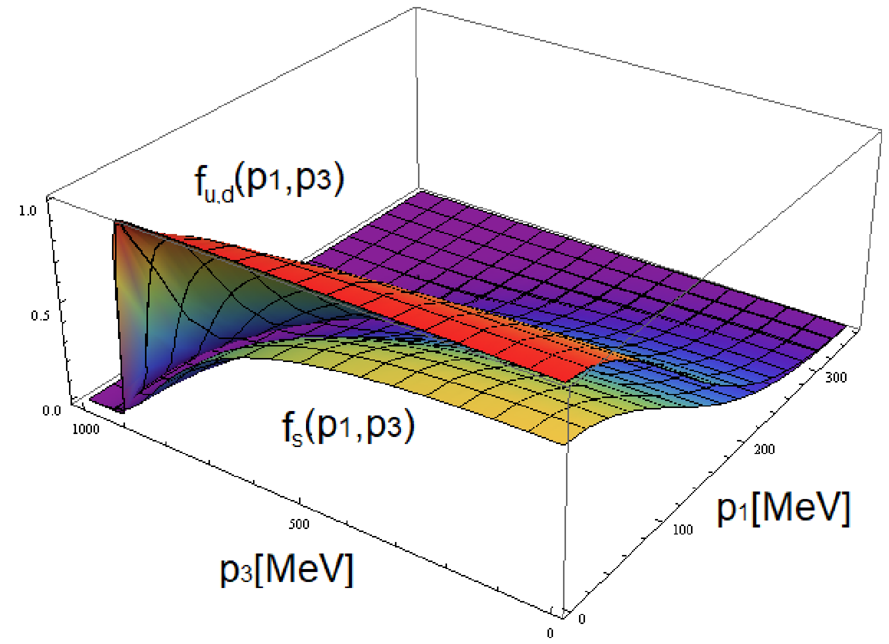
With a further decrease in the duration of the pulse of the color field, qualitative changes are already observed.
A strange flavour disappears and only u and d quarks are created. But even for them, the population of states falls below the level of saturation.
And there is an advantage for the upper quark in the number of pairs produced.

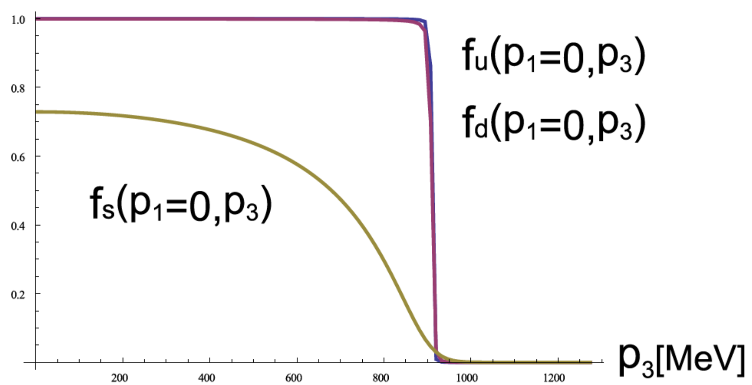


In considering the examples given, we limited ourselves to a single point in momentum space $p_1 = p_2 = p_3 = 0$. To obtain general estimates it is necessary to have data for the distribution functions in the whole range of possible values $\vec{p}$.
To save computational resources, we will not further include a triplet of heavy quarks. In the model under consideration, the direction of action of the field is independent of time. Because of this, the problem has a symmetry, which manifests itself in the behavior of the distribution functions in momentum space. Therefore, in order to obtain complete information about the distribution function, it is sufficient to calculate its values only in a two-dimensional domain $0 \leq p_1 \leq p_{1max}$, $p_2 = 0$ and $0 \leq p_3 \leq p_{3max}$.

The details of the time evolution of the distribution function are not of interest for estimating the observed collision characteristics. Therefore, in what follows we shall consider only the final spectrum of particles formed to the time of the termination of the action of the color field. Here is an example of such a distribution at the starting conditions $\tau = 2$ fm/c.

For light flavors, the distribution is strongly anisotropic. The direction of the field action is priority and populated to an energy level of more than 900 MeV.

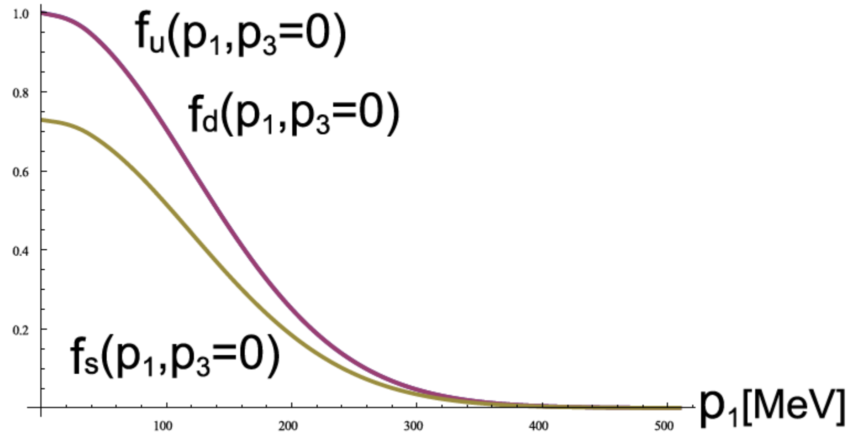




Despite the twofold difference in masses between u and d quarks, the behavior of their distribution functions is almost identical.

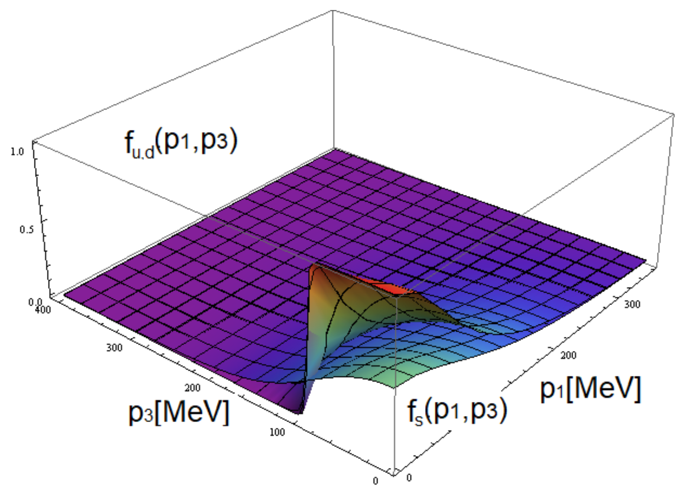
The behavior of the distribution function of s quarks is softer. It is not thermal.

Here we show the dependence of the distribution function on the transverse momentum.
The behavior of u and d quarks is also identical. The population of states is s quarks lower by about a quarter. But, unlike the longitudinal distribution, this proportionality is practically constant.

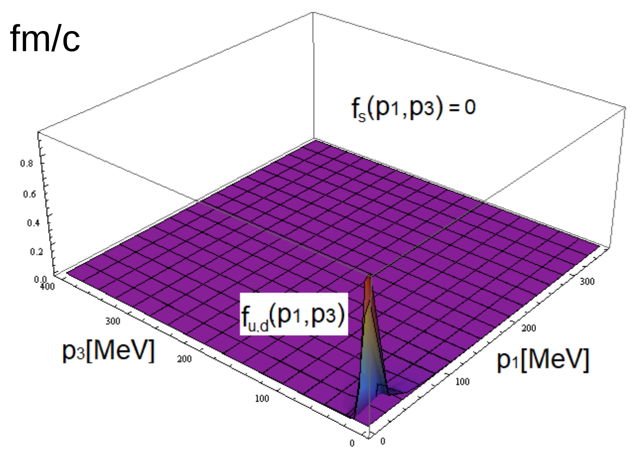


The general form of the distribution functions for short pulses :

τ = 0.2 fm/c



τ = 0.02 fm/c



The density and energy density at different pulse durations:

$\tau = 2.0\, fm/c$:

$$n_\Sigma = n_u + n_d + n_s = 2.86\, fm^{-3} + 2.86\, fm^{-3} + 1.89\, fm^{-3} = 7.61\, fm^{-3}$$

$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 3.63 GeV/fm^3 + 3.62 GeV/fm^3 + 2.35 GeV/fm^3 = 9.6\, GeV/fm^3$$

$\tau = 0.2\, fm/c$:

$$n_\Sigma = n_u + n_d + n_s = 0.62\, fm^{-3} + 0.62\, fm^{-3} + 0.54\, fm^{-3} = 1.78\, fm^{-3}$$

$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.21 GeV/fm^3 + 0.21 GeV/fm^3 + 0.20 GeV/fm^3 = 0.62\, GeV/fm^3$$

$\tau = 0.02\, fm/c$:

$$n_\Sigma = n_u + n_d + n_s = 0.01\, fm^{-3} + 0.01\, fm^{-3} + 0.0088\, fm^{-3} = 0.0288\, fm^{-3}$$

$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.0034 GeV/fm^3 + 0.0034 GeV/fm^3 + 0.0034 GeV/fm^3 \simeq 0.01\, GeV/fm^3$$

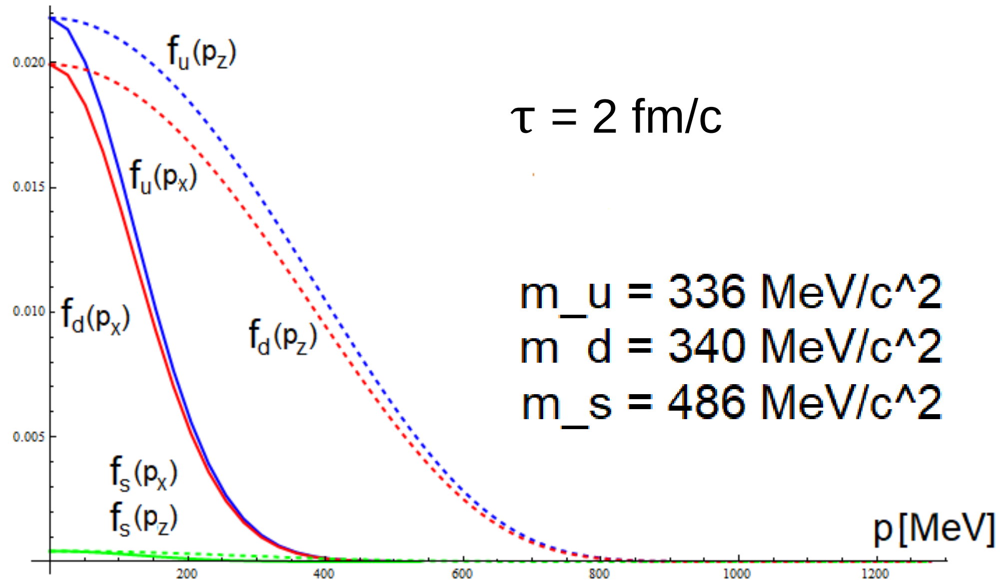
$$\boldsymbol{\varepsilon_u = \varepsilon_d = \varepsilon_s} \text{ !!!}$$

Critical energy density for the formation of quark-gluon plasma $\varepsilon_{cr} \simeq 1.0\, GeV/fm^3$

$$\boldsymbol{\varepsilon_{2.0} > \varepsilon_{cr} \simeq \varepsilon_{0.2} > \varepsilon_{0.02}}$$

The behavior of the model with constituent values for the quark masses.

The maximum value of the distribution function of u quark is only 0.022.
The asymmetry along the directions of the field and across remains, bat not so pronounced.



$$\varepsilon_{2.0} \ll \varepsilon_{cr} \simeq \varepsilon_{0.2} > \varepsilon_{0.02}$$

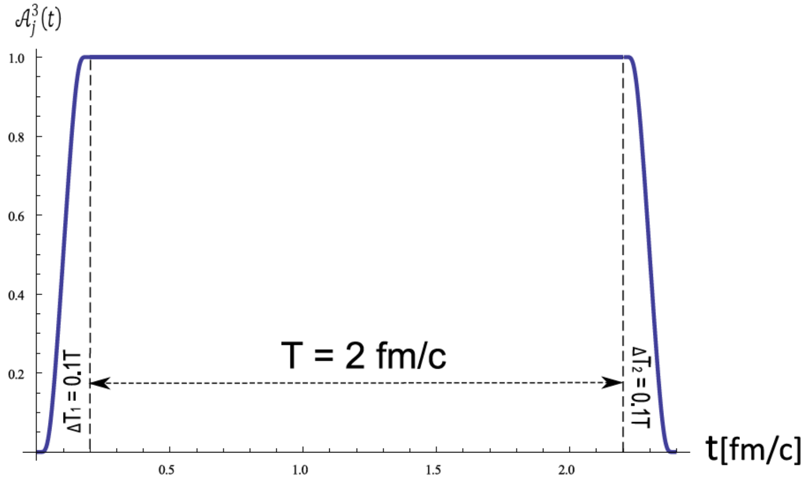
$$\tau = 2.0\, fm/c:$$
$$n_\Sigma = n_u + n_d + n_s = 0.039\, fm^{-3} + 0.036\, fm^{-3} + 0.00069\, fm^{-3} = 0.075\, fm^{-3}$$
$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.047 GeV/fm^3 + 0.043\, GeV/fm^3 + 0.00084 GeV/fm^3 = 0.090\, GeV/fm^3$$

$$\tau = 0.2\, fm/c:$$
$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.32\, fm^{-3} + 0.32\, fm^{-3} + 0.18\, fm^{-3} = 0.82\, fm^{-3}$$
$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.21 GeV/fm^3 + 0.21 GeV/fm^3 + 0.14 GeV/fm^3 = 0.57\, GeV/fm^3$$

$$\tau = 0.02\, fm/c:$$
$$n_\Sigma = n_u + n_d + n_s = 0.018\, fm^{-3} + 0.018\, fm^{-3} + 0.016\, fm^{-3} = 0.052\, fm^{-3}$$
$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.017 GeV/fm^3 + 0.017 GeV/fm^3 + 0.016 GeV/fm^3 = 0.05\, GeV/fm^3$$

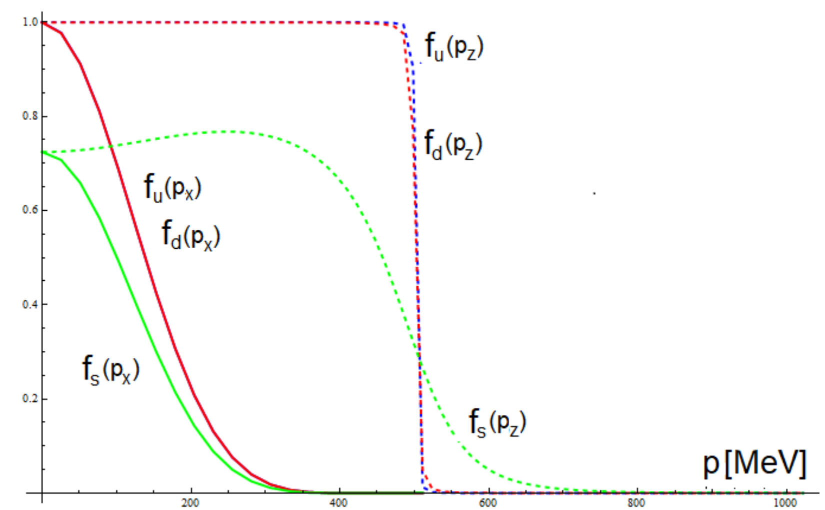
Field model with constant string tension

Field enable time ΔT₁=0.2 fm/c
The duration of a constant field T=2 fm/c
Field Off Time ΔT₂=0.2 fm/c
Total time of field action 2.4 fm/c
For the correct operation of the model, it is necessary to ensure continuity and smoothness of the field dependence on time.



$f_{s\ max}$ ($p_z \simeq 300$ MeV)

$$n_\Sigma = n_u + n_d + n_s$$
$$= 2.83\ fm^{-3} + 2.83\ fm^{-3} + 2.23\ fm^{-3}$$
$$= 7.89\ fm^{-3}$$
$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s$$
$$= 2.31 GeV/fm^3 + 2.31\ GeV/fm^3$$
$$+ 1.86 GeV/fm^3 = 6.48\ GeV/fm^3$$

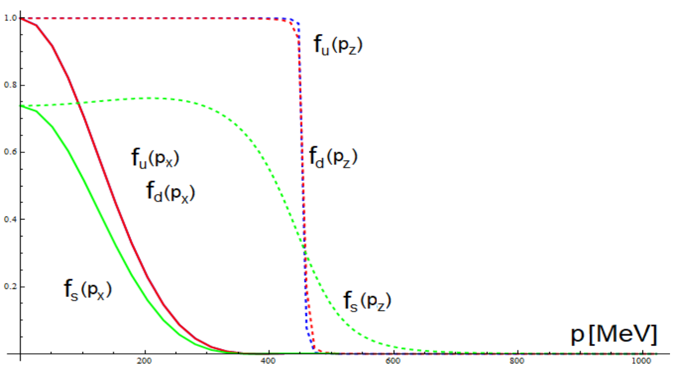


Field model with constant string tension at large values of the Lorentz factor

Field enable time ΔT₁=0.002 fm/c
The duration of a constant field T=2 fm/c
Field Off Time ΔT₂=0.002 fm/c
Total time of field action 2.004 fm/c

$$n_\Sigma = n_u + n_d + n_s = 2.65\ fm^{-3} + 2.65\ fm^{-3} + 2.10\ fm^{-3} = 7.40\ fm^{-3}$$

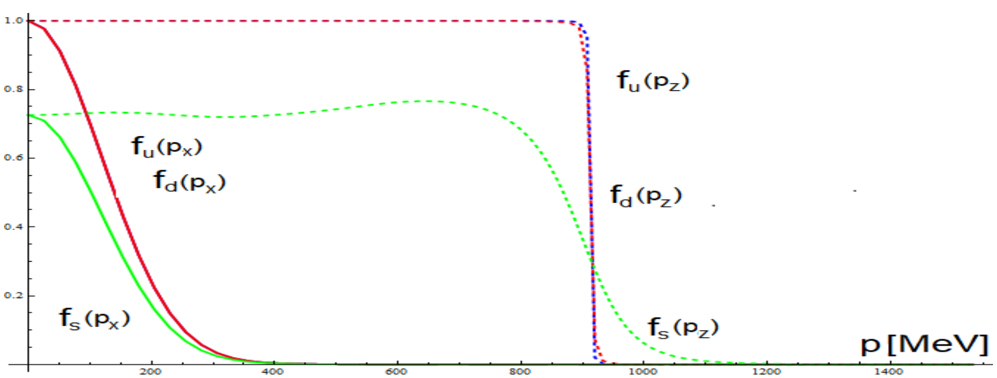
$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 1.99 GeV/fm^3 + 1.99\ GeV/fm^3 + 1.62 GeV/fm^3 = 5.60\ GeV/fm^3$$



Field enable time ΔT₁=0.002 fm/c
The duration of a constant field T=4 fm/c
Field Off Time ΔT₂=0.002 fm/c
Total time of field action 2.004 fm/c

$$n_\Sigma = n_u + n_d + n_s = 4.9\ fm^{-3} + 4.9\ fm^{-3} + 3.8\ fm^{-3} = 13.6\ fm^{-3}$$

$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 7.13 GeV/fm^3 + 7.12\ GeV/fm^3 + 5.62 GeV/fm^3 = 19.87\ GeV/fm^3$$



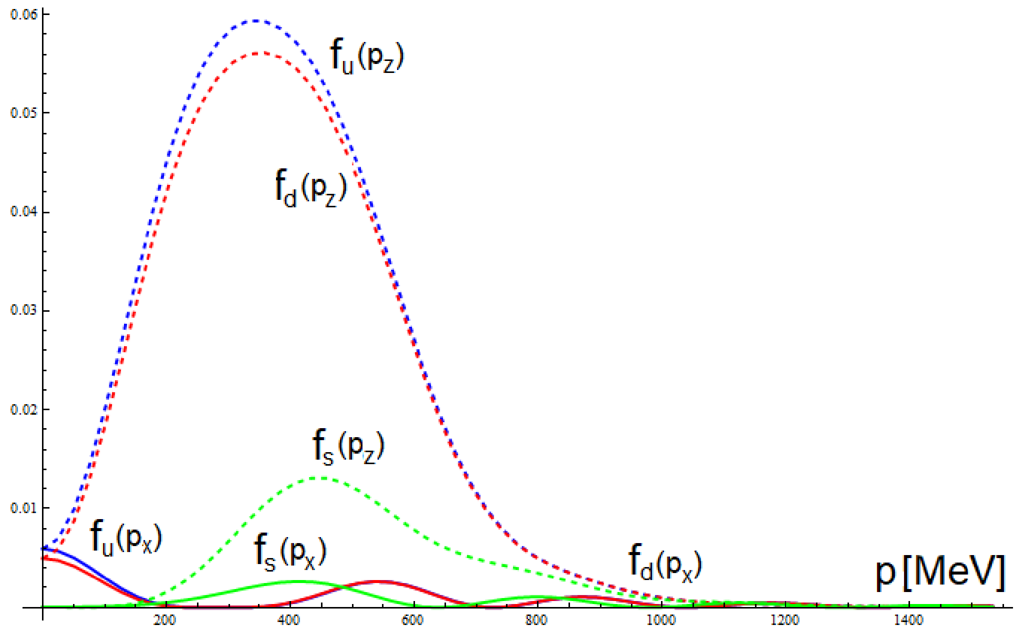
Field model with constant string tension and constituent values for the quark masses

Field enable time ΔT₁=0.2 fm/c
The duration of a constant field T=2 fm/c
Field Off Time ΔT₂=0.2 fm/c
Total time of field action 2.4 fm/c

$M_{current} \rightarrow M_{constituent}$

$$n_\Sigma = n_u + n_d + n_s = 0.48\ fm^{-3} + 0.47\ fm^{-3} + 0.25\ fm^{-3} = 1.20\ fm^{-3}$$

$$\varepsilon_\Sigma = \varepsilon_u + \varepsilon_d + \varepsilon_s = 0.57 GeV/fm^3 + 0.55\ GeV/fm^3 + 0.33 GeV/fm^3 = 1.45\ GeV/fm^3$$



Conclusions.

The presented model is based on nonperturbative kinetic theory in the quasiparticle representation for the abelian projection of QCD. It allows to obtain a system of equations describing the production of different flavors quarks in a strong color field by Schwinger mechanism. The possibility of its use is considered for the process of multiple creation of quarks at the initial stage of the collision of relativistic nuclei.

Using this approach we are able to investigate in detail the dependence of the composition and spectrum of all quark flavors on the dynamics of the formation and evolution of the color field. It is shown that when choosing the parameters of the color field and some assumptions about the characteristics of the produced pairs, it is possible to ensure the proximity of the generated quark plasma to thermal equilibrium. On the other hand, this makes it possible, according to available experimental data, to draw some conclusions about the processes occurring immediately after the primary parton interaction.

It seems correct to take into account these new possibilities when adapting or developing the event generating software for NIKA detectors.

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Thank you for attention!