Memory of D.N.Zubarev,
teacher and friend

Nonperturbative Kinetic Theory of the Field Induced Phase Transition in Graphene

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1. Introduction

Universality in description of the vacuum particle production in some the simplest situations.
Basic kinetic equation (KE)

$$\dot{f}(\bar{p},t) = \frac{1}{2}\lambda(\bar{p},t)\int_{-\infty}^{t} dt'\lambda(\bar{p},t')[1 \mp 2f(\bar{p},t')]\cos\theta(\bar{p};t,t'),$$

$$\theta(\bar{p};t,t') = 2\int_{t'}^{t} d\tau\varepsilon(\bar{p},\tau),$$

$\varepsilon(\bar{p},\tau)$ **- quasienergy.**
Historical reference:

1. **S.Schmidt, D.Blaschke, G.Röpke, S.A.Smolyansky, A.V.Prozorkevich, A.D.Toneev (1998) Standard scalar and spinor QED,** $A^n\ \left(0,0,0,A^3 = A(t)\right)$
   Spin degree of freedom are latent.
   Quasiparticle representation.
2. **I.Bialynichy-Birula, P.Cornicki, J.Rafelski (1991)**
   Standard QED, linear polarized electric field.
   The Wigner representation.

Other systems: $\bar{E} \uparrow\uparrow \bar{H}$
FRW cosmology (scalar, spinor, vector fields)
two zones solid state

2. Formalism

The Dirac-type equation for the low-energy excitations in graphene in the above described time dependent electric field is

$$i\hbar\dot{\Psi}\left(\vec{x},t\right)=v_F\hat{\vec{P}}\vec{\sigma}\Psi\left(\vec{x},t\right),$$

where $\hat{P}_k=-i\hbar\nabla_k-(e/c)A_k(t)$ **is the quasi-momentum** $(k=1,2)$, σ_k **are the Pauli matrices corresponding to the pseudospin structure of graphene.**
The Hamiltonian of the theory

$$H(t)=\frac{i\hbar}{2}\int d^2x\left[\Psi^\dagger(\vec{x},t)\dot{\Psi}(\vec{x},t)-\dot{\Psi}^\dagger(\vec{x},t)\Psi(\vec{x},t)\right]$$

is the 00 component of the corresponding stress-tensor [n+1] and it can be transformed with help of the equation of motion to the form

$$H(t)=v_F\int d^2x\Psi^\dagger(\vec{x},t)\hat{\vec{P}}\vec{\sigma}\Psi(\vec{x},t).$$

Here we dropped the spin indexes.
The wave function here is a two-component spinor permitting the decomposition

$$\Psi^T\left(\vec{x},t\right)=\frac{1}{\left(2\pi\hbar\right)^2}\int d^2p\left(\Psi_{\vec{p}}^{(1)}\left(t\right),\Psi_{-\vec{p}}^{(2)}\left(t\right)\right)e^{i\vec{p}\vec{x}/\hbar}.$$

which translates the Hamiltonian function in the momentum representation.

For the physical interpretation of the model it is appropriate to go over to the quasiparticle representation, where the Hamiltonian of the theory is diagonal.
It is reached with help of the unitarv transformation

$$U^{\dagger}(t)v_F\vec{P}\vec{\sigma}U(t) = \varepsilon(\vec{p},t)\sigma_3 = H_{\vec{p}}(t),$$

and $\Phi = U^{\dagger}\Psi$ **with the unitary matrix**

$$U(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} \exp(-i\varkappa/2) & \exp(-i\varkappa/2) \\ \exp(i\varkappa/2) & -\exp(i\varkappa/2) \end{pmatrix}.$$

The function $\varkappa = P^2/P^1$, $P^k = p^k - (e/c)A^k(t)$ **. The quasienergy** $\varepsilon(\vec{p},t)$ **determined by the dispersion relation in the vicinity of the Dirac points**

$$\varepsilon(\vec{p},t) = v_F\sqrt{P^2} = v_F\sqrt{(P^1)^2 + (P^2)^2}.$$

Then

$$i\hbar\dot{\Phi} = H_{\vec{p}}(t)\Phi + \frac{1}{2}\lambda\hbar\sigma_1\Phi,$$

and

$$\lambda(\vec{p},t) = \dot{\varkappa} = \frac{ev_F^2[E_1P_2 - E_2P_1]}{\varepsilon^2(\vec{p},t)}.$$

Introducing the notation

$$\Phi(\vec{p},t) = \begin{bmatrix} a(\vec{p},t) \\ b^{\dagger}(-\vec{p},t) \end{bmatrix},$$

The Hamiltonian function can be rewritten in the quasiparticle form ($[dp] = d^2p(2\pi\hbar)^{-2}$)

$$H(t) = \int [dp]\varepsilon(\vec{p},t)\Phi^{\dagger}(\vec{p},t)\,\sigma_3\Phi(\vec{p},t)| = \int [dp]\varepsilon(\vec{p},t)\left[a^{\dagger}(\vec{p},t)a(\vec{p},t) - b(-\vec{p},t)b^{\dagger}(-\vec{p},t)\right]$$

At this stage one can go over to the occupation number representation and replace the amplitudes $a^{\dagger}(t)$,$a(t)$ and $b^{\dagger}(t)$, $b(t)$ by the corresponding creation and annihilation operators for electrons and holes considered as quasiparticles.
These operators are defined on the in-vacuum state $|\mathrm{in}\rangle$ with vector potential $\vec{A}_{\mathrm{in}}$ and satisfy the canonical anti-commutation relations (other elementary anti-commutators are equal to zero):

$$\begin{aligned}\left\{a(\vec{p},t),a^{\dagger}(\vec{p}',t)\right\}_{+} &= \left\{b(\vec{p},t),b^{\dagger}(\vec{p}',t)\right\}_{+}\\ &= (2\pi)^{2}\delta(\vec{p}-\vec{p}').\end{aligned}$$

The equations of motion of the Heisenberg type for the description of the unitary evolution of the creation and annihilation operators:

$$\begin{aligned}\dot{a}(\vec{p},t) &= \frac{i}{\hbar}\left[H(t),a(\vec{p},t)\right]-\frac{i}{2}\lambda\left(\vec{p},t\right)b^{+}(-\vec{p},t)\\ &= \frac{i}{\hbar}\left[H_{tot}(t),a(\vec{p},t)\right],\\ \dot{b}(\vec{p},t) &= \frac{i}{\hbar}\left[H(t),b(-\vec{p},t)\right]+\frac{i}{2}\lambda\left(\vec{p},t\right)a^{+}(\vec{p},t)\\ &= \frac{i}{\hbar}\left[H_{tot}(t),b(-\vec{p},t)\right],\end{aligned}$$

The Fock space is constructed on the time dependent vacuum state.

$$H_{tot}=H+H_{pol},$$

$$H_{pol}(t) = \frac{\hbar}{2}\int[dp]\lambda(\vec{p},t)[a^{\dagger}(\vec{p},t)b^{\dagger}(-\vec{p},t) - b(-\vec{p},t)a(\vec{p},t)]$$

describes the dynamics of vacuum polarization.

Now one can obtain the KE. Let us introduce the distribution functions for the electrons and the holes,

$$f^e(\vec{p},t) = \langle \mathrm{in}|a^+(\vec{p},t)a(\vec{p},t)|\mathrm{in}\rangle,$$
$$f^h(\vec{p},t) = \langle \mathrm{in}|b^+(-\vec{p},t)b(-\vec{p},t)|\mathrm{in}\rangle.$$

The averaging procedure here is carried out under the in-vacuum state $|in>$. Differentiation with respect to time results in

$$\dot{f}^e(\vec{p},t) = \frac{i\lambda}{2}(\vec{p},t)\left\{f^{(+)}(\vec{p},t) - f^{(-)}(\vec{p},t)\right\},$$

where anomalous averages have been introduced

$$f^{(+)}(\vec{p},t) = \langle \mathrm{in}|a^+(\vec{p},t)b^+(-\vec{p},t)|\mathrm{in}\rangle,$$
$$f^{(-)}(\vec{p},t) = \langle \mathrm{in}|b(-\vec{p},t)a(\vec{p},t)|\mathrm{in}\rangle\ .$$

The equations of motion for these functions have the form

$$\dot{f}^{(+)}(\vec{p},t) = \frac{2i}{\hbar}\varepsilon(\vec{p},t)f^{(+)}(\vec{p},t) - \frac{i\lambda(\vec{p},t)}{2}[1-2f(\vec{p},t)]$$
$$\dot{f}^{(-)}(\vec{p},t) = \frac{-2i}{\hbar}\varepsilon(\vec{p},t)f^{(-)}(\vec{p},t) + \frac{i\lambda(\vec{p},t)}{2}[1-2f(\vec{p},t)].$$

Here it was assumed that $f^e = f^h = f$ holds as a consequence of the electroneutrality condition.

Final integral form of the KE of non-Markovian type:

$$\dot{f}(\vec{p},t) = \frac{1}{2}\lambda(\vec{p},t)\int\limits_{t_0}^{t} dt'\lambda(\vec{p},t')\left[1-2f(\vec{p},t')\right]\cos\theta(t,t'),$$

where

$$\theta(t,t') = \frac{2}{\hbar}\int\limits_{t'}^{t} dt''\varepsilon(\vec{p},t'').$$

This KE is written for zero initial condition, $f(t_0) = 0.$ For the first time a KE of such type was obtained in the QED for the description of vacuum creation of electron-positron pairs under action of a time dependent spatially homogeneous linearly polarized electric field. This method is based on the usage of unitary nonequivalent canonical transformations for the transition to the quasiparticle representation. In the considered situation this approach is applicable and leads to the KE that has the same mathematical structure as in the D=3+1 QED case.
An advantage of the unitary approach is also the possibility of a generalization of this method to the case of a two-dimensional electric field with the vector potential $A^k(t)(k = 1,2)$.

Let us remark that the transition from the one-dimensional electric field (linear polarization) to two or three field dimensions (arbitrary polarization) in D=3+1 QED is connected with the necessity to take into account a larger number of spin degrees of freedom and is accompanied with a significant increase in the number of necessary KE's.
The main feature consider the KE is the absence of an energy gap in the quasienergy. Such kind of models have been investigated long ago. Below this feature will be investigated in the situation when the e-h-system in graphene is exposed to a time dependent electric field. In the presence of the external field the Dirac points $\varepsilon_0(p) = 0$ re transformed to a family of Dirac lines which depend parametrically on time.

For the numerical analysis of the KE for different field models it is appropriate to rewrite it in the form of an equivalent system of ordinary differential equations

$$\dot{f} = \frac{1}{2}\lambda u, \quad \dot{u} = \lambda\left(1 - 2f\right) - \frac{2\varepsilon}{\hbar}v, \quad \dot{v} = \frac{2\varepsilon}{\hbar}u,$$

with the corresponding initial conditions $f(t_0) = u(t_0) = v(t_0) = 0.$ **The auxiliary functions u v describe polarization effects and can be expressed via the anomalous averages:**

$$u = \frac{i}{2}\left[f^{(+)} - f^{(-)}\right], v = \frac{1}{2}\left[f^{(+)} + f^{(-)}\right].$$

For this system of equations one readily obtains the integral of motion

$$(1 - 2f)^2 + u^2 + v^2 = 1,$$

which is compatible with the initial conditions.

3. Observables

It is straightforward to write expressions for the pair number density

$$n(t) \;=\; N \int [dp] f(\vec{p}, t).$$

The factor N corresponds to number of species (or flavors) of quasiparticles in graphene: N = 4 in the low energy model and N = 2 in the tight binding model.
The current density consists of two components, the conductivity and polarization current densities,

$$j_k(t) = j_k^{\rm cond}(t) + j_k^{\rm pol}(t).$$

These currents are defined by the distribution function f and the polarization function u, correspondingly. Firstly we consider currents in the low-energy model. On the basis of the standard definition of the current density

$$j_k(t) \;=\; -e\frac{\delta H(t)}{\delta A_k(t)}$$

one can obtain from Hamiltonian with taken into account of the flavor number

$$j_k(t) \;=\; 4ev_F \int d^2x \Psi^* \left(\vec{x}, t\right) \sigma_k \Psi \left(\vec{x}, t\right).$$

Going over to the quasiparticle representation with help of the unitary operator we obtain

$$j_k(t) = 4ev_F \int [dp] \Phi^\dagger \left(\vec{p}, t\right) U^\dagger(t) \sigma_k U(t) \Phi \left(\vec{p}, t\right).$$

This expression can be divided into the conductivity and polarization currents.

$$j_i^{\rm cond}(t) = 8ev_F^2 \int [dp] f(\vec{p},t) \frac{P_i}{\varepsilon(\vec{p},t)},$$

$$j_i^{\rm pol}(t) = 8ev_F^2 \int [dp] u(\vec{p},t) \frac{1}{\varepsilon(\vec{p},t)} \begin{cases} -P_2, i=1, \\ P_1, i=2. \end{cases}$$

In weak fields in the Markov approximation one can show that the polarization effects dominate

$$|j^{pol}(t)| \gg |j^{cond}(t)|$$

This conclusion is corroborated also by direct numerical calculations.
Let us note, that the conductivity and polarization currents are not collinear in general case. In the case of the linear polarized electric field collinearity of the currents rebuilds.

Function u determines vacuum polarization current. Physical meaning of the other polarization function v is revealed, if to write the total energy density of quasiparticles including the polarization energy:

$$E_{tot} = E + E_{pol},$$

where

$$E(t) = 4 \int [dp] \varepsilon(\vec{p},t) f(\vec{p},t),$$

$$E_{pol}(t) = 4 \int [dp] \hbar\lambda(\vec{p},t) v(\vec{p},t).$$

3. Order parameter

Specific features in description of the vacuum particle creation^

1) **t – dependent vacuum state (t –non invariant vacuum show)**
2) **Vacuum polarization effects described by the $< a^{+}b^{+} >, < ba >$ and the corresponding vacuum polarization function u, v.**

It has been put forward to use these functions as the universal order parameter in the combination $\Phi = u + iv$ for description of the vacuum production.

We write the corresponding equation of motion

$$\dot{\Phi} - \frac{2i\varepsilon}{\hbar}\Phi = \lambda(1-2f)$$

The formal solution of this equation with the zero initial condition is

$$\Phi(t) = \int_{t_0}^{t} dt' \lambda(t') \left[1 - 2f(t')\right] \exp\left[\frac{2i}{\hbar}\int_{t_,}^{t} d\tau \varepsilon(\tau)\right].$$

Let us consider now a finite electric field which is shut down at the point of time t_{off}, i.e. $E(t > t_{off}) = 0$ and hence $\lambda(t > t_{off}) = 0$. Then, for $t > t_{off}$ it follows that the order parameter is different from zero and oscillates with the frequency $2\,\varepsilon_{out}/\hbar$, i.e.

$$\Phi(t > t_{off}) = \Phi_{out}(\vec{p}) \left[\frac{2i\varepsilon_{out}}{\hbar}(t - t_{off})\right],$$

where the asymptotical value of the quasienergy

$$\varepsilon_{out} = \varepsilon(t \to \infty) = v_F \sqrt{(\vec{p} - \frac{e}{c}\vec{A}_{out})^2}, \quad A^k_{out} = \lim_{t\to\infty} A^k(t).$$

Statement

$$\Phi_{out}(\vec{p}) = \int_{t_0}^{t_{off}} dt' \lambda(t') \left[1 - 2f(t')\right] \exp\left[\frac{2i}{\hbar} \int_{t_,}^{t_{off}} d\tau \varepsilon(\tau)\right]$$

is defined by all prehistory of the system evolution in a given external field.
The presence of such residual oscillations of the order parameter is a prerequisite of analogical behavior of the polarization current. Thus,

$$|\Phi(t > t_{off})|^2 = |\Phi_{out}(\vec{p})|^2 = const$$

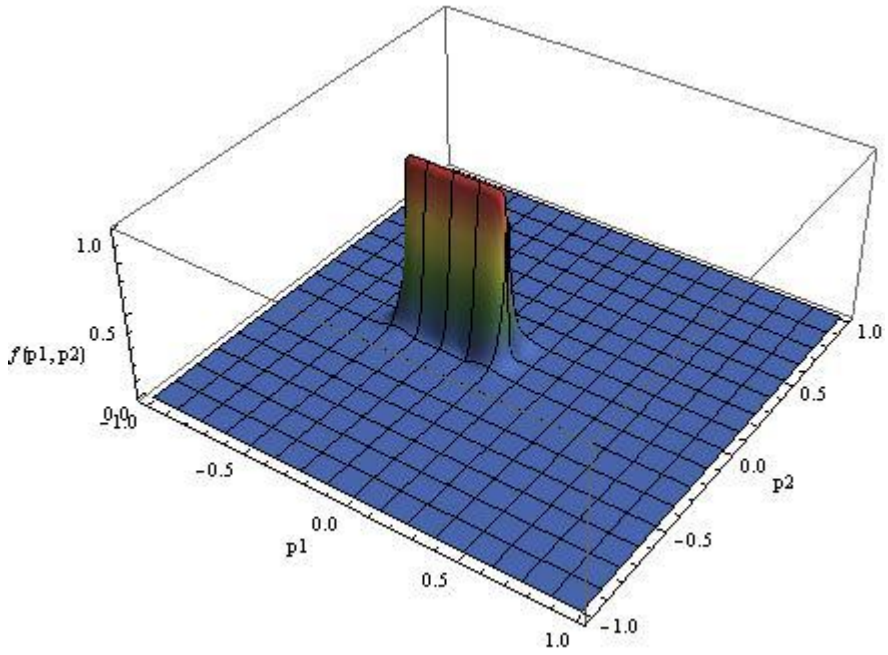
after switching off the external field, i.e. the long-range order is formed.
The amplitude of oscillations of the order parameter in the residual state can be defined from the integral of motion rewriting in the form

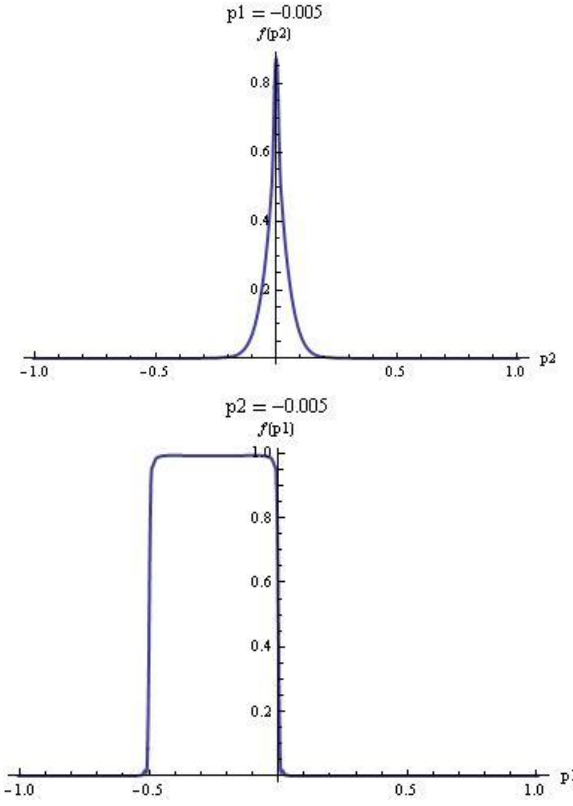
$$(1 - 2f_{out})^2 + |\Phi_{out}|^2 = 1.$$

The order parameter is among of characteristic fields considered as an peculiar field induced phase transition.

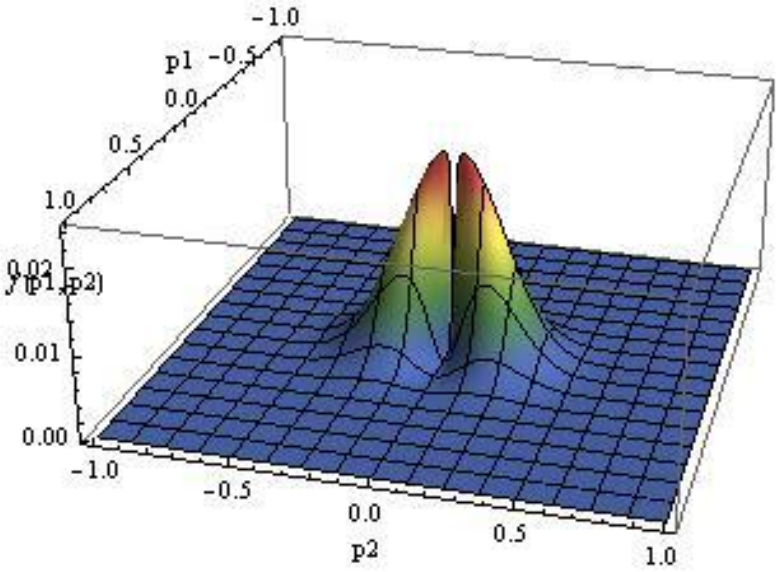
5. Numerical results
5.1 Distribution unction

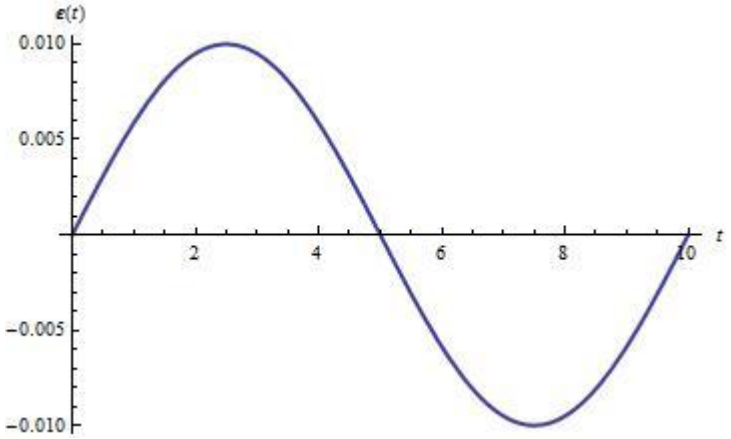
We start with the case of a constant field. The figures shows the distribution function of carriers formed as a result of the action of a strong constant field ~1000000 V/cm along the axes x1 in the course of ~10^-14 s

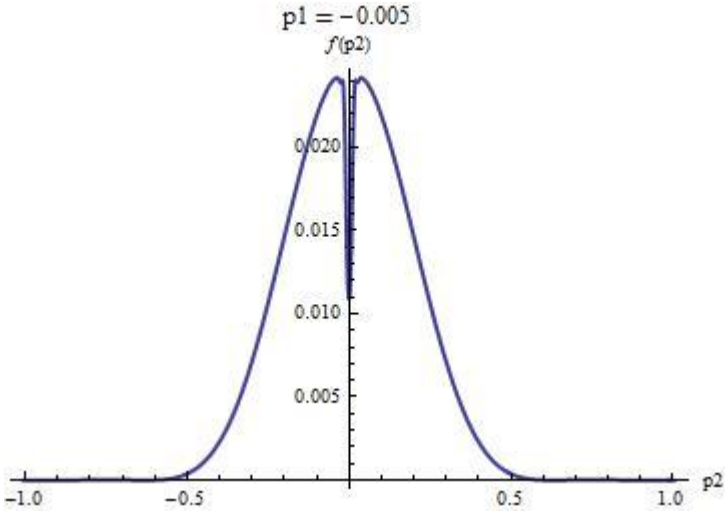




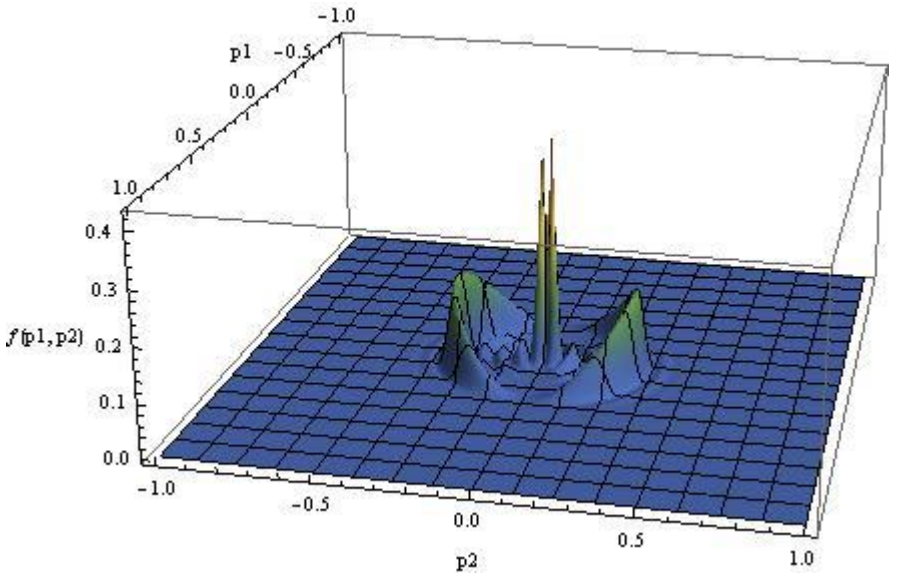
Here is the result of the action of one harmonic bipolar pulse.
The amplitude and direction of the field is the same as in the previous case.
The pulse duration is 20% of the duration of the action of the constant field in the previous case.

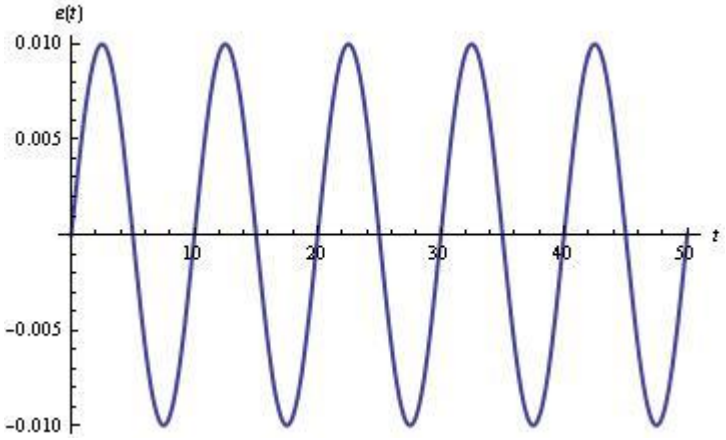


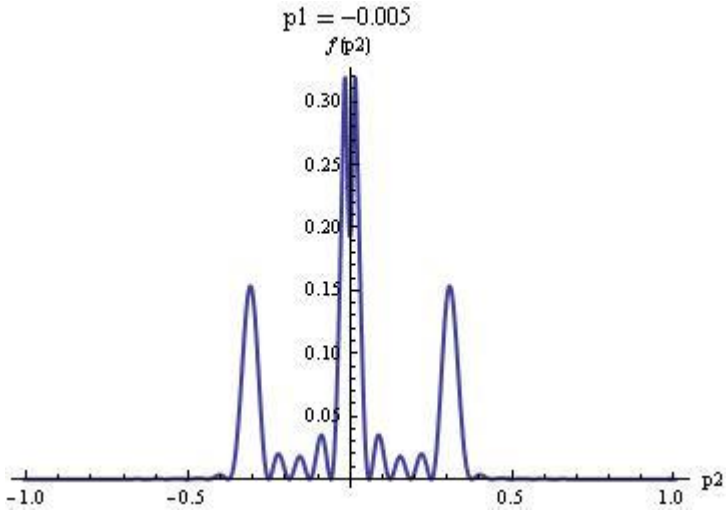




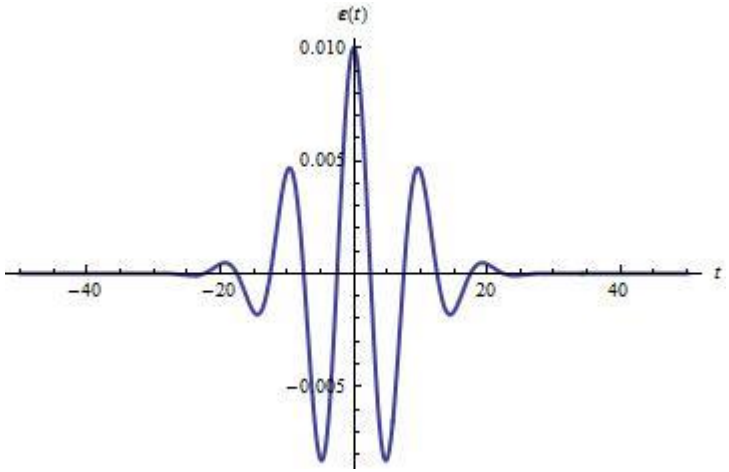
Increase the exposure time.
Now it's five full periods.
The maximum values of the distribution function increased approximately fifteen times.
There is a redistribution of carriers in momentum space.
They concentrate on a circle equidistant from the Dirac point.
But the distribution as before anisotropic.
The direction along the active field is preferable.

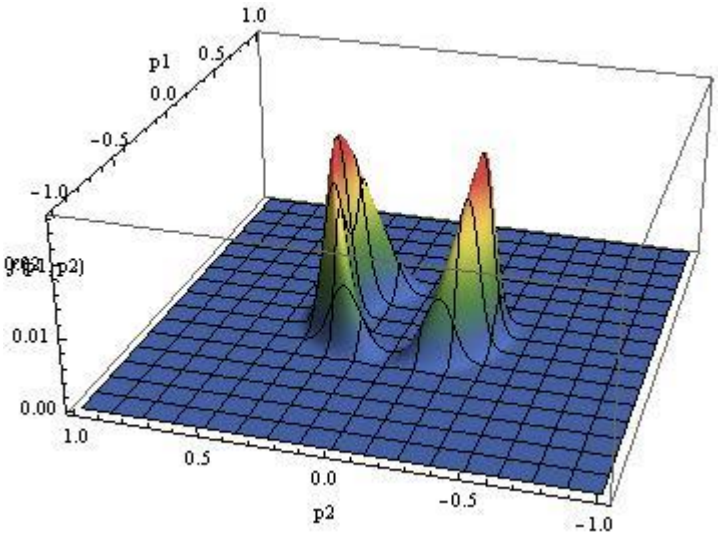


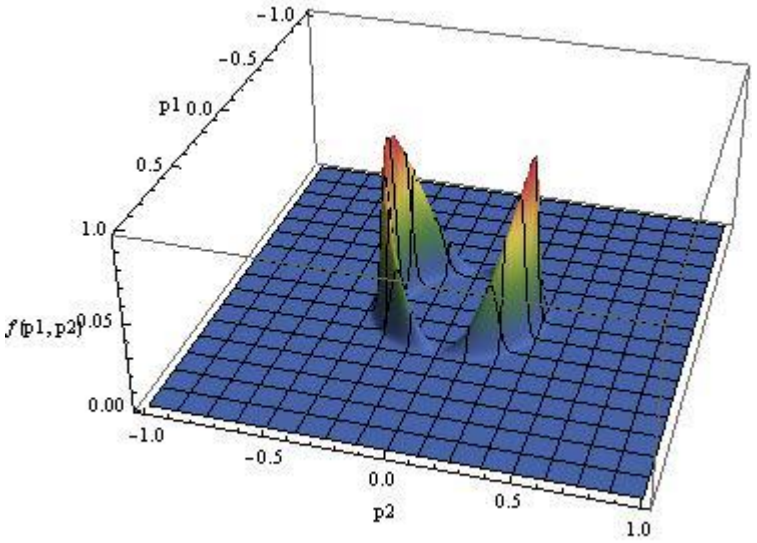




The frequency and amplitude of the electric field oscillations have been preserved, but we will format the pulse with Gaussian cutoff symmetrically around the maximum value.
The presented dependence of the field on time and the upper 3D figure correspond to the Gaussian cutoff parameter σ = 5 ($\sigma = \omega\tau$).
For the bottom 3D figure σ = 10.







The above results were obtained for a constant direction of the electric field.
In the 2 + 1 dimensional model under consideration, both components of the field enter equally into KE.
This allows us not to limit ourselves to the study of fields that preserve the direction of their action.
We can look at the result of the action of the field with changing magnitude and direction of the action.
Or consider a field of constant magnitude with a rotating direction of action.
The latter variant corresponds to a fall on a graphene layer of a circularly polarized wave.
To complete the picture, let us trace completely the transition from linear to circular polarization. To do this, we define the components of the electric field in the form:

$$\begin{aligned} E_x(t) &= E_a \cos(\omega t) \exp(-t^2/2\tau^2), \\ E_y(t) &= E_a \cos(\omega t - \phi) \exp(-t^2/2\tau^2) \end{aligned}$$

Then by changing the value of ϕ from zero to half of Pi, we can smoothly go from linear polarization to circular. Gaussian trimming is designed for a realistic model.

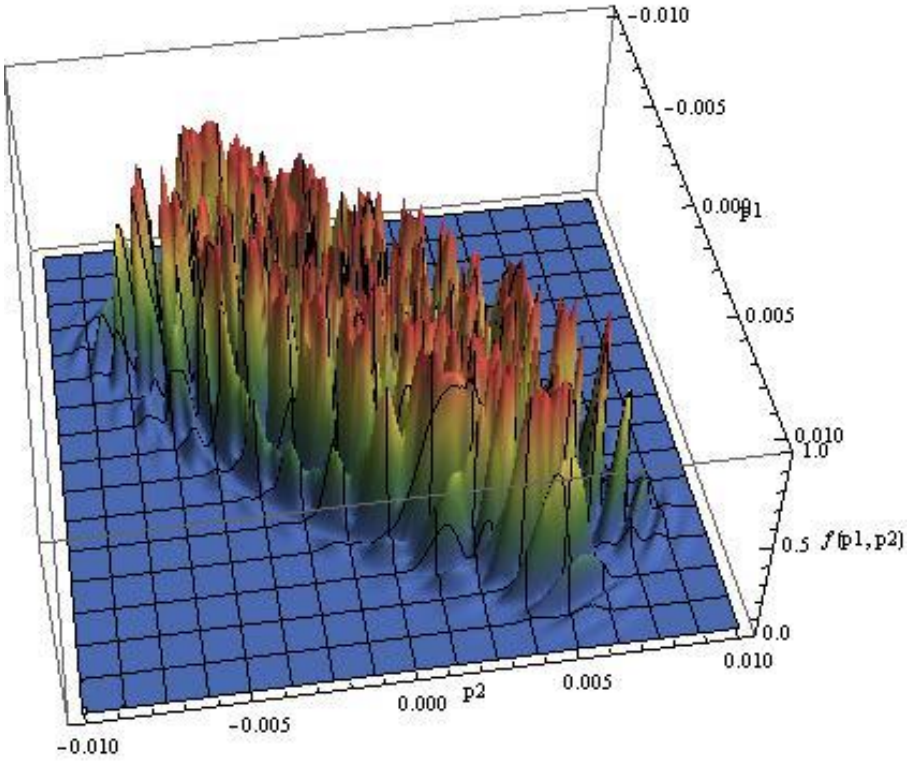
The results for the field with the following parameters are presented:

Frequency $\omega/2\pi$ **= 1 THz.**

The amplitude of each component 1000 V/cm.

Gaussian cutoff parameter $\sigma = \omega\tau$ **= 10.**

These are quite realistic parameters.



The figure shows the form of the distribution function for ϕ **= 0. This is linear polarization. The field is directed diagonally between the x and y axes and does not change its direction in time. The direction indicated by the polarization of the field is also represented in the momentum space. Although the structure of the distribution function is complex, the diagonal direction is clearly visible. We note that for the chosen pulse parameters the distribution function is concentrated in the immediate vicinity of the Dirac point. In such conditions, the applicability of the model used is undeniable.**

Already at

$$\phi = 0.3\pi/2$$

the distribution function has noticeable changes (upper figure).

The lower figure is the result of the action of a pulse with circular polarization at

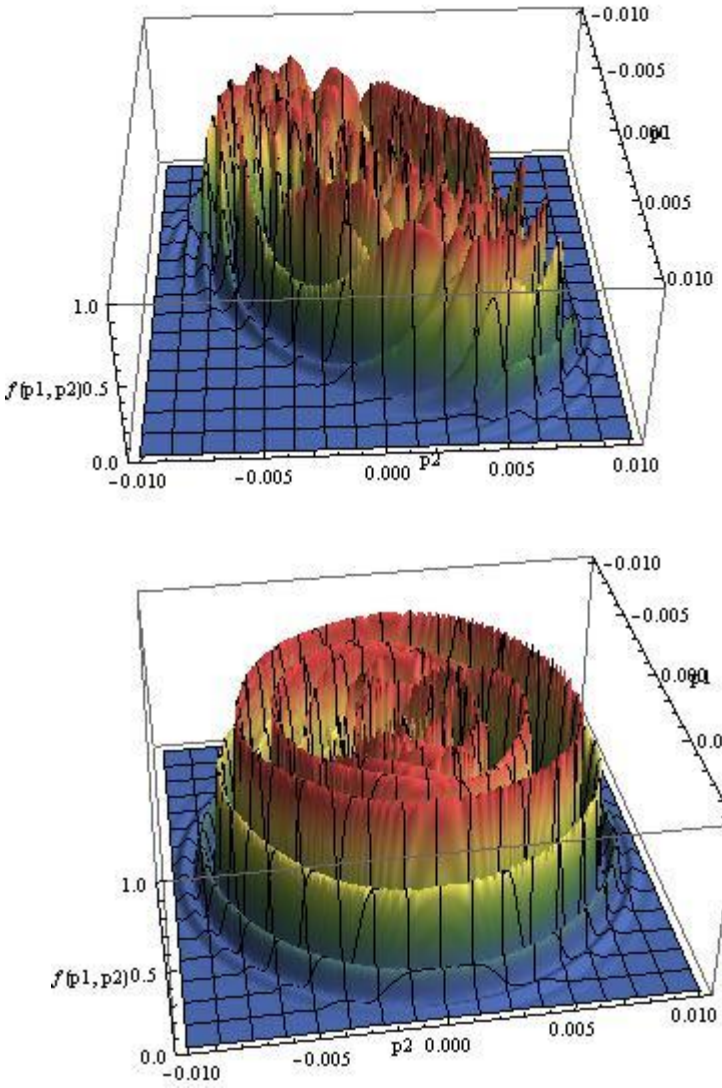
$$\phi = \pi/2$$

Another type of symmetry of the active field is reflected in the momentum space. A radial symmetry of the distribution function around the Dirac point is observed.

This symmetry is not strict. The reason is the presence of a Gaussian cutoff, which leads to a non-equivalence of the evolution of the field components in the presence of a phase shift between them.

The localization of the distribution function in the neighborhood of the Dirac point is preserved.

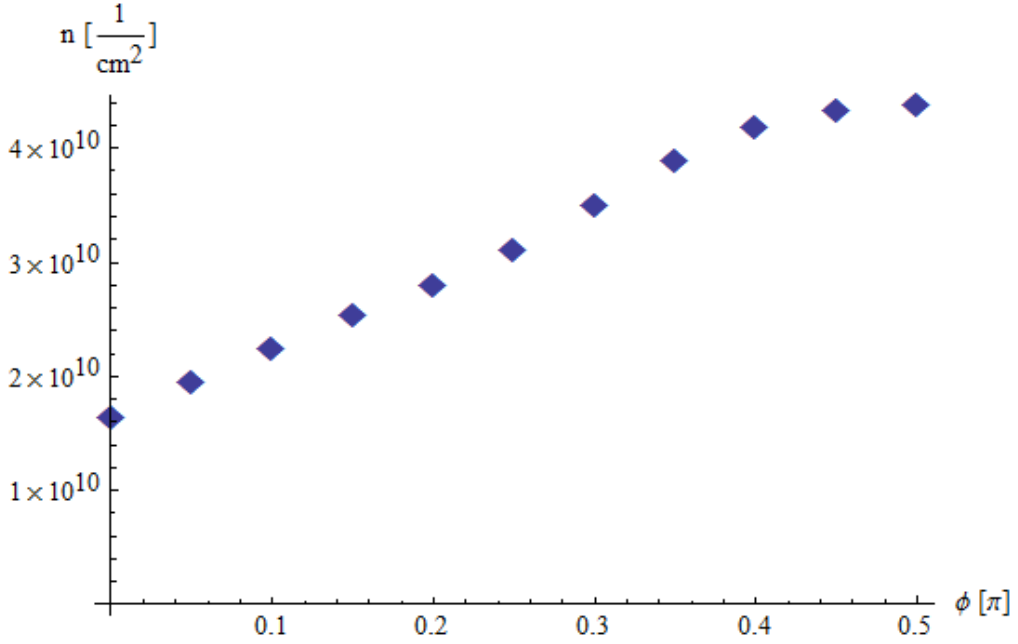
In all the cases considered, the maximum values of the distribution function reach saturation.



It should be noted that similar calculations are extremely difficult in the standard QED. In the case under consideration the presence of detailed information about the distribution function of carriers makes it possible to determine the observed characteristics of the model. One of the most important is the density of carriers:

$$n = \frac{4}{(2\pi\hbar)^2}\int d^2p f(\vec{p})$$

We present here the results of calculations of the dependence of the density of residual carriers on the phase difference between the electric field components E_x and E_y in the interval $0 \le \phi \le \pi/2$. The remaining parameters are the same. The transition to "circular" polarization increases the production efficiency of carriers by about 2.7 times.



5.2 Residual currents

Above we showed the distribution function in the final state after the termination of an external electric field. Hence in this case we are talking about the residual density of carriers. I recall that in this model we use as the initial state a vacuum in which carriers are absent. All carriers in the final state are created during the time of the field action.

But the dynamics of the processes during the action of the field may also be of interest. First of all, this refers to currents generated by an external electric field. The currents can be related both to the motion of the created carriers and to the polarization of the physical vacuum in the external field:

$$j_i^{\rm cond}(t) = \frac{8ev_F}{(2\pi\hbar)^2}\int d^2p\ f(\vec{p},t)\begin{cases} v_F P_1/\varepsilon, i=1 \\ v_F P_2/\varepsilon, i=2 \end{cases}$$

$$j_i^{\rm pol}(t) = \frac{8ev_F}{(2\pi\hbar)^2}\int d^2p\ u(\vec{p},t)\begin{cases} -v_F P_2/\varepsilon, i=1 \\ v_F P_1/\varepsilon, i=2 \end{cases}$$

For clarity, we consider a unipolar super-Gaussian pulse.
Such a pulse should generate a certain number of charge carriers with an asymmetric distribution in momentum space. What should give some final value of the conduction current after the field is complete. Since there is no dissipation in our model (ballistic conductivity), after removal of the field, the conduction current must retain its value.
Of particular interest is the behavior of the polarization current.

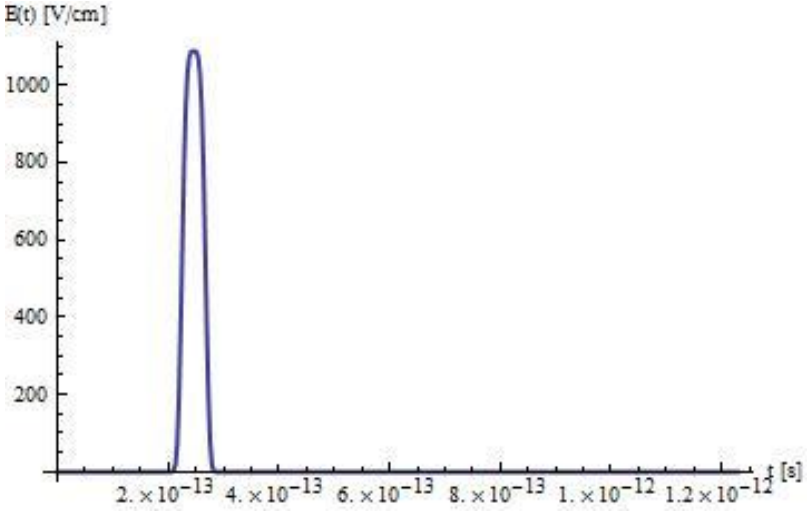
Consider super-Gaussian pulse with a coefficient n = 4 and:

$$E_0 = 1.088 * 10^3 \ {}^{V}/_{cm}$$

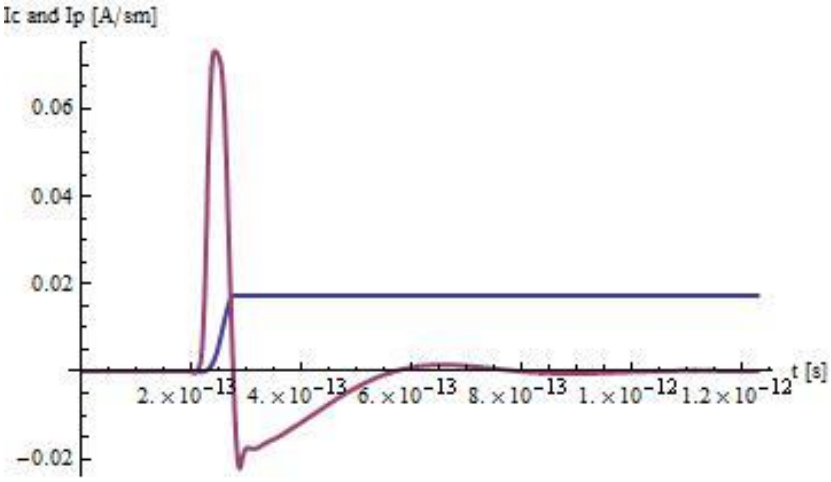
$$t_m = 2.46 * 10^{-13} s$$

$$\tau = 1.23 * 10^{-13} s$$

(upper figure)



In the next figure, the blue line shows the behavior of the conduction current. It is quite expected.
The behavior of the polarization current is indicated by a purple line.
It is not so trivial.
This demonstrates the complexity of the processes of the reaction of a physical vacuum to the action of an external electric field.



6. Tight binding model

It is not difficult to obtain now the analogous KE in the D = 2 + 1 tight binding model of the nearest neighbor interaction. The Hamiltonian function in the moment representation is equal in this case

$$H_{\vec{p}}(t) = \begin{pmatrix} 0 & h_{\vec{p}}(t) \\ h^*_{\vec{p}}(t) & 0 \end{pmatrix} = h'_{\vec{p}}(t)\sigma_1 - h''_{\vec{p}}(t)\sigma_2,$$

where

$$h_{\vec{p}}(t) = h'_{\vec{p}}(t) + ih''_{\vec{p}}(t) = -\gamma \sum_{\alpha} \exp \frac{i}{\hbar}\vec{P}\vec{\delta}_{\alpha} \quad ,$$

with $\gamma \approx 2.7\ eV$ is the hopping energy, and

$$\vec{\delta}_1 = \frac{a}{3}(0, \sqrt{3}),\ \vec{\delta}_2 = \frac{a}{3}(\pm 3/2, -\sqrt{3}/2)$$

A the locations of the nearest neighbors, $a \approx 3$ Å .
An external electric field is introduced here according to the rule

$$\vec{p} \rightarrow \vec{P} = \vec{p} - e/c\vec{A}(t).$$

Such theory belongs to the class of theories with the highest derivatives.
Defined here Hamiltonian function have the same pseudospin structure as in low energy model. It allows to conserve also the way of derivation of KE. The quasienergy and the amplitude of the transitions undergo change only with the following formal substitutions:

$$\varepsilon(\vec{p}, t) = \sqrt{h^*_{\vec{p}}(t)h_{\vec{p}}(t)} = |h_{\vec{p}}(t)|, \qquad \lambda(\vec{p}, t) = \frac{1}{|h_{\vec{p}}(t)|^2}\left\{\dot{h}''_{\vec{p}}(t)h'_{\vec{p}}(t) - \dot{h}'_{\vec{p}}(t)h''_{\vec{p}}(t)\right\}.$$

Analogical KE can be obtained also for the case of multilayer graphene model.

The conductivity and polarization currents have the following form (the flavor number in the given model is equal to 2):

$$j_k^{cond}(t) = -4e\gamma \int [dp] f(\vec{p},t)[F_k^{(1)}(\vec{p},t)\cos\chi \ + F_k^{(2)}(\vec{p},t)\sin\chi],$$

$$j_k^{pol}(t) = -4e\gamma \int [dp] f(\vec{p},t)[-F_k^{(1)}(\vec{p},t)\sin\chi \ + F_k^{(2)}(\vec{p},t)\cos\chi]$$

with the vector form factors

$$F_k^{(1)}(\vec{p},t) = \sum_\alpha \delta_\alpha^{(k)} \sin\,(\frac{1}{\hbar}\vec{P}\vec{\delta}_\alpha),$$

$$F_k^{(2)}(\vec{p},t) = \sum_\alpha \delta_\alpha^{(k)} \cos\,(\frac{1}{\hbar}\vec{P}\vec{\delta}_\alpha)$$

and χ is angle of the unitary rotation in the matrix of the type

$$\chi = -h''_{\vec{p}}(t)/h'_{\vec{p}}(t).$$

Taking this into account, we obtain expressions

$$j_k^{cond}(t) = -4e\gamma \int [dp] f(\vec{p},t) \sum_\alpha \delta_\alpha^{(k)} \sin\,(\chi + \frac{1}{\hbar}\vec{P}\vec{\delta}_\alpha),$$

$$j_k^{pol}(t) = -4e\gamma \int [dp] u(\vec{p},t) \sum_\alpha \delta_\alpha^{(k)} \cos\,(\chi + \frac{1}{\hbar}\vec{P}\vec{\delta}_\alpha).$$

7. Conclusion

1) **The kinetic theory of the eh – excitations in graphene has been constructed in the frameworks of the low energy and tight binding models.**

2) **The obtained KE's belong to the general class of the strong field theories described by the basic KE.**

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THANK YOU
FOR ATTENTION