Quantum statistical approach to permittivity of metallic plasmas

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Motivation of the work:

- Femtosecond optical measurements of complex reflection coefficient or coefficient of self-absorption is effective tool for WDM & ultrafast processes diagnostic [1]!
- Interpretation of modern experiments on intense energy fluxes action on matter requires to know WDM permittivity $\varepsilon(\omega)$ in wide range of frequencies: from $\omega \to 0$ till X-ray, wide range of e^- & ion temperatures T_e, T_i, wide range of densities ϱ.
- QS operator approach can be used to derive model for $\varepsilon(\omega)$ both for relatively high $(T_i \gg T_{melt})$ [2] and relatively low $(T_i \leq T_{melt})$ [3] ion temperatures

[1]Agranat M B, Andreev N E, Ashitkov S I, Veysman M E et al, JETP Lett. **85**, 271 (2007)
[2]M. Veysman, G. Röpke, M. Winkel, H. Reinholz, PRE **94**, 013203 (2016).
[3]M. Veysman, G. Röpke, H. Reinholz, J. Phys. Conf. Ser. **946**, 012012 (2018); ibid, to be published (2018-2019)

Motivation of the work:

Wide-range over $\omega, T_e, T_i, \varrho$ model for permittivity $\varepsilon(\omega)$ is necessary !

Figure from [Price D F et al, PRL **75**, 252 (1995)]:

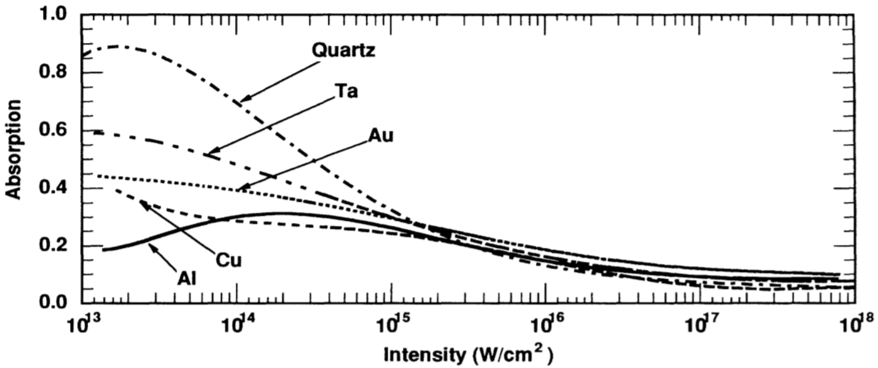

FIG. 1. Absorption fraction vs peak laser intensity for aluminum, copper, gold, tantalum, and quartz targets. In Figs. 1, 3, 4, and 5 laser intensity is the temporal and spatial peak value of the laser intensity.

Linear response theory with *interband transitions*

① **relevant statistical operator**, introduced as generalized Gibbs ensemble, derived from the principle of maximum of entropy:

$$\hat{\rho}_{rel}(t) = Z_{rel}(t)^{-1} \exp\left[-\beta(\hat{H} - \mu\hat{N}) + \sum_n F_n(t)\hat{B}_n\right],$$

$$Z_{rel}(t) = \mathrm{Tr}\left[-\beta(\hat{H} - \mu\hat{N}) + \sum_n F_n(t)\hat{B}_n\right],$$

Lagrange parameters β, μ and $F_n(t)$ are introduced to fix given averages:

$$\mathrm{Tr}\left\{\hat{B}_n\rho(t)\right\} = \langle\hat{B}_n\rangle = \mathrm{Tr}\left\{\hat{B}_n\rho_{rel}(t)\right\},$$

$\{\boldsymbol{B}_n\}, n = 1\ldots N$ is the chosen set of observables in the form of **momentum of density matrix** **for different electron levels**:

$$\boldsymbol{B}_n = \boldsymbol{P}_n = \sum_{\boldsymbol{p}} \hbar\boldsymbol{p}_n n_{\boldsymbol{p},n},$$

where $n_{\boldsymbol{p},n} = a^{+}_{\boldsymbol{p},n}a_{\boldsymbol{p},n}$, $\boldsymbol{p}_n = m\partial E_{p,n}/\partial p$, $E_{p,n} = \boldsymbol{p}^2/(2m_n) + E_n$

Linear response theory with *interband transitions*

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$$Z_{rel}(t) = \mathrm{Tr}\left[-\beta(\hat{H} - \mu\hat{N}) + \sum_n F_n(t)\hat{B}_n\right],$$

Note: more general case, with L-th moments of density operator: $\{\boldsymbol{B}_n^L\}, n = 1 \dots N$ is the chosen set of observables in the form of **momentum of density matrix for different electron levels**:

$$\boxed{\boldsymbol{B}_n^L = \boldsymbol{P}_n^L = \sum_{\boldsymbol{p}} \hbar \boldsymbol{p}_n (E_{p,n}/T)^{(L-1)/2} n_{\boldsymbol{p},n}},$$

where $n_{\boldsymbol{p},n} = a^+_{\boldsymbol{p},n} a_{\boldsymbol{p},n}$, $\boldsymbol{p}_n = m\partial E_{p,n}/\partial p$, $E_{p,n} = \boldsymbol{p}^2/(2m_n) + E_n$

Linear response theory with *interband transitions*

② Non-equilibrium statistical operator $\hat{\rho}(t)$ is determined by the dynamical evolution of the system with Hamiltonian $\hat{H}_{tot} = \hat{H} + \hat{H}_{ext}(t)$ and relevant statistical operator $\hat{\rho}_{rel}(t)$

$$\hat{\rho}(t) = \lim_{\epsilon \to 0} \epsilon \int_{-\infty}^{t} dt' e^{-\epsilon(t-t')} \hat{U}(t,t') \rho_{rel}(t') \hat{U}^{\dagger}(t,t'),$$

$$i\hbar \partial_t \hat{U}(t,t') = \hat{H}_{tot} \hat{U}(t,t'), \qquad \hat{H}_{tot} = \hat{H} + \hat{H}_{ext},$$

$$\hat{H}_{ext}(t) = -e\hat{\boldsymbol{R}}\boldsymbol{E}(t),\ \hat{\boldsymbol{R}} = \sum\nolimits_i \hat{\boldsymbol{r}}_i,\ \dot{\hat{\boldsymbol{R}}} = \sum\nolimits_{n} \hat{\boldsymbol{P}}_{n}/m.$$

If response parameters $F_n(t)$ are small $\Rightarrow$ expansion of $\hat{\rho}_{rel}(t)$ and $\hat{\rho}_{irrel}(t) = \hat{\rho}(t) - \hat{\rho}_{rel}(t)$ over $F_n(t)$ $\Rightarrow$ linear response equations

$$\boxed{\sum_m \left[\mathfrak{C}_{nm} - i\omega_* \eta_{nm} \right] \mathcal{F}_m = \sum_m \eta_{nm}},$$

$$\boxed{\langle \hat{P}_n \rangle = eE/\omega_{a.u.} \sum\nolimits_m \eta_{nm} \mathcal{F}_m}$$

for dimensionless response parameters $\mathcal{F}_m$ and average momentums $\langle \hat{P}_n \rangle$, **expressed in terms of dimensionless correlation functions:**

dimensionless response parameters $\mathcal{F}_m$ and correlation functions η_{nm}, $\mathfrak{C}_{nm}$:

$$\mathcal{F}_m = F_m \frac{mT\omega_{a.u.}}{e\boldsymbol{E}},\ \eta_{nm} = \frac{(\hat{\boldsymbol{P}}_n;\hat{\boldsymbol{P}}_m)}{mT},\ \ \mathfrak{C}_{nm} = \frac{\langle\dot{\hat{\boldsymbol{P}}}_n;\dot{\hat{\boldsymbol{P}}}_m\rangle_{\omega+i0}}{mT\omega_{a.u.}}; \omega_* = \frac{\omega}{\omega_{a.u.}};$$

$$(\hat{A};\hat{B}) = \int_0^\beta d\tau\,\mathrm{Tr}\left\{\hat{A}(-i\hbar\tau)\hat{B}^+\rho_0\right\},\ \langle\hat{A};\hat{B}\rangle_z = \int_0^\infty dt e^{izt}\left(\hat{A}(t);\hat{B}\right)$$

③ current $\boldsymbol{J} = e\sum_n\langle\hat{\boldsymbol{P}}_n\rangle$ and permittivity ε are expressed via response parameters and correlation functions:

$$\boxed{\boldsymbol{J} = \frac{ne^2}{m\omega_{a.u.}}\boldsymbol{E}\sum_m \eta_{nm}\mathcal{F}_m} \quad\Rightarrow\quad \boxed{\varepsilon(\omega) = 1 - \frac{\omega_p^2/\omega^2}{1+i\nu(\omega)/\omega}},$$

with complex effective collision frequency:

$$\boxed{\nu(\omega) = \omega_{a.u.}\left[i\omega_* + Q_\omega^{-1}\right],\ Q_\omega = \sum_n S_n\mathcal{F}_n,\ S_n = \sum_m \eta_{mn},\ \eta_{mn} = \delta_{mn}N_m/N_\Sigma}$$

In two-level system:

$$\boxed{Q_\omega = \left[S_1^2U_{22} + S_2^2U_{11} - 2S_1S_2U_{21}\right]\Big/\left[U_{22}U_{11} - U_{21}^2\right],\ U_{nm} = \mathfrak{C}_{nm} + i\omega_*\eta_{nm}}$$

Correlation functions for e-e & e-i interactions:

$$H = \sum_p E_p \hat{a}_p^\dagger \hat{a}_p + \sum_{pk} V_{\rm ei}(k) \hat{a}_{p+k}^\dagger \hat{a}_p + \frac{1}{2} \sum_{p_1 p_2 k} V_{\rm ee}(k) \hat{a}_{p_1+k}^\dagger \hat{a}_{p_2-k}^\dagger \hat{a}_{p_2} \hat{a}_{p_1} \Rightarrow$$

correlation functions in screened Born approximation [1-3]: $\mathfrak{C}_{nm} = \mathfrak{C}_{nm}^{\rm ee} + \mathfrak{C}_{nm}^{\rm ei}$,

$$\mathfrak{C}_{nm}^{\rm ei}(\omega) = \frac{iZ}{3\pi^2} \int_0^\infty f_{\rm scr}^{\rm i}(y) dy \int_{-\infty}^{\infty} \frac{dx}{x} \frac{R_{nm}^{\rm ei}(x,y)}{w + i\delta - x} \ln \left[\frac{1 + e^{\epsilon_\mu - (x/y - y)^2}}{1 + e^{\epsilon_\mu - (x/y + y)^2}} \right],$$

$$\mathfrak{C}_{nm}^{\rm ee}(\omega) = \frac{i}{3\sqrt{2}\pi^2} \int_0^\infty f_{\rm scr}^{\rm e}(y) dy \int_{-\infty}^{\infty} \frac{dx}{x} \frac{R_{nm}^{\rm ee}(x,y)}{w + i\delta - x} \ln \left[\frac{1 + e^{\epsilon_\mu - (x/y - y)^2}}{1 + e^{\epsilon_\mu - (x/y + y)^2}} \right],$$

$f_{\rm scr}^{\rm e(i)}(y)$ are screening functions.

[1]M. Veysman, G. Röpke, M. Winkel, and H. Reinholz, Phys. Rev. E **94**, 013203 (2016).
[2]H. Reinholz and G. Röpke, Phys. Rev. E **85**, 036401 (2012).
[3]H. Reinholz, G. Röpke, S. Rosmej, and R. Redmer, Phys. Rev. E **91**, 043105 (2015).

Correlation functions for e-ph interactions:

with Frohlich Hamiltonian:

$$\widehat{H} = \sum_{k,n} E_{k,n}\widehat{a}^{+}_{k,n}\widehat{a}_{k,n} + \sum_{q,\lambda} \hbar\omega_{q,\lambda}\widehat{b}^{+}_{q,\lambda}\widehat{b}_{q,\lambda} + \sum_{k,q,n,n',\lambda} g_k(q,n,n',\lambda)\widehat{a}^{+}_{k+q,n}\widehat{a}_{k,n'}(\widehat{b}^{+}_{q,\lambda} + \widehat{b}_{-q,\lambda})$$

$$\begin{aligned}\mathfrak{C}_{nm} = \frac{-i/\hbar}{m\omega_{a.u.}} \sum_{p,q,\lambda} \Big\{ g_{p-q} g_p(q,m,n) \Big[& K^{p,m}_{p-q,n+} \left[n_{p-q,n}(1-n_{p,m}) + (n_{p-q,n} - n_{p,m})N_{q,\lambda} \right] \\ & - K^{p,m}_{p-q,n-} \left[n_{p,m}(1-n_{p-q,n}) - (n_{p-q,n} - n_{p,m})N_{q,\lambda} \right] \Big] p_z(p_z - q_z) \\ - \delta_{mn} \sum_i g_{p-q} g_p(q,i,n) \Big[& K^{p+q,i}_{p,n+} \left[n_{p,n}(1-n_{p+q,i}) + (n_{p,n} - n_{p+q,i})N_{q,\lambda} \right] \\ & - K^{p+q,i}_{p,n-} \left[n_{p+q,i}(1-n_{p,n}) - (n_{p,n} - n_{p+q,i})N_{q,\lambda} \right] \Big] p_z^2 \Big\},\end{aligned}$$

$$K^{p+q,m}_{p,n,\pm} = \frac{1}{\widetilde{\Delta}^{p+q,m}_{p,n,\pm}} \left[\frac{1}{\omega + \Delta^{p+q,m}_{p,n,\pm}} + \frac{1}{\omega - \Delta^{p+q,m}_{p,n,\pm}} \right], \quad \begin{aligned} \Delta^{p+q,m}_{p,n,\pm} &= \frac{E_{p+q,m} - E_{p,n}}{\hbar} \pm \omega_{q,\lambda}, \\ \widetilde{\Delta}^{p+q,m}_{p,n,\pm} &= \frac{E_{p+q,m} - E_{p,n}}{\hbar} \pm \omega_{q,\lambda} \frac{T_e}{T_{ion}}, \end{aligned}$$

$$n_{p,n} = \left[1 + e^{(E_{p,n}-\mu)/T_e} \right]^{-1}, \quad N_{q,\lambda} = \left[e^{\omega_{q,\lambda}/T_{ion}} - 1 \right]^{-1}, \quad E_{p,n} = \frac{p^2}{2m_n} + E_n$$

Correlation functions for e-ph interactions:

With e$^-$-phonon coupling function for longitudinal optical phonons, $\omega_q = \omega_{\mathsf{LO}}$,
$g^2 = g_{\rm LO}(q)^2 = 2\pi e^2 \hbar\omega_{\mathsf{LO}} \varepsilon_{\infty,0}/[q^2\Omega_0]$, [G. D. Mahan, *Many Particle Physics*, 2000]:

$$\mathfrak{C}_{nn} = \frac{-im_*^2}{\kappa}\int_0^\infty y\,dy \int_{-\infty}^\infty dx\, X_{nn}(x)\Delta F_{nn}(x,y) - \sum_{i\neq n}\mathfrak{C}^0_{in},$$

$$\mathfrak{C}^0_{mn} = \frac{im_*^2}{4\kappa}\int_0^\infty dy\int_{-\infty}^\infty dx \Big\{ \Big[\Big(\frac{x^2}{y^3}+\frac{2x}{y}+y\Big)\Delta F_{mn}(x,y) + \frac{m_*^{-1}}{y}\Delta\widetilde{F}_{mn}(x,y)\Big] X_{mn}(x)$$
$$+\Big[\Big(\frac{x^2}{y^3}-\frac{2x}{y}+y\Big)\Delta F_{nm}(x,y) + \frac{m_*^{-1}}{y}\Delta\widetilde{F}_{nm}(x,y)\Big] X_{nm}(x)\Big\},$$

$$X_{mn}(x) = \left[\frac{1/\varphi(x,\alpha)}{w+i\delta+\varphi(x,1)} + \frac{1/\varphi(x,\alpha)}{w+i\delta-\varphi(x,1)}\right]\left[\frac{1}{e^{4x+4w_{nm}}-1} - \frac{1}{e^{4\alpha w_{\rm LO}}-1}\right]$$

$$\varphi(x,t) = x + w_{nm} - t w_{\rm LO},$$

$$\Delta F_{mn}(x,y) = F_1(A_-^m(x,y)) - F_1(A_+^n(x,y)), \quad F_1(t) = \ln\left(1+\tfrac{1}{t}\right),$$
$$\Delta\widetilde{F}_{mn}(x,y) = F_2(A_-^m(x,y)) - F_2(A_+^n(x,y)), \quad F_2(t) = \ln^2(t) + 2\,\mathrm{Li}_2(-t),$$
$$A_\pm^m(x,y) = \exp\left[(x/y \pm y)^2 + (E_m-\mu)/T_e\right],$$

$w = \frac{1}{4}\hbar\omega/T_e$, $w_{\rm LO} = \frac{1}{4}\hbar\omega_{\mathsf{LO}}/T_e$, $w_{nm} = \frac{1}{4}(E_n - E_m)/T_e$, $\alpha = T_e/T_{ion}$

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$$\mathfrak{C}_{mn}^{m\neq n} = \frac{i m_*^2}{4\kappa} \int_0^{\infty} dy \int_{-\infty}^{\infty} dx \Bigg\{ \Bigg[\left(\frac{x^2}{y^3} - y \right) \Delta F_{mn}(x,y) + \frac{m_*^{-1}}{y} \Delta \widetilde{F}_{mn}(x,y) \Bigg] X_{mn}(x) + \text{the same with } m \leftrightarrow n \Bigg\},$$

$$X_{mn}(x) = \left[\frac{1/\varphi(x,\alpha)}{w + i\delta + \varphi(x,1)} + \frac{1/\varphi(x,\alpha)}{w + i\delta - \varphi(x,1)} \right] \left[\frac{1}{e^{4x+4w_{nm}} - 1} - \frac{1}{e^{4\alpha w_{\mathrm{LO}}} - 1} \right]$$

$$\varphi(x,t) = x + w_{nm} - t w_{\mathrm{LO}},$$

$$\Delta F_{mn}(x,y) = F_1(A_-^m(x,y)) - F_1(A_+^n(x,y)), \quad F_1(t) = \ln\left(1 + \tfrac{1}{t}\right),$$

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$$A_\pm^m(x,y) = \exp\left[(x/y \pm y)^2 + (E_m - \mu)/T_e \right],$$

$w = \frac{1}{4}\hbar\omega/T_e$, $w_{\mathrm{LO}} = \frac{1}{4}\hbar\omega_{\mathrm{LO}}/T_e$, $w_{nm} = \frac{1}{4}(E_n - E_m)/T_e$, $\alpha = T_e/T_{ion}$

Simplified expression for real part of $\mathfrak{C}_{11}$

$$\mathfrak{C}'_{11} = \frac{4/(3\pi^2)}{\widetilde{\omega}_{a.u.}^{1/2}\varepsilon_{\rm eff}} \frac{m_*^2 w_{\rm LO}}{n_e \lambda_e^3} \sum_{\sigma=\pm 1} \frac{(e^{4[w_{\rm LO}+\sigma w]}-1)^{-1} - (e^{4w_{\rm LO}\alpha}-1)^{-1}}{w_{\rm LO}(\alpha-1)-\sigma w}$$
$$\times \int_0^\infty y dy \ln\left[\frac{1+\exp\left\{\mu/T_e - \left[y-(w_{\rm LO}+\sigma w)/y\right]^2\right\}}{1+\exp\left\{\mu/T_e - \left[y+(w_{\rm LO}+\sigma w)/y\right]^2\right\}}\right].$$

$w = \frac{1}{4}\hbar\omega/T_e$, $\widetilde{\omega}_{au} = \hbar\omega_{au}/T_e$, $\alpha = T_e/T_{ion}$,

$w_{\rm LO} = \frac{1}{4}\hbar\omega_{\rm LO}/T_e$ is the frequency of longitudinal optical phonons.

Without contribution of interband transitions and in single-moment approximaion	$\nu(\omega) = \omega_{a.u.}\mathfrak{C}_{11}$

Some asymptotics

The case of small phonon frequencies, $\hbar\omega_{\rm LO} \ll T_e$, $\hbar\omega_{\rm LO} \ll T_i$

1) for **high** laser frequencies, $\hbar\omega \gg T_e$:

$$\boxed{\mathfrak{C}'_{11} \approx \frac{m_*^2/3}{\pi^{3/2}\varepsilon_{\rm eff}} \frac{T_{ion}}{\sqrt{27.2\,\text{eV}\cdot\hbar\omega}}},$$

2) for **low** laser frequencies, $\omega \le \omega_{\rm LO}$:

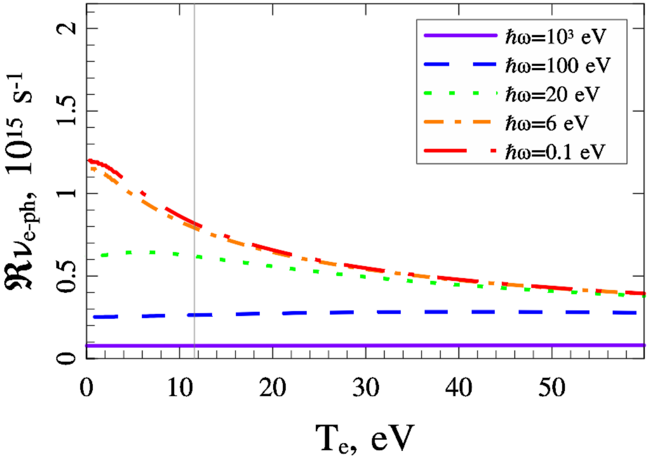
$$\boxed{\mathfrak{C}'_{11} = \frac{m_*^2}{2\pi^{3/2}\varepsilon_{\rm eff}} \frac{T_{ion}}{\sqrt{27.2\,\text{eV}\cdot T_e}} (E_F/T_e)^{-3/2} \ln(1+e^{\mu/T_e}) \times \begin{cases} 1, & \omega \ll \omega_{\rm LO}, \\ 9/4, & \omega = \omega_{\rm LO} \end{cases}}$$

or

$$\boxed{\mathfrak{C}'_{11} \sim T_{ion}/\sqrt{27.2\,\text{eV}\cdot E_F}}, \quad T_e \ll E_F,$$

$$\boxed{\mathfrak{C}'_{11} \sim T_{ion}/\sqrt{27.2\,\text{eV}\cdot T_e}}, \quad T_e \gg E_F.$$

Numerical example: without Umklapp proceses



$\mathrm{Re}\,\{\nu\}\,(T_e)$, for solid-density Al with average ion charge $Z = 3$, ion temperature $T_i = 1$eV, for different laser frequencies $\hbar\omega$.

Correlation function for Umklapp processes:

with Hubbard Hamiltonian:

$$\widehat{H} = \sum_{k,\sigma} E_k \widehat{a}^+_{k,\sigma} \widehat{a}_{k,\sigma} + \frac{U}{2N} \sum_{k,k',q,g,\sigma} \widehat{a}^+_{k+q-g,\sigma} \widehat{a}^+_{k'-q,-\sigma} \widehat{a}_{k',-\sigma} \widehat{a}_{k,\sigma} \Rightarrow$$

$$\mathfrak{C}'_{11} = C_U \frac{(m_*/m)^2 U^2 T_e^2}{E_F^3 E_{a.u.}} J_\omega,$$

$$J_\omega = \frac{4}{W} \int_{-2\epsilon_\mu}^{2\epsilon_\Delta - 2\epsilon_\mu} dt \left[\frac{1}{e^{t-W} - 1} - \frac{1}{e^t - 1} \right] \ln\left[\frac{e^{t/2} + e^{-B/2}}{e^{t/2-B/2} + 1} \right] \ln\left[\frac{e^{t/2-W/2} + e^{-B/2}}{e^{t/2-B/2-W/2} + 1} \right],$$

$$\epsilon_\Delta = \Delta_E/T_e\ , \quad \epsilon_\mu = \mu/T_e, \quad B = \epsilon_\mu + \epsilon_\Delta, \quad W = \hbar\omega/T_e$$

Low-temperature asymptotics: $E_F/T_e \gg 1$, $\mu \approx E_F$, $\Delta_E/T_e \gg 1$:

$$J_\omega = \frac{2\pi^2}{3}\left[1 + \frac{\hbar^2\omega^2}{4\pi^2 T_e^2}\right], \quad \mathfrak{C}'_{11} \sim \frac{U^2}{E_F^3 E_{a.u.}}\left[T_e^2 + \frac{\hbar^2\omega^2}{4\pi^2}\right] \quad \text{for } \hbar\omega \ll E_F,$$

$$J_\omega = \frac{E_F + \Delta_E}{\hbar\omega}\left[\frac{\pi^2}{3} + 2\frac{\Delta_E^2}{T_e^2}\right], \quad \mathfrak{C}'_{11} \sim 2\frac{U^2(E_F + \Delta_E)}{E_F^3 E_{a.u.}\hbar\omega}\left[T_e^2\frac{\pi^2}{6} + \Delta_E^2\right] \quad \text{for } \hbar\omega \gg E_F$$

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with Hubbard Hamiltonian:

$$\widehat{H} = \sum_{k,\sigma} E_k \widehat{a}^+_{k,\sigma}\widehat{a}_{k,\sigma} + \frac{U}{2N}\sum_{k,k',q,g,\sigma} \widehat{a}^+_{k+q-g,\sigma}\widehat{a}^+_{k'-q,-\sigma}\widehat{a}_{k',-\sigma}\widehat{a}_{k,\sigma} \Rightarrow$$

$$\mathfrak{C}'_{11} = C_U \frac{(m_*/m)^2 U^2 T_e^2}{E_F^3 E_{a.u.}} J_\omega,$$

$$J_\omega = \frac{4}{W}\int_{-2\epsilon_\mu}^{2\epsilon_\Delta - 2\epsilon_\mu} dt \left[\frac{1}{e^{t-W}-1} - \frac{1}{e^t - 1}\right] \ln\left[\frac{e^{t/2}+e^{-B/2}}{e^{t/2-B/2}+1}\right] \ln\left[\frac{e^{t/2-W/2}+e^{-B/2}}{e^{t/2-B/2-W/2}+1}\right],$$

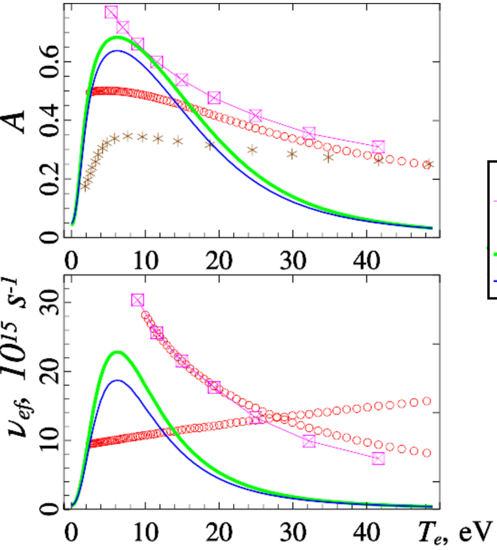
$$\epsilon_\Delta = \Delta_E/T_e\ , \quad \epsilon_\mu = \mu/T_e, \quad B = \epsilon_\mu + \epsilon_\Delta, \quad W = \hbar\omega/T_e$$

High-temperature asymptotics: $E_F/T_e \ll 1$, $\Delta_E/T_e \ll 1$:

$$J_\omega = \frac{32}{9\pi}\frac{E_F^3(E_F+\Delta_E)^3}{T_e^6}, \quad \mathfrak{C}'_{11} \sim \frac{U^2(E_F+\Delta_E)^3}{T_e^4 E_{a.u.}}, \quad \text{for } \hbar\omega \ll E_F, |\mu|,$$

$$J_\omega = 2\frac{(E_F+\Delta_E)^3}{\hbar\omega T_e^2}, \quad \mathfrak{C}'_{11} \sim \frac{U^2(1+\Delta_E/E_F)^3}{\hbar\omega E_{a.u.}} \quad \text{for } \hbar\omega \gg E_F, |\mu|$$

Numerical example: with Umklapp proceses



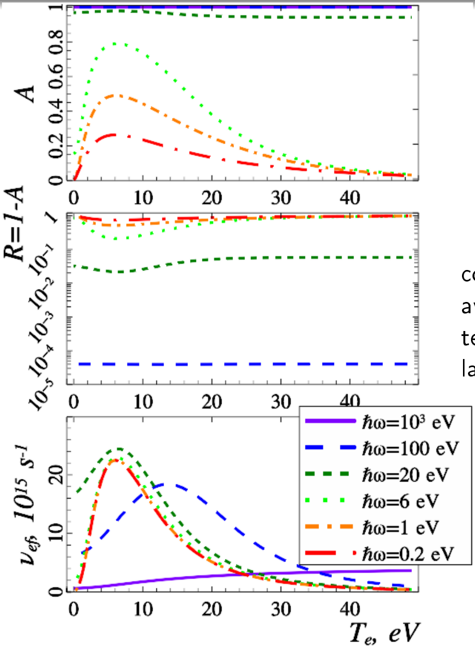
$\mathrm{Re}\{\nu\}(T_e)$, Absorption & Reflection coefficients for solid-density Al with average ion charge $Z = 3$, different ion temperatures T_i, for laser wavelength

- $*$ Cauble *et al* [PRE **52**, 2974, 1995]
- Veysman *et al* [PRE **94**, 013203, 2017]
- Povarnitsyn *et al* [App. Surf. Sci. **258**, 9480, 2012]
- $T_i = 0.04$ eV
- $T_i = T_{melt} = 0.083$ eV

Note: Umklapp & electron-phonon interactions could occure at $T_i > T_{melt}$, if

$$t < \tau_m = \frac{r_a}{v_s} \approx 7.5\text{fs}\frac{A_{at}^{5/6}}{(ZT_e[\text{eV}])^{1/2}\varrho^{1/3}}$$

Numerical example: with Umklapp proceses



$\text{Re}\,\{\nu\}\,(T_e)$, Absorption & Reflection coefficients for solid-density Al with average ion charge $Z = 3$, ion temperature $T_i = 0.04\text{eV}$, for different laser frequencies $\hbar\omega$.

Conclusions

- Simple wide-range (over ω) expressions for correlation functions, complex collision frequency $\nu(\omega)$ and permittivity $\varepsilon(\omega)$ are obtained using QS approach, LRT, Frohlich Hamiltonian for e^--phonon and Habbard Hamiltonian for e^-–e^- interactions
- The model gives dependencies of ν & ε due to e^--phonon interactions on ω, T_e and T_i. For $T_e, T_i > \hbar\omega_{\mathrm{LO}}$ usual dependence $\mathrm{Re}\{\nu\} \sim T_i$ follows. Dependence on T_e and dependence on T_i at $T_e < \hbar\omega_{\mathrm{LO}}$ can be more complex.
- The model permits one to take into account *interband* transitions. Similar approach for accounting *interband* transitions at higher electron temperatures can be done with Hamiltonian containing e-e and e-i interactions instead of e-ph interaction.
- Model accounts for *Umklapp* processes and their dependence on ω

The material is partly contained at [M.Veysman H.Reinholz G.Roepke, J. Phys., Conf. Ser., to be published, 2018]