Peculiarities of Buzdin and Chimera Steps in the IV-Curve of Superconductor Ferromagnetic φ_0 Josephson Junction

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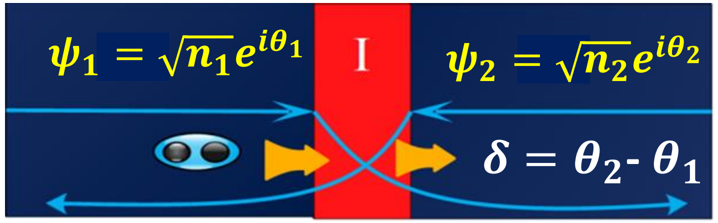
MMCP
20-25 October 2024
Yerevan Armenia

Outline

- Introduction to Josephson effect
- Anomalous Josephson Junctions
- Dynamical equations for φ_0 -JJ
- **Shaprio, Buzdin and chimera steps in φ_0 -JJ**

Josephson Effect



B. D. Josephson Phys. Lett. (1962).

DC supercurrent

$$I_s = I_C \sin[\delta]$$

I_C:critical current

AC current

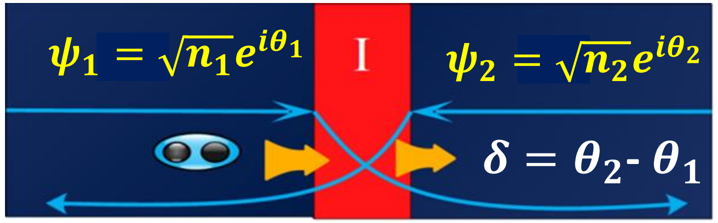
$$V = \frac{\hbar}{2e}\frac{d\delta}{dt}$$

1μV,
ω_J=483.6 *MHz*

$$I = I_C \sin\left[\frac{2\pi}{\Phi_0} V\, t + \delta_0\right]$$

W.C.Stewart, Appl.Phys.Lett.12,277(1968)
D.E.McCumber,J.Apple.Phys.39,3113(1968)
J. A. Blackburn et. al., Physics Reports, 611, (2016).

Josephson Effect



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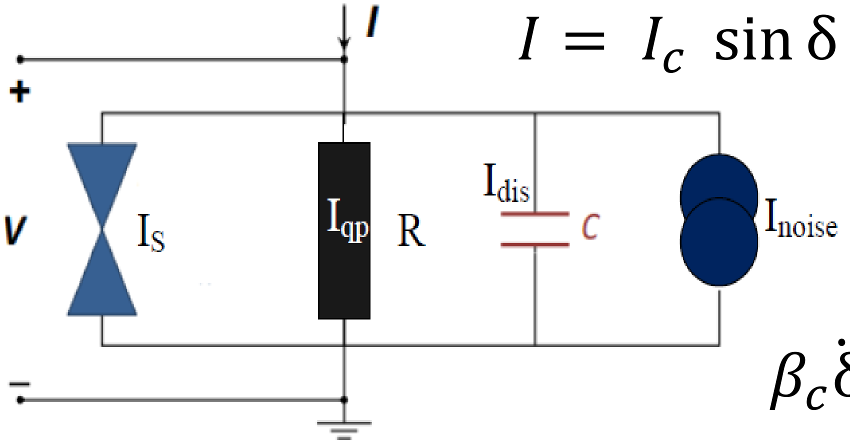
Resistivity and capacitively shunted junction(RCSJ)



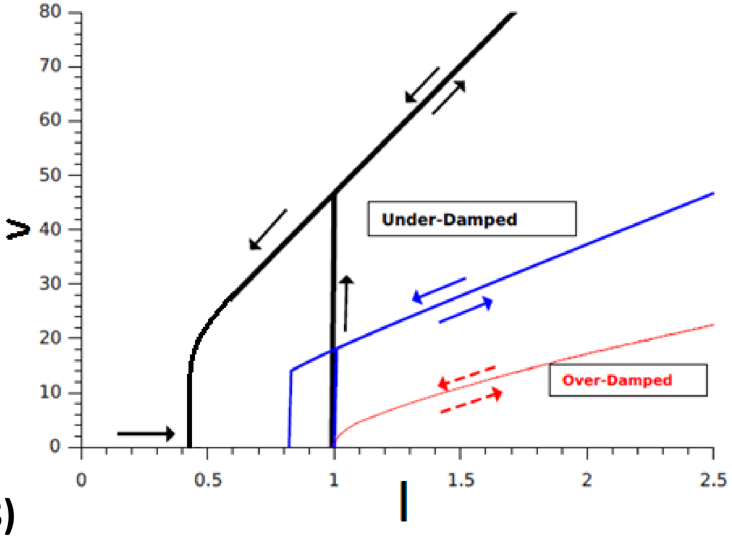
$$I = I_c \sin\delta + \frac{\hbar}{2eR}\dot{\delta} + \frac{\hbar C_J}{2e}\ddot{\delta} + I_{noise}$$

R: is the normal resistance.
C_J: natural capacity of the JJ.

$$\beta_c\ddot{\delta} + \dot{\delta} + \sin\delta + I_{noise} = I$$

McCumber parameter

$$\beta_c = \frac{2eI_cR^2C}{\hbar}$$

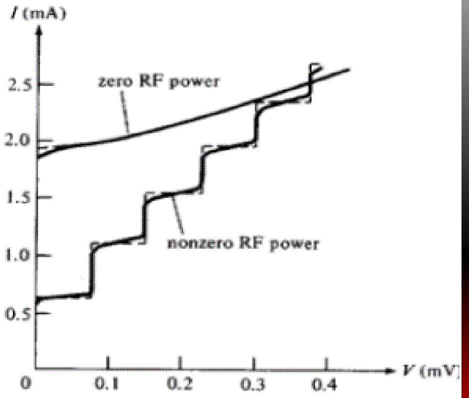


Shapiro step

$$I + I_{ac}\sin[\omega\, t]$$

$$V_0 = \frac{n\hbar}{2e}\omega,$$

n=1,2,3

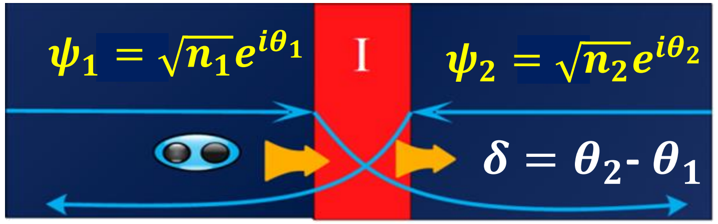


S. Shapiro, PRL, 11(2):80, 1963.

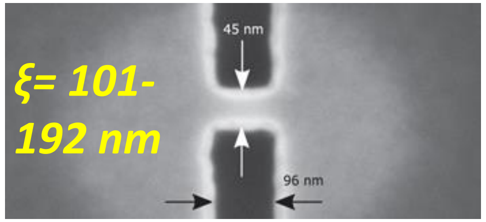
Josephson Effect

Josephson Energy

$$E_J = \int_0^t I_s V\, dt = \frac{\hbar I_c}{2e}(1 - \cos(\delta))$$



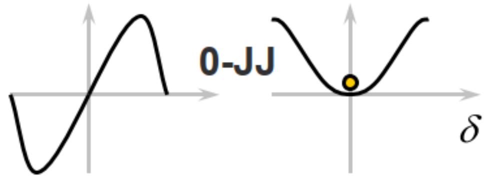
B. D. Josephson ***Phys. Lett. (1962).***



C. D. Shelly et al.,
SUST, 30 095013 (2017)

CPR

Josephson energy



$I_s = I_0 \sin(\delta)$

G.S: $\delta = 0$

Ic	$\varepsilon_J = \hbar I_c/(2e)$
1mA	2eV
1μA	2meV
1nA	2μeV

Josephson Effect

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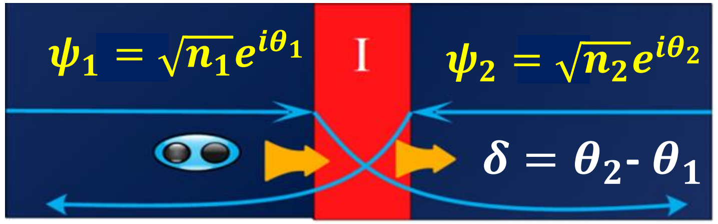
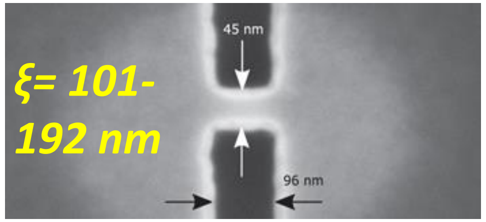
$$E_J = \int_0^t I_s V\, dt = \frac{\hbar I_c}{2e}(1 - cos(\delta))$$



B. D. Josephson ***Phys. Lett. (1962).***



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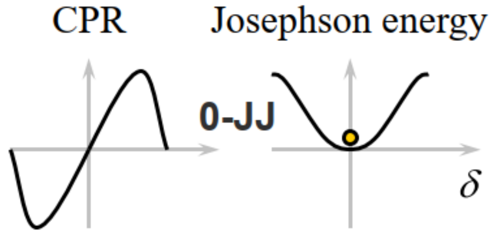

$I_s = I_0 \sin(\delta)$ G.S: $\delta = 0$

Ic	$\varepsilon_J = \hbar I_c/(2e)$
1mA	2eV
1μA	2meV
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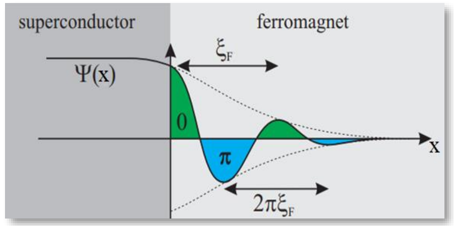
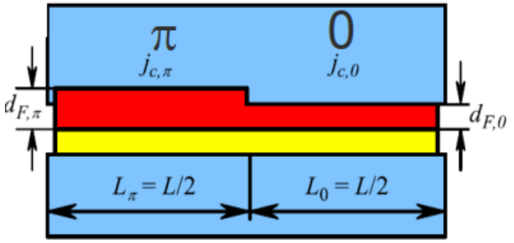
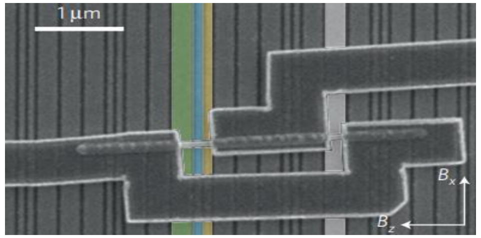


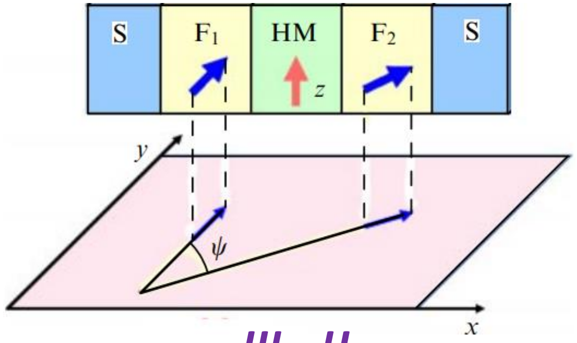
V. V. Ryazanov et al., PRL. 86, 2427 (2001)



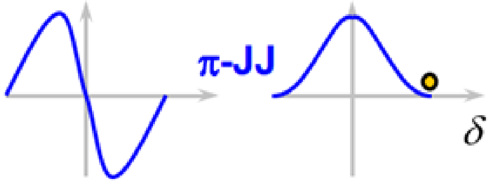
H. Sickinger et al., PRL. 109, 107002 (2012)



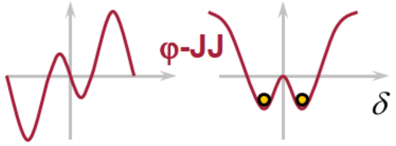
D. B. Szombati et al., Nat. Phys. 12, 568 (2016)



Ψ - JJ

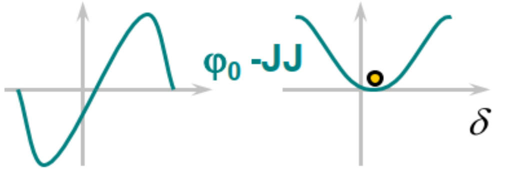


$I_s = I_0 \sin(\delta - \pi)$ G.S: $\delta = \pi$



$I_s = I_{0,1}\sin(\delta) + I_{0,2}\sin(2\delta)$ G.S: $\delta = \pm\varphi$, $0 < \varphi < \pi$

$(\varphi = \arccos(-\frac{I_{0,1}}{2I_{0,2}})$ for $-I_{0,2} > \frac{I_{0,1}}{2})$



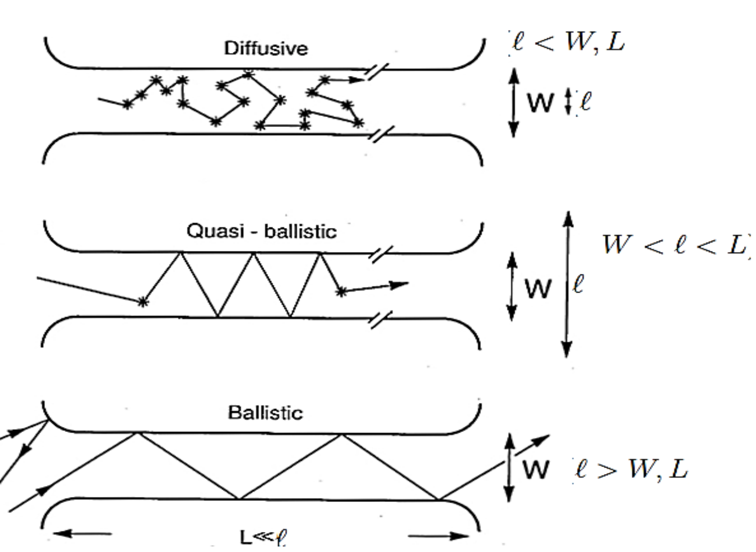
$I_s = I_0 \sin(\delta - \varphi_0)$ G.S: $\delta = \varphi_0$
(with $0 < \varphi_0 < \pi$)

$I(\delta) = I_c \sin(\delta - \psi)$

S. V. Mironov et al., PRB. 104, 134502 (2021) → ψ junction

Slide from D. Koelle (2018)

Anomalous φ_0-JJ



φ_0	$= \frac{4\alpha_R L E_Z}{(\hbar v_F)^2} \approx 0.01\pi$ (Ballistic regime -SFS)
E_Z	$= g\mu_B B/2$ Zeeman energy, (B=100 mT)
L	Barrier length (150 nm).
v_F	Fermi velocity (3.2×10^5 m/s)
α_R	Rashba coupling (0.4 eV Å)

Rev. Lett. 101, 107005 (2008).

ℓ:mean free path, W:width, L: length

φ_0	$= \frac{\tau m^{*2} E_Z (\alpha L)^3}{3\hbar^6 D} \approx 0.0028\pi$ (Diffusive regime -SNS)
τ	elastic scattering time (0.13 ps)
$D=\frac{1}{3}\tau v_F^2$	diffusion constant (40 cm^2 /s)
m*	= 0.25 m_e effective electron mass

EPL 110, 57005 (2015).

Anomalous φ_0-JJ

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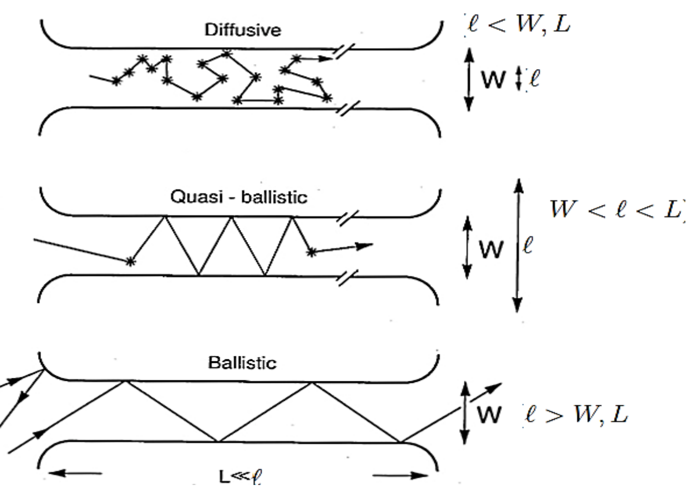

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φ_o	$= \frac{4\alpha_R L E_Z}{(\hbar v_F)^2} \approx 0.01\pi$ (Ballistic regime -SFS)
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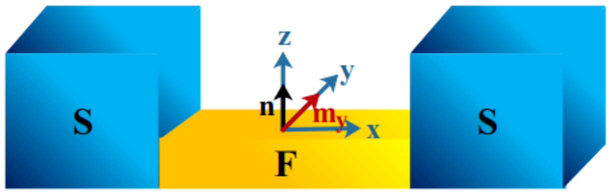
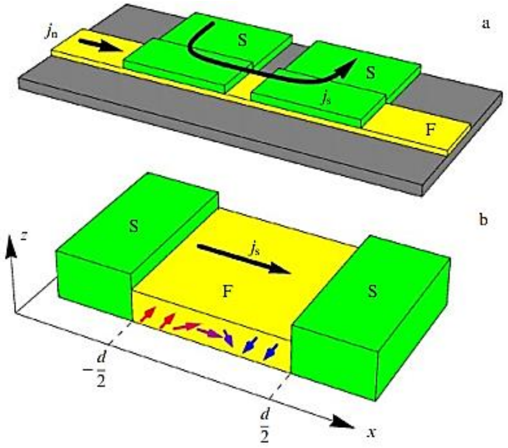
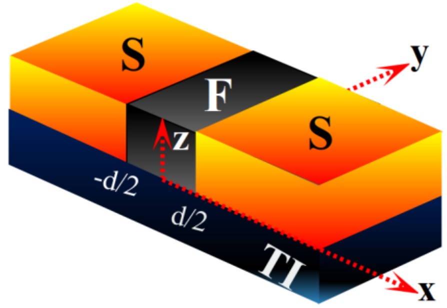


φ_o	$r\, M_y/M_o$
I_s	$I_c \sin(\varphi-\varphi_o)$
r	strength of spin-orbit.

Rev. Lett. 101, 102, 017001 (2009).





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EPL 110, 57005 (2015).

φ_o (t)	$-2\pi\beta x_o(t)/d_w$
d_w	Domain wall width
β	SOC constant

φ_o	$2h_y d/v_F$
I_s	$I_c(h_x)\sin(\varphi-\varphi_o)$

Phys. Rev. B. 100, 054506 (2019)

Yu. M. Shukrinov. Phys.-Usp. 65 317 (2022).

Phys. Rev. Lett. 123 207001 (2019)

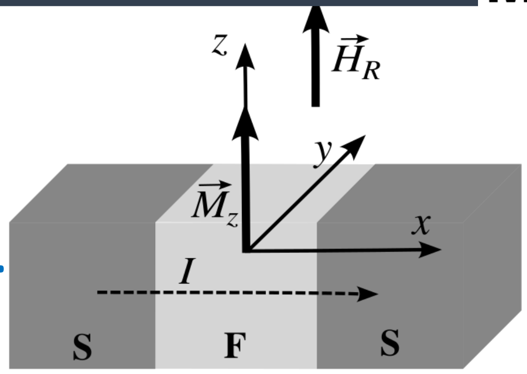
Dynamical Equations

❑ LLG equation

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_0}\left(\mathbf{M} \times \frac{d\mathbf{M}}{dt}\right)$$

$$H_{eff} = -\frac{1}{V_F}\frac{\delta E}{\delta \mathbf{M}},$$

- E: total energy.
- V_F: volume of ferromagnet.
- γ: gyromagnetic ratio.
- A: Gilbert damping.



Magnetic anisotropy

$$\boldsymbol{H}_{\boldsymbol{eff-ani}} = K_{an}\frac{M_{\boldsymbol{z}}}{M_0^2}\hat{\boldsymbol{e}}_z$$

K_{an}: anisotropic constant

M_o: saturation magnetization

Ac. M.F

$$\boldsymbol{H}_{eff-ext} = \boldsymbol{H}_R \sin(\omega_R t)\,\hat{\boldsymbol{e}}_{\boldsymbol{y}}$$

JJ - Energy

$$\boldsymbol{H}_{eff-JJ} = \boldsymbol{r}\, E_J\left(\sin\left(\varphi - \boldsymbol{r}\frac{M_{\boldsymbol{y}}}{M_0}\right)\right)\hat{\boldsymbol{e}}_{\boldsymbol{y}}$$

Dynamical Equations

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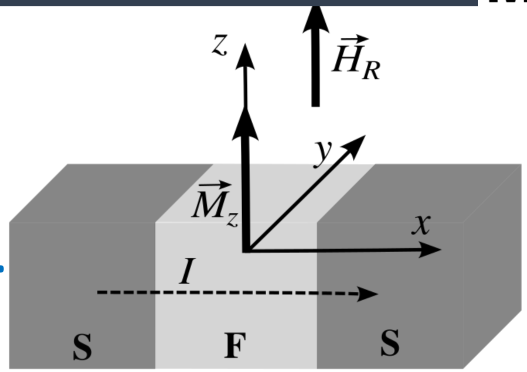

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❑ Normalizations

- $t \rightarrow t/\omega_c$ & $\omega_c = 2\pi I_c R/\Phi_0$
- $\omega_J = \Omega_J/\omega_c$ & $\Omega_J = 2e\,v/\hbar$
- $\omega_F = \Omega_f/\omega_c$ & $m_i = M/M_0$, i: x,y,z
- $h_{eff} = H_{eff}/K_{an}V_F$ & $G = \frac{E_J}{K_{an}v}$ G can be <<1 or >>1 (20-100)

$$\frac{d\mathbf{m}}{dt} = -\frac{\omega_F}{(1+\alpha^2)}\left[\mathbf{m}\times\mathbf{h}_{eff} + \alpha\left(\mathbf{m}\times(\mathbf{m}\times\mathbf{h}_{eff})\right)\right]$$

LLG equation

Dynamical Equations

- LLG equation

$$\frac{d\mathbf{m}}{dt} = -\frac{\omega_F}{(1+\alpha^2)}\left[\mathbf{m}\times\mathbf{h}_{eff} + \alpha\left(\mathbf{m}\times(\mathbf{m}\times\mathbf{h}_{eff})\right)\right]$$

- RCSJ equation

$$\dot{V} = \left[I + A\sin(\omega_R t) - V(t) + r\dot{m}_y - \sin(\varphi - r m_y)\right]$$

$$\dot{\varphi} = V(t)$$

Initialize: I=0, V=0.

Start iteration on Current

Start iteration on time

i- Solve the coupled Eq.s
ii-Check that $\|M_s(t)\| = 1$.
iii- Find V(t), $m_i(t)$, i=x,y,z.

$\langle V\rangle = \frac{1}{T_f - T_i}\int_{T_i}^{T_f}\frac{\hbar}{2e}\frac{d\varphi}{dt}dt$. & $m_i^{(av)}$(I).

The final Voltage and m_i(I) are the initial for the next current step.

After reaching maximum current we reverse current (0--> I^{max} --> 0)

Dynamical Equations

- **LLG equation**

$$\frac{d\mathbf{m}}{dt} = -\frac{\omega_F}{(1+\alpha^2)}\left[\mathbf{m}\times\mathbf{h}_{eff} + \alpha\left(\mathbf{m}\times(\mathbf{m}\times\mathbf{h}_{eff})\right)\right]$$

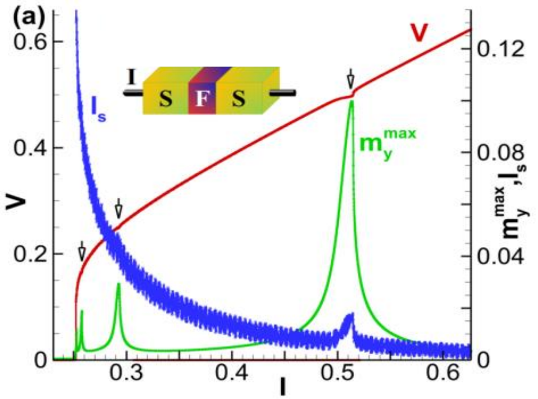
- **RCSJ equation**

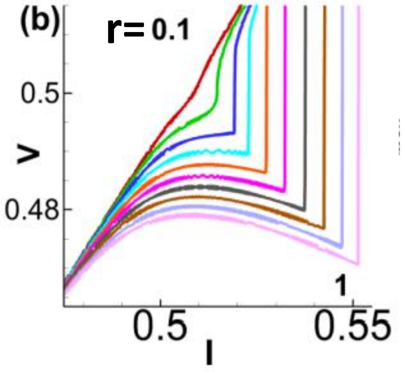
$$\dot{V} = \left[I + A\sin(\omega_R t) - V(t) + r\dot{m}_y - \sin(\varphi - r m_y)\right]/\beta_c$$

$$\dot{\varphi} = V(t)$$

- ***Some features of φ_0 −JJ***

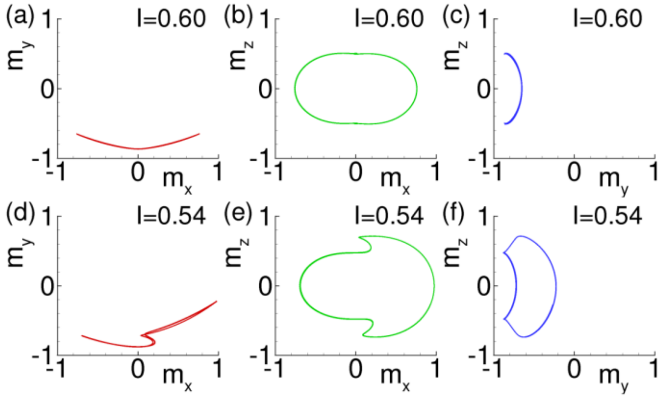
Negative differential resistance





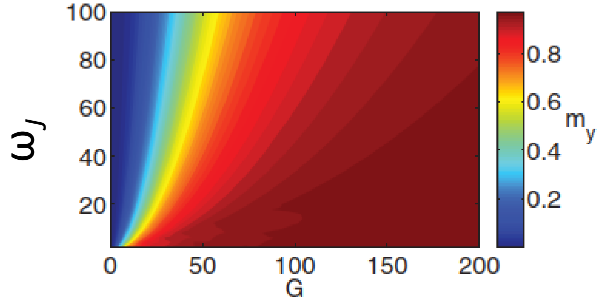
PRB 106, 014505 (2022).

Magnetization trajectory



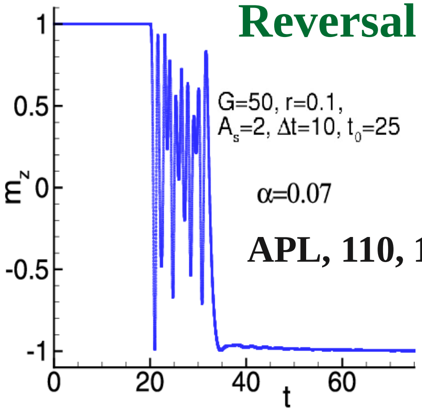
PRB 99, 224513 (2019)

Re-orientation of easy axis



EPL, 122 (2018) 37001.

Reversal



APL, 110, 182407 (2017).

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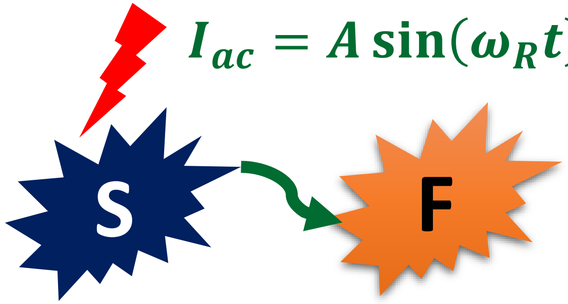
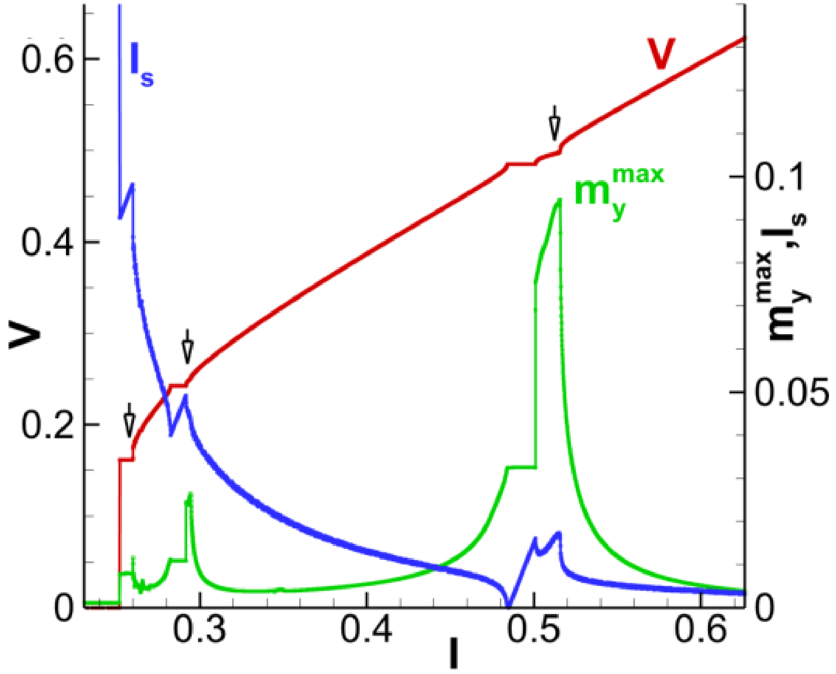
Yu . M. Shukrinov, *Phys. Usp.* 65 317–354 (2022)

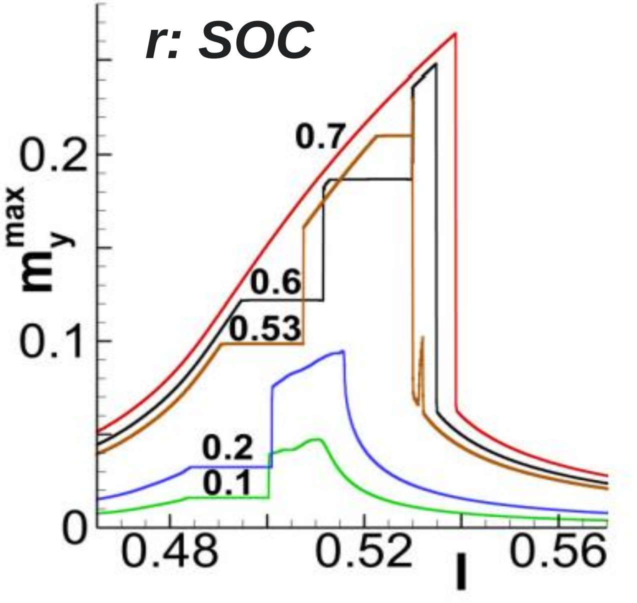
Shapiro and Locking steps

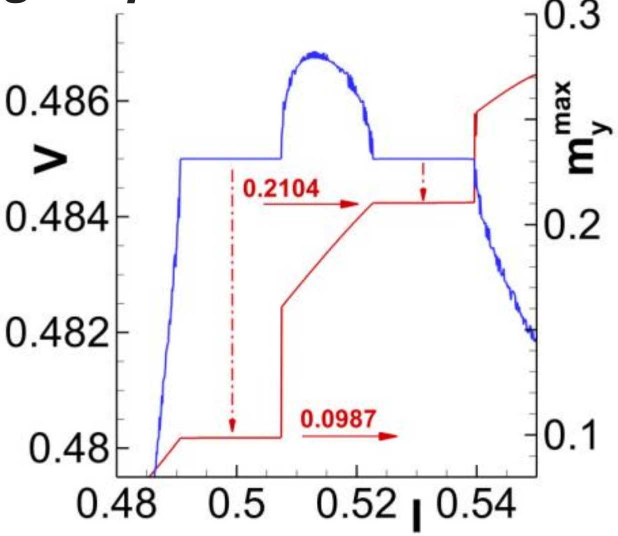
$$I_{ac} = A\sin(\omega_R t)$$

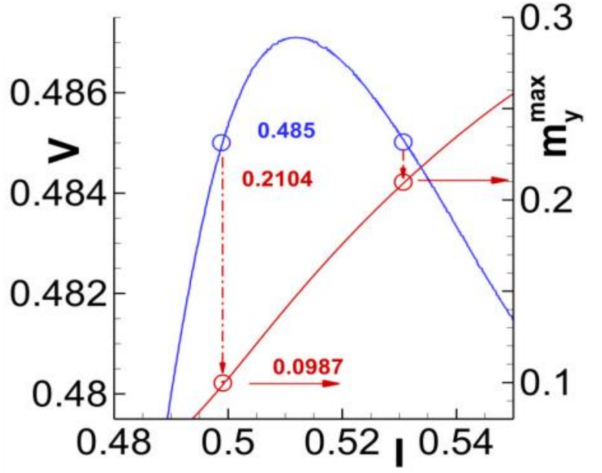




Manifestation of locking step







Position of locking step

PRB 106, 014505 (2022).

φ_o	r	l	G	α	w_c	I	V	M_o (sat.Mag)	w_F	β_c
rm_y	lv_{so}/v_F	$4h_{ex}L/(\hbar v_F)$	EJ/(K V)	0.01- 0.1	$2eRI_c/\hbar$ ~ GHz	~mA	~ µV	8 x 10^5 A/m	0.5	25

Buzdin step

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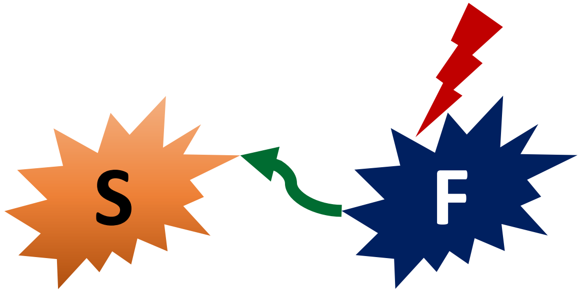
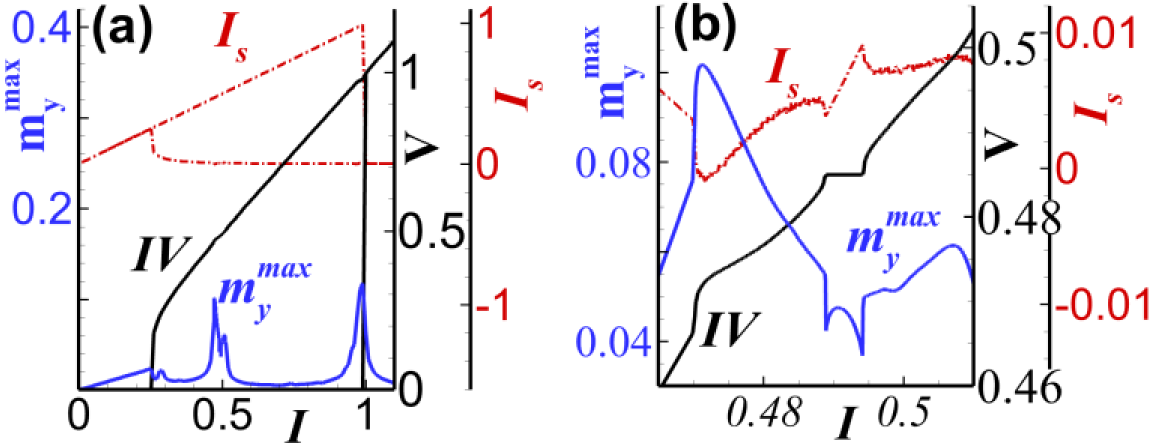
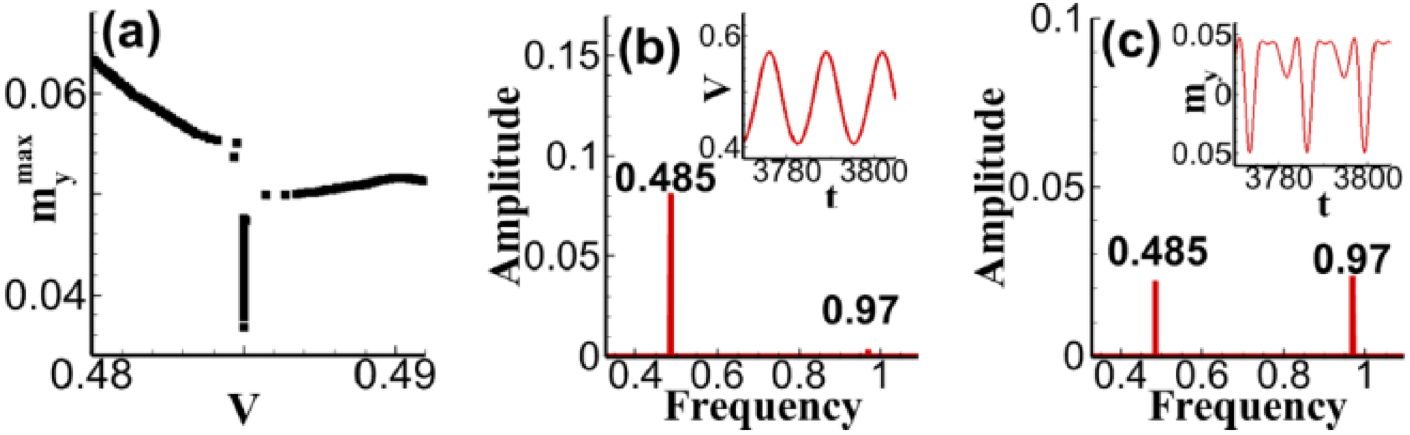
$$H_{ac} = h_{ac} \sin(\omega_R t)$$



A = 0, hR =1, r = 0.5, G = 0.01,
α = 0.01, ω_R = 0.485, ω_F = 0.5;

Locking of Josephson oscillations by the magnetic component of external radiation (MCR).





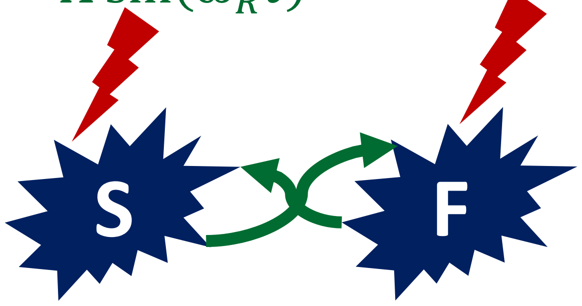
m_y^{max} as a function of V ; (b) and (c) The time dependence of V and m_y , and the corresponding FFT analysis in the center of the bubble, respectively.

Yu. Shukrinov, et. al., PRB 109, 024511 (2024)

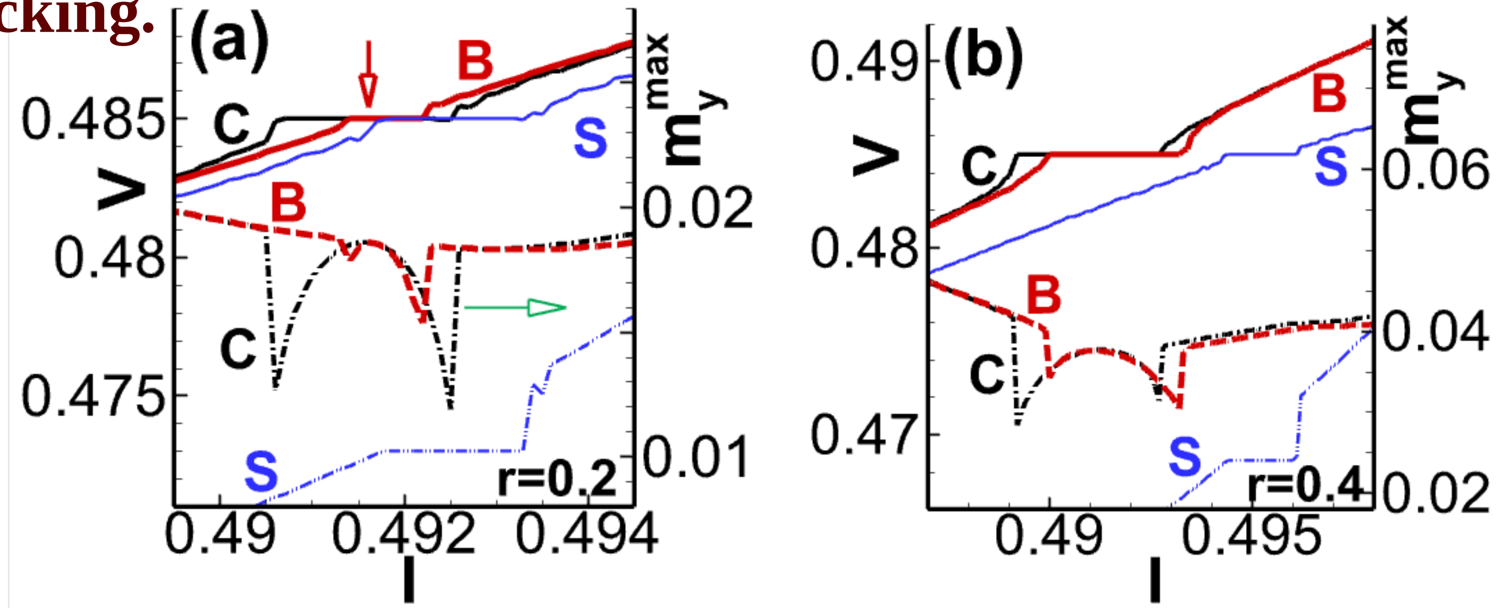
Chimera Step

Two different mechanisms of locking.

$I_{ac} = A \sin(\omega_R t)$ $\quad H_{ac} = h_{ac} \sin(\omega_R t)$



JJ. parameter	
Area	0.1 *0.1 μm^2
I_c	10 μA
R	1 Ω
ω_c	30 GHz
Power	~ 10^{-10} Watt
ω_F	15 GHz
M_0	8 x 10^5 A/m
γ	1.76 *10^{11} Hz/Tesla



The effects of both radiation components , h_R = 1, and A = 0.01.

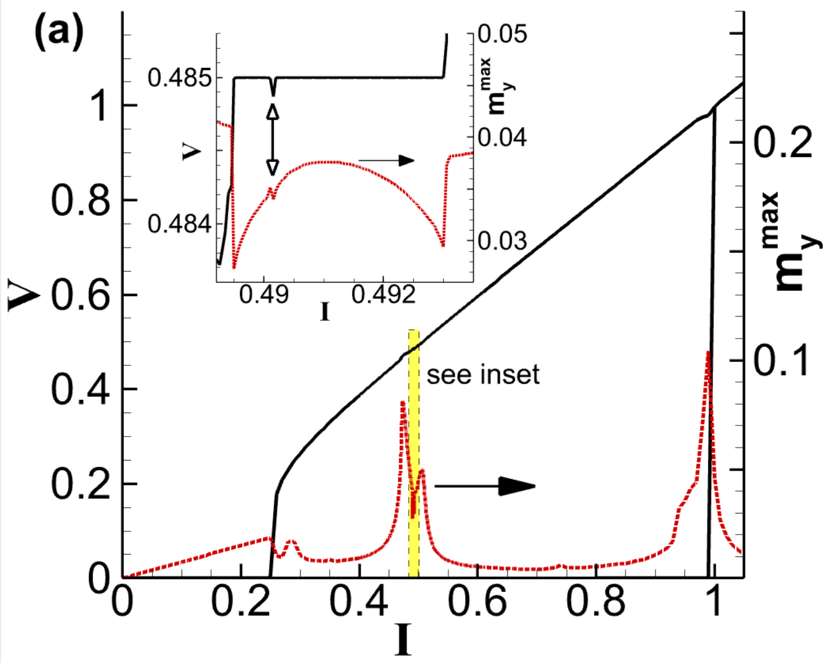
$$h_R = \frac{\gamma\, I_c\, A}{\upsilon^{3/2}\omega_c}\sqrt{\frac{2R}{S\epsilon}}$$

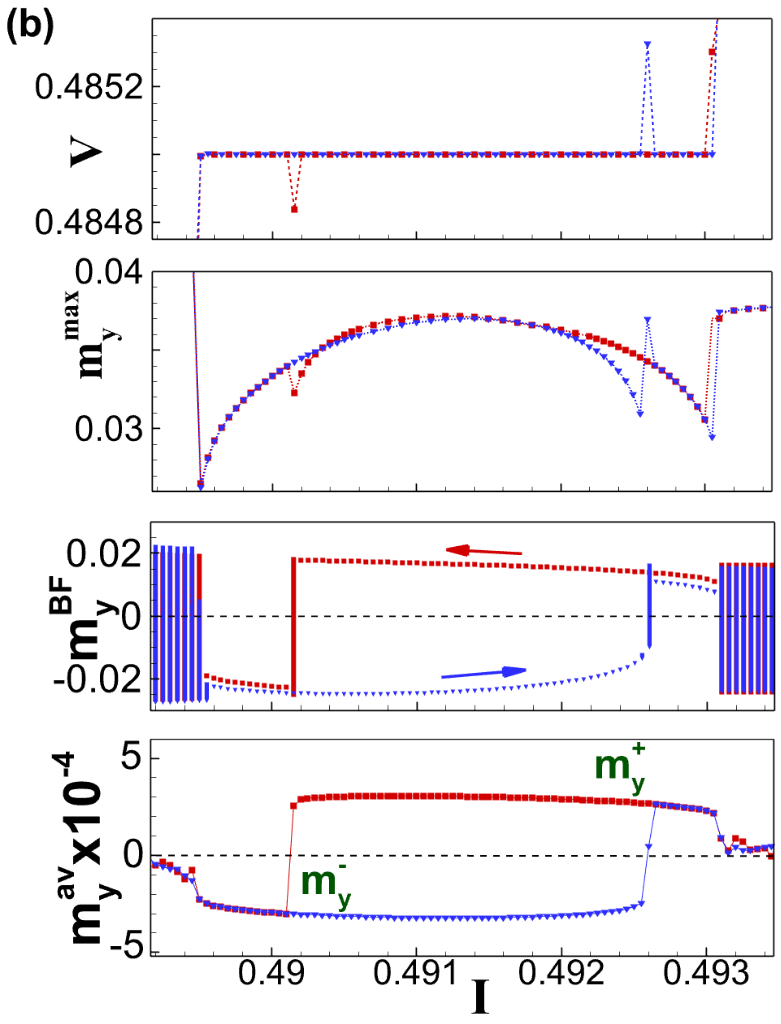
- $\upsilon = \frac{1}{\sqrt{\mu\epsilon}}, \mu: permiability, \epsilon: permitivity.$
- $\omega_c: charateristic\ frequency\ of\ the\ JJ.$
- $R: resistance\ of\ the\ JJ.$
- $S: area\ of\ the\ JJ.$

Yu. Shukrinov, et. al., PRB 109, 024511 (2024)

Manifestation of two dynamical states in φ_0-junction

Manifestation of chimera step in the IV and m_y^{max} (I), m_y^{BF} (I), m_y^{av} (I) under external electromagnetic radiation with A =0.005 and hR = 1.

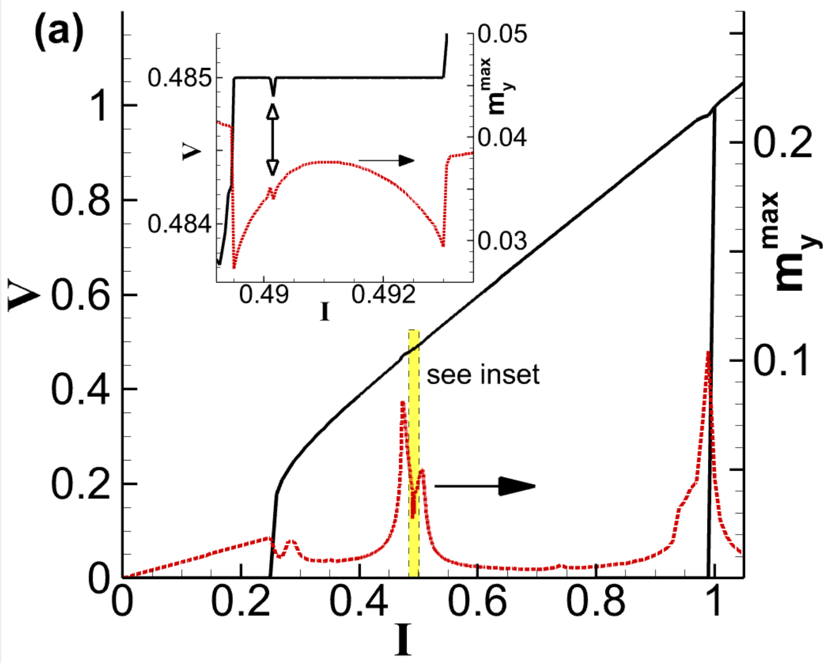


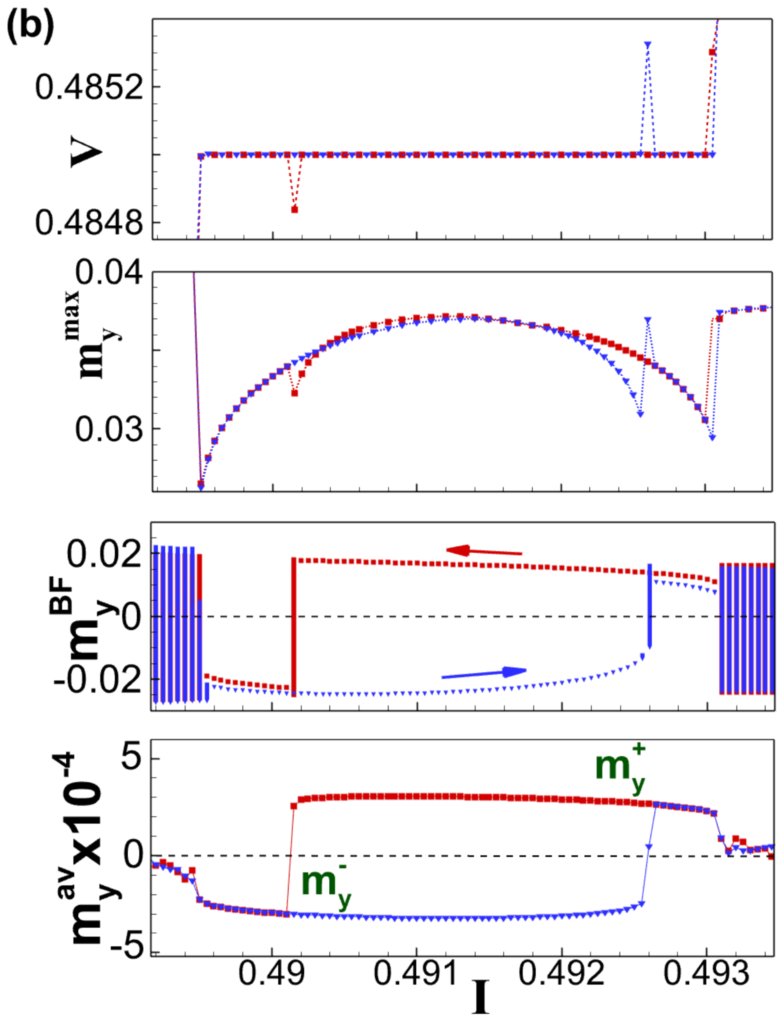


M. Nashaat, et. al., PRB 110, 024510 (2024)

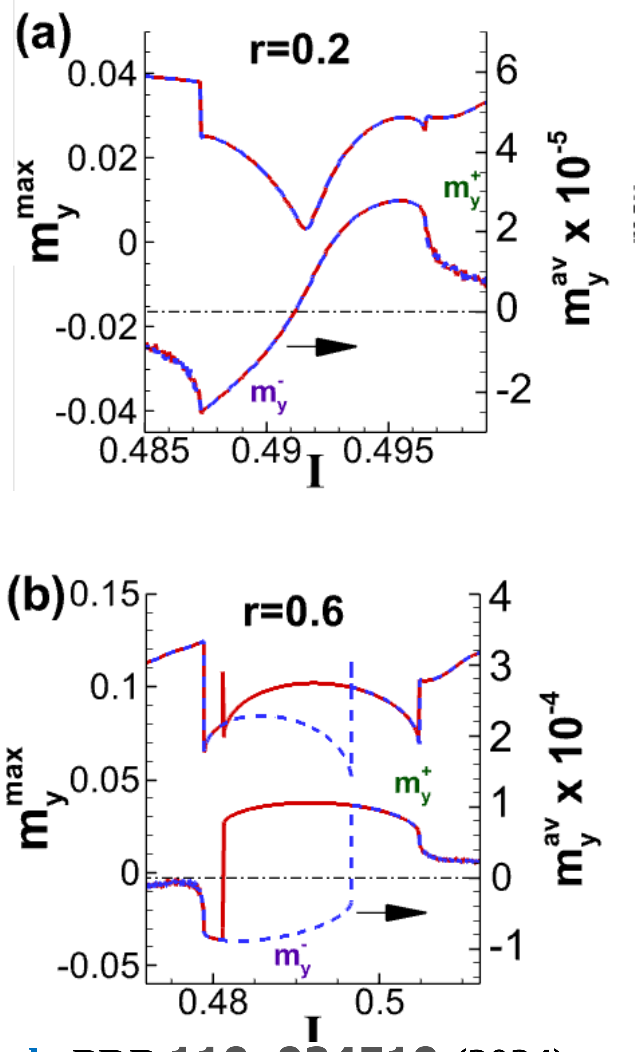
Manifestation of two dynamical states in φ_0-junction

Manifestation of chimera step in the IV and m_y^{max} (I), m_y^{BF} (I), m_y^{av} (I) under external electromagnetic radiation with A =0.005 and hR = 1.





Magnetic hysteresis



M. Nashaat, et. al., PRB 110, 024510 (2024)

Manifestation of two dynamical states in φ_0-junction

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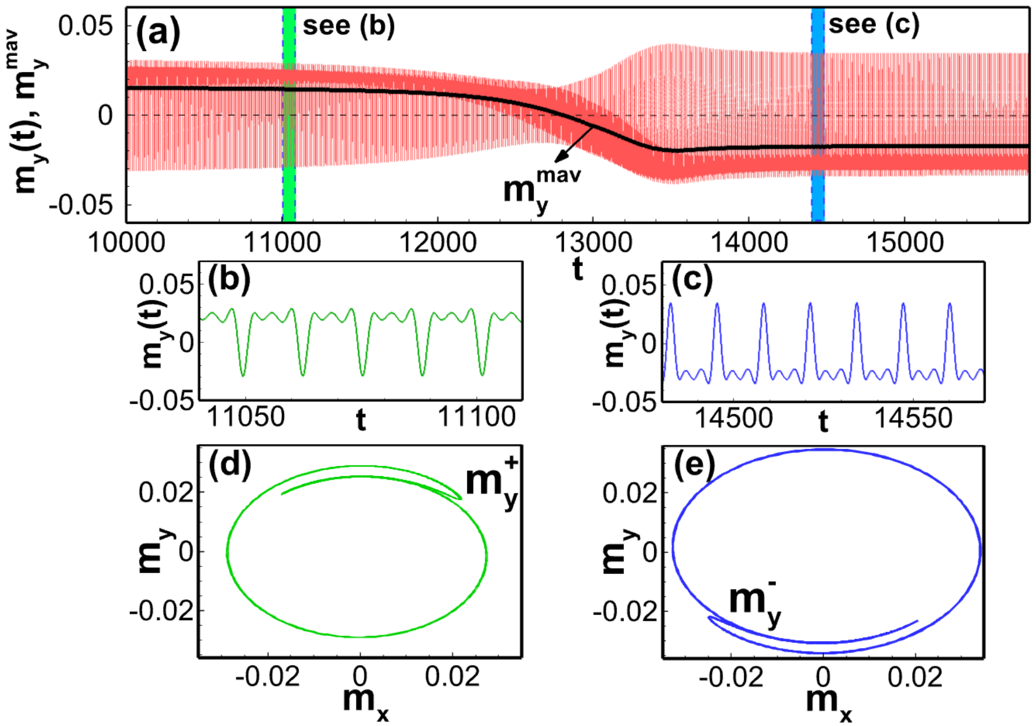
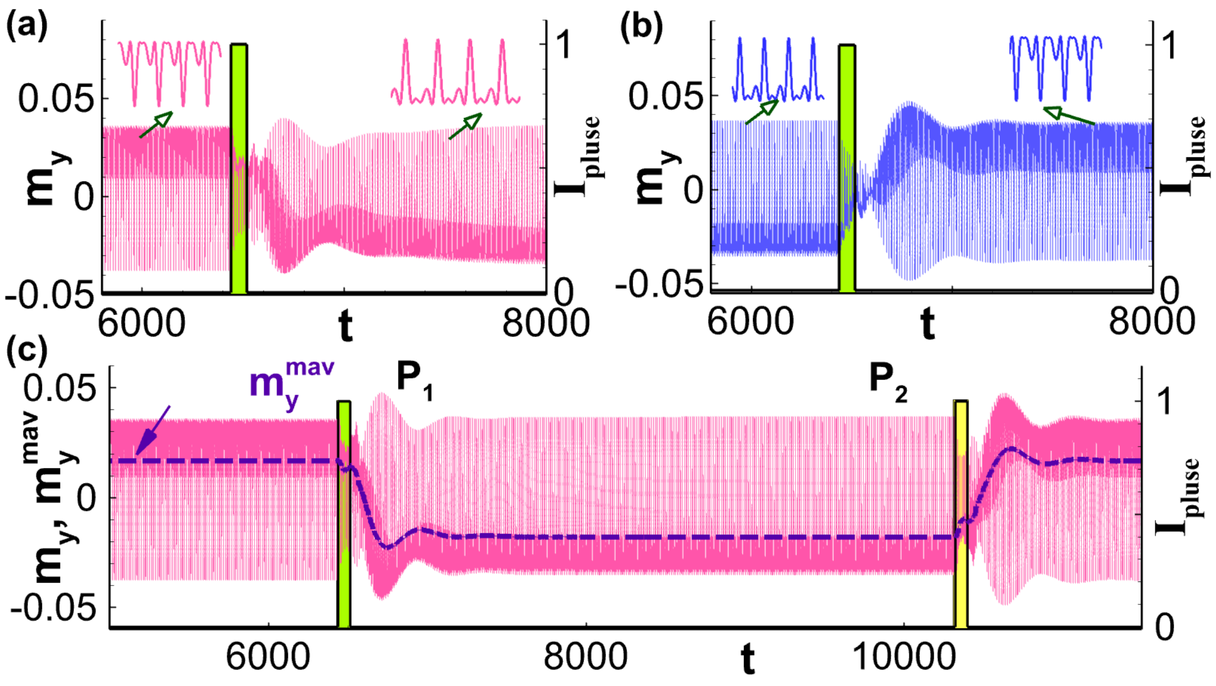


(a) The change in the m_y component dynamics in the case of the transition $m_y^+ \rightarrow m_y^-$; along with moving average; (b) and (c) show m_y (t) before the transition (state m_y^+) and after (state m_y^-); (d) and (e) magnetization trajectories in the plane x – y .

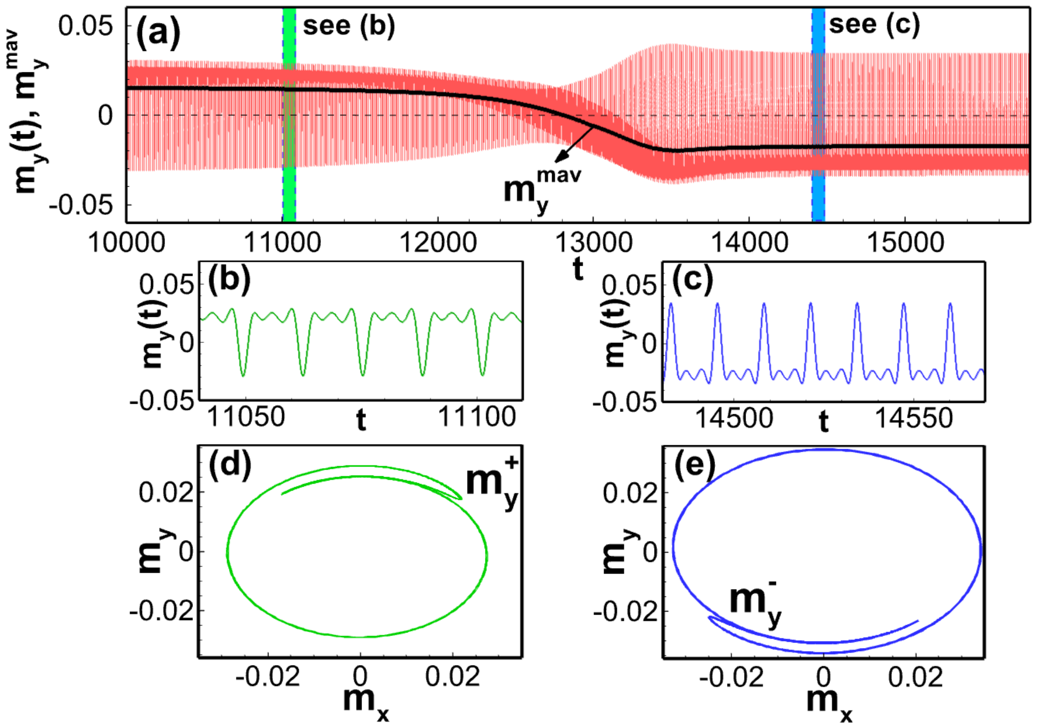
Switching between dynamical states

Magnetization dynamics for m_y under rectangular pulse signal (a) decreasing current; (b) increasing current; (c)decreasing current with two successive pulses, the dashed line shows the moving average during these switching process



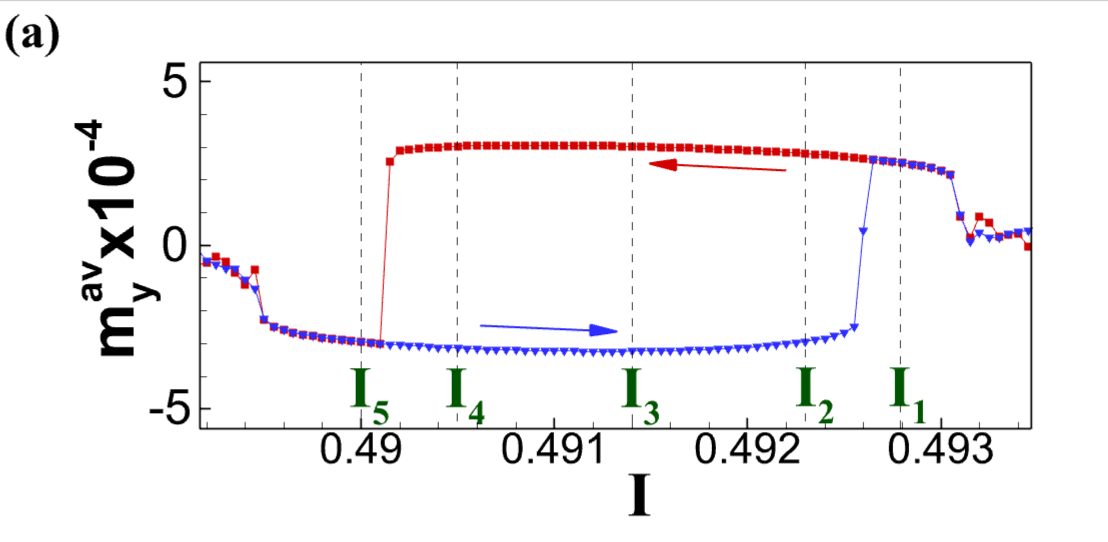
M. Nashaat, et. al., PRB **110, 024510** (2024)

Manifestation of two dynamical states in φ_0-junction



(a) The change in the m_y component dynamics in the case of the transition $m_y^+ \rightarrow m_y^-$; along with moving average; (b) and (c) show m_y (t) before the transition (state m_y^+) and after (state m_y^-); (d) and (e) magnetization trajectories in the plane x – y .

Phase shift between dynamical states



Pair	$I_s(t)$		V(t)	
	P_s	PC	P_s	PC
I_1^d, I_1^i	0^o	1	0^o	1
$I_1^{i(d)}, I_2^i$	176^o	-0.888	179^o	-0.898
I_3^d, I_3^i	177^o	-0.932	181^o	-0.931
$I_4^d, I_5^{i(d)}$	175^o	-0.872	179^o	-0.876
I_5^d, I_5^i	0^o	1	0^o	1

(Ps): Phase shift in degree and (PC): Pearsons correlation

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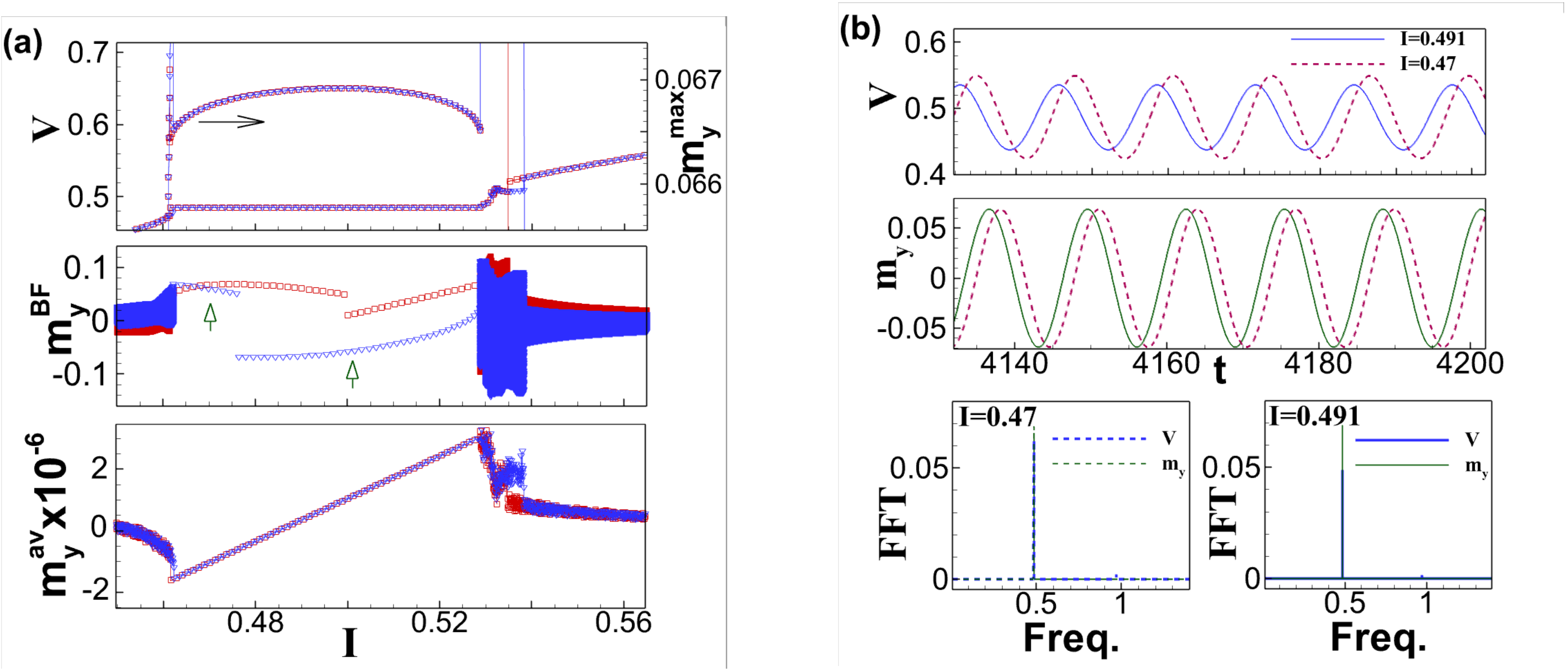
Shapiro Step & Manifestation of dynamical states



(a) Enlarged part for Shapiro step with two loop calculation, and m_y^{BF} (I), m_y^{av} (I) at A = 0.4 and h_R = 0. All panels are done at r = 0.4, ω = 0.485. The green hollow arrows indicate the value of current at which the time dependence are calculated. (b) Temporal dependence and FFT for V (t) and my(t) component at I = 0.47 and I = 0.491 for increasing current direction (see hollow arrows in (a)) .

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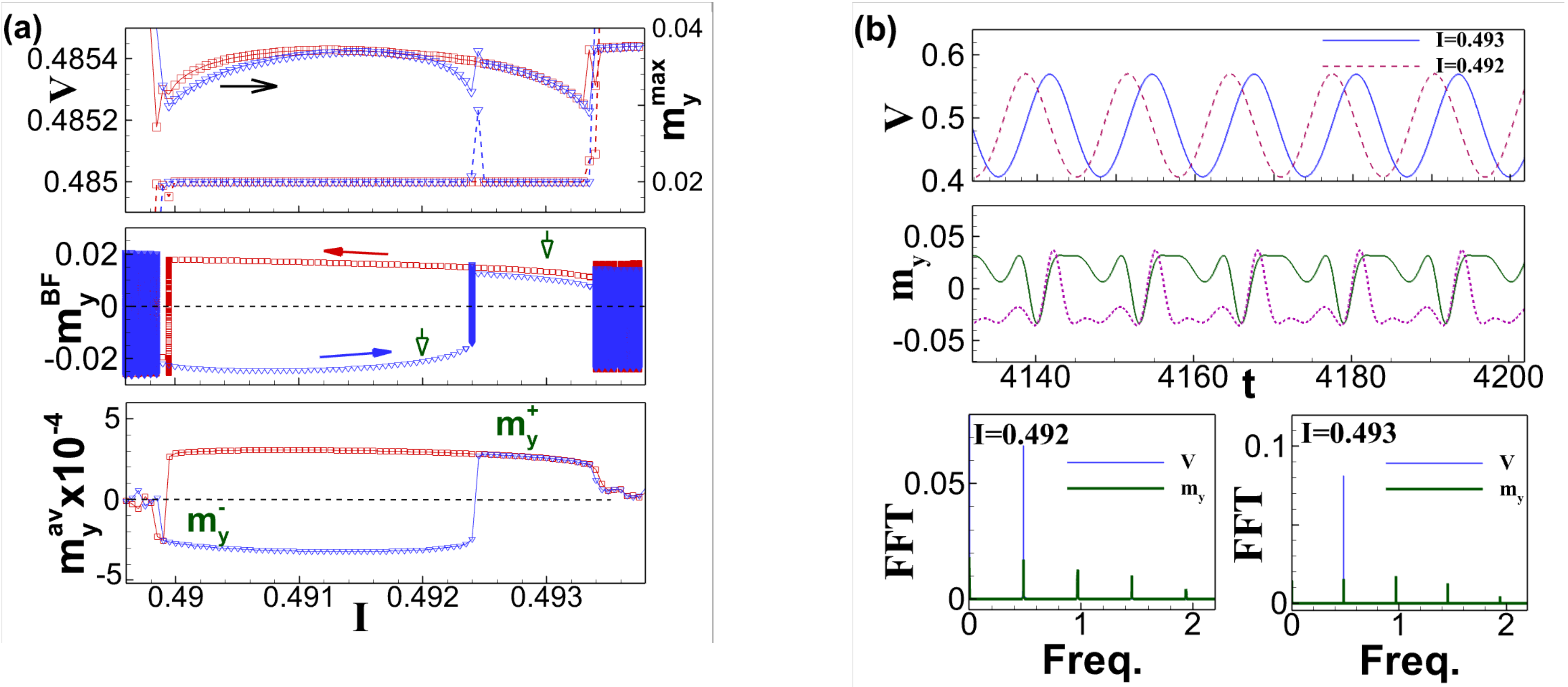
Buzdin step & Manifestation of dynamical states



(a) Enlarged part for Shapiro step with two loop calculation, and m_y^{BF} (I), m_y^{av} (I) at A = 0 and h_R = 1. All panels are done at r = 0.4, ω = 0.485. The green hollow arrows indicate the value of current at which the time dependence are calculated. (b) Temporal dependence and FFT for V (t) and my(t) component at I = 0.47 and I = 0.491 for increasing current direction (see hollow arrows in (a)) .

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Conclusions

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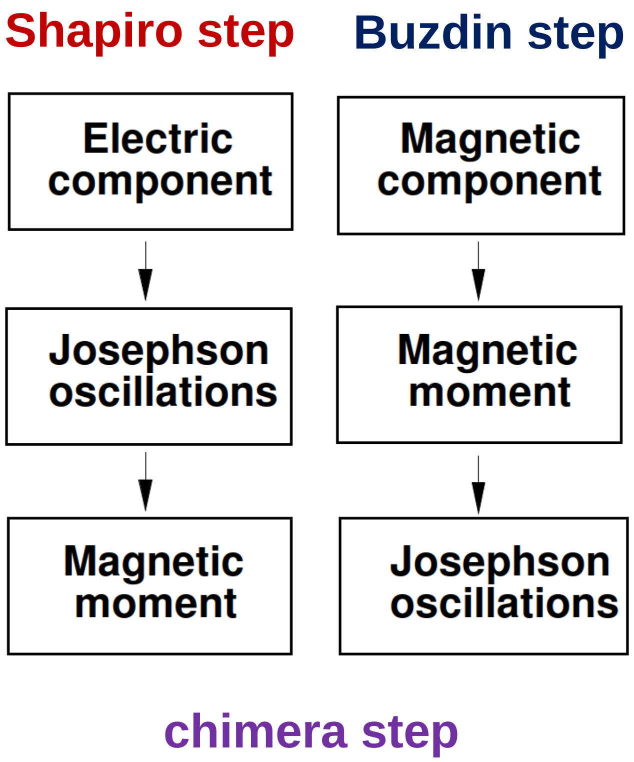
- Unique locking phenomena in the superconductor-ferromagnet-superconductor φ_0 Josephson junction under external electromagnetic radiation are demonstrated.

- Unlike the Shapiro steps where the magnetic moment remains constant along the step, in Buzdin and chimera steps it changes though the system is locked.

- Implementation of two types of dynamical states of magnetization is demonstrated which have a phase shift of π in the synchronization region of magnetic precession and Josephson oscillations.

- Transitions between these states with increasing and decreasing bias current show hysteresis, which is reflected in the bifurcation diagram and as spikes in the current-voltage characteristics



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Thank you