Stochastic voter model driven by avalanche-like perturbations

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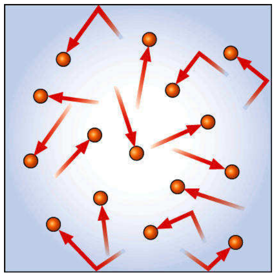
From statistical physics to social sciences

Statistical physics



Subject of studies:
- complex systems consisting of a large number of interacting elements

Differences:
- nature of the elements and mechanisms of interactions

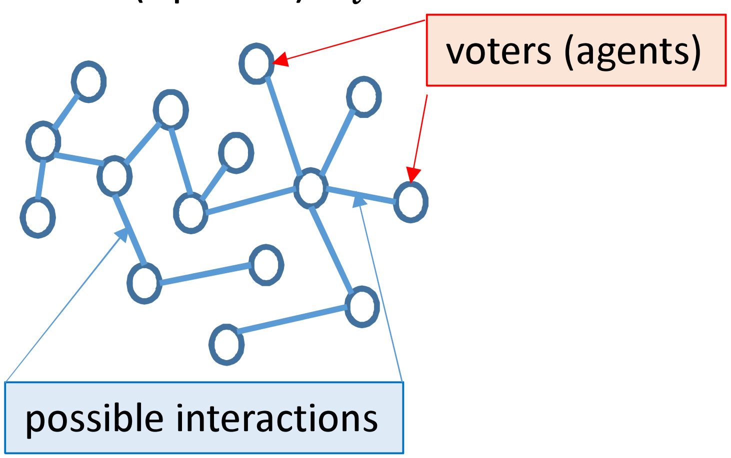
Common ground:
-collective phenomena emerging from the interactions of individuals

Sociology

!!! universality allows to hope that models and methods developed in statistical physics can be useful in describing how social systems behave

VOTER MODELS

state (opinion): $s_i = \pm 1$



$w_i^{\pm}$ - probability of a voter changing its opinion=

to copy the opinion of one of its neighbors

R. A. Holley and T. M. Liggett, The annals of probability , 3(4) (1975),643.
DOI: 10.1214/aop/1176996306

to copy the opinion of one of its neighbor
+ to switch the opinion independently

Toral R. at al., The noisy voter model on complex networks, 2016

Motivation and objectives

number of variants of the basic model
(Sidney Redner, Reality Inspired Voter Models: A Mini-Review, C.D.Physique, 20(4)(2019)275)

ATTENTION!

-agent who changes its opinion at this time step
-way of change
are RANDOMLY! chosen

MOTIVATION: take into account the REASONS that make

the agent change its opinion
INFORMATION STRESS

and

choose one of the ways
INTERACTS with neighbors OR NOT

OBJECTIVES:

1) to construct the model

- the changes in the binary state (opinion) of the voters (agents) are caused by the avalanche-like dynamics of the threshold variables (stresses);
- if an agent interacts with its environment it copies the state of its more informed neighbor
- if an agent is unlinked with its neighbors but is informed enough it switches its state independently

Examples: - two-way voting process in a community of people influenced by their environment

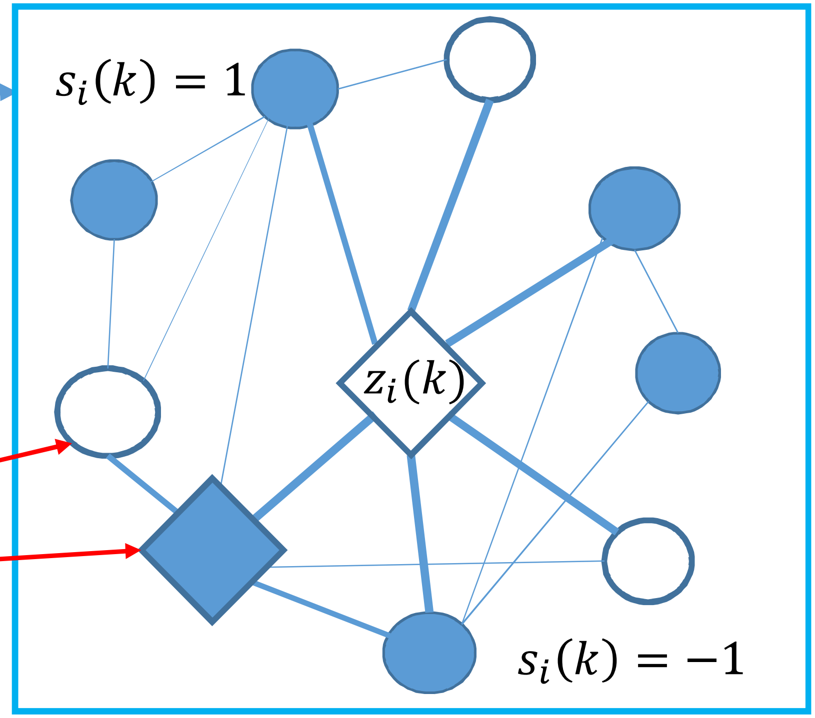
- decision-making process in a group of traders under market pressure

2) to study the dynamical regimes arising in the model

Description of the model

MULTI-ELEMENT SYSTEM:

-scale-free network with $P(l) \sim l^{-\gamma}, \gamma \approx 3$
-node=agent with opinion $s_i = \pm 1$
- link=possibility for agents to interact with each other



FIRST MODIFICATION:

- each agent can be active at each time step (k) with probability a_i.
- inactive agent is unlinked with its environment
- active agent activates all its links

in the network to connect with its neighbors

SECOND MODIFICATION:

- $z_i(k)$ threshold dynamical variable denotes the value of the information stress for the agent
- $\{z_i(k)\}$ changes according to the OFC-Earthquake model (Olami Z., Feder H., Christensen K., *Self-organized criticality in a continuous, nonconservative cellular automaton modeling earthquakes*, Phys. Rev. Lett. 68, 1992) , that is adapted for activity-driven network (Savitskaya N.E., Fedorova T.A. *"Model of opinion dynamics caused by information pressure in multi-agent system with stochastic activation of links"*/Physica Scripta, 99, 025007(2024),doi 10.1008/1402-4896/ad1859)

Model. Avalanches

1. Addition rule: $\forall i$: $z_i(k^n_{beg}) = z_i(k^n_{beg} - 1) + \Delta z; \Delta z \in]0, 2/L];$

2. Activation rule: with probability a_i each agent becomes active and activates its links

$$\begin{cases} \delta_i(k^n_{beg}) = 1 \; probability \; a_i \\ \delta_i(k^n_{beg}) = 0 \;\; probability \; 1 - a_i \end{cases}$$

3. Toppling rules:

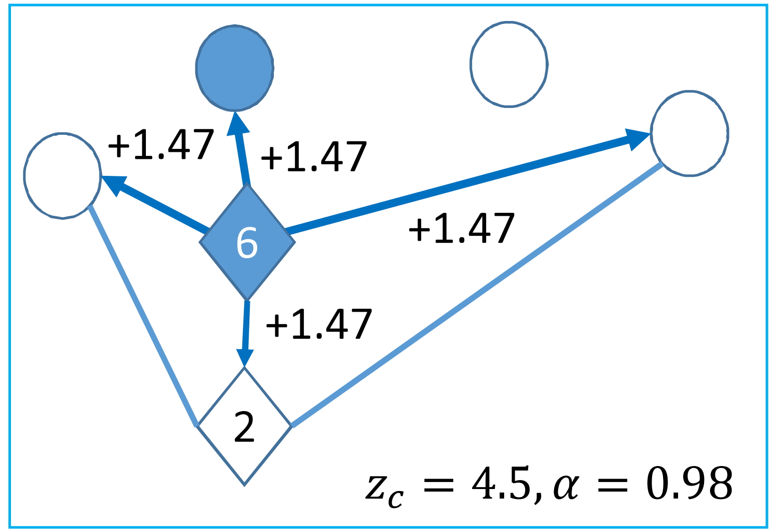
If $\delta_i(k) = \delta_i(k^n_{beg}) = 1$ and $z_i(k) \geq z_c$,

$then \; z_i(k) \rightarrow z_i(k+1) = 0;$

$z_{j \in nn(i)}(k) \rightarrow z_{j \in nn(i)}(k+1) = z_{j \in nn(i)}(k) + \alpha z_i(k)/l_i;$

4. Avalanche: repeat step 3 up to for all active agents $z_i(k^n_{end}) < z_c$

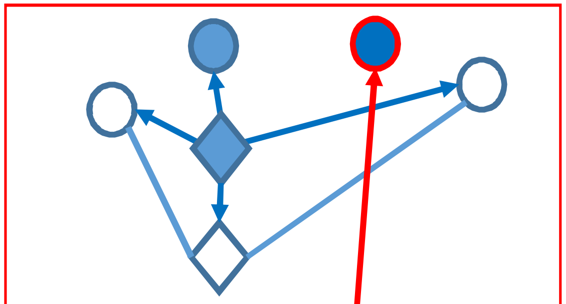
5. All links are canceled and steps (1-5) are repeated

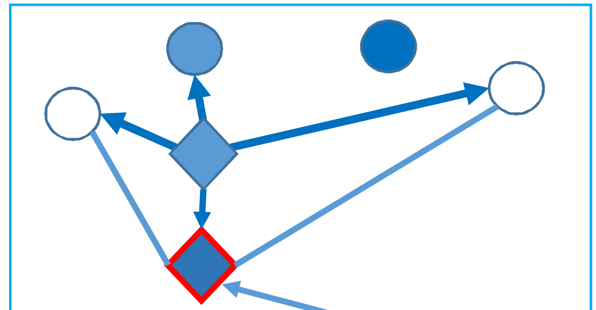


Size of n-th avalanche (total number of topplings during the avalanche)

$$s(n) = \frac{1}{L} \sum_{k=k^n_{beg}}^{k^n_{end}} \sum_{i \in G_n} \delta_i(k) \theta[z_i(k) - z_c]$$

Model. Opinion dynamics





- At the beginning of an avalanche, inactive overcritical agents switch their opinions :
- $if\ \delta_i(k_{beg}^n) = 0$ and $z_i > z_c, then\ s_i(k_{beg}^n) \to s_i(k_{beg}^n + 1) = -s_i(k_{end}^{n-1})$

- During an avalanche active agents copy the opinion of their toppling neighbors
- If $z_i(k)$ topples then $\forall j \in nn(i)$ if $\delta_j(k) = 1$, then $s_j(k) \to s_j(k+1) = s_i(k)$

$$w_i^{\pm}(s_i = \mp 1 \to \pm 1) = \frac{1}{2}(1-a_i)\rho_i^a(1 \mp s_i) + a_i \frac{1}{2}\sum_j A_{ij}(1 - s_j s_i) a_j \rho_j^a;$$

ρ_i^a- probability for an agent being "overcritical" ($z_i \geq z_c$)

$A_{ij} = \begin{cases} 0\ if\ \nexists l_{ij} \\ 1\ if\ \exists l_{ij} \end{cases}$ - adjacent matrix elements

DYNAMIC PROPERTIES

- stationary state of the model is not static
- system migrates through the ensemble of metastable states
- mode of system dynamics depends on relation between the probabilities of two competing processes

Dynamical regimes. Fokker-Plank equation

$m(k_{end}^n) = \frac{1}{L}\sum s_i(k_{end}^n)$ - system averaged opinion at the end of n-th avalanche

$\rho(m,t)$ - probability density for *m=m* at moment *t*

$m \to m + \Delta m$ with probability $\omega^+ = \sum_i w_i^+$

$m \to m - \Delta m$ with probability $\omega^- = \sum_i w_i^-$

$$\rho(m,t+\Delta t) = (1-w^-(m)-w^+(m))\rho(m,t)+w^+(m-\Delta m)\rho(m-\Delta m,t)+w^-(m+\Delta m)\rho(m+\Delta m,t)$$

$$\frac{\partial}{\partial t}\rho(m,t) = -\frac{\partial}{\partial m}\frac{\Delta m}{\Delta t}(w^+(m)-w^-(m))\rho(m,t)+\frac{1}{2}\frac{\partial^2}{\partial m^2}\frac{\Delta m^2}{\Delta t}(w^+(m)+w^-(m))\rho(m,t)$$

$$\rho^{st}(m) = C\exp(-L\int\frac{-F(m')+D'(m')/L}{D(m')}dm')$$

$$F(m) = w^+ - w^-; D(m) = w^+ + w^-$$

Mean-field approach for a heterogeneous network

$a_i = a;$

$a_i\rho_i^a =< s >= \frac{1}{N}\sum_n s(n)$ – probability of agent toppling, averaged over an ensemble of N avalanches

$\rho_i^a = \rho_a =< s > \frac{1-a}{a}$ – probability of agent being "overcritical" at the end of an avalanche

$A_{ij} \to P(l_i, l_j) = \frac{l_i l_j}{<l>L}$ - probability of linking for agents with degrees l_i and l_j

$w_i^\pm \to w_l^\pm$; $\sum_i w_i^\pm \to \sum_l w_l^\pm$ from node with number *i* to node of degree *l*

Dynamical regimes. Analytical results

$$\rho^{st}(m) \sim \frac{1}{(a<s><l>(1-m^2)+(1-a)\rho_a)^{1-P}};$$

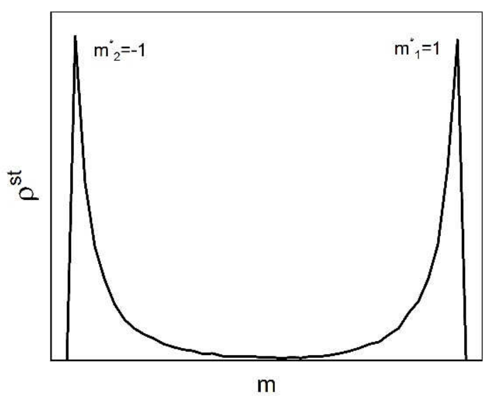
$$P = L\frac{(1-a)^2}{a^2<l>}; <l> = 2l_{min}$$

Activity is a parameter we are able to control even in real systems:

$$a_c = \frac{1}{1+\sqrt{2l_{min}/L}}$$

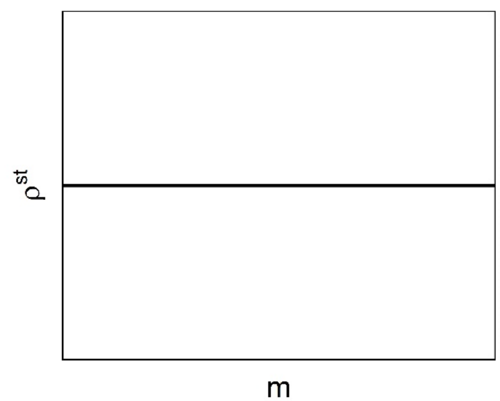
FINITE-SIZE SYSTEM

$a > a_c$



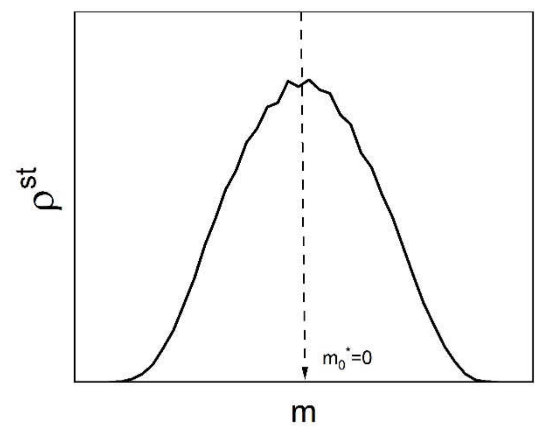
Bimodal Mode
-the system spends most of its time in consensus states.

$a = a_c$



Critical mode
- all values for m occur with the same probability

$a < a_c$

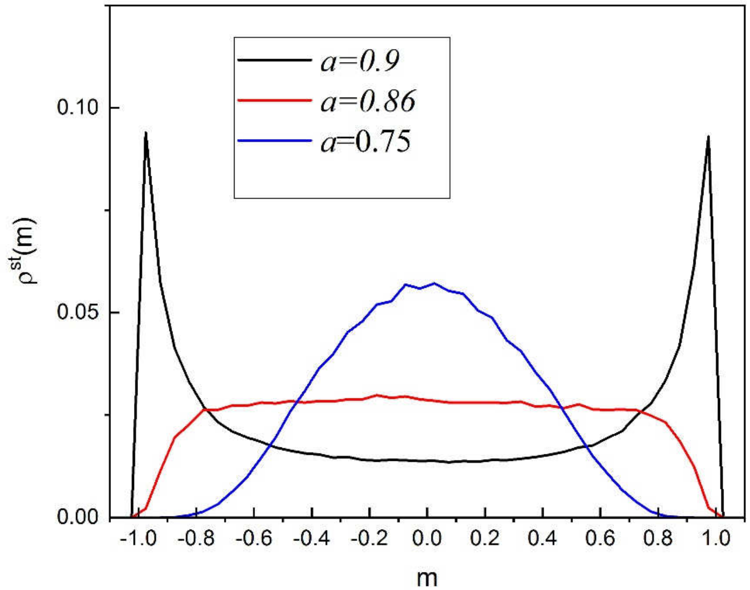


Unimodal Mode
- opposite values of s coexist and averaged m equals zero

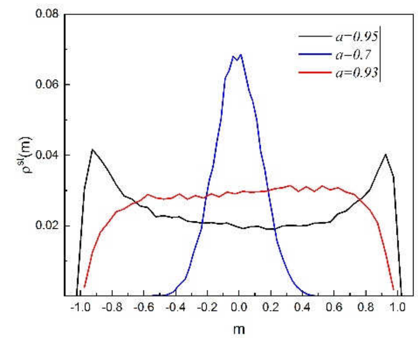
Dynamical regimes. Numerical results

L=2500

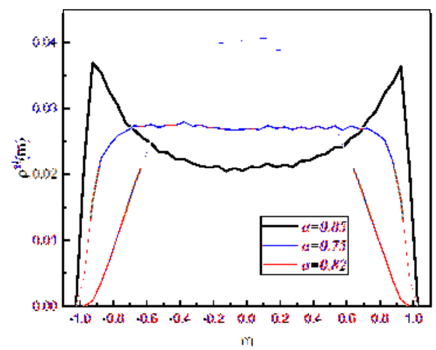
$l_{min} = 5\,, a_c \approx 0.86$



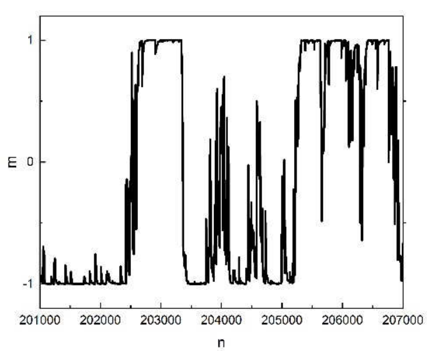
$l_{min} = 2, a_c \approx 0.93$



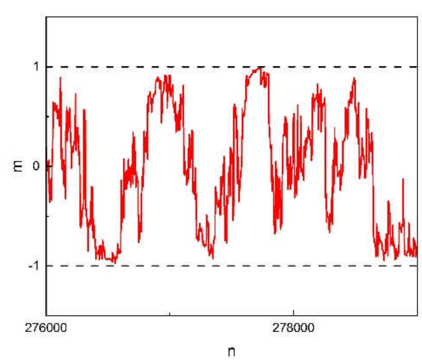
$l_{min} = 10, a_c \approx 0.82$



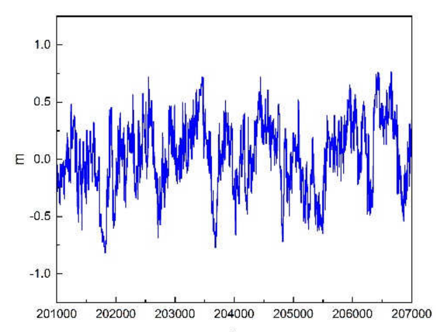
$a > a_c$



$a = a_c$



$a < a_c$



Physics and society

$a > a_c$
- group of agents who actively communicate with each other and tend to trust the opinion of a more informed neighbor. The most natural state for such a system is consensus, where the collective opinion coincides with the opinion of the most informed agent.

$a < a_c$
-group of agents who are mostly isolated from each other and tend to make decisions independent of the opinions of their environment. In this case the system tends to be in state of the dynamical opinion balance.

Conclusions

The opinion dynamics model, which takes into account the reasons for the agent to change its opinion and the way it chooses to do so, is constructed.

The mode of opinion dynamics is completely determined by the introduced agent characteristic - "activity". If "activity" is larger than its critical value, the system switches between two consensus states. In the opposite case, the system tends to the state where opposite opinions coexist and the system-averaged opinion is zero. The critical value of "activity" defines the state where all values of the common opinion occur with equal probability.

- The control parameter (activity) is a quantity with clear physical meaning. In addition, in real systems, the activity is easy to manipulate.

- The obtained analytical and numerical results reflect the intuitive ideas about the behavior of multi-agent systems and processes occurring in them.

- The modification of the model gives the possibility to model the process of opinion formation under conditions never considered before. For example, we can study the influence of the lifetime of the activated links on the behavior of the system, or the case of time-varying activities, and so on.

Thank you for attention!

Noisy voter model

(Toral R. at al., The noisy voter model on complex networks, 2016)

agent has a possibility

to copy the opinion of its neighbor

to switch the opinion independently

$$w_i^{\pm} = \frac{h_i^{copy}}{2l_i} \sum_j A_{ij} \,(1 - s_i s_j) + h_i^{ind\pm}$$

system migrates through a set of metastable states

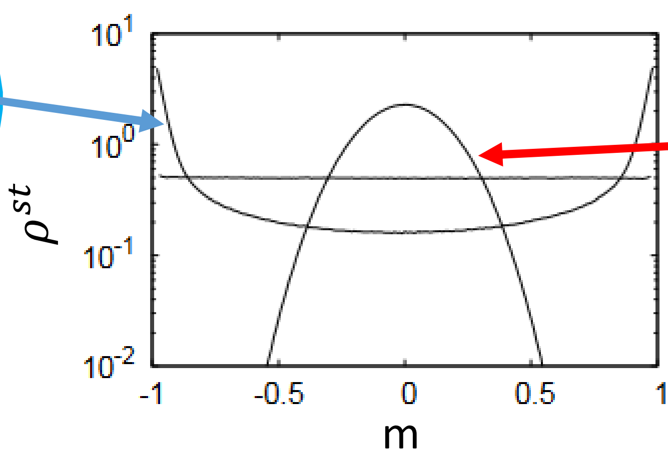
consensus

dynamical balance

$$P \sim h^{copy} / h^{ind}$$

$m = \frac{1}{L} \sum s_i$ -system averaged opinion

$P > 1$ system switches between two consensus state



$P < 1$ opposite opinions coexist