Lattice study of rotating QCD properties

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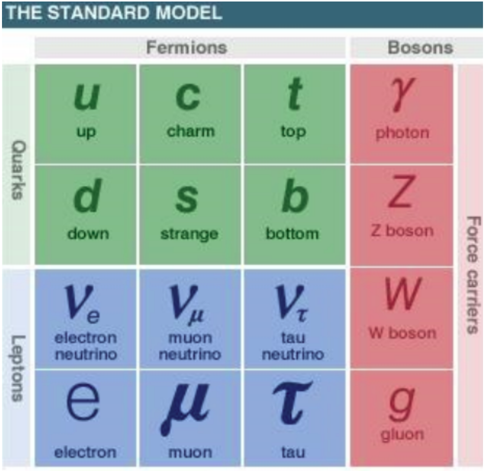
Outline:

- Introduction
- Lattice simulation of theory of strong interaction (QCD)
- Few applications
- Lattice study of rotating QCD properties
- Conclusion

Table of elementary particles



We consider quarks, gluons and their interactions

Theory of strong interactions (QCD)

- Similar to electrodynamics (gauge theory)
- Maxwell's equations
  - QED: linear
  - QCD: nonlinear
- Coupling constants ($V(r) \sim \frac{\alpha}{r}$)
  - QED: $\alpha_{em} = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$
  - QCD: $\alpha_s = \frac{g^2}{4\pi\hbar c} \sim 1$
- QCD is strongly interacting nonlinear theory
- A lot of applications: nuclear physics, astrophysics, cosmology, elementary particles...

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One of the most complicated physical systems even without quarks and very important theory!

Theory of strong interactions (QCD)

The Lagrangian of QCD

- Degrees of freedom
  - Quark q (similar to electron)
  - Gluon A, F_a (similar to photon)
- Building Lagrangian of QCD

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_{f=u,d,s,\ldots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}\gamma^\mu \hat{A}_\mu q$$

- QCD Lagrangian is well known but one cannot calculate observables

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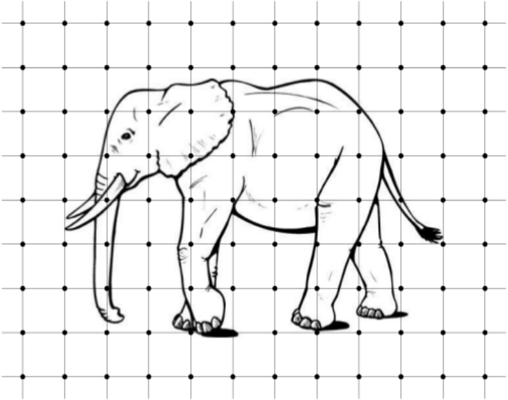
- QCD Lagrangian is well known but one cannot calculate observables
  - In particular: **Derivation of confinement from Lagrangian is a millenium problem**

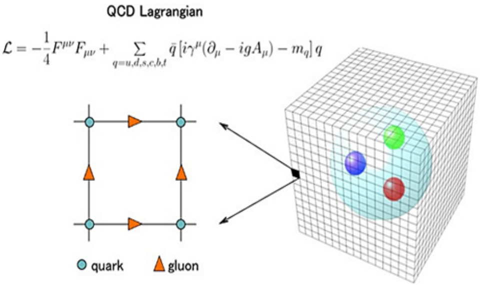
Lattice QCD



Lattice simulation

- Allows to study strongly interacting systems
- Based on the first principles of quantum field theory
- Powerful due to modern supercomputers and algorithms

Building lattice QCD



- Introduce regular cubic four dimensional lattice $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing–a
- Degrees of freedom
  - **Gluon fields:** 3x3 matrices $U \in SU(3)$, live on links
  - **Quarks fields:** column $q, \bar{q}$, live on sites

Building lattice QCD

- We study QCD in thermodynamic equilibrium
- Partition function Z
  $E = T^2 \frac{\partial \log Z}{\partial T}, \quad p = T \frac{\partial \log Z}{\partial V}, \quad S = -\frac{\partial T \cdot \log Z}{\partial V}$
- QCD partition function
  $Z = \int DU \exp\left(-S_G(U)\right) \times \prod_{i=u,d,s\ldots} \det\left(\hat{D}_i(U) + m_i\right)$
- In continuum lattice partition function exactly reproduces QCD partition function
  - Gluon contribution: $S_G(U)\Big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
  - Quark contribution:
    $\bar{q}(\hat{D}(U) + m)q\Big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$

Lattice simulation of QCD

Properties

- We calculate partition function
  $$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s...} \det\left(\hat{D}_i(U) + m_i\right)$$
- Carry out continuum extrapolation $a \to 0$
- Uncertainties (discretization and finite volume effects) can be systematically reduced
- The first principles based approach. No assumptions!
- Parameters: α_s and masses of quarks

Lattice simulation of QCD

- We calculate partition function

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s...} \det\left(\hat{D}_i(U) + m_i\right) = \int DU e^{-S_{eff}(U)}$$

  - 96×48^3
  - Variables: $96 \cdot 48^3 \cdot 4 \cdot 8 \sim 300 \cdot 10^6$
  - Matrices: $100 \cdot 10^6 \times 100 \cdot 10^6$
- What about Monte Carlo approach?
- However, the $S_{eff}(U)$ distribution is **very localized**;
- Field configurations $U(x)$ away from action minima are **significantly suppressed**;
- How to efficiently sample from this distribution?

Lattice simulation of QCD

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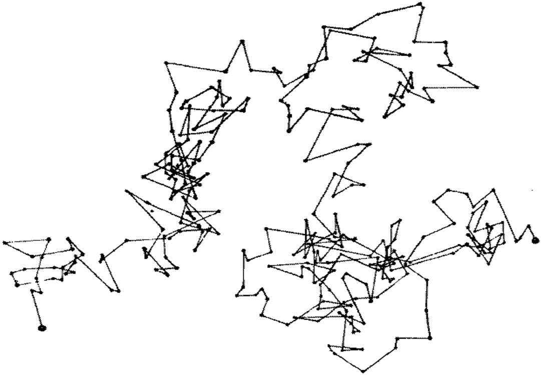
One applies the Hybrid Monte Carlo algorithm

The Hybrid Monte Carlo algorithm



- HMC can be considered as Brownian motion of the system
- Accept/reject step at the end of the trajectory
  - if $S_{eff}(U_{n+1}) < S_{eff}(U_n)$ the U_{n+1} is accepted
  - otherwise U_{n+1} is accepted with $p \sim e^{-[S_{eff}(U_{n+1}) - S_{eff}(U_n)]}$
- Simulation of quantum system!
- For large number of the trajectories $p(U) \sim e^{-S_{eff}(U)}$

The Hybrid Monte Carlo algorithm

Introduce conjugate momenta $\pi(x)$ and consider Hamiltonian

$$H(U,\varphi,\pi) = \int d^4x \left(S_G(U(x)) + S_F(U(x),\varphi(x)) + \frac{1}{2}\pi^2(x) \right).$$

Steps of the algorithm:

- Generate conjugate momenta from Gaussian distribution $\pi(x) \sim N(0,1)$;
- Introduce fictions *molecular dynamics* time τ;
- Run Hamiltonian evolution of fields and conjugate momenta

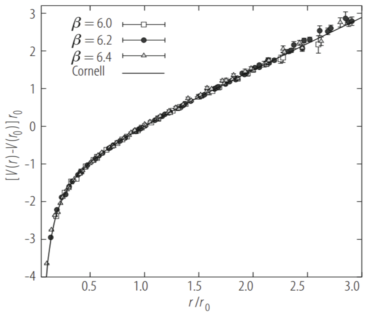
$$\frac{\partial U(x,\tau)}{\partial \tau} = \frac{\partial H(x,\tau)}{\partial \pi(x,\tau)} \quad \frac{\partial \pi(x,\tau)}{\partial \tau} = -\frac{\partial H(x,\tau)}{\partial U(x,\tau)}.$$

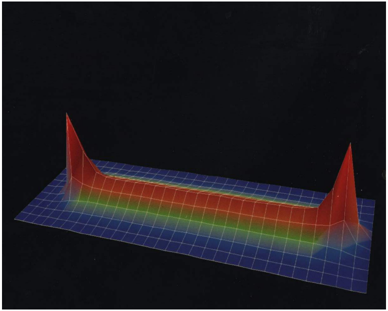
- Accept the final field configuration with the probability $\min\left(1, \exp\left(H' - H\right)\right)$ — Metropolis accept/reject procedure.

Applications

- Spectroscopy
- Matrix elements and correlations functions
- Thermodynamic properties of QCD
- Transport properties of QCD
- Phase transitions
- Nuclear physics
- Properties of QCD under extreme conditions
  - High temperature
  - Huge magnetic field
  - Large baryon density
  - Relativistic rotation
  - ...
- Vacuum structure and topological properties
- Beyond the Standard Model at strong coupling
- ...

Confinement in lattice simulations



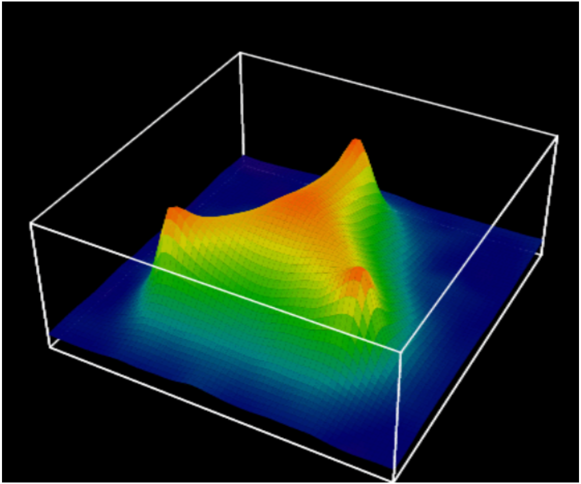
, ,

- Millenium problem – confinement
- Can be solved for one hour at modern laptop

What are we made of?

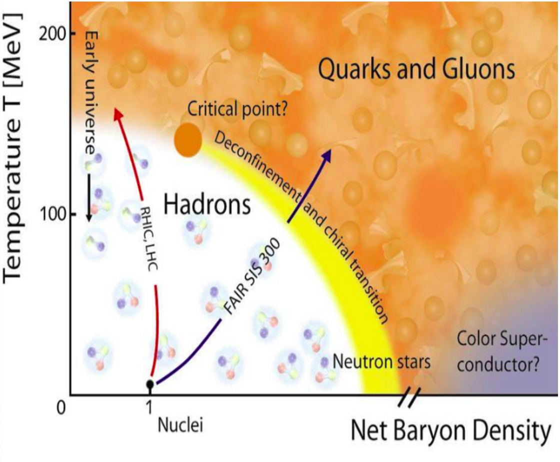
- In the world around us we observe the law
  $M \simeq \sum_i M_i$
- but in QCD
  $p(uud) \quad M_p c^2 = 938 \text{ MeV} \gg (m_u + m_u + m_d)c^2 = 12 \text{ MeV}$
  $n(udd) \quad M_n c^2 = 940 \text{ MeV} \gg (m_u + m_d + m_d)c^2 = 15 \text{ MeV}$
- Where is the rest of the mass?

Chromoelectric fields in proton



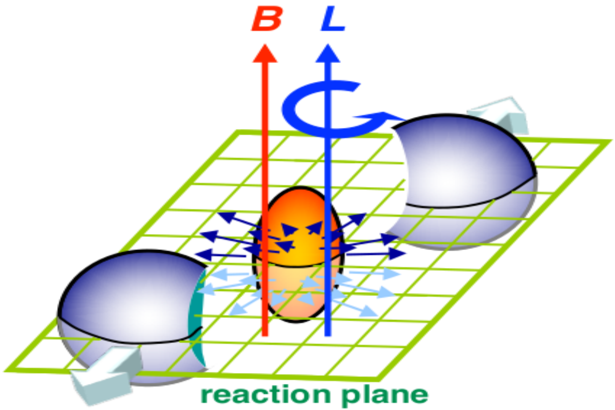
- We are made of gluons to >90%!

Phase transitions in QCD



- Confinement/deconfinement phase transition at temperatures $T \sim 150$ MeV$(\sim 1.5 \cdot 10^{12}\ K)$

Rotation of QGP in heavy ion collisions

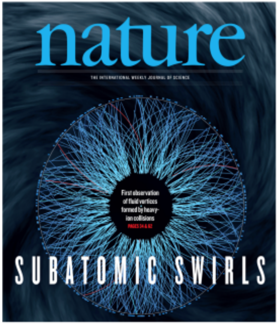


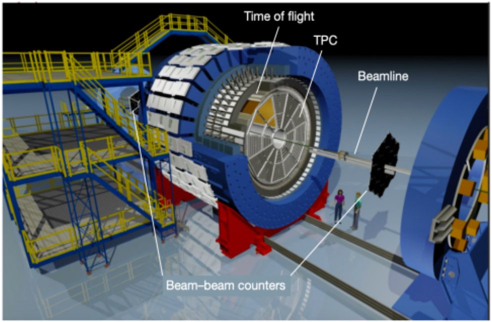
- QGP is created with non-zero angular momentum in non-central collisions

The most vortical fluid ever observed

The experimental result for the vorticity:

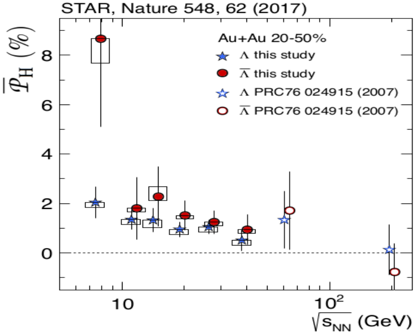
$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$





The STAR Collaboration, Nature 62, 548 (2017)

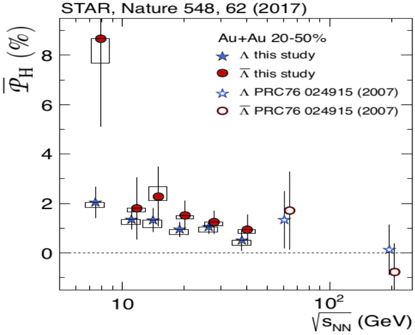
Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 10^{22} s^{-1}$)
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions



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How relativistic rotation influences QCD?

Study of rotating QGP

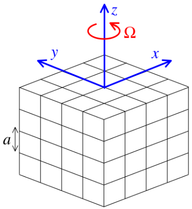
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

Sign problem

$$S_G = \frac{1}{2g^2}\int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau\hat\varphi})^2 + (F^a_{\tau z})^2 + \\ + (F^a_{rz})^2 + \left(1-(\Omega r)^2\right)(F^a_{\hat\varphi z})^2 + \left(1-(\Omega r)^2\right)(F^a_{r\hat\varphi})^2 + \\ + 2ir\Omega (F^a_{r\hat\varphi}F^a_{\tau r} - F^a_{\hat\varphi z}F^a_{\tau z}) \Big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω

Lattice study of rotating gluodynamics/QCD

- Critical temperature in rotating gluodynamics
- Critical temperatures in rotating QCD
- Equation of state
- Moment of inertia
- Inhomogeneous phase transition
- ...

EoS of rotating gluodynamics

- Free energy of rotating QCD ($F = -T \log Z$)

$$F(T, R, \Omega) = F_0(T, R) + C_2 \Omega^2 + ...$$

- The moment of inertia

$$C_2 = -\frac{1}{2} I_0(T, R), \quad I_0(T, \Omega) = -\frac{1}{\Omega} \left(\frac{\partial F}{\partial \Omega} \right)_{T, \Omega \to 0}$$

- Instead of $I_0(T, R)$ we calculate $K_2 = -\frac{I_0(T,R)}{F_0(T,R) R^2}$
- Sign of K_2 coincides with the sign of $I_0(T, R)$

EoS of rotating gluodynamics

- Classical moment of inertia

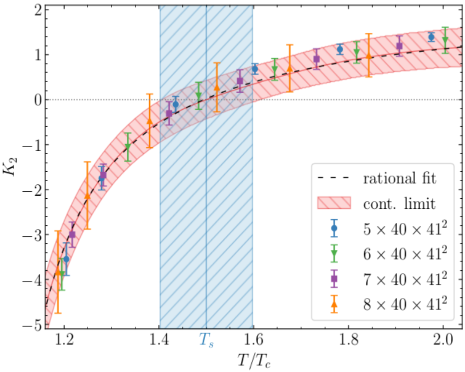
$$I_0(R) = \int_V d^3x x_\perp^2 \rho_0(x_\perp)$$

- Related to the trace of EMT $\quad T^\mu_\mu = \rho_0(x_\perp)c^2$
- Generation of mass scale in QCD and scale anomaly

$$T^\mu_\mu \sim \langle G^2 \rangle \sim \langle H^2 + E^2 \rangle$$

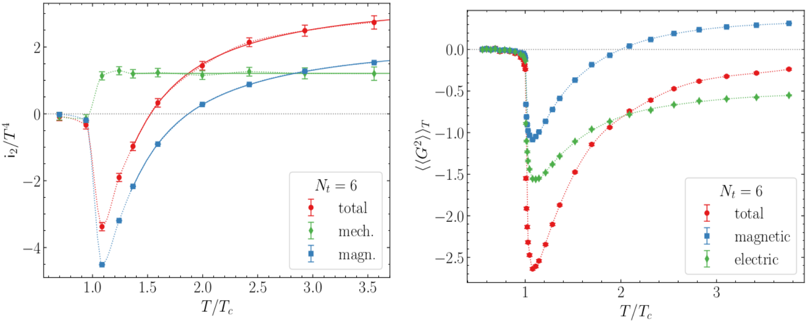
- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?
- $I_0 = I_{mech} + I_{magn}$ *valid for QCD!*
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2 \sim \langle {S_1}^2 \rangle$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle \sim \langle S_2 \rangle$

Moment of inertia of gluon plasma



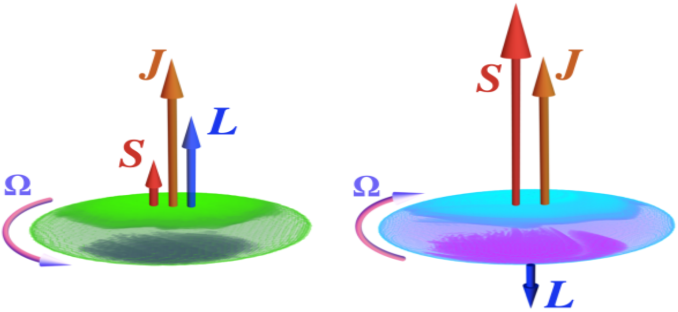
- $I(T,R) = -F_0(T,R)K_2R^2$
- $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- $I < 0$ is related to magnetic condensate and the scale anomaly
- We believe that the same is true for QCD

Moment of inertia of gluon plasma



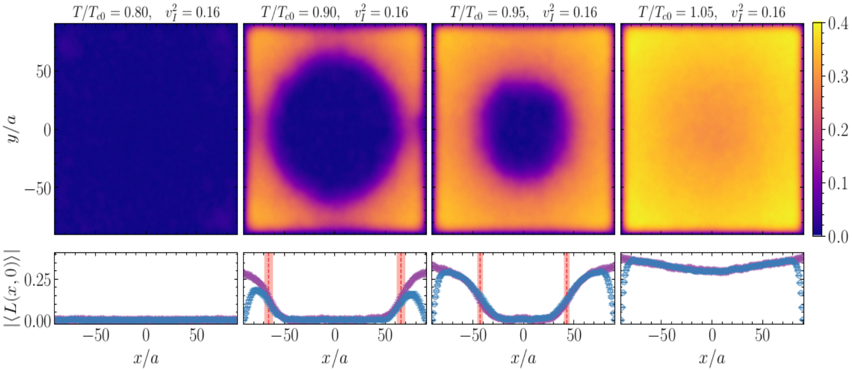
- $i_2 = \frac{I_0}{VR_\perp^2}, \quad I_0 = I_{mech} + I_{magn}$
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle$
- Gluon condensate: $\langle G^2 \rangle = \langle E^2 \rangle + \langle H^2 \rangle$

Negative Barnett effect(?)



- $J = I_2\Omega = -\left(\frac{\partial F}{\partial \Omega}\right)_T$, $\mathbf{J} = \mathbf{L} + \mathbf{S}$
- $\mathbf{L} \uparrow\uparrow \mathbf{\Omega}$, $\mathbf{S} \uparrow\downarrow \mathbf{\Omega}$ might lead to $\mathbf{J} \uparrow\downarrow \mathbf{\Omega}$ and $I_2 < 0$

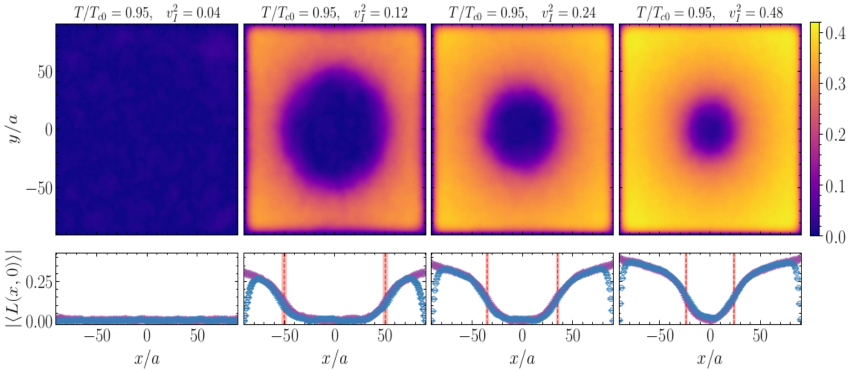
Inhomogeneous phase transition in QGP



- Huge lattices are required for simulations
- Cylindrical Symmetry is restored
- The results for PBC and OBC coincides in the bulk
- Confinement in the center and deconfinement in the periphery

  In disagreement with Ehrenfest–Tolman law
- Inhomogeneous phase takes place below T_c

Inhomogeneous phase transition in QGP



- The phase transition is induced by rotation

Conclusion

- Understanding of QCD properties is interesting for us and important for applications
- QCD is very complicated theory and it can be studied reliably only within lattice simulation
- Lattice study of rotating gluodynamics and QCD have been carried out
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- We observed inhomogeneous phase transition in GP: deconfinement in the central and confinement in the periphery regions

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THANK YOU!