Three-Body Problem and Precision Physics

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Yerevan, October, 2024

Content

1. Quantum Three-Body Problem
2. Nonrelativistic Quantum Mechanics (NRQED)
3. Physics of exotic atoms
4. Precision Spectroscopy of the Hydrogen molecular ions

Variational expansion

Variational Principle for bound states

We solve a stationary Schrödinger equation,

$$H\Psi = E\Psi,$$

We assume that Hamiltonian H is a selfadjoint operator in a Hilbert space, which satisfies

$$H \geq cI, \tag{1}$$

where c is some constant, not necessarily positive.
Let us define a functional

$$\Phi(\Psi) = \frac{(\Psi, H\Psi)}{(\Psi, \Psi)}\ ,$$

This functional is bound from below by c. Stationary points of the functional (1) determine energies and WF of bound states.

Exponential expansion

This basis has a long history, probably the first explicit formulation of the method has been done by Power and Somorjai[2]

The wave function is taken in the form

$$\Psi_{LM}(\mathbf{r}_1,\mathbf{r}_2) = \sum_{l_1+l_2=L \text{ or } L+1} C_{l_1 l_2 n}\, \mathcal{Y}^{l_1 l_2}_{LM}(\mathbf{r}_1,\mathbf{r}_2) e^{-\alpha_n r_1 - \beta_n r_2 - \gamma_n r_{12}},$$

where α_n, β_n, and γ_n are randomly generated parameters:

$$\alpha_i = \left[\left\lfloor \frac{1}{2} i(i+1)\sqrt{p_\alpha} \right\rfloor (A_2 - A_1) + A_1\right] + \\ +i\left\{\left[\left\lfloor \frac{1}{2} i(i+1)\sqrt{q_\alpha} \right\rfloor (A'_2 - A'_1) + A'_1\right]\right\},$$

$\lfloor x \rfloor$ designates the fractional part of x, p_α and q_α are some prime numbers. Parameters β_i and γ_i are obtained in a similar way.

[2] J.D. Power and R.L. Somorjai, Phys. Rev. A **5** (1972) 2401

Exponential expansion. Main properties

One of the merits of the method is a very high convergence rate (in common).

Demerits of the method:

- Fast degeneracy of the basis set with increase of the basis. In a double precision arithmetics already for $N \sim 200$ calculations become unstable. That may be cured by the use of multiprecision packages, which allows to work with arbitrary number of significant digits.
- For the helium atom ground state for large N rate of convergence become rapidly to slow down.
- Slow convergence for systems with two heavy particles as H_2^+.

Examples. Helium Atom

Helium atom

PHYSICAL REVIEW A **98**, 012510 (2018)

Nonrelativistic energy levels of helium atoms

D. T. Aznabaev,[1,2,3] A. K. Bekbaev,[1,4] and Vladimir I. Korobov[1,5]

TABLE II. Nonrelativistic energies of the S, P, D, and F states of a helium atom. N is the number of basis functions. The two lines represent two consecutive calculations with the largest basis sets to show convergent digits. The third line presents calculations by Drake and Yan [23].

State	N	E_{nr}	State	N	E_{nr}
1^1S	18000	−2.90372 43770 34119 59831 11592 45194 40432	4^1S	14000	−2.03358 67170 30725 44743 92926 44363 64
1^1S	22000	−2.90372 43770 34119 59831 11592 45194 40443	4^1S	18000	−2.03358 67170 30725 44743 92926 44363 87
2^1S	18000	−2.14597 40460 54417 41580 50289 75461 918	4^3S	14000	−2.03651 20830 98236 29958 03780 71617 853
2^1S	22000	−2.14597 40460 54417 41580 50289 75461 921	4^3S	16000	−2.03651 20830 98236 29958 03780 71617 874
	[23]	−2.14597 40460 5443(5)			
2^3S	14000	−2.17522 93782 36791 30573 89782 78206 81124	4^1P	18000	−2.03106 96504 50240 71475 89314 36090 3
2^3S	16000	−2.17522 93782 36791 30573 89782 78206 81125	4^1P	22000	−2.03106 96504 50240 71475 89314 36094 1
	[23]	−2.17522 93782 367912(1)		[23]	−2.03106 96504 5024(3)
2^1P	18000	−2.12384 30864 98101 35924 73331 42354	4^3P	18000	−2.03232 43542 96630 33195 38824 67087
2^1P	22000	−2.12384 30864 98101 35924 73331 42374	4^3P	22000	−2.03232 43542 96630 33195 38824 67103
	[23]	−2.12384 30864 98092(8)		[23]	−2.03232 43542 9662(2)
2^3P	16000	−2.13316 41907 79283 20514 69927 63793	4^1D	22000	−2.03127 98461 78684 99621 39438 073
2^3P	18000	−2.13316 41907 79283 20514 69927 63806	4^1D	26000	−2.03127 98461 78684 99621 39438 143
	[23]	−2.13316 41907 7927(1)		[23]	−2.03127 98461 78687(7)
3^1S	18000	−2.06127 19897 40908 65074 03499 37089 2816	4^3D	18000	−2.03128 88475 01795 53802 34920 591
3^1S	22000	−2.06127 19897 40908 65074 03499 37089 2824	4^3D	22000	−2.03128 88475 01795 53802 34920 630
				[23]	−2.03128 88475 01795(3)
3^3S	14000	−2.06868 90674 72457 19199 65329 11291 75048	4^1F	18000	−2.03125 51443 81748 60863 20824 071
3^3S	16000	−2.06868 90674 72457 19199 65329 11291 75049	4^1F	22000	−2.03125 51443 81748 60863 20824 079
				[23]	−2.03125 51443 81749(1)

Other examples

system		E
$^4\mathrm{He}^+\bar{p}\,(L\!=\!34,\,v\!=\!1)$	Kino	-2.996335432
	Korobov	$-2.9963354479662700(5)$
$\mathrm{H}_2^+\,(L\!=\!0,\,v\!=\!19)$	Moss	-0.49973123063
	Korobov	$-0.49973123065581\,2(2)$

The last example is of special interest since that is the last vibrational S-state. The wave function of this state has 19 nodes(!), and a binding energy is 3.390939346×10^{-6} a.u., that is *five orders* less than the binding energy of the ground state.

Nonrelativistic QED

Concept of NRQED

QED

$$\mathcal{L}_{\rm QED} = \bar{\psi}\left[(i\partial - e\,A)\,\gamma - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

$$\Downarrow$$

Nonrelativistic QED

$$\mathcal{L}_{\rm NRQED}$$

$$\Downarrow$$

Effective Hamiltonian

$$H_{\rm eff} = \sum_i \frac{\mathbf{P}_i^2}{2m_i} + e^2\sum_{j>i}\frac{Z_iZ_j}{r_{ij}} + \text{higher order corrections}$$

(Here $\mathbf{P}_i = \mathbf{p}_i + e\mathbf{A}$)

Nonrelativistic QED Lagrangian

The Lagrangian for NRQED is built out nonrelativistic (two-component) Pauli spinor fields ψ for each of the electron, positron, muon, proton, etc. Photons are treated in the same fashion as in QED.

$$
\begin{aligned}
L_{\rm eff} = -\tfrac{1}{2}(E^2-B^2) + \psi_e^* \left(i\partial_t - e\varphi + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + \ldots \right) \psi_e \\
+\psi_e^* \left(c_F \frac{e}{2m}\boldsymbol{\sigma}\mathbf{B} + c_D \frac{e}{8m^2}\left[\mathbf{DE}\right] + c_S \frac{e}{8m^2}\left\{\boldsymbol{\sigma}\cdot[i\mathbf{D}\times\mathbf{E}]\right\} \right)\psi_e \\
+ \text{higher order terms} + \text{muon, proton, etc.} \\
-\frac{d_1}{m_e m_\ell}\left(\psi_e^*\boldsymbol{\sigma}_e\psi_e\right)\left(\psi_\ell^*\boldsymbol{\sigma}_\ell\psi_\ell\right) + \frac{d_2}{m_e m_\ell}\left(\psi_e^*\psi_e\right)\left(\psi_\ell^*\psi_\ell\right) + \ldots
\end{aligned}
$$

where $\mathbf{D} = \boldsymbol{\nabla} - ie\mathbf{A}$.

$$
\begin{aligned}
c_D &= 1 + 2\kappa + \frac{\alpha}{\pi}\frac{8}{3}\left[\ln\left(\frac{m}{2\Lambda}\right) + \frac{5}{6} - \frac{3}{8}\right], \\
c_S &= 1 + 2\kappa, \\
c_F &= 1 + \kappa, \\
d_1 &= (Z\alpha)^2 \frac{2}{m_e^2 - m_\ell^2} \ln\left(\frac{m_e}{m_\ell}\right), \\
d_2 &= (Z\alpha)^2 \left\{ \frac{7}{3} - 2\ln\left(\frac{m}{2\Lambda}\right) + \frac{2}{m_e^2 - m_\ell^2}\left[m_e^2 \ln\left(\frac{m_\ell}{\mu}\right) - m_\ell^2 \ln\left(\frac{m_e}{\mu}\right) \right] \right\}.
\end{aligned}
$$

NRQED requirements

Requirements for the NRQED Lagrangian interaction terms:

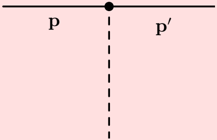
- Hermiticity;
- Gauge invariance. We use covariant derivatives: $\mathbf{D} = \boldsymbol{\nabla} - ie\mathbf{A}$;
- Parity. The Lagrangian should be parity even;
- Time reversal. The Lagrangian should be even under time reversal transformation.
- Coupling constants are determined by requiring that predictions of QED and NRQED agree to a desired order in (v/c);
- Contributions from QED that involve relativistic loop momenta are absorbed into NRQED in a form of various local interactions.

Basic interactions and perturbation theory

Examples of basic interactions in NRQED. Vertices.

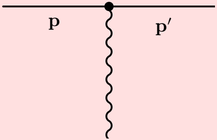
Coulomb

$$e$$



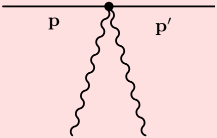
"dipole"

$$-e\left[\frac{\mathbf{p}'+\mathbf{p}}{2m}\right]$$



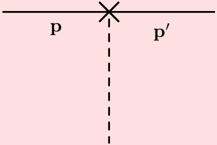
$\mathbf{A}^2$

$$e^2\left[\frac{\delta^{ij}}{2m}\right]$$



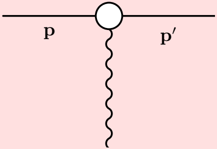
Darwin's

$$-e\left[\frac{1}{8m^2}\right]\mathbf{q}^2$$



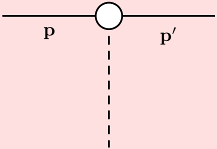
Fermi's

$$e\left[\frac{i}{2m}\right](\mathbf{q}\times\boldsymbol{\sigma})$$



spin–orbit

$$e\left[\frac{i}{4m^2}\right](\mathbf{p}'\times\mathbf{p})\cdot\boldsymbol{\sigma}$$



Here $\mathbf{q}=\mathbf{p}'-\mathbf{p}$ is a transferred impulse of the particle.

NRQED propagators

A natural choice of a gauge for the electromagnetic field is the Coulomb gauge ($\mathbf{kA} = 0$)

$$\begin{cases} G^{00} = \dfrac{1}{\mathbf{k}^2}\,, & \text{— the Coulomb photon propagator,} \\[2ex] G^{ij} = \dfrac{\delta_{ij} - k_i k_j/\mathbf{k}^2}{k^2 + i\varepsilon}\,, & \text{— the transverse photon propagator,} \\ G^{0i} = G^{i0} = 0, \qquad i,j = 1,2,3. & \end{cases}$$

For exchange photons $k_0 \approx m\alpha^2$ and

$$G^{ij} \approx -\frac{1}{\mathbf{k}^2}\left[\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right].$$

Propagators for massive particles

$$\frac{1}{E - \mathbf{p}^2/(2m) + i\varepsilon}\,.$$

Zero-order approximation and perturbation theory

Zero-order approximation is the nonrelativistic Hamiltonian

$$H_0 = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + \frac{e^2}{4\pi} \sum_{j>i} \frac{Z_i Z_j}{r_{ij}}$$

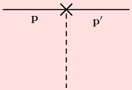
Its Green's function is defined as follows

$$K_0(2,1) = e^{-iH_0(t_2-t_1)}, \qquad [i\partial/\partial t_2 - H_0(2)]\, K_0(2,1) = i\delta(2,1).$$

Let $H = H_0 + V$, then we can expand K in increasing powers of V:

$$K(2,1) = K_0(2,1) + K^{(1)}(2,1) + K^{(2)}(2,1) + \dots$$

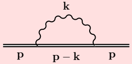
For instantaneous interaction:



$$K^{(1)}(2,1) = -i \int K_0(2,3) V(3) K_0(3,1)\, d\tau_3,$$

For a transverse photon:

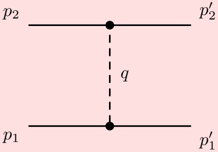
$$K^{(1)}(2,1) = -i \int K_0(2,4) V(4) G(4,3) K_0(4,3) V(3) K_0(3,1) d\tau_3 d\tau_4,$$



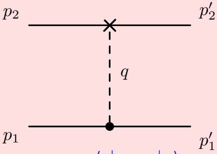
Functions $V(3)$ and $V(4)$ are some vertex functions of interaction with the electro-magnetic field.

Leading order relativistic and radiative contributions.

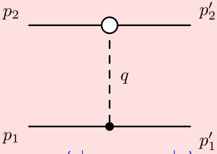
Breit-Pauli Hamiltonian



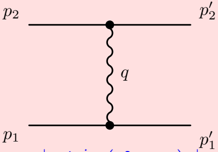
$$e^2 \left\langle i \left| \frac{1}{\mathbf{q}^2} \right| f \right\rangle$$



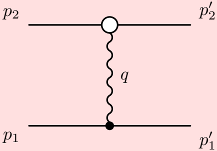
$$-e^2\, c_D^{(2)} \left\langle i \left| \frac{1}{8m_2^2} \right| f \right\rangle$$



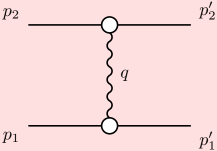
$$e^2\, c_S^{(2)} \left\langle i \left| \frac{i\boldsymbol{\sigma}_2[\mathbf{q}\times\mathbf{p}_2]}{4m_2^2\mathbf{q}^2} \right| f \right\rangle$$



$$-e^2 \left\langle i \left| \frac{p_1^i p_2^j}{m_1 m_2} \left(\frac{\mathbf{q}^2 - q_i q_j}{\mathbf{q}^4} \right) \right| f \right\rangle$$



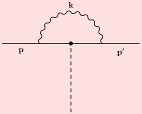
$$e^2\, c_F^{(2)} \left\langle i \left| \frac{i\boldsymbol{\sigma}_2[\mathbf{q}\times\mathbf{p}_1]}{2m_1 m_2\mathbf{q}^2} \right| f \right\rangle$$



$$-e^2\, c_F^{(1)}\, c_F^{(2)} \left\langle i \left| \frac{[\mathbf{q}\times\boldsymbol{\sigma}_1][\mathbf{q}\times\boldsymbol{\sigma}_2]}{4m_1 m_2\mathbf{q}^2} \right| f \right\rangle$$

Electron self-energy. Low energy approximation

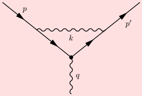
$k < \Lambda$



We use a quasi-relativistic approximation to the self-energy contribution

$$\Gamma_{(1)} = ie^2 \int \frac{d^4k}{(2\pi)^4}\, \frac{\mathbf{p}+(\mathbf{p}+\mathbf{k})}{2m} \times \frac{1}{E+k^0-(\mathbf{p}-\mathbf{k})^2/2m}\, \frac{1}{E+k^0-(\mathbf{p}'-\mathbf{k})^2/2m} \times \frac{\mathbf{p}'+(\mathbf{p}'+\mathbf{k})}{2m}\, \frac{1}{k^2-\lambda_{min}^2} \left(\delta^{ij} - \frac{k^ik^j}{\mathbf{k}^2+\lambda_{min}^2}\right).$$

$k \geq \Lambda$



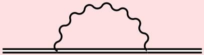
In a limit of small q and taking into account a new regularization parameter Λ

$$\Gamma^{\nu}_{(1)}(p,p') = \gamma^{\nu}\frac{\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{mc}{2\Lambda} - \frac{3}{8} + \frac{5}{6}\right) + \frac{i}{2m}\frac{\alpha}{2\pi}\sigma^{\nu\mu}q_{\mu},$$

$$\sigma^{\nu\mu} = \frac{i}{2}\left(\gamma^{\nu}\gamma^{\mu} - \gamma^{\mu}\gamma^{\nu}\right).$$

Self-energy correction in the NRQED. Low energy.

$k < \Lambda$



The ultrasoft scale contribution may be expressed:

$$E_L = \frac{2\alpha}{3\pi m^2}\int_0^\Lambda k\,dk\left\langle \mathbf{p}\left(\frac{1}{E_0-H-k}\right)\mathbf{p}\right\rangle - \delta m\,\langle\psi_0|\psi_0\rangle.$$

The integrand may be further rearranged using the operator identity

$$(E_0-H-k)^{-1} = -1/k - \frac{1}{k^2}(E_0-H) + \frac{1}{k^2}\frac{(E_0-H)^2}{E_0-H-k}$$

that results in

$$E_L = \frac{2\alpha}{3\pi m^2}\left[-\left\langle\mathbf{p}^2\right\rangle\Lambda + \langle\mathbf{p}\,[H,\mathbf{p}]\rangle\ln\Lambda + \int\frac{dk}{k}\left\langle\mathbf{p}\frac{(E_0-H)^2}{E_0-H-k}\mathbf{p}\right\rangle\right] - \delta m\,\langle\psi_0|\psi_0\rangle.$$

Self-energy correction in the NRQED. Low energy.

Thus, the remaining part may be split onto a finite nonlogarithmic contribution

$$E_L^{(0)} = \frac{2\alpha}{3\pi m^2}\int_0^{E_h} k\,dk \left\langle \mathbf{p}\left(\frac{1}{E_0-H-k}+\frac{1}{k}\right)\mathbf{p}\right\rangle + \frac{2\alpha}{3\pi m^2}\int_{E_h}^{\infty}\frac{dk}{k}\left\langle \mathbf{p}\frac{(E_0-H)^2}{E_0-H-k}\mathbf{p}\right\rangle$$

and the divergent part

$$E_L^{(1)} = \frac{2\alpha}{3\pi m^2}\left(\int_{E_h}^{\Lambda}\frac{dk}{k}\right)\left\langle \mathbf{p}\,[H,\mathbf{p}]\right\rangle = \frac{\alpha}{3\pi m^2}\ln\frac{\Lambda}{E_h}\;\left(4\pi Z\alpha\langle\delta(\mathbf{r})\rangle\right)$$

which results in appearance of the logarithmic term, the cut-of parameter is later canceled out by the logarithmic contribution from the high energy part. Here E_h is the Hartree energy and $E_h = m\alpha^2$.

Self-energy correction in the NRQED. High Energy

Let us consider the Darwin term in the NRQED Lagrangian

$$c_D \frac{e}{8m^2}\left[\mathbf{DE}\right]$$

For an electron the coefficients c_D is defined as follows

$$\begin{aligned} c_D &= 1 + 2a_e + 8m^2 F_1'(0) \\ &= 1+2a_e+\frac{\alpha}{\pi}\frac{8}{3}\left[\ln\left(\frac{m}{2\Lambda}\right)+\frac{5}{6}-\frac{3}{8}\right] + \left(\frac{\alpha}{\pi}\right)^2 A_1^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 A_1^{(3)}, \end{aligned}$$

where a_e is the anomalous magnetic moment of an electron, Λ is a NRQED cutoff parameter.

Self-energy correction in the NRQED. High energy.

Here we take the $m\alpha^5$ order contribution from the NRQED Lagrangian Darwin term:

$$E_H = -\frac{c_D^{(5)}}{8m^2} 4\pi Z\alpha\langle\delta(\mathbf{r})\rangle, \qquad c_D^{(5)} = 2\frac{\alpha}{2\pi} + \frac{\alpha}{\pi}\frac{8}{3}\left[\ln\left(\frac{m}{2\Lambda}\right) + \frac{5}{6} - \frac{3}{8}\right].$$

Then we get for the self-energy contribution for S states

$$E_H = \frac{\alpha}{3\pi m^2}\left[\ln\alpha^{-2} + \ln\frac{E_h}{\Lambda} - \ln 2 + \frac{5}{6}\right] 4\pi Z\alpha\langle\delta(\mathbf{r})\rangle.$$

Summing up the E_L and E_H contributions we see that the cutoff parameter Λ cancels out and we've got a finite expression for the self-energy contribution.

Self-energy correction for a bound state

Replacing $E_h \to 2R_\infty$, we arrive at the well-known expression[1]

$$\Delta E_{se} = \frac{4\alpha(Z\alpha)}{3m^2}\left[\ln\alpha^{-2} - \ln[k_0(n,l)/R_\infty] + \frac{5}{6}\right]\langle\psi|\delta(\mathbf{r})|\psi\rangle + \frac{\alpha(Z\alpha)}{2\pi m^2}\left\langle\psi\left|\frac{\mathbf{r}\times\mathbf{p}}{r^3}\cdot\frac{\boldsymbol{\sigma}}{2}\right|\psi\right\rangle.$$

where $\ln(k_0/R_\infty)$ is the Bethe logarithm

$$\ln\left[k_0(n,l)/R_\infty\right] = \sum_n \frac{\mathbf{p}_{0n}\mathbf{p}_{n0}(E_n - E_0)\ln(|E_n - E_0|/R_\infty)}{\mathbf{p}_{0n}\mathbf{p}_{n0}(E_n - E_0)},$$

[1] H.A. Bethe and E.E. Salpeter, *Quantum mechanics of one– and two–electron atoms*, Plenum Publishing Co., New York, 1977.

Applications

Physics of exotic atoms

Muon Catalysed Fusion

Reactions:

$$d + t \longrightarrow {}^4He + n + 17{,}6\ \text{МэВ},$$

$$d + d \longrightarrow {}^3He + n + 3{,}3\ \text{МэВ},$$

$$d + d \longrightarrow t + p + 4{,}0\ \text{МэВ},$$

$$p + d \longrightarrow {}^3He + \gamma + 5{,}5\ \text{МэВ}.$$

Muon Catalysis Fusion Cycle

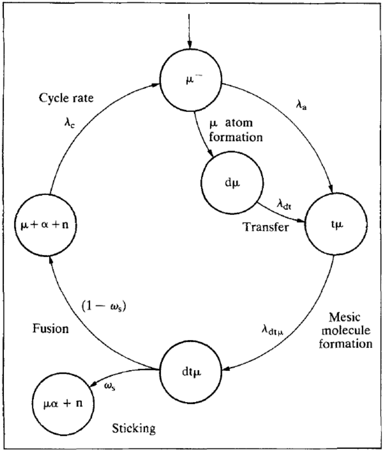
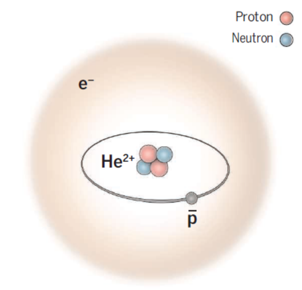


Figure 1. The principal muon catalysis fusion cycle in a deuterium and tritium mixture.

Antiprotonic Helium. Experiment of year 2010.



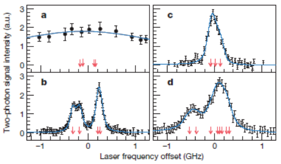


Figure 2 | **Profiles of sub-Doppler two-photon resonances. a**, Doppler- and power-broadened profile of the single-photon resonance (36, 34)→(35, 33) of $\bar{p}^4He^+$. **b**, Sub-Doppler two-photon profile of (36, 34)→(34, 32) involving the same parent state. **c**, **d**, Profiles of (33, 32)→(31, 30) of $\bar{p}^4He^+$ (**c**) and (35, 33)→(33, 31) of $\bar{p}^3He^+$ (**d**). Black filled circles indicate experimental data points with 1-s.d. error bars, blue lines are best fits of theoretical line profiles (see text) and partly overlapping arrows indicate positions of the hyperfine lines. a.u., arbitrary units.

[M. Hori et al. *Nature* **475**, *484 (2011)]*

Isotope	Transition $(n,l)\to(n-2,l-2)$	Transition frequency (MHz)	
		Experiment	Theory
$\bar{p}^4He^+$	(36, 34)→(34, 32)	1,522,107,062(4)(3)(2)	1,522,107,058.9(2.1)(0.3)
	(33, 32)→(31, 30)	2,145,054,858(5)(5)(2)	2,145,054,857.9(1.6)(0.3)
$\bar{p}^3He^+$	(35, 33)→(33, 31)	1,553,643,100(7)(7)(3)	1,553,643,100.7(2.2)(0.2)

Experimental values show respective total, statistical and systematic 1-s.d. errors in parentheses; theoretical values (ref. 3 and V. I. Korobov, personal communication) show respective uncertainties from uncalculated QED terms and numerical errors in parentheses.

$$A_r(e) = 0.000\,548\,579\,909\,1(7) \quad [1.4\times10^{-9}]$$

Antiprotonic Helium in Russian Media

Lenta.ru: Главное: | Lenta.ru: Прогресс: Физик | как сделать снимок экра | Как сделать снимок экра

lenta.ru/news/2011/07/28/antiproton/

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28.07.2011, 14:46:17 — Версия для печати | PDA/КПК

Антипротонный деселератор, который ученые использовали для работы. Фото CERN

Физики взвесили антипротоны в атомкулах

Группа ученых Масаки Хори (Masaki Hori) из Института квантовой оптики общества Макса Планка провела наиболее точное измерение массы антипротона, улучшив известное значение на несколько порядков. Исследователи в очередной раз подтвердили, что массы протона и антипротона совпадают. Статья ученых появилась в журнале *Nature*, а ее краткое изложение приводит physicsworld.com.

В рамках исследования физики изучали так называемые атомкулы - экзотические молекулы гелия, в которых один из электронов замещен антипротоном. Эти объекты, способные существовать несколько микросекунд и получившие название антипротонного гелия, были открыты в 1991 году японскими физиками.

Последние новости

17.10 18:48 Удальцова отпустили под подписку о невыезде

17.10 18:26 Организаторы выборов в КС заявили о сохранности денег участников МММ

17.10 18:46 Власти Москвы проследят за митингами оппозиции

17.10 19:46 Госдума одобрила законопроект об образовании

17.10 19:13 Пивоваров обвинил Мамонтова в искажении смысла "Срока"

17.10 20:19 Посольство США в Стокгольме возобновило работу после эвакуации

17.10 19:59 В Сирии подбитый вертолет взорвался в воздухе

Аутсайд

Розовый жираф: Нобелевский успех закоренелого двоечника и его мамы
Нынешний нобелевский лауреат сэр Джон Гёрдон, оказывается, был закоренелым двоечником.

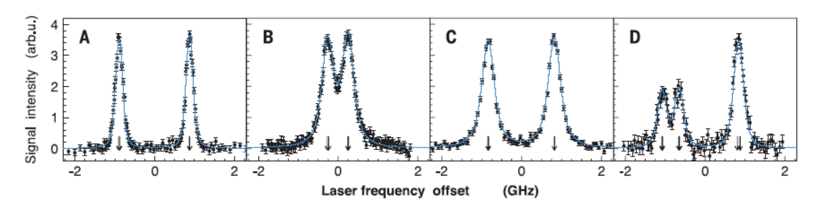
Science: Dance Your Ph.D. Finalists Announced
Финалисты конкурса "Станцуй свою диссертацию", объявленного престижным научным журналом, пытаются... станцевать свою диссертацию.

BBC News: Bloodhound land speed rocket test roars over Newquay
Захватывающее видео подготовки к тесту аппарата Bloodhound, похожего скорее на ракету, чем на суперкар.

New Scientist: Reality revealed: The ultimate fabric of the universe
NewScientist пытается за две с половиной минуты рассказать, из чего состоит реальность и приходит к

Modern status. Experiment

Masaki Hori *et al.* Science **354**, 610 (2016)



$$m_{\bar{p}}/m_e = 1836.152\,6734(15) \quad [8\times 10^{-10}]$$

Precision Spectroscopy of HD^+

CODATA18 values and new experiments

The CODATA18 constants:

Rydberg constant	$R_\infty = 10\,973\,731.568\,160(21)$ m^{-1}	$1.2\cdot10^{-12}$
deuteron mass	$m_d = 2.013\,553\,212\,745(40)$ u	$2.0\cdot10^{-11}$
electron mass	$m_e = 5.485\,799\,090\,65(16)\cdot10^{-4}$ u	$2.9\cdot10^{-11}$

Electron-to-proton mass ratio:

	m_p/m_e	m_d/m_p
CODATA18	1836.15267343(11)	1.99900750139(10)
Blaum[1]	1836.152673358(55)	1.999007501228(59)
Myers[2]	1836.152673435(55)	1.999007501274(38)

[1]) S. Rau *et al.* Nature **585**, 43 (2020).
[2]) D.J. Fink, E.G. Myers. Phys. Rev. Lett. **124**, 013001 (2020).

HD^+. Theory and experiment

Theoretical and experimental spin-averaged transition frequencies (in kHz). CODATA18 values of fundamental constants were used in the calculations.

$(L,v)\to(L',v')$	theory	experiment
$(0,0)\to(1,0)$	1 314 925 752.932(19)	1 314 925 752.910(17)
$(0,0)\to(1,1)$	58 605 052 163.9(0.5)	58 605 052 164.24(86)
$(3,0)\to(3,9)$	415 264 925 502.8(3.3)	415 264 925 501.8(1.3)

Results

Reduced mass $\mu = \dfrac{m_p m_d}{(m_p+m_d)m_e}$ inferred from the HD^+ ion spectroscopy

	μ
CODATA18	$1223.899\,228\,722(51)$
$(0,0) \to (0,1)$	$1223.899\,228\,743(16)_{exp}(17)_{th}$
$(0,0) \to (1,1)$	$1223.899\,228\,707(17)_{exp}(17)_{th}$
$(3,0) \to (3,9)$	$1223.899\,228\,730(04)_{exp}(17)_{th}$
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Relative uncertainty: $u_r(\mu) = 1.4 \times 10^{-11}$.

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	$1223.899\,228\,730(04)_{exp}(17)_{th}$

Relative uncertainty: $u_r(\mu) = 1.4 \times 10^{-11}$.

Mass ratios from spectroscopy and Myers' experiment:

$$m_p/m_e = 1836.152673436(44), \qquad m_d/m_e = 3670.482967763(88),$$

and the new CODATA22 value is $m_p/m_e = 1836.152673426(32)$.

Thank you for your attention!