Some new algorithms for Monte-Carlo event generators

Yahor Dydyshka[a,b,c]
SANC group

[a] *Dzhelepov Laboratory of Nuclear Problems, JINR, Dubna, Russia*
[b] *Institute for Nuclear Problems, Belarusian State University, Minsk, Belarus*
[c] *Dubna State University, Russia*

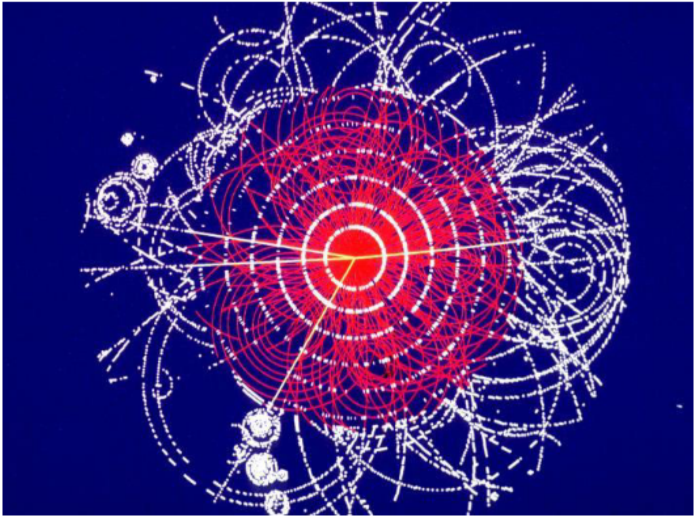
MMCP-2024

Outline

- Motivation
- Generator vs. Integrator
- Adaptive “foam”
- Smooth functions
- Conclusion and plans

Motivation

Collider experiments



Generators vs. Integrators

Integration with MC

$$I = \int_a^b f(x)dx = (b-a)\langle f \rangle, \qquad \langle f \rangle \approx \frac{1}{N}\sum_{i=1}^{N} f(\xi_i), \qquad \xi_i \in [a,b]$$

error estimation $\delta I \approx \sqrt{\frac{\mathsf{D}[f]}{N}}$, with dispersion $\mathsf{D}[f] = \langle f^2 \rangle - \langle f \rangle^2$

Generation of events

$$I = \int_a^b f(x)dx = \int_a^b \int_0^{f_{max}} \theta[y < f(x)]dxdy \approx \frac{1}{N}\sum_{i=1}^{N} \theta[f(\xi_i) < \eta_i] = \frac{N_{acc}}{N}, \quad \begin{array}{l} \xi_i \in [a,b], \\ \eta_i \in [0, f_{max}] \end{array}$$

Additional task: to find f_{max} global on $[a,b]$.

Real world conditions

Integrand properties

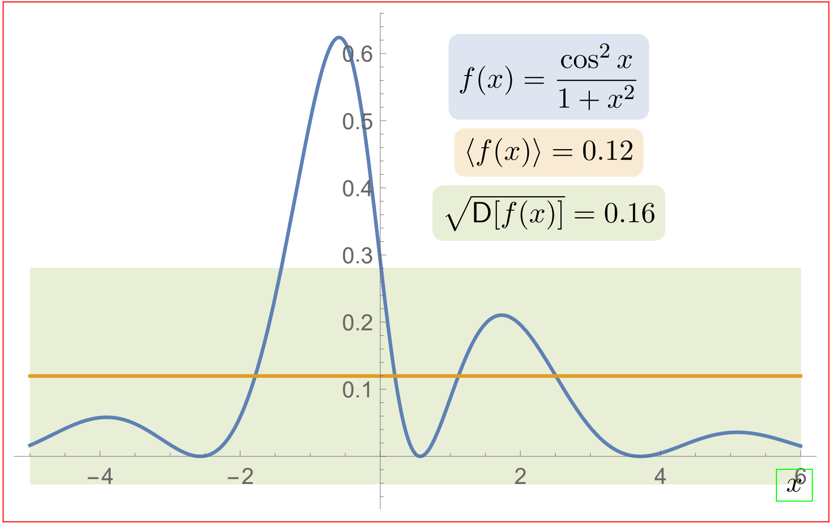
- Matrix element squared is positive and analytic function
- Experimental cuts make integrand piecewise analytic
- Typical singularities are poles and branch-points outside of integration contour

Typical dimensions

- Born-level integral is from 1- to 3-dimensional for $2 \to 2$ processes.
- Each additional particle increases dimension by 3.
- Markov Chain Parton showers allow variable-dimension events

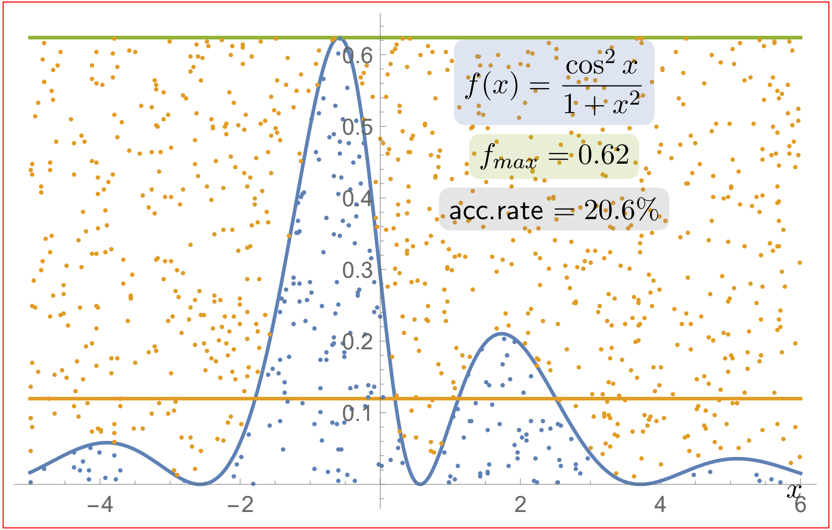
Interval arithmetic approach

1D example

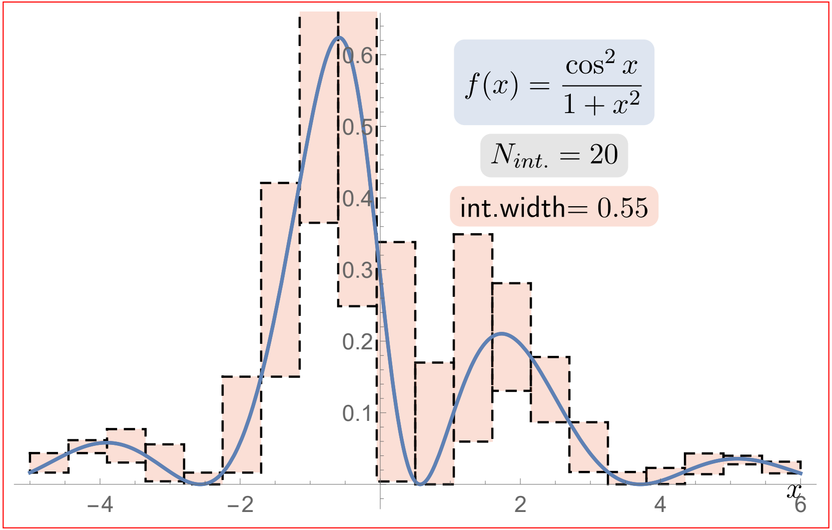


$f(x) = \frac{\cos^2 x}{1 + x^2}$
$\langle f(x) \rangle = 0.12$
$\sqrt{\mathrm{D}[f(x)]} = 0.16$
0.6
0.5
0.4
0.3
0.2
0.1
−4
−2
2
4
6
x

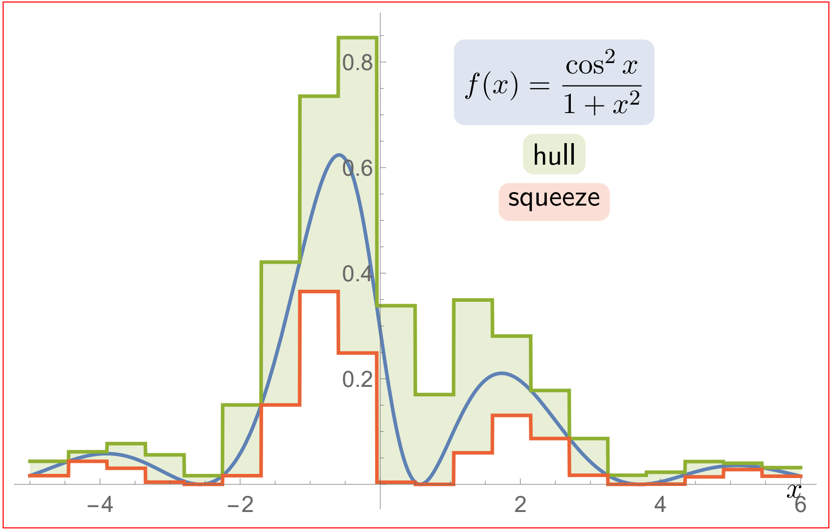
1D example



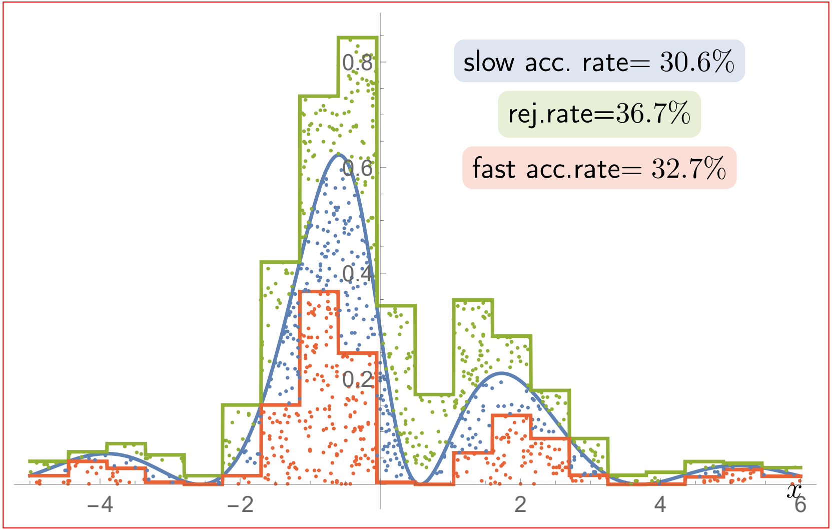
1D example



1D example



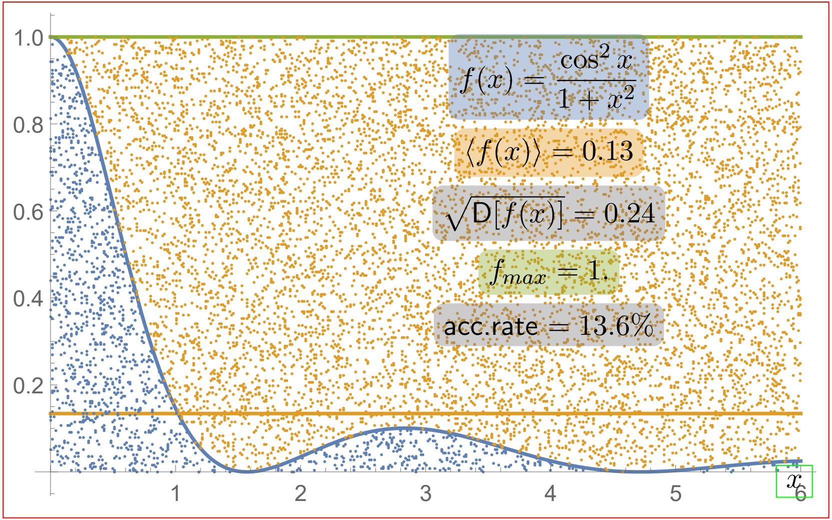
1D example



slow acc. rate= 30.6%
rej.rate=36.7%
fast acc.rate= 32.7%
0.8
0.6
0.4
0.2
−4
−2
2
4
6
x

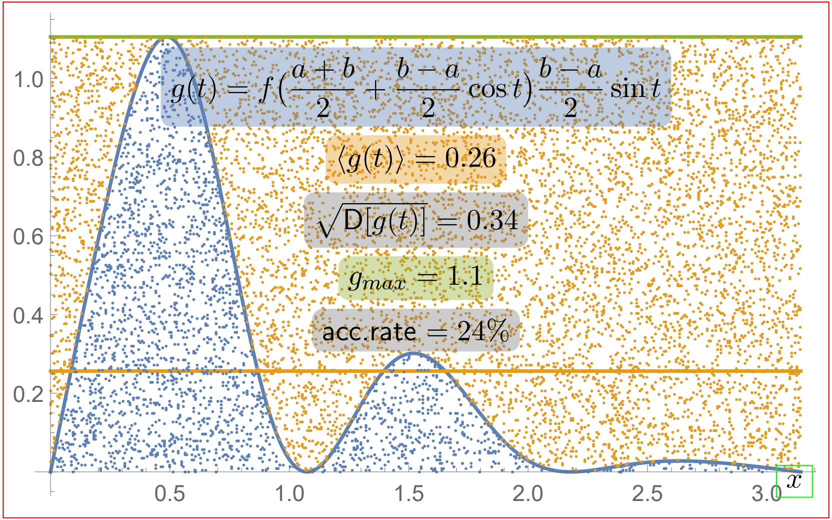
Mirroring technique

1D example

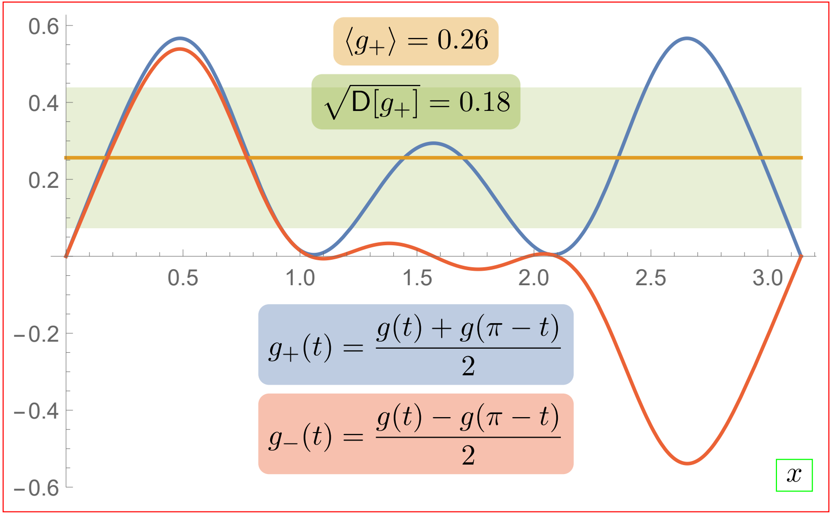


1.0
0.8
0.6
0.4
0.2
1
2
3
4
5
6
x
$f(x) = \frac{\cos^2 x}{1 + x^2}$
$\langle f(x) \rangle = 0.13$
$\sqrt{\mathrm{D}[f(x)]} = 0.24$
$f_{max} = 1.$
acc.rate = 13.6%

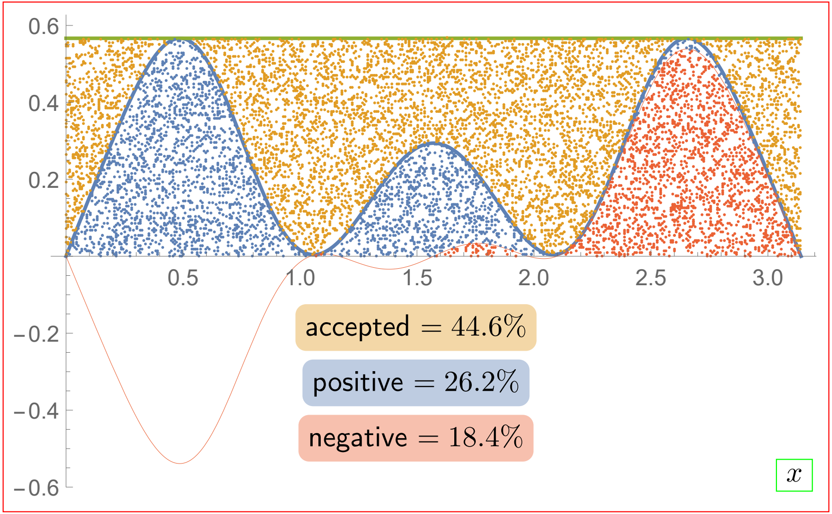
Chebyshev change of variable



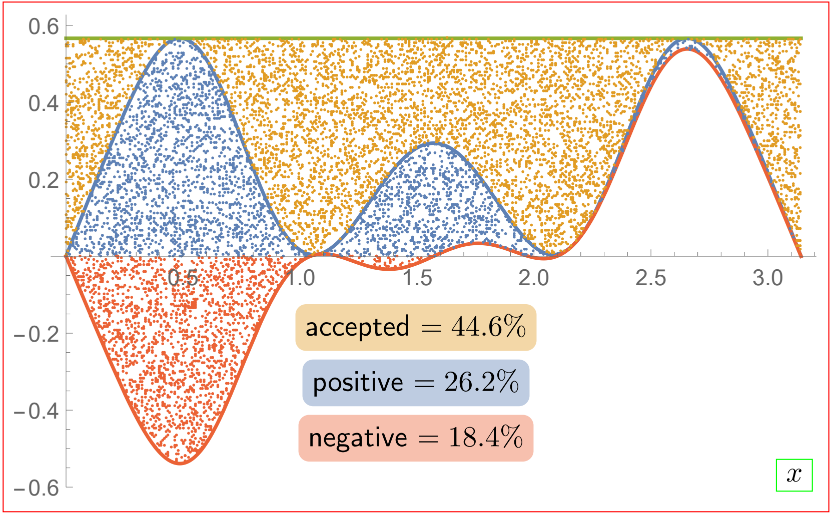
Symmetric and antisymmetric parts



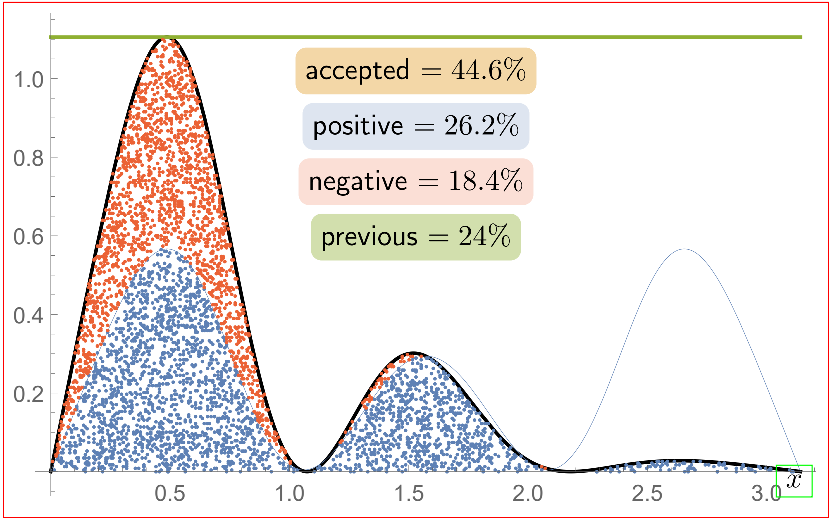
Symmetric and antisymmetric parts (sampling)



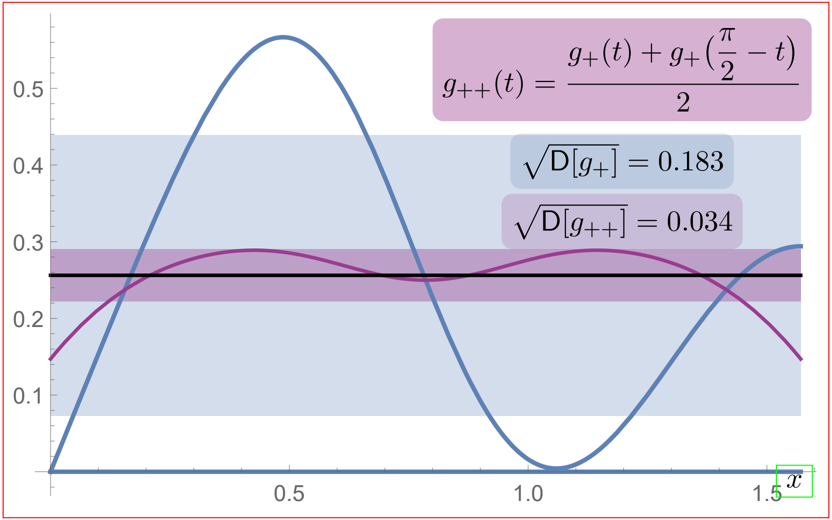
Symmetric and antisymmetric parts (mirroring)



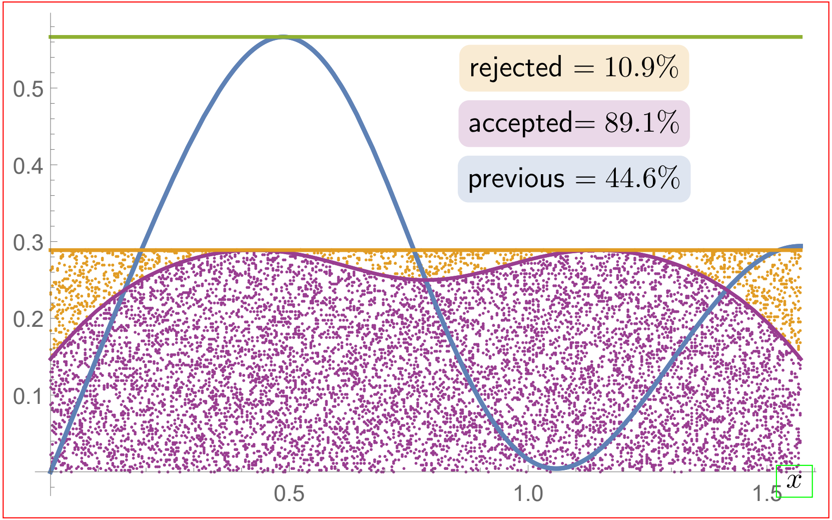
Symmetric and antisymmetric parts (mirroring)



Symmetrize once more



Symmetrize once more (generate)



Fourier expansion

$$g(t) = \frac{a_0}{2} + a_1 \cos t + a_2 \cos 2t + a_3 \cos 3t + a_4 \cos 4t + a_5 \cos 5t + \dots,$$

Symmetrization

$$g_+(t) \equiv \frac{g(t) + g(\pi - t)}{2} = \frac{a_0}{2} + a_2 \cos 2t + a_4 \cos 4t + a_6 \cos 6t + \dots,$$

$$g_{++}(t) \equiv \frac{g_+(t) + g_+\left(\frac{\pi}{2} - t\right)}{2} = \frac{a_0}{2} + a_4 \cos 4t + a_8 \cos 8t + a_{12} \cos 12t + \dots,$$

Hadamar transofrm

$$\begin{aligned} g_{++}(t) &= g(t) + g_1(t) + g_2(t) + g_3(t), \\ g_{+-}(t) &= g(t) + g_1(t) - g_2(t) - g_3(t), \\ g_{-+}(t) &= g(t) - g_1(t) + g_2(t) - g_3(t), \\ g_{--}(t) &= g(t) - g_1(t) - g_2(t) + g_3(t) \end{aligned}$$

with

$$\begin{aligned} g_1(t) &= g(\pi - t)/4 \\ g_2(t) &= g\left(\frac{\pi}{2} - t\right)/4 \\ g_3(t) &= g\left(\frac{\pi}{2} + t\right)/4 \end{aligned}$$

Conclusion

- Interval arithmetics allows construction of efficient MC generator resolving *global maximum* problem.
- Adaptive algorithms based on interval arithmetic can be build
- Smoothness properties of integrand can be benefited by with Hadamar symmetrization.

The research is supported by grant of the Russian Science Foundation
(project No. 22-12-00021)