Iterative solution of DGLAP equations in QED

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based on works with U. Voznaya:
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Electron is as inexhaustible as atom

Outline

Motivation

- Development of physical programs for future high-energy HEP colliders
- Having high-precision theoretical description of basic e^+e^- and other HEP processes is of crucial importance
as for solving problems of the Standard Model
as well as for new physics searches
- Two-loop calculations are still in progress, and higher-order QED corrections are also important
- The formalism of QED parton distribution functions gives a fast estimate of the bulk of higher-order effects
- Parallels between QCD and QED

To do list for QED

- Compute 2-loop QED radiative corrections to differential distributions of key processes: Bhabha scattering, muon decay, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \pi^+\pi^-$, $e^+e^- \to ZH$ etc.
- Estimate higher-order contributions within some approximations
- Account for interplay with QCD and electroweak effects
- Construct a reliable Monte Carlo code(s)

Leading and next-to-leading logs in QED

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc. for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$ and $\alpha/(2\pi) \approx 0.001$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

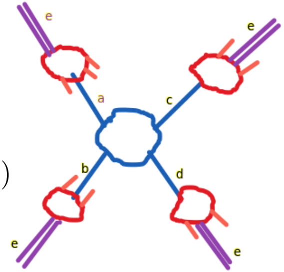
with at least $n = 3,\ 4$ are required for future e^+e^- colliders

In the collinear approximation we can get them within the NLO QED structure function formalism

- F.A.Berends, W.L. van Neerven, G.J.Burgers, NPB'1988
- A.A., K.Melnikov, PRD'2002; A.A. JHEP'2003

QED NLO master formula

The NLO Bhabha cross section reads



$$d\sigma = \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\rm str}_{ae}(z_1)\mathcal{D}^{\rm str}_{b\bar{e}}(z_2)$$

$$\times \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right]$$

$$\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\rm frg}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\rm frg}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right)$$

$$+\mathcal{O}\left(\alpha^n L^{n-2}, \frac{m_e^2}{s}\right)$$

$\alpha^2 L^2$ and $\alpha^2 L^1$ terms are completely reproduced [A.A., E.Scherbakova, JETP Lett. 2006; PLB 2008] || $\bar{e} \equiv e^+$

QED NLO DGLAP evolution equations

$$\mathcal{D}_{ba}\left(x, \frac{\mu_R}{\mu_F}\right) = \delta_{ab}\delta(1-x) + \sum_{c=e,\gamma,\bar{e}} \int\limits_{\mu_R^2}^{\mu_F^2} \frac{dt}{t} \int\limits_{x}^{1} \frac{dy}{y} P_{bc}(y,t) \mathcal{D}_{ca}\left(\frac{x}{y}, \frac{\mu_R^2}{t}\right)$$

μ_F is a factorization (energy) scale

μ_R is a renormalization (energy) scale

D_{ba} is a parton density function (PDF)

P_{bc} is a splitting function or kernel of the DGLAP equation

N.B. In QED $\mu_R = m_e \approx 0$ is the natural choice

QED splitting functions

The perturbative splitting functions are

$$P_{ba}(x, \bar{\alpha}(t)) = \frac{\bar{\alpha}(t)}{2\pi} P_{ba}^{(0)}(x) + \left(\frac{\bar{\alpha}(t)}{2\pi}\right)^2 P_{ba}^{(1)}(x) + \mathcal{O}(\alpha^3)$$

$$\text{e.g.} \quad P_{ee}^{(0)}(x) = \left[\frac{1+x^2}{1-x}\right]_+$$

They come from direct loop calculations, see, e.g., review “Partons in QCD” by G. Altarelli. For instance, $P_{ba}^{(1)}(x)$ comes from 2-loop calculations.

The splitting functions can be obtained by reduction of the ones known in QCD to the abelian case of QED.

$\bar{\alpha}(t)$ is the QED running coupling constant in the $\overline{\text{MS}}$ scheme

$\mathcal{O}(\alpha)$ matching

The expansion of the master formula for ISR gives

$$d\sigma^{(1)}_{e\bar{e}\to\gamma^*} = \frac{\alpha}{2\pi}\left\{2LP^{(0)} \otimes d\sigma^{(0)}_{e\bar{e}\to\gamma^*} + 2d^{(1)}_{ee} \otimes d\sigma^{(0)}_{e\bar{e}\to\gamma^*}\right\} + d\,\bar{\sigma}^{(1)}_{e\bar{e}\to\gamma^*} + \mathcal{O}\left(\alpha^2\right)$$

We know the massive $d\sigma^{(1)}$ and massless $d\,\bar{\sigma}^{(1)}$ ($m_e \to 0$ with $\overline{\text{MS}}$ subtraction) results in $\mathcal{O}(\alpha)$. E.g.

$$\frac{d\sigma^{(1)}_{e\bar{e}\to\gamma^*}}{d\sigma^{(0)}_{e\bar{e}\to\gamma^*}} = \frac{\alpha}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \left(\ln\frac{s}{m_e^2} - 1\right) + \delta(1-z)(\ldots), \quad z \equiv \frac{s'}{s}$$

Scheme dependence comes from here

Factorization scale dependence is also from here

N.B. "Massification procedure”

Iterative solution

Analytic expressions for NLO "e in e" and "γ in e" PDFs and fragmentation functions [A.A., U.Voznaya, JPG 2023]

$$
\begin{aligned}
\mathcal{D}_{ee}(x,\mu_F,m_e) &= \delta(1-x) + \frac{\alpha}{2\pi} L P^{(0)}_{ee}(x) + \frac{\alpha}{2\pi} d^{(1)}_{ee}(x,m_e,m_e) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left(\frac{1}{2}P^{(0)}_{ee}\otimes P^{(0)}_{ee}(x) + \frac{1}{2}P^{(0)}_{ee}(x) + \frac{1}{2}P^{(0)}_{e\gamma}\otimes P^{(0)}_{\gamma e}(x)\right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^2 L \left(P^{(0)}_{e\gamma}\otimes d^{(1)}_{\gamma e}(x,m_e,m_e) + P^{(0)}_{ee}\otimes d^{(1)}_{ee}(x,m_e,m_e) - \frac{10}{9}P^{(0)}_{ee}(x) + P^{(1)}_{ee}(x)\right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^3 \left(\frac{1}{6}P^{(0)}_{ee}\otimes P^{(0)}_{ee}\otimes P^{(0)}_{ee}(x) + \frac{1}{6}P^{(0)}_{e\gamma}\otimes P^{(0)}_{\gamma\gamma}\otimes P^{(0)}_{\gamma e}(x) + \dots\right) \\
&+ \left(\frac{\alpha}{2\pi}\right)^3 L^2 \left(\frac{539}{27} + \frac{11}{3z} - 8\ln^3(1-z)\frac{1+z^2}{1-z} + \dots\right) \\
&+ \mathcal{O}(\alpha^2 L^0, \alpha^3 L^1, \alpha^4 L^4)
\end{aligned}
$$

The large logarithm $L \equiv \ln\frac{\mu_F^2}{\mu_R^2}$ with factorization scale $\mu_F^2 \sim s$ or $\sim -t$; and renormalization scale $\mu_R = m_e$.

N.B. A mistake in $\mathcal{O}(\alpha^3 L^3)$ is corrected.

ISR corrections to $e^+e^- \to Z(\gamma^*)$ ($\sqrt{s} = M_Z{+}10$ GeV)

LO $\mathcal{O}(\alpha^n L^n)$ and NLO $\mathcal{O}(\alpha^n L^{n-1})$ ISR corrections in % above the Z-peak for $z_{\min} = 0.1$

Type / n	1	2	3	4	5
LO γ	185.3272	−22.0460	−0.3575	0.2192	−0.0169
NLO γ	−6.9498	3.1530	0.0780	−0.0459	
LO pair	—	1.8440	0.4155	−0.0072	0.0054
NLO pair	—	−0.8311	−0.1513	0.0196	
Σ	178.3774	−17.8801	−0.0153	0.1857	−0.0115

PRELIMINARY NUMBERS

ISR corrections to $e^+e^- \to Z(\gamma^*)$ ($\sqrt{s} = M_Z$)

LO $\mathcal{O}(\alpha^n L^n)$ and NLO $\mathcal{O}(\alpha^n L^{n-1})$ ISR corrections in % at the Z-peak for $z_{\min} = 0.1$

Type / n	1	2	3	4	5
LO γ	−32.7365	4.8843	−0.3776	0.0034	0.0032
NLO γ	2.0017	−0.5952	0.0710	−0.0019	
LO pair	—	−0.3057	0.0875	0.0016	−0.0001
NLO pair	—	0.1585	−0.0460	0.0038	
Σ	−30.7348	4.1419	−0.2651	0.0069	0.0031

N.B. $\mathcal{O}(\alpha^2 L^0)$ ISR corrections are known [Berends; Blümlein]

Impact of new corrections on LEP results?!

PRELIMINARY NUMBERS

Applications

- ISR in electron-positron annihilation $e^+e^- \to \gamma^*, Z^*$
  "Higher-order NLO initial state radiative corrections to e^+e^- annihilation revisited" [A.A., U.Voznaya, PRD'2024]
- $\mathcal{O}(\alpha^3 L^2)$ corrections to muon decay spectrum: relevant for future experiments [A.A., U.Voznaya, PRD'2024]
- Implementation into ZFITTER, production of benchmarks, tuned comparisons with KKMC which uses YFS exponentiation for ISR
- Application to different e^+e^- annihilation channels and asymmetries within the SANC project
- $\mathcal{O}(\alpha^3 L^2)$ corrections to muon-electron scattering for MUonE experiment (in progress)

QED PDFs vs. QCD ones

Common properties:

- QED splitting functions = abelian part of QCD ones
- The same structure of DGLAP evolution equations
- The same Drell-Yan-like master formula with factorization
- Factorization scale and scheme dependence

Peculiar properties:

- QED PDFs are calculable
- QED PDFs are less inclusive
- QED renormalization scale $\mu_R = m_e$ is preferable
- QED PDFs can (do) lead to huge corrections
- Massification procedure

Outlook

- Parton picture is there also in QED
- QED PDF are similar to QCD ones, but with some differences
- QED cross-checks QCD
- Having high theoretical precision for the normalization processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, and $e^+e^- \to 2\gamma$ is crucial for future e^+e^- colliders, especially for the Tera-Z mode
- We need complete two-loop QED results, but (sub)leading higher order corrections are also numerically important
- New Monte Carlo codes are required
- Semi-analytic codes are relevant for estimates and benchmarks