The current status of the SANC project

A. Arbuzov (for L. Kalinovskaya)
on behalf of the SANC team
(supported by RSF grant N 22-12-00021)

JINR Dzhelepov Laboratory of Nuclear Problems

MMCP-2024, Yerevan, Armenia
21-25 October, 2024

SANC team, status 2015-2024

**A. Arbuzov, S. Bondarenko, I. Boyko, Ya. Dydyshka, U. Voznaya,
L. Kalinovskaya,
A. Kampf, L. Rumyantsev, R. Sadykov, V. Yermolchyk, U. Yermolchyk.**

Outline

- **Colliders, Universal tools, Theoretical support of HEP experiments**
- **Precision, steps to improving the theoretical precision**
- **Present status SANC:**
  analitical basement: precomputation, structures, form factors, helicity amplitudes; framework of SANC tools
- **Basic processes of the SM for e^+e^- annihilation in SANC,**
  example di-boson production relevant for CEPC
- **Future Opportunities**
- **Conclusion**

Colliders

Colliders that fall within our area of interest.

Past:

LEP – Z factory; 91 – 209 GeV (c.m.s.);
SLC – Z factory; 50 – 200 GeV (c.m.s.);
TEVATRON (RunII) – W, Z factory; thousands of t events; each beams of 980 GeV (max. energy);
LHC – W, Z, t factory; 7 – 14 TeV (c.m.s.);

Present:

LHC – W, Z, t factory; 7 – 14 TeV (c.m.s.);
RHIC – pp, ion-collisions; polarized; up to 510 GeV (c.m.s.);

Far future:

ILC – e^+e^-; polarized; up to 500 GeV;
CLIC – e^+e^-; polarized; up to 3 TeV;
FCC-ee – ee; up to 350 GeV (c.m.s);
FCC-hh – hh; up to 100 TeV (c.m.s);
μ-colliders – H, Z-factory; billions of Z-events;
LHeC – e^+e^- (up to 60 GeV) (c.m.s), pp (up to 7 TeV) (c.m.s);

Upcoming projects:

NICA – protons and deuterons, ion-collisions; polarized; up to $\sim$ 20 GeV (c.m.s);
EIC – ep, pp ,ee, e + light ion-collisions; polarized; up to $\sim$ 275 GeV (c.m.s);
eRHIC – ep, pp ,ee, e + light ion-collisions; polarized; up to $\sim$ 30 GeV (c.m.s);
Genie (Qini) – photon-photon; up to 2 MeV;
CEPC – H, Z-factory; billions of Z-events, precision $\delta \sim 10^{-4}$; polaraized ?(propose 2023); up to 240 GeV (c.m.s);

Electron is as inexhaustible as atom

Universal tools

* **Universal tools** — MC generators, theoretical precision $\sim 10\%$:
  - **PHOTOS** — **the code**, for given "process" generates "process + photon(s)"
  - **CompHEP** — **the system**, **any process** up to $2 \to 6$, tree level
  - **PYTHIA** — **the code**, **many processes**, lowest perturbative order
* **Precision tools** — semi-analytic approach, inclusive observables, theoretical precision $\leq .1\%$:
  - **ZFITTER/DIZET**, **TOPAZ0** — fitters of LEP era, Z resonance
  - **FeynArts** — practically any process up to $2 \to 2$, only one-loop contributions
* **Precision MC generators**
  - **Special purpose:**
    KORALZ, KKMC, BHLUMI, BHWIDE, TAUOLA, TauSpinner
    — event distributions with one-loop corrections, exponentiation, LL-higher orders; precision below .1%.
  - **General purpose: one-loop level, precision well below 1%**

    — integrators
    - **DYNNLO**
    - **FEWZ**

    — event generators
    - **POWHEG**
    - **GRACE-loop** — practically **any process** up to $\mathbf{2 \to 2}$, several $\mathbf{2 \to 3}$
    - **ReneSANCe, MCSANC** — aimed at **any process** up to $\mathbf{2 \to 3}$

Tools for theoretical support

Basic requirements:

- MC generator/integrator
- precision (per mille level) – better then experimental (i.e. 1+2 loops RC)
- ISR
- full phase space
- arbitrary polarization
- massive case (even in loops)

State-of-the-art accuracy for EW correction: NNLO+NNNLL, including threshold resummation

Precision MC event generator for the luminosity process at future e^+e^- colliders

Forecast [1] of the total (physical) theoretical uncertainty for the FCCee_{350}, ILC_{1000} and CLIC_{3000} luminosity calorimetric detectors:

	FCCee_{350}	ILC_{1000}	CLIC_{3000}
total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

[1] arXiv:2211.14230, B.F.L. Ward, S. Jadach et. al.,"Overview of theoretical precision of the luminosity at future electron-positron colliders"

SANC project

- Present status SANC tools:
  MC integrator `MCSANC` – pp, $p\bar{p}$ and e^+e^- modes,
  MC integrator `SANCphot` – $\gamma\gamma$ modes,
  MC generator `ReneSANCe` – pp, e^+e^- and $\gamma\gamma$ modes
- One-loop corrections:
  a) calculation of one-loop amplitudes, their renormalisation, the regularisation of IR divergencies, and the combination of virtual as well as real-emission contributions in order to cancel them.
  b) Helicity approach for Born, virt, soft, hard.
- accounting up to two loops (first steps)
  The perturbative expansion of the squared matrix element M takes the following form:

$$
\begin{aligned}
|M|^2 = \quad & |M_{(0)}|^2 \qquad (\mathbf{LO}, \mathbf{BORN}) \\
+ \quad & 2\mathcal{R}eM^*_{(0)}M_{(1)} \qquad (\mathbf{NLO}) \\
+ \quad & |M_{(1)}|^2 + 2\mathcal{R}eM^*_{(0)}M_{(2)} \qquad (\mathbf{NNLO})
\end{aligned}
$$

  where the subscript indicates the loop order.
- accounting polarization
- codes are achieved both for hadronic, leptonic and $\gamma\gamma$ collisions.

Analitical calculations in SANC

FORM language[2]

Key Steps:

⟶ Precomputation gauge invariant subsets

⟶ Calculation Covariant Amplitude (structure findings)

⟶ Singling out Form Factors

⟶ Calculation Helicity Amplitudes
(**method Vega-Vudka** for virtual part and
spinor formalizm for hard contribution)

[2] B. Ruijl T. Ueda and J. Vermaseren, *FORM version 4.2*, arXiv:1707.06453, 2017

SANC tools, Framework

- Analytical expressions are obtained for the form-factors and amplitudes of generalized processes and stored as the **FORM** language expressions.
- The latter are translated to the **Fortran**/**C++** modules for specific parton level processes with NLO QCD and EW corrections.
- The modules are utilising **Looptools** [3] and **SANClib** packages for loop integrals evaluation.
- To build a Monte Carlo code one convolutes the partonic cross sections from the modules with the parton density functions and feeds the result as an integrand to any Monte Carlo algorithm implementation, e.g. **FOAM** or **Vegas** from **Cuba** library [4].

P.S. The module's procedures for partonic cross sections are significantly unified and allow to calculate fully differential hadronic cross sections.

[3] **LoopTools** is a software package designed for the evaluation of scalar and tensor one-loop integrals (in Fortran, C++, and provides a Mathematica interface for the scalar one-loop functions

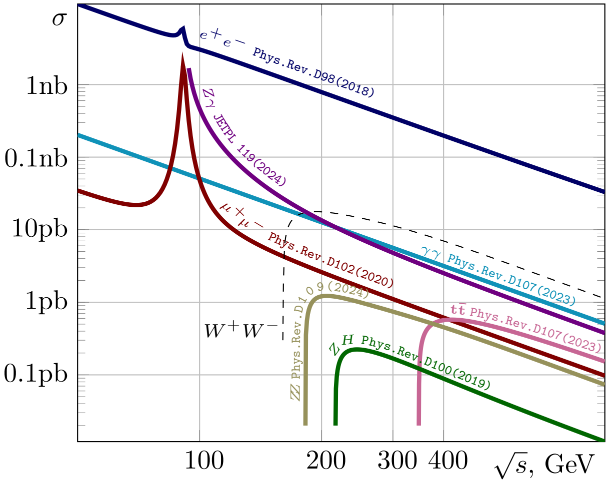
[4] **Cuba** – a library for multidimensional numerical integration The Cuba library offers a choice of four independent routines for multidimensional numerical integration: Vegas, Suave, Divonne, and Cuhre.

Basic processes of the SM for e^+e^- annihilation

Basic processes of the SM for e^+e^- annihilation:

- $\to e^+e^-$
- $\to \mu^+\mu^-$
- $\to \gamma\gamma$
- $\to t\bar{t}$
- $\to ZH$
- $\to ZZ$
- $\to Z\gamma$
- $\to W^+W^-$

The cross sections at the Born level are given for polar angles between $10^o < \theta < 170^o$ in the final state.



Today's Applications, one-loop level

- four fermion processes, i.e.
  - LUMU for lepton colliders
    - $e^+e^- \to e^+e^-$
    - $e^+e^- \to \mu^+\mu^-$
    - $e^+e^- \to \gamma\gamma$
    - $e^-e^- \to e^-e^-$
  - μe scattering — experiment MUONE, muon colliders
    - $\mu^+e^- \to e^-\mu^+$, $\mu^+\mu^- \to \mu^-\mu^+$
    - $\mu^-e^- \to e^-\mu^-$, $\mu^-\mu^- \to \mu^-\mu^-$
- lepton colliders — di-boson production
  - $e^+e^- \to ZH(Z\gamma, ZZ)$
- accompanying mode — $\gamma\gamma$ collisions for $\gamma\gamma$ colliders

Review SANC in MMCP2024

Report about

- development basement SANC
  Yahor Dydyshka —
  Some new algorithms for Monte-Carlo event generators
  https://indico.jinr.ru/event/4467/contributions/28842/
- $\gamma\gamma \to \gamma\gamma$ ($\gamma Z, ZZ, e^+e^-, \nu\bar{\nu}$) processes in SANC
  Renat Sadykov —
  Polarized photon-photon collisions in ReneSANCe Monte Carlo generator
  https://indico.jinr.ru/event/4467/contributions/28843/
- linuminosity in SANC
  Vitaly Yermolchyk —
  ReneSANCe *event generator for precise luminosity determination*
  https://indico.jinr.ru/event/4467/contributions/28844/
- extension basement higher orders RC
  Serge Bondarenko —
  Two-loop corrections to gamma gamma -> gamma process
  https://indico.jinr.ru/event/4467/contributions/28845/
- extension basement in QED part
  Andrej Arbuzov —
  Iterative solution of DGLAP equations in QED
  https://indico.jinr.ru/event/4467/contributions/28846/
- possible application calculation SANC to RHIC programm
  Alexey Kampf —
  Radiative corrections to $W^{\pm}$ boson hadroproduction with longitudinal polarization of initial states
  https://indico.jinr.ru/event/4467/contributions/28847/
- SANC status and possible application to experiments
  this report —
  SANC for higher calculation: application to e^+e^- -factories, $\gamma\gamma$-colliders, $pp(p\bar{p})$-accelerators
  https://indico.jinr.ru/event/4467/contributions/29095/

Cross section, one-loop level

$$\sigma^{\text{one-loop}}_{\chi_1\chi_2} = \sigma^{\text{Born}}_{\chi_1\chi_2} + \sigma^{\text{virt}}_{\chi_1\chi_2}(\lambda) + \sigma^{\text{soft}}_{\chi_1\chi_2}(\lambda, \bar{\omega}) + \sigma^{\text{hard}}_{\chi_1\chi_2}(\bar{\omega}).$$

σ^{Born} — the Born cross section,
$\sigma^{\text{virt}} = \sigma^{\text{QED}} + \sigma^{\text{weak}}$ — the contribution of virtual (loop) corrections,
$\sigma^{\text{soft(hard)}}$ — the soft (hard) photon emission contribution
(the hard photon energy $E_\gamma > \bar{\omega} = \omega\sqrt{s}/2$).

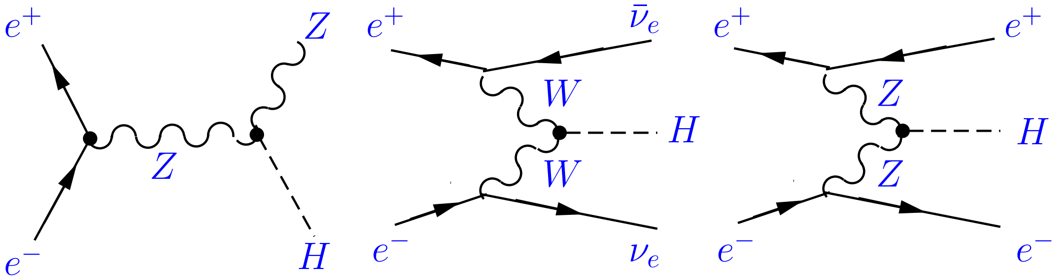
We divide the virtual part into two gauge invariant subsets: σ^{QED} and σ^{weak}. To the QED contribution we refer all diagrams in which there is an exchange of at least one photon. The rest is the weak part. **Auxiliary parameters** λ ("photon mass") and $\bar{\omega}$ (soft-hard separator) **are canceled** after summation. The cancellation is controlled numerically by calculating the cross section at several values of the λ and $\bar{\omega}$ parameters. When calculating the emission of real photons **we keep the electron masses to regularize the collinear divergences**.

Di-boson production $e^+e^- \to 2b$

The investigation of di-boson production represents a crucial target for the e^+e^- Colliders. These processes **provide a direct probe of the non-abelian character of electroweak interactions** in the Standard Model as well as of **possible deviations from standard triple-gauge-boson-coupling interactions** due to new physics. On top of this, **the study of the weak boson polarization supplies a probe of the mechanism of electroweak symmetry breaking**, which is responsible for the appearance of a longitudinal polarization mode of massive gauge bosons.

Higgs production, CEPC as a Higgs factory

The main Higgs production processes at the e^+e^- colliders are:



Higgsstrahlung: $e^+e^- \to ZH$ WW fusion: $e^+e^- \to \bar{\nu}\nu H$ ZZ fusion: $e^+e^- \to e^+e^-H$

Higgsstrahlung $e^+e^- \to ZH$: one-loop cross sections

P_{e^-}	P_{e^+}	σ^{hard}, fb	σ^{Born}, fb	$\sigma^{\text{one-loop}}$, fb	δ, %
0	0	82.0(1)	225.59(1)	206.77(1)	-8.3(1)
-0.8	0	96.7(1)	266.05(1)	223.33(2)	-16.1(1)
-0.8	-0.6	46.3(1)	127.42(1)	111.67(2)	-12.4(1)
-0.8	0.6	147.1(1)	404.69(1)	334.99(1)	-17.2(1)

Hard ($E_\gamma > 1$ GeV), Born and one-loop cross sections in fb and relative corrections for the c.m. energy $\sqrt{s} = 250$ GeV and various polarization degrees of the initial particles. The relative correction δ (%) is defined as follows:

$$\delta = \frac{\sigma^{\text{one-loop}}(P_{e^-}, P_{e^+})}{\sigma^{\text{Born}}(P_{e^-}, P_{e^+})} - 1.$$

$e^+e^- \to HZ$: higher-order contributions of the initial state radiation in the QED structure function formalism

ISR corrections of different order of $\mathcal{O}(\alpha^n L^n), n = 2-4$ in the LLA approximation for the c.m.s. energies $\sqrt{s} = 240$ GeV and 250 GeV in the $\alpha(0)$ EW scheme. We provide numbers in two points because these energies are particular for the process under consideration. Namely, the Born-level cross section has a peak at ~ 240 GeV while the present plans of future e^+e^- colliders envisage operation at 250 GeV where the counting rate of the signal is higher.

	$\mathcal{O}(\alpha^2L^2)$ γ	$\mathcal{O}(\alpha^2L^2)$ e^+e^-	$\mathcal{O}(\alpha^2L^2)$ $\mu^+\mu^-$	$\mathcal{O}(\alpha^3L^3)$ γ	$\mathcal{O}(\alpha^3L^3)$ e^+e^-	$\mathcal{O}(\alpha^3L^3)$ $\mu^+\mu^-$	$\mathcal{O}(\alpha^4L^4)$ γ	$\sum_{n=2}^{4} \mathcal{O}(\alpha^nL^n)$
$\delta\sigma_{\text{LLA}}$, fb	1.128(1)	−0.368(1)	−0.218(1)	0.176(1)	0.019(1)	0.011(1)	−0.023(1)	0.727(1)
δ_{LLA}, %	0.500(1)	−0.163(1)	−0.097(1)	0.078(1)	0.008(1)	0.005(1)	−0.010(1)	0.322(1)

$\sqrt{s} = 240$ GeV. No cuts are imposed. The Born cross section is $\sigma_0 = 225.74(1)$ fb.

	$\mathcal{O}(\alpha^2L^2)$ γ	$\mathcal{O}(\alpha^2L^2)$ e^+e^-	$\mathcal{O}(\alpha^2L^2)$ $\mu^+\mu^-$	$\mathcal{O}(\alpha^3L^3)$ γ	$\mathcal{O}(\alpha^3L^3)$ e^+e^-	$\mathcal{O}(\alpha^3L^3)$ $\mu^+\mu^-$	$\mathcal{O}(\alpha^4L^4)$ γ	$\sum_{n=2}^{4} \mathcal{O}(\alpha^nL^n)$
$\delta\sigma_{\text{LLA}}$, fb	−0.223(1)	−0.268(1)	−0.159(1)	0.211(1)	−0.010(1)	−0.006(1)	−0.016(1)	−0.468(1)
δ_{LLA}, %	−0.099(1)	−0.119(1)	−0.070(1)	0.094(1)	−0.004(1)	−0.003(1)	−0.007(1)	−0.207(1)

$\sqrt{s} = 250$ GeV. No cuts are imposed. The Born cross section is $\sigma_0 = 225.59(1)$ fb.

No cuts are imposed. $\delta_{\text{ISR LLA}} \equiv \delta\sigma_{\text{ISR LLA}}/\sigma_0$.

Photonic corrections are large but they are mainly due to collinear and soft singularities which are known in QED at any perturbative order, hence can be resummed.

Main background: $e^+e^- \to ZZ(Z\gamma)$

Integrated Born and one-loop cross sections σ in pb and relative corrections δ (%) for unpolarized and polarized initial beams at the c.m.s. energies, $\alpha(0)$ scheme.

P_{e^+}, P_{e^-}	$0, 0$	$-1, +1$	$+1, -1$	$0.3, -0.8$	$-0.3, 0.8$	$0, -0.8$	$0, 0.8$
$\sqrt{s} = 250$ GeV							
$e^+e^- \to Z\gamma$							
σ^{Born}, pb	4.094(1)	6.353(1)	10.025(1)	6.087(1)	4.067(1)	4.829(1)	3.360(1)
σ^{NLO}, pb	4.489(1)	7.572(1)	10.364(1)	6.332(1)	4.796(1)	5.048(1)	3.931(1)
δ^{NLO}, %	9.63(1)	19.19(1)	3.38(1)	4.03(1)	17.92(1)	4.53(1)	16.98(1)
δ^{QED}, %	7.63(1)	7.52(1)	7.50(1)	7.57(1)	7.60(1)	7.61(1)	7.65(1)
δ^{weak}, %	2.01(1)	11.68(1)	−4.12(1)	−3.55(1)	10.31(1)	−3.08(1)	9.32(1)
$e^+e^- \to ZZ$							
σ^{Born}, pb	1.0198(1)	1.2070(1)	2.8722(1)	1.7225(1)	0.80661(1)	1.3529(1)	0.68675(1)
σ^{NLO}, pb	1.0087(1)	1.4717(1)	2.5625(1)	1.5508(1)	0.95079(3)	1.2270(1)	0.79067(3)
δ^{NLO}, %	−1.08(1)	21.93(1)	−10.78(2)	−9.97(1)	17.88(1)	−9.30(1)	15.14(1)
δ^{QED}, %	−1.36(1)	−1.39(1)	−1.39(1)	−1.38(1)	−1.37(1)	−1.37(1)	−1.34(1)
δ^{weak}, %	0.29(2)	23.32(1)	−9.39(1)	−8.59(1)	19.24(2)	−7.93(1)	16.48(2)

SANC: Future Opportunities

Exponentiation, soft photon resummation

- BHLUMI — the standard YFS-based approach done with the squared amplitudes.
- BabaYaga — uses soft-photon resummation, however based on the parton shower method.
- Another foreseeable approach, KKMC – based on a more sophisticated exponentiation done at the level of spin amplitudes.
- SANC — proposed the same approach as in BabaYaga

SANC: Future Opportunities

Application for selected processes

- **EW, EW/QCD, two loops**
  all processes for LUMI: Bhabha, two photon production, two muon production
- **QCD, one-loop**
  - Project RHIC $\longrightarrow$ polarized CC&NC Drell-Yan
  - Project NICA $\longrightarrow$ Prompt Photons, J/ψ
  - as a consequence: Application for Polarized PDF, i.e. extraction of spin dependent parton densities and their uncertanties
- **EW**
  Project CEPC: e^+e^- and $\gamma\gamma$ modes

Conclusion

Status SANC project:
A good **platform has been created for theoretical support** relevant for LHC physics, e^+e^- and $\gamma\gamma$ colliders and moreover for polarized experiment RHIC and NICA (near future, November, SPD meeting)