Method for Alignment of Multilayer 2D Coordinate Detectors

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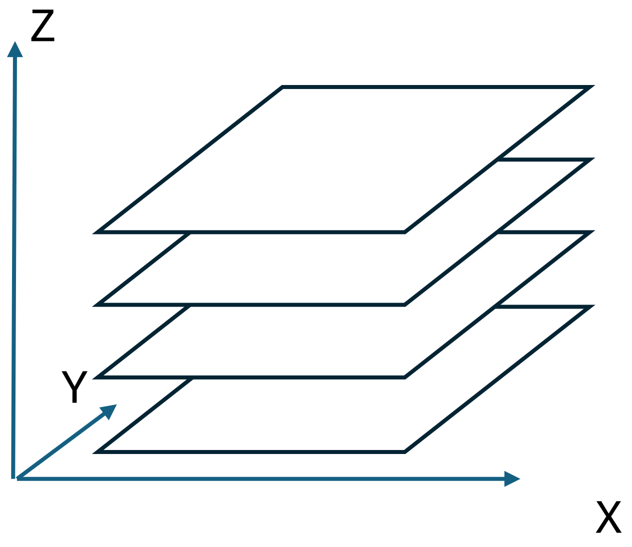
2D coordinate charged particle detectors



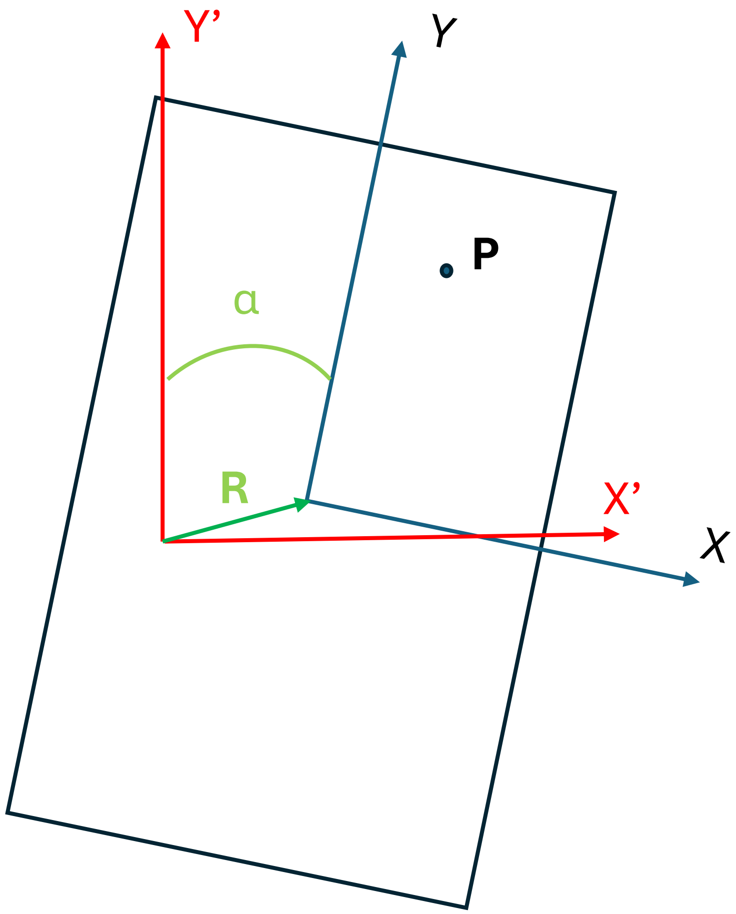
Charged particle coordinate detectors get signals from passing high energy charged particles. They have some media for primary ionization and some mechanism to amplify those primary signals. These detectors can be based on gas or solid state.
They can produce coordinates of the particle tracks by getting electrical signals from electrodes. These signals are produced during electron amplification process.
We don't care in this report about the nature of the coordinates production.

Detector consist of multiple layers of 2D coordinates particle detectors.
Each layer can detect coordinates (X,Y) of the charged particle signal within the layer.
Having track coordinates from all the layers we can estimate relative layer positions.

Alignment of the layers:

1. **Affects spatial resolution of the detector**
2. **Improve track reconstruction quality**
3. **Manufacturing quality control**

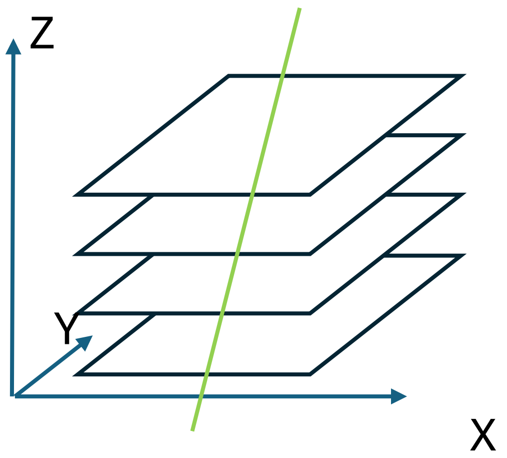
Layer misalignment



Let's assume we have common reference frame X'Y' for all layers
Each layer have local reference frame XY.
Each layer can misalignment in common RF: i.e. can be shifted by vector **R** and rotated by angle α, which are specific for each layer

Each point P of the track is measured within a layer within local XY RF
To get a track point in common RF we can make transformations:

$$x_z{}' = x\cos(\alpha_z) - y\sin(\alpha_z) + w_z$$
$$y_z{}' = x\sin(\alpha_z) + y\cos(\alpha_z) + h_z$$



We can now reconstruct a track using those coordinates from all layers

Further we assume tracks to be straight lines:

$$x_t = az + b$$
$$y_t = cz + d$$

Functional to minimize

$$\varphi = \sum_{i=1}^{N}\sum_{j=1}^{M}\{\frac{(x_{ij}cos(\alpha_j) - y_{ij}sin(\alpha_j) + w_j - a_i z_j - b_i)^2}{\sigma_x^2} + \frac{(x_{ij}sin(\alpha_j) + y_{ij}cos(\alpha_j) + h_j - c_i z_j - d_i)^2}{\sigma_y^2}\}$$

Here N is a total number of tracks, M is a number of layers
α_j, w_j, h_j – layer alignment parameters
a_i, b_i, c_i, d_i – track parameters
σ_x and σ_y are spatial resolution of detector in corresponding directions

In general, we need to minimize this functional for both alignment and track variables. I’ve tried to do this but stuck at some point and decided to make some simplifications:

1. Use even number of layers (I use 6, as I plan to apply this computation to CMS CSCs)
2. Choose Z=0 in between inner layers (between L3 and L4), so that Z1=-Z6, Z2=-Z5, Z3=-Z6. This will be used later.
3. Break solution into two steps:

- Assume we have ideal alignment and find track parameters.
- Having track coordinates find alignment parameters.

Find track parameters

$$\varphi'_{a_i} = -\frac{2}{\sigma_x^2}\sum_{j=1}^{6} z_j(x_{ij} - a_i z_j - b_i)$$

$$\varphi'_{b_i} = -\frac{2}{\sigma_x^2}\sum_{j=1}^{6} (x_{ij} - a_i z_j - b_i)$$

Solving these equations equal to 0, one can see highlighted terms are cancelled as sum of Zs is 0.

Similar formulas are for Y part of the track

$$x_{ij}^t = z_j \frac{\sum_{m=1}^{6} z_m x_{im}}{\sum_{m=1}^{6} z_m^2} + \frac{\sum_{m=1}^{6} x_{im}}{6}$$

$$y_{ij}^t = z_j \frac{\sum_{m=1}^{6} z_m y_{im}}{\sum_{m=1}^{6} z_m^2} + \frac{\sum_{m=1}^{6} y_{im}}{6}$$

We will use these track positions further

Alignment parameters: shifts

$$\varphi'_{w_j} = \frac{2}{\sigma_x^2}\sum_{i=1}^{N}\left(x_{ij}\cos(\alpha_j) - y_{ij}sin(\alpha_j) + w_j - x_{ij}^t\right) = 0$$

Similar equation is for Y shift - h_j

$$w_j = -\cos(a_j)\frac{\sum_{i=1}^N x_{ij}}{N} + \sin(a_j)\frac{\sum_{i=1}^N y_{ij}}{N} + \frac{\sum_{i=1}^N x_{ij}^t}{N}$$

$$h_j = -\cos(a_j)\frac{\sum_{i=1}^N y_{ij}}{N} - \sin(a_j)\frac{\sum_{i=1}^N x_{ij}}{N} + \frac{\sum_{i=1}^N y_{ij}^t}{N}$$

Note: if we do not break solution into 2 steps (tracks and alignment) we will get here w_j interweaved between all the layers

Equation for angle α

$$\varphi'_{\alpha_j} = \frac{2}{\sigma_x^2}\sum_{i=1}^{N}(x_{ij}cos(\alpha_j) - y_{ij}sin(\alpha_j) + w_j - x_{ij}^t)(-x_{ij}sin(\alpha_j) - y_{ij}cos(\alpha_j)) +$$
$$\frac{2}{\sigma_y^2}\sum_{i=1}^{N}(x_{ij}sin(\alpha_j) + y_{ij}cos(\alpha_j) + h_j - y_{ij}^t)(x_{ij}cos(\alpha_j) - y_{ij}sin(\alpha_j)) = 0$$

Here we have all layers separated as w_j and h_j also calculated separately.
So, we have single trigonometric equation for each layer. After simplification, the result equation gets the following form:

$$A_j cos(2\alpha_j) + B_j sin(2\alpha_j) + C_j sin(\alpha_j) + D_j cos(\alpha_j) = 0$$

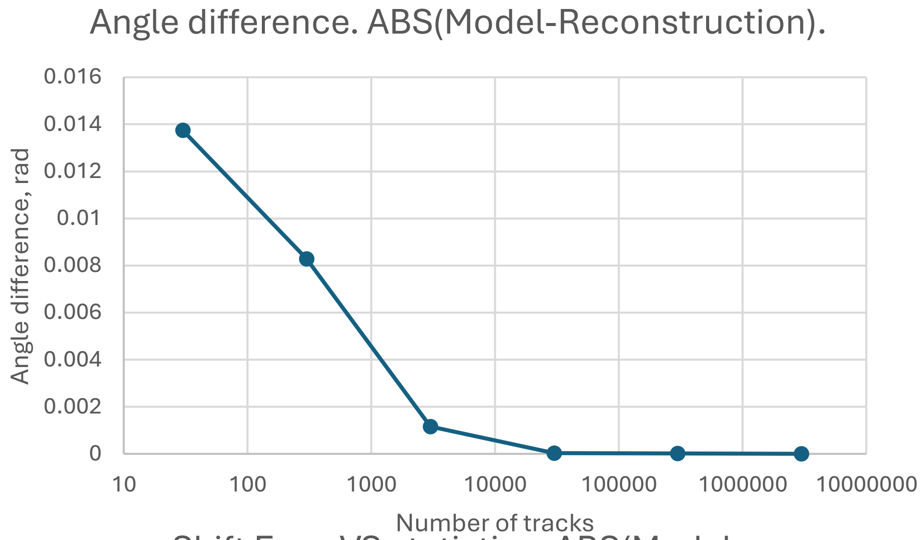
Here A, B ,C ,D are coefficients consisting of different sums of x and y measurements.

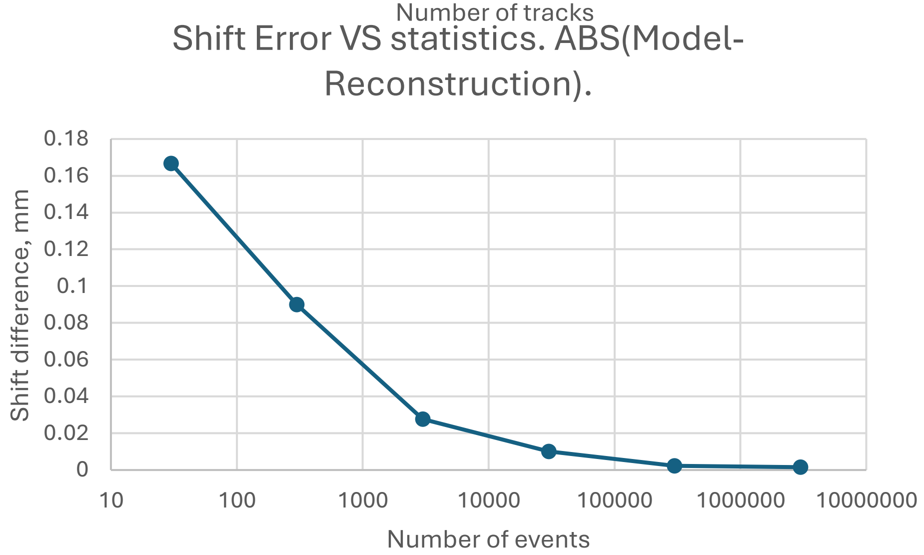
Statistics influence

Model system used:
$\sigma_x = \sigma_y = 0.1$ mm Gaus

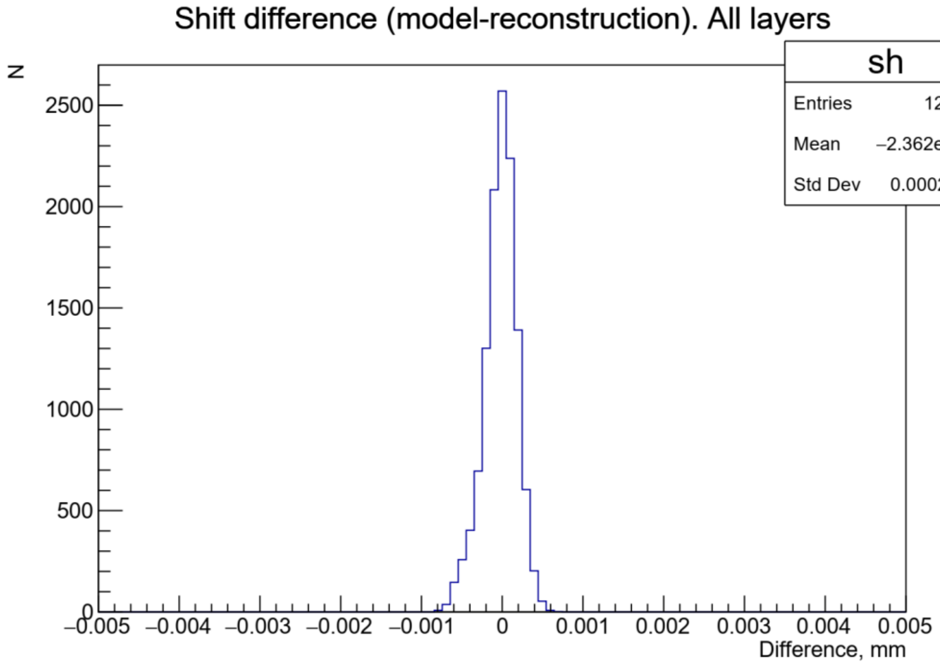
	x-shift	yshift	angle
L1	0.124658	-0.73376	-0.00079
L2	0.103642	-0.59627	0.000021
L3	0.107752	0.429028	0.000241
L4	0.020742	0.460305	0.000716
L5	-0.14898	0.549223	0.00016
L6	-0.20781	-0.10852	-0.00035

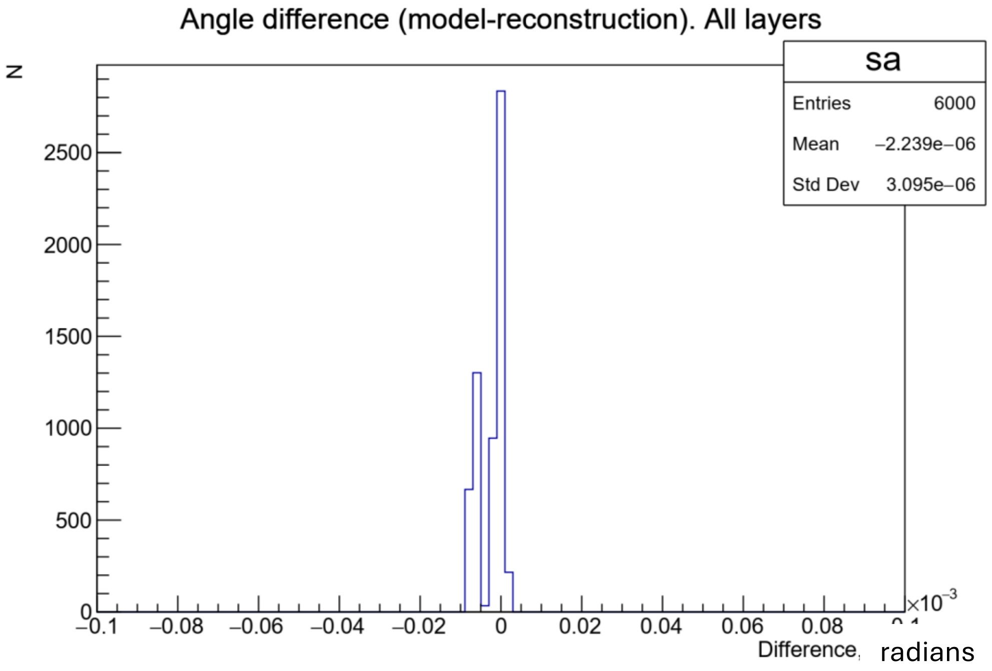
Random shifts and rotations of 6 layers are generated.
Shifts are within 0.5 mm, Angle is within 0.001 radians.
Layer size is 500x500 mm^2
Layer spacing 22mm, track maximum inclination ~0.5 rad.
Averages are set to 0.
Plots are for sums of shifts and angles from all layers.





Input data influence





With model setup from previous slide 1000 sets of 300000 tracks were generated.
Left plot has X and Y shift difference for all 6 layers for each dataset.
Right plot has angle difference for each layer for each dataset.
There is no significant influence of the input data on calculation results.

Conclusion

- Algorithm for alignment of layers of particle detectors is presented.
- Model simulations show that the method is independent on the input dataset.
- Influence of statistics is investigated on the model. We can predict the required amount of data.
- Work in progress now on real CMS CSC data.