Modified transport approach for description of fragmentation reactions in heavy-ion collisions .

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in collaboration with

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Outline

- Motivation.
- Description of heavy-ion collisions with transport-statistical approach (BNV-SMM) and its numerical implementation.
- Comparison with experimental data and other model calculations
- Attempt to explain ratio of isotope yields on heavy and light targets
- Conclusions

Transport theory: Boltzmann-Nordheim-Vlasov (BNV) approach

time evolution of the one-body phase space density: f(r,p;t)

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla} f - \vec{\nabla} U \, \vec{\nabla}_p f = I_{coll}[f,\sigma]$$

Physical input:

mean field potential U (equation of state)

and in-medium elastic cross section

F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189

V. Baran, M. Colonna, M. Di Toro, Phys. Rep., **410** (2005**)** 335

Density functional

U(ρ(r)) = Nuclear Mean Field + Symmetry terms + Coulomb

$$U(\rho) = A\left[\frac{\rho}{\rho_0}\right] + B\left[\frac{\rho}{\rho_0}\right]^d + C(-1)^k\left(\rho_n - \rho_p\right)/(\rho_n + \rho_p) + U_{\mathrm{coul}}$$

$$A = -356 MeV, B = 303 MeV, d = 7/6, k = 1(p), 2(n), C = 36 MeV$$

Collision term

$$I_{coll}[f_1,\sigma] = \frac{g}{h}\int d^3p_2\, d^3p_3\, d^3p_4\, \sigma(12,34)\,\delta(\vec{p}_1+\vec{p}_2-\vec{p}_3+\vec{p}_4)\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3+\varepsilon_4)\left[\bar{f}_1\bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3\bar{f}_4\right]$$

Pauli blocking factors for final state
g degeneracy

$$(1-f(r,v_i;t)) \equiv (1-f_i) := \bar{f}_i$$

Collision term: treatment by stochastic simulation

1. Select in each time step d*t* TPs with distance $d \le \sqrt{\sigma/\pi}$
2. Collide with probability $P = \sigma_{el}/\sigma_{max}$ with random scattering angle
3. Check Pauli blocking of final state in phase space

Computationally most expensive part of calculation

Step 1:

density function initialization for each nuclei separately.

$f(\vec{r},\vec{p},t) = \sum_{j=1}^{KA} g(\vec{r}-\vec{r}_j(t))\tilde{g}(\vec{p}-\vec{p}_j(t))$

A-number of nucleons K – number of test-particles per nucleon,

r_j, p_j – coordinates of test-particles

energy minimization in Woods-Saxon potential $V = \frac{V_0}{1+\exp(\frac{\vec{r}_j - R}{a})}$, $R = r_0 A^{1/3}$

Add coordinates R_1, R_2 and moments P_1, P_2 of nuclei in space and

Step 2 start dynamical calculations

Newton equations: $\frac{\partial \vec{p}_i(t)}{\partial t} = -\nabla_r U(\vec{r}_i, t)$; $\frac{\partial \vec{r}_i(t)}{\partial t} = \frac{\vec{p}_i(t)}{m}$

with

$$U(\rho) = A_{Sk}\left[\frac{\rho}{\rho_0}\right] + B_{Sk}\left[\frac{\rho}{\rho_0}\right]^d + C_{sym}\frac{\rho_n - \rho_p}{\rho_n + \rho_p} + U_{Coul}$$

A_{Sk} = -356 МэВ, B_{Sk} = 303МэВ, d = 7/6 и C_{sym}= 36МэВ

Leap-frog method:

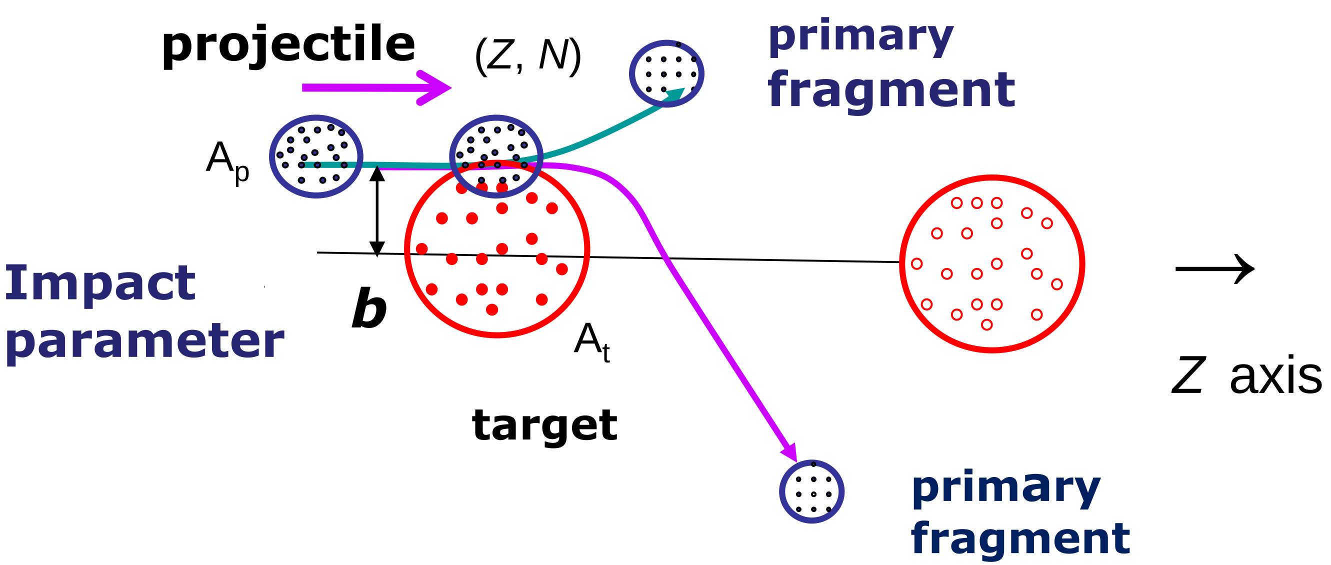
$$\vec{p}_i\left(t+\frac{1}{2}\Delta t\right) = \vec{p}_i(t) - \frac{1}{2}\Delta t \nabla_r U(\vec{r}_i, t)$$

$$\vec{r}_i(t+\Delta t) = \vec{r}_i(t) + \Delta t\ \vec{p}_i(t+\frac{1}{2}\Delta t)/m$$

time step 1 fm/c

Schematic view of the collision of two heavy ions

30MeV/nucleon < $E_{projectile}$ < 100MeV/nucleon



Peripheral reactions, large *b* → **Deep-inelastic + Direct reactions**

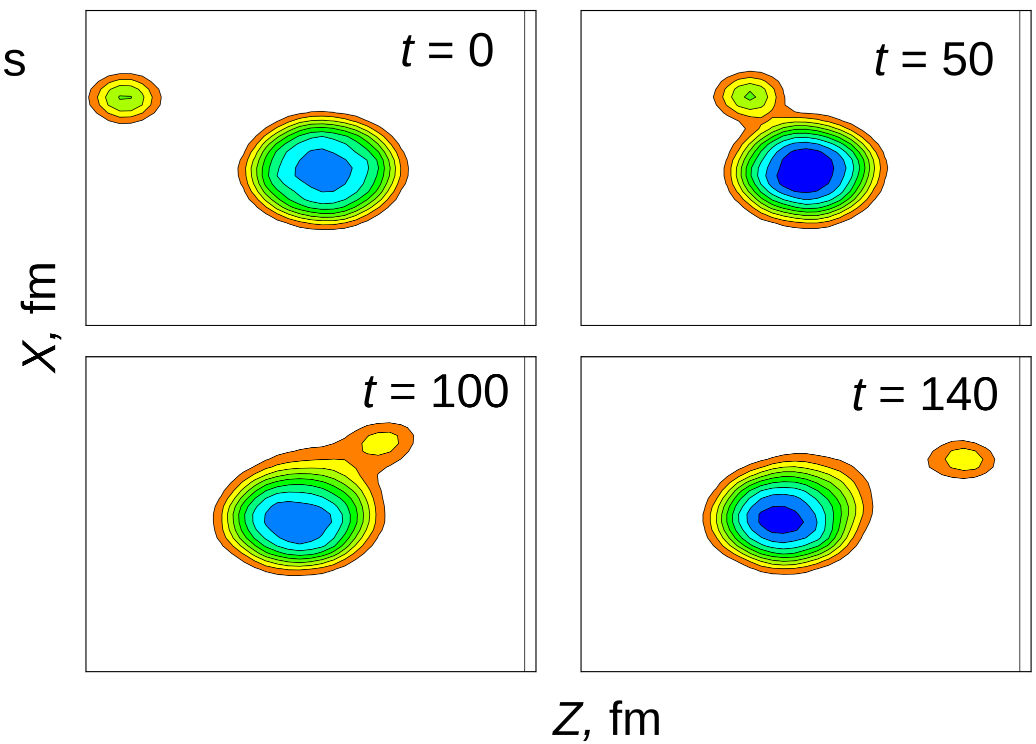
Evolution in time in the mean field until freeze-out time

Input:

mass numbers A_1, A_2 of colliding ions
charge numbers Z_1, Z_2
Projectile energy E_0 in lab. system

Output:

mass A_i, charge Z_i,
coordinates R_i, moments P_i
and excitation energy of the of the moving forward fragment



Density contour plots
$^{18}O(35\ A\ MeV) + ^{181}Ta$
at $b = 9$ fm
(t=0, 50, 100, 140 fm / c
(10 fm/c = $3.3 \cdot 10^{-23}$ c))
t = 140 fm/ c - freeze-out time
Stop calculations
and adjust Coulomb trajectories

$$\rho(t) = \int d\vec{p}\, f(\vec{r},\vec{p},t)$$
$$A(t) = \int d\vec{r}\, \rho(\vec{r},t)$$
$$Z(t) = \int d\vec{r}\, I\, \rho(\vec{r},t)$$
$$P(t) = \int d\vec{r}\, p(\vec{r},t)\rho(\vec{r},t)$$

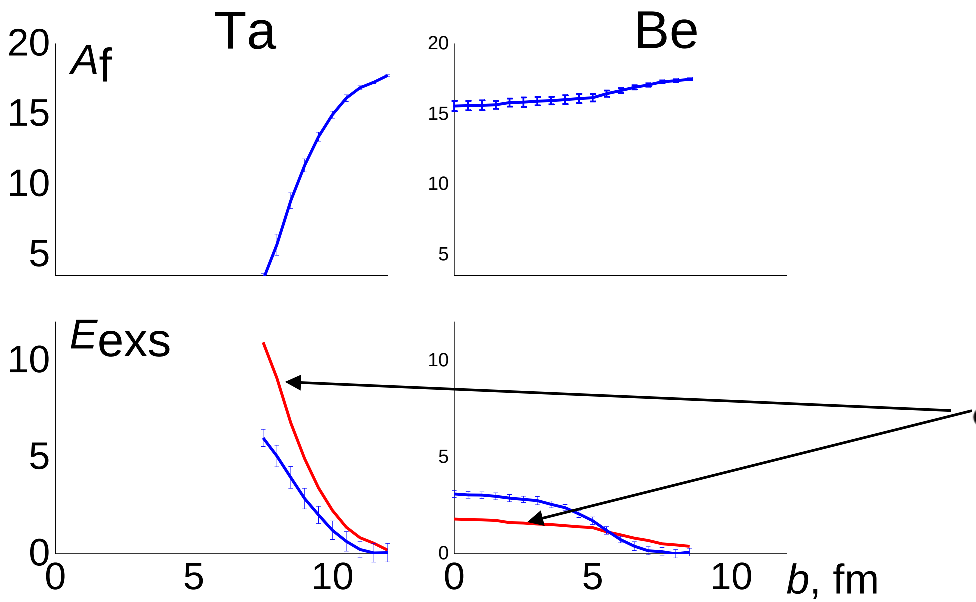
$$\int \Longrightarrow \sum$$

$$E_{int}(t) = \frac{1}{2m_{t.p.}} \int d\vec{r}\, (p(\vec{r},t) - P(t))^2 \rho(\vec{r},t)$$

$$E_{exc} = (E_{pot}+E_{int})(t=t_{freeze-out}) - (E_{pot}+E_{int})(t=0)$$

Results of transport model (BNV) calculations for two reactions: projectile ^{18}O (35 Mev/ nucleon) on ^{181}Ta and ^{9}Be targets

d*b* = 0.5 fm,
b = 7.5 ,…,12.5 fm



d*b* = 0.5 fm,
b = 0. ,…,8. fm

δE = 13.3 d*A*

Number of test-particles per nucleon *K*=240

Mass number A_f and excitation energy E_{exc} of moving forward fragments as a function of impact parameter *b*

Calculations for each impact parameter are repeated 50 times

$$\sigma = \sum_{i=1}^{L} b_i \, db_i \, \sigma_i$$

Decreasing step in impact parameter *b*

^{18}O (35 Mev/ nucleon) on ^{181}Ta

Results of BNV calculations for isotope cross-sections

σ (N, Z)	*N*	H	He	Li	Be	B	C	N	O
b= 8 fm	1	0	0	0	0	0			
		0,00309	0,0646	0,00535	0	0			
	3	0,00232	0,303	0,273	6,03E-4	0	0	0	0
		0	0,0649	0,262	0,00156	0	0	0	0
	5	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
9 fm	1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	5	0	0	0	0,0075	0,0213	0	0	0
		0	0	0	0,0515	0,676	0,05	0	0
	7	0	0	0	0,00333	0,156	0,0345	0	0
		0	0	0	0	6,67E-5	1E-4	0	0
	9	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
10 fm	1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0,00176	0,00124	0
		0	0	0	0	0	0,202	0,55	0,00169
	9	0	0	0	0	0	0,0531	0,189	0,00214
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0

Results of BNV calculations for isotope angle θ

^{18}O (35 Mev/ nucleon) on ^{181}Ta

Θ(N, Z)	*N*	H	He	Li	Be	B	C	N	O
b= 8 fm	1	0	0	0	0	0			
		30,1	30,7	31,5	0	0			
	3	30,7	31,4	30,7	29	0	0	0	0
		0	30,5	30	28,6	0	0	0	0
	5	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
9 fm	1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	5	0	0	0	13,4	13,4	0	0	0
		0	0	0	13,4	13,2	12,9	0	0
	7	0	0	0	13,4	13,1	13,5	0	0
		0	0	0	0	15	15	0	0
	9	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
10 fm	1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	7	0	0	0	0	0	4	4	0
		0	0	0	0	0	3,59	3,48	3
	9	0	0	0	0	0	3,45	3,36	3
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0

8 fm: 30

9 fm: 13

10 fm: 3.5

Results of BNV calculations for isotope velocities v

^{18}O (35 Mev/ nucleon) on ^{181}Ta

b=	$v(N, Z)/v_p$ *N*	H	He	Li	Be	B	C	N	O
8 fm	1	0	0	0	0	0	[illegible]	[illegible]	[illegible]
		0,683	0,678	0,675	0	0	[illegible]	[illegible]	[illegible]
	3	0,683	0,667	0,667	0,675	0	0	0	0
		0	0,666	0,668	0,675	0	0	0	0
	5	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
9 fm	1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	5	0	0	0	0,824	0,824	0	0	0
		0	0	0	0,819	0,817	0,82	0	0
	7	0	0	0	0,808	0,813	0,804	0	0
		0	0	0	0	0,775	0,775	0	0
	9	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
10 fm	1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0,925	0,925	0
		0	0	0	0	0	0,921	0,921	0,925
	9	0	0	0	0	0	0,92	0,92	0,925
		0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0

8 fm: 0.7

9 fm: 0.81

10 fm: 0.925

STEP 3: Calculating cold evaporation residues:

Input parameters: A_{fr}, $Z_{fr,}$ $\underline{E}_{exc}$ R, P from BNV calculation

To be able to compare with experimental data continuous value of impact parameter *b* should be used. This is achieved by the use of interpolation procedure making it possible to come to smaller step in *b* .

To find intermediate values of velocity *v* (and angle θ) the substitution was made $v_{average}(b)$ ($\theta_{average}(b)$ instead of ‘0’ values)

Velocity (*v*) distribution, ^{181}Ta - target, *b*=9

1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0,824	0,824	0	0	0
6	0	0	0	0,819	0,817	0,82	0	0
7	0	0	0	0,808	0,813	0,804	0	0
8	0	0	0	0	0,775	0,775	0	0
9	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0

1	0,81	0,81	0,81	0,81	0,81	0,81	0,81	0,81
2	0,81	0,81	0,81	0,81	0,81	0,81	0,81	0,81
3	0,81	0,81	0,81	0,81	0,81	0,81	0,81	0,81
4	0,81	0,81	0,81	0,81	0,81	0,81	0,81	0,81
5	0,81	0,81	0,81	0,824	0,824	0,81	0,81	0,81
6	0,81	0,81	0,81	0,819	0,817	0,82	0,81	0,81
7	0,81	0,81	0,81	0,808	0,813	0,804	0,81	0,81
8	0,81	0,81	0,81	0	0,775	0,775	0,81	0,81
9	0,81	0,81	0,81	0	0,81	0,81	0,81	0,81
10	0,81	0,81	0,81	0	0,81	0,81	0,81	0,81
11	0,81	0,81	0,81	0	0,81	0,81	0,81	0,81
12	0,81	0,81	0,81	0	0,81	0,81	0,81	0,81

Different approaches can also be used to predict isotope distributions produced in nuclear collisions at Fermi energies

- **Transport approaches: QMD** (quantum molecular dynamics) *J. Aichelin, Phys. Rep.* ***202****, 233 (1991).***,**
  AMD(antisymmetrized/fermionic molecular dynamics)
  A. Ono and H. Horiuchi, Prog. Part. Nucl. Phys. ***53****, 501 (2004);*
- **EPAX (**an Empirical PArametrization of fragmentation CROSS sections)
  K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000).
- **Abration-Ablation model*****,*** *Bowman J.D., Swiatecki W.J., Tsang C.F. // LBL Report. 1973. LBL-2908.*
- **HIPSE (**Heavy-Ion Phase-Space Exploration) *D. Lacroix et al.,Phys Rev. C 69 054604 (2004)*
- ***etc*.**

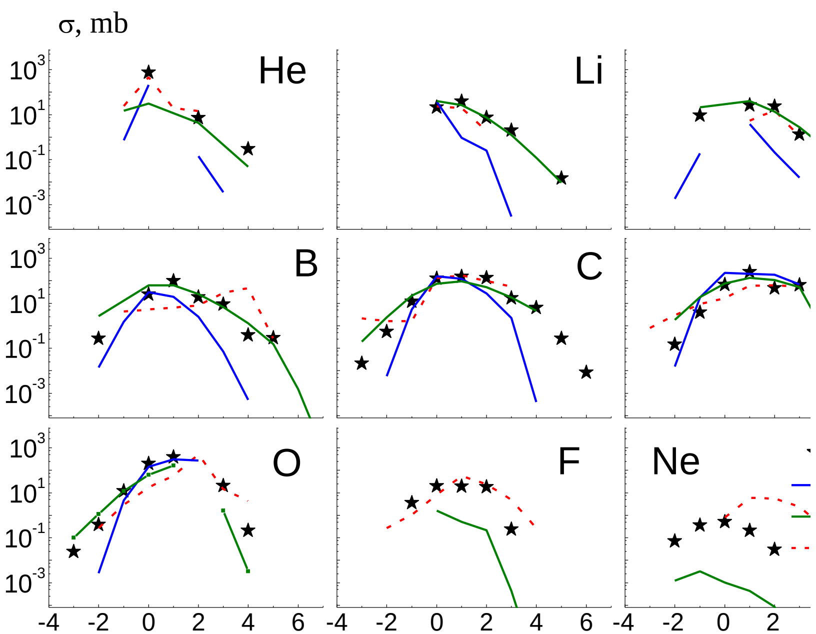
STEP 3: Isotope distributions of fragments as a function of *N-Z*

Calculating cold evaporation residues:

SMM code, P. Bondorf, et al.,Phys. Rep. 257, 133 (1995)

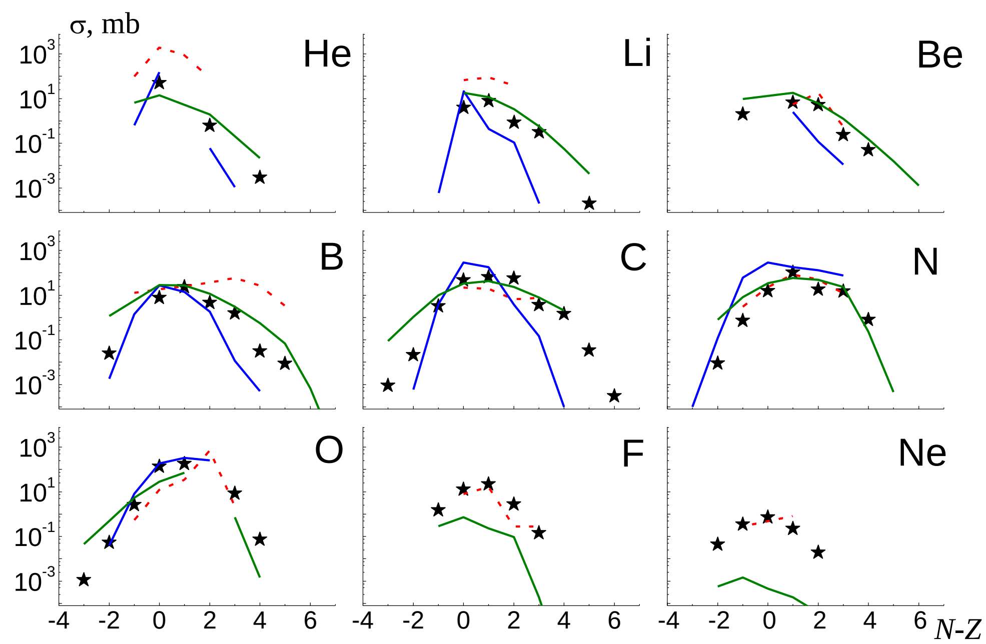
Input parameters: A_{fr}, Z_{fr}, E_{exc}, R, P from BNV calculation

^{18}O(35 *A*Mev)+^{181}Ta



d*b* = 0.0625 fm,
b = 7.5 ,…,12. fm

^{18}O(35 *A*Mev)+^{9}Be



d*b* = 0.125 fm,
b = 0 ,…,8 fm

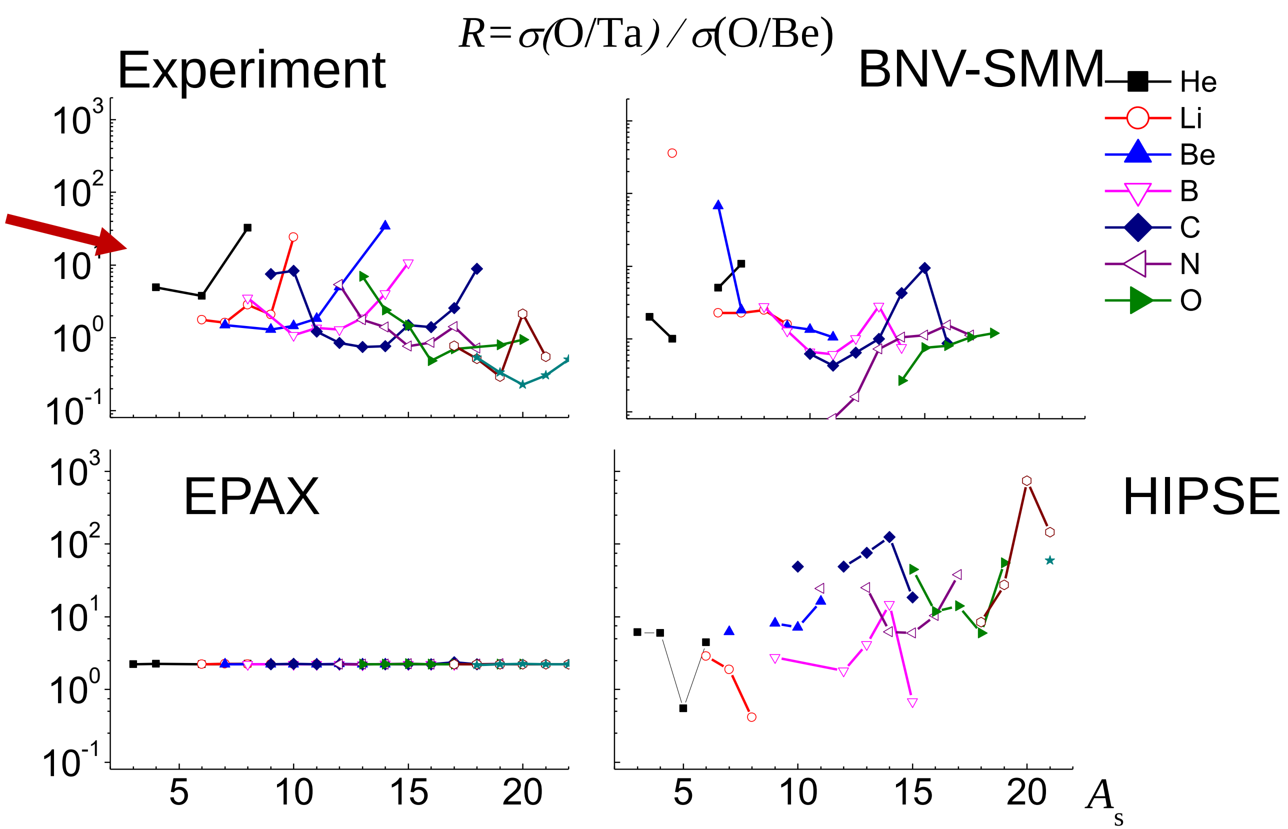
Experiment Artukh, A.G, et al. Phys. Part. Nuclei Lett. 18, 19 (2021).

Isotope ratio for reactions with the same projectile and different targets

1 Excess of neutron reach isotopes in reactions on heavy target

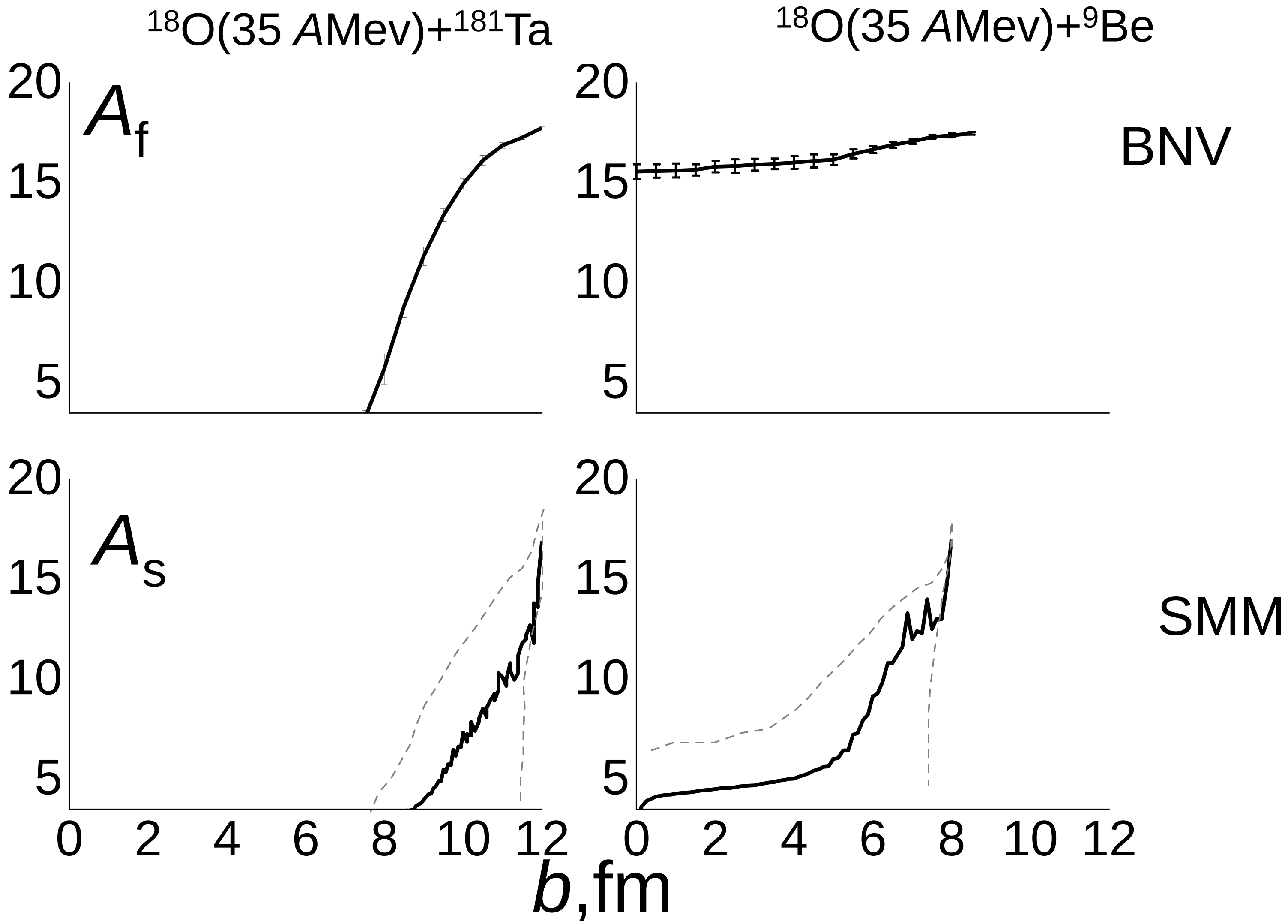
2 The minima of the ratios diminishes with increasing fragment mass

How to explain this behavior?

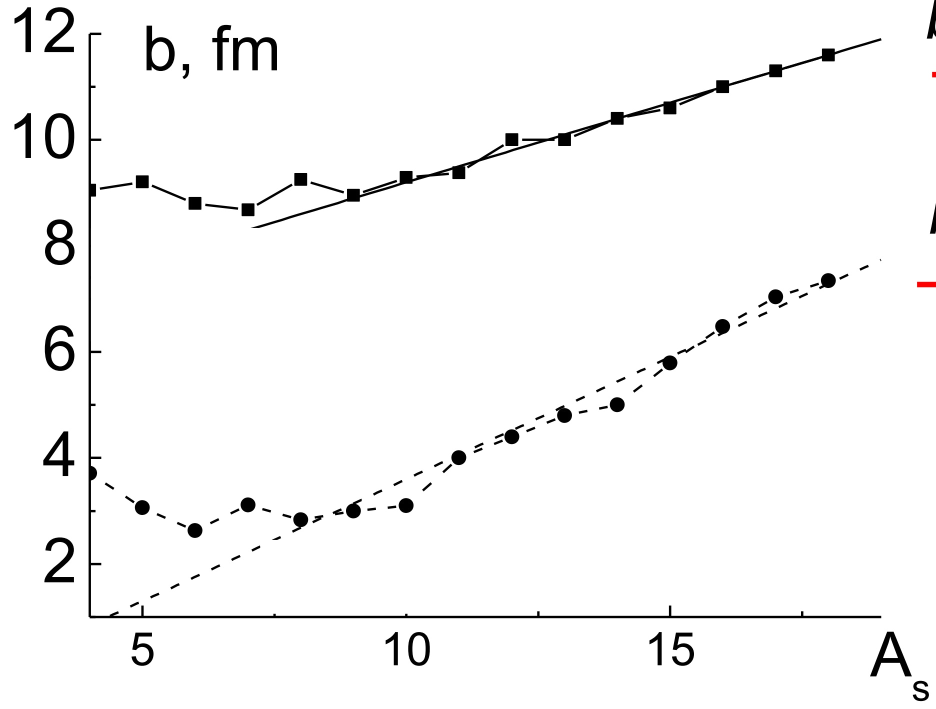


Possible explanation

Fragment mass number A_f as a function of impact parameter b



Linear fit of impact parameter *b* dependence on the mass number of secondary fragment A_s

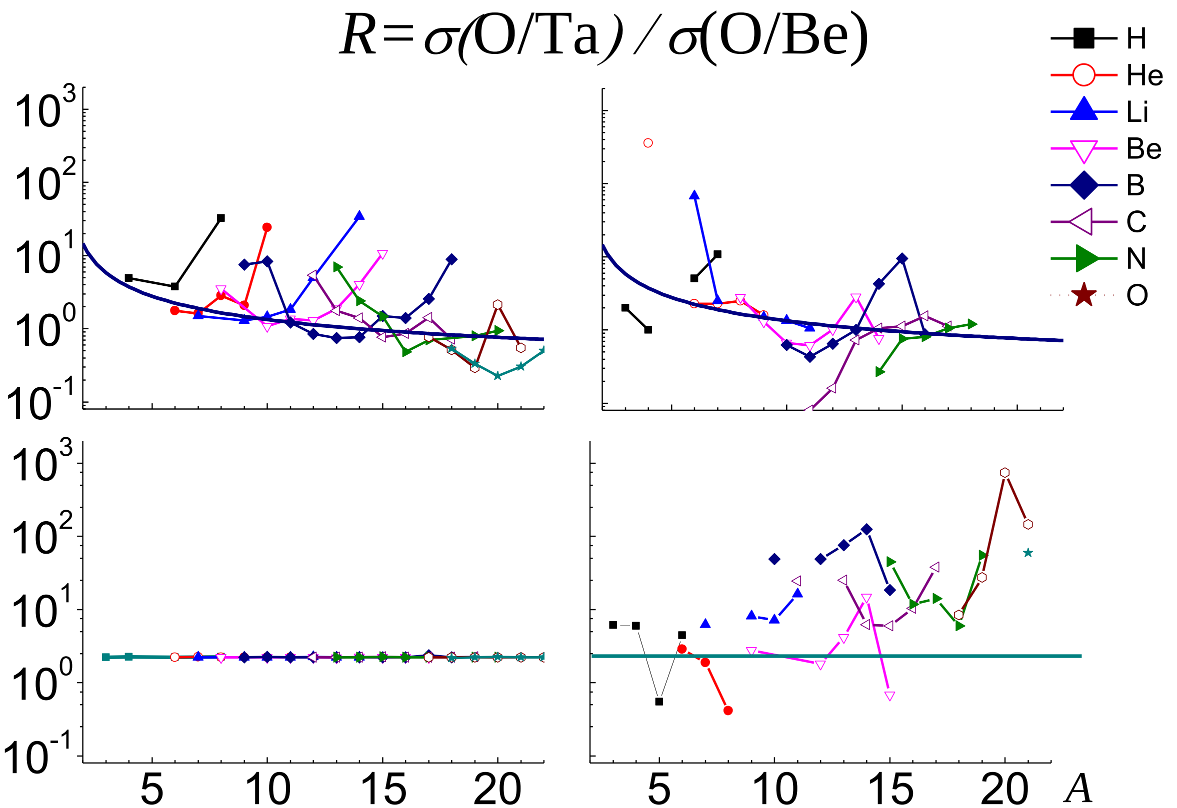


$b_{Ta} = 7.4 + 0.23 * A_s$

$b_{Be} = -0.6 + 0.44 * A_s$

Isotope Ratio

- $$R_{\mathrm{EPAX}} = \frac{\sigma_{\mathrm{Ta}}}{\sigma_{\mathrm{Be}}} = \frac{(A_{\mathrm{O}}^{\frac{1}{3}} + A_{\mathrm{Ta}}^{\frac{1}{3}} - 2.38)}{(A_{\mathrm{O}}^{\frac{1}{3}} + A_{\mathrm{Be}}^{\frac{1}{3}} - 2.38)} = 2.03$$
- $$R_{\mathrm{BNV-SMM}} = b_{\mathrm{Ta}} * db_{\mathrm{Ta}} / (b_{\mathrm{Be}} * db_{\mathrm{Be}})$$



Conclusions

Heavy-ion fragmentation reactions are interesting for physics and important for applications. A microscopic understanding is desirable.

In this work we use transport theory. The solution of the highly non-linear transport equation is mathematically challenging. To de-excite hot fragments produced in transport calculations we use SMM code. We compared the results with experimental data of COMBAS set-up (FLNR JINR): ^{18}O (35 *A* MeV) on targets ^{181}Ta and on ^{9}Be

We also compared calculations in combined transport-statistical model with two well known models: EPAX and HIPSE models

We found that our calculations describe fragments close to *N*-*Z* = 0 rather well, but for neutron reach isotopes our calculations give smaller values than the experiment

The target dependence (the ratio of isotope yields produced in the reactions with the same projectile on heavy and light targets) can be explained from the dependence of mass fragments A_s on impact parameter *b* of the collision

From our calculations it follows that the excess of neutron-reach nuclides in the reactions on heavy target is due to the fact that in collision on heavy target the exotic fragments could be produced at first stage of the reaction (before evaporation), while in the reaction on light target the projectile nucleus looses only few nucleons.

Thank you
for
attention