ITMO UNIVERSITY

Saint Petersburg, Russia

Симуляторы квантовоподобных вычислений на основе распределенных физических систем

Алоджанц Александр

Поддержан грантом РНФ № 23-22-00058

Plan of the talk

ITMO UNIVERSITY

AI-generated image of a quantum robotic Schrödinger's cat that reads quantum machine learning review paper.

A. Melnikov, M. Kordzanganeh, A. Alodjants & Ray-Kuang Lee, Quantum machine learning: from physics to software engineering, Advances in Physics: X, 8:1 (2023)

- **Quantum inspired algorithms /simulators for NP-hard problem solution;**
- **Photonic transport in 2D structures enhanced by complex networks;**
- **Random walks on graphs and quantum speedup problem.**

Give answer on:

How we can minimize computational overheads and improve speedup?

Vital Problem at NISQ Era

ITMO UNIVERSITY

John Preskill

We need Quantum, or, Quantum-like (quantum inspired, cognitive, Fussy, etc.) computation?

NISQ - Noisy Intermediate-Scale Quantum technology

John Preskill, Quantum 2, 79 (2018)

Quantum & Quantum Inspired Hardware

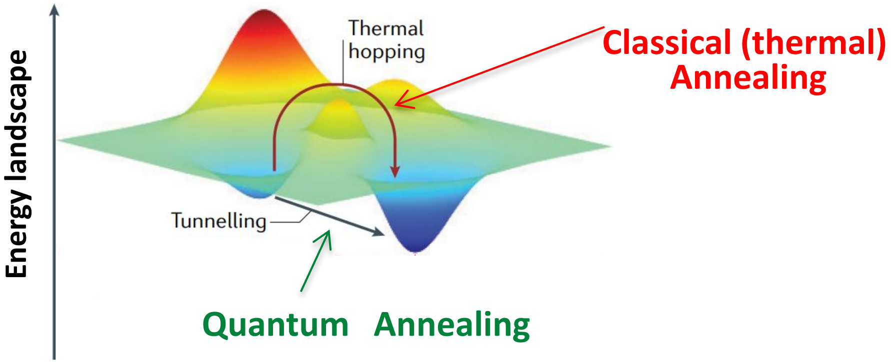
ITMO UNIVERSITY

	Definition	Type	Qubits	Players
Quantum inspired emulators and simulators	*they are classical computers, simulating quantum algorithms. They are slower than quantum computers.*	Ising machines used for optimisation	none	TOSHIBA, FUJITSU, HITACHI Inspire the Next, NTT, Microsoft
Quantum annealer	they use "average quality" qubits and only part of quantum algorithms are processed.	Ising machines used for optimisation	Superconductors	D-Wave The Quantum Computing Company
NISQ « Noisy Intermediate-Scale Quantum »	50-100 qubits – more performing than HPC but still limited	Quantum processor – can solve any problem	Superconductors	Microsoft, IBM, Google, rigetti, Raytheon
Universal quantum computer	> 100 qubits	Quantum processor – can solve any problem	Superconductors Photons Spin qubits Quasi particles NV centers Trapped ions Cold atoms	Microsoft, 阿里云, Honeywell, PASQAL, Silicon Quantum Computing, IONQ, ANAMETRIC, intel

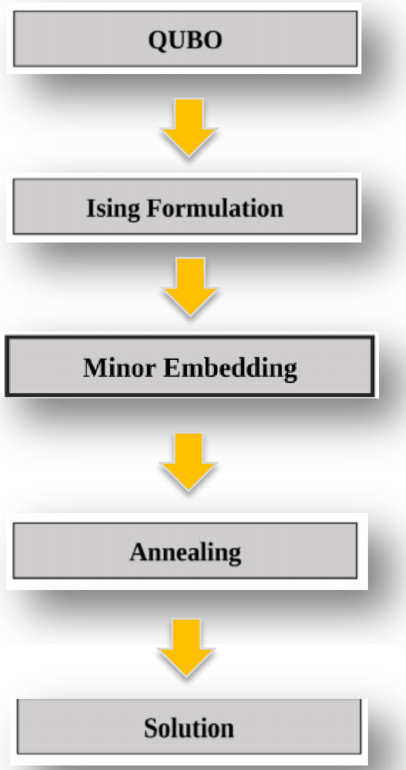
YOLE Développement

Workflow for solving Quadratic Unconstrained Binary Optimization problems

Quantum annealing vs Classical annealing



D-Wave 2000Q annealing



The Hamiltonian of the Ising model in a transverse field

$$\mathscr{H}(t) = \underbrace{\sum_{i \in \mathrm{V}(G)} h_i(t)\sigma_i^z + \sum_{ij \in \mathrm{E}(G)} J_{ij}(t)\sigma_i^z\sigma_j^z}_{\text{Final Hamiltonian}} + \underbrace{\sum_{i \in \mathrm{V}(G)} \Delta_i(t)\sigma_i^x}_{\text{Transverse field}}$$

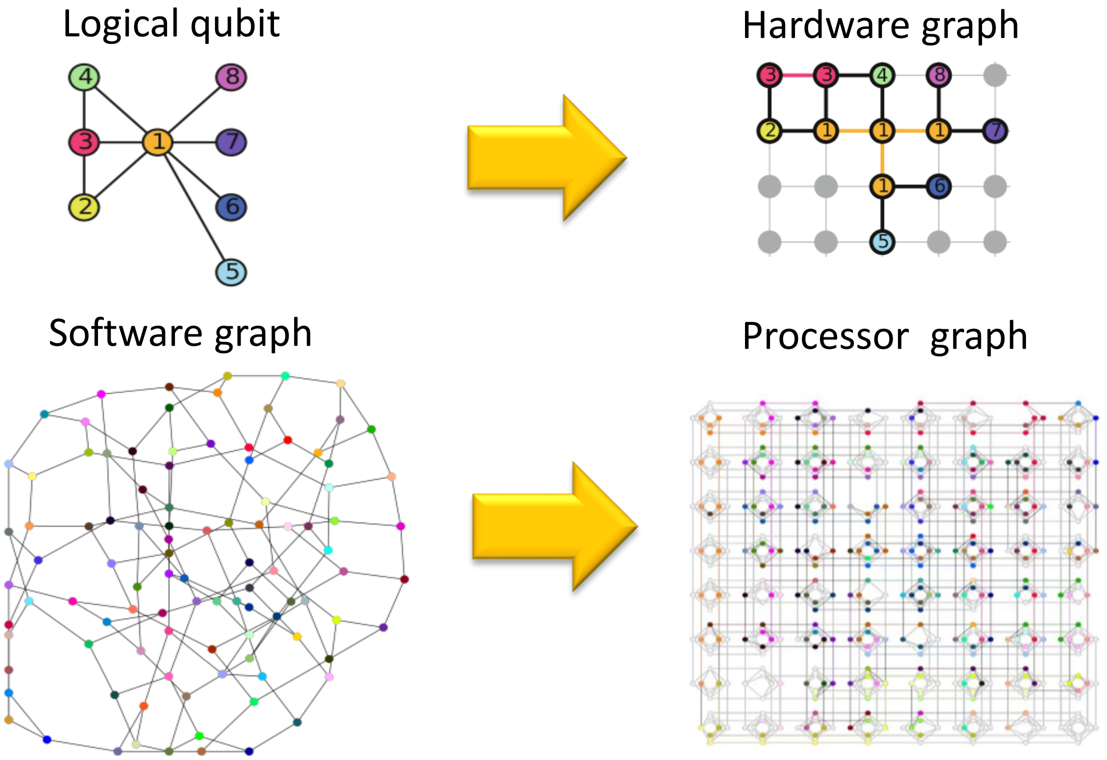
Final Hamiltonian

Transverse field

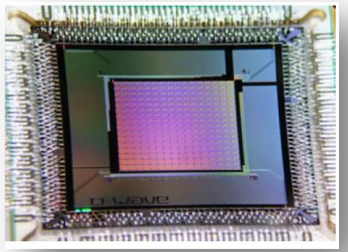
Vicky Choi, Quantum Inf . Process (2008)

Minor Embedding as Computational Overhead

ITMO UNIVERSITY



Chimera (graph) C16 D-Wave 2000Q processor



ME allows to map abstract (Ising) graph to physical lattice device

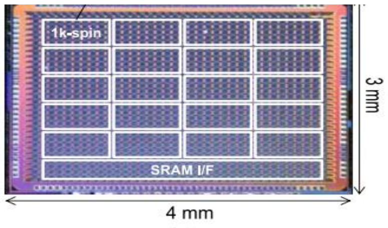
Embedding introduces considerable overhead relative to the fully connected model:

for N logical qubits, $\sim N^2$ physical qubits are required

QUBO-solvers

ITMO UNIVERSITY

Spin chip (CMOS) , Hitachi

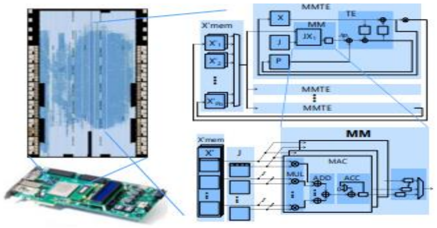


2*20,000 spins

ASIC Digital МИ, Fujitsu

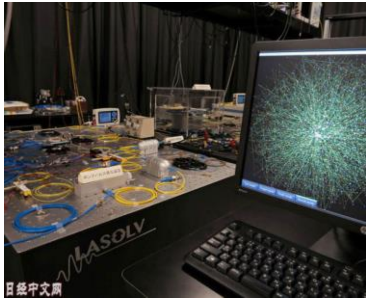
8192 spins

FPGA Bifurcation machine, BM, Toshiba



4096 spins

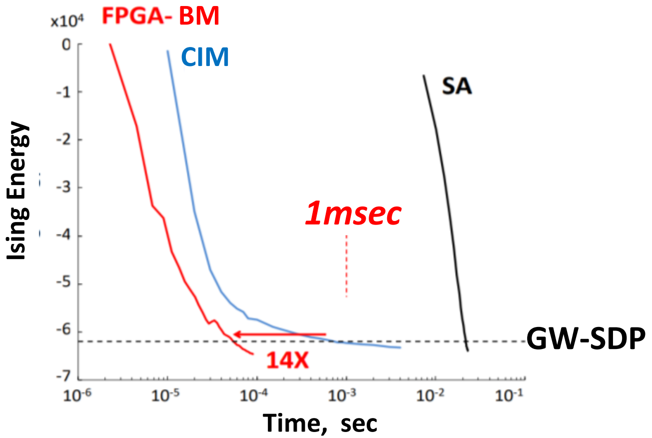
Coherent Ising Machine, (CIM)



100 000 spins

Mohseni, N., et al,
Nat Rev Phys 4, 363 (2022)

MAX CUT на полно-связном графе из 2000 спинов



SA (simulated annealing) , CPU (Intel Core i9-9900K, 3.60 GHz with 64-gigabyte RAM

FPGA ресурсы позволяют работать с матрицами $O(N^2)$

Goemans-Williamson semi-definite programming (GW-SDP) $O(N^3)$

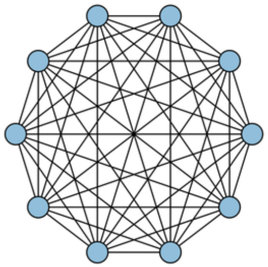
Some Network Models

Any network node *i* characterized by node degree k_i that determines number of links coupled for this node.

Statistical properties of any network may be characterized by its node degree distribution function p_k.

p_k is probability that node has exactly *k* links, *k = 0, 1, 2, ...*

Regular graph

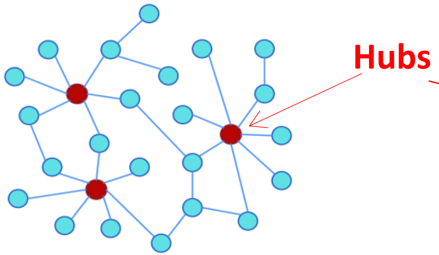


$$p(k) = \delta(k - k_0)$$

Erdős–Rényi graph

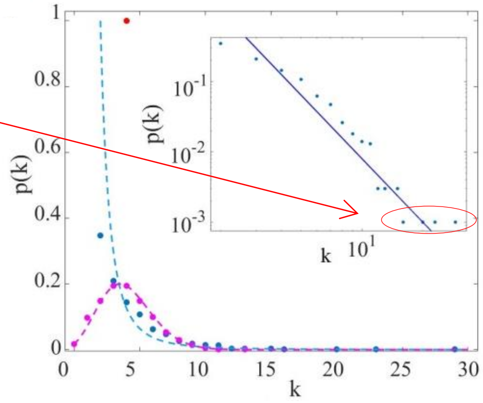
$$p_k = \frac{\langle k \rangle^k e^{-\langle k \rangle}}{k!}$$

Scale-free graph vs SG !



$$p_k \propto \frac{1}{k^\gamma}$$

Node degree distribution p_k



Statistical properties determined by

- Average node degree $\langle k \rangle$,
- Normalized node degrees correlation function $\zeta = \frac{\langle k^2 \rangle}{\langle k \rangle}$

- is robust against links removing;
- represents strongly interacting (disordered) system in Nature.

Simulator Model with Light-Qubit Coupling in 2D Microstructures

ITMO UNIVERSITY

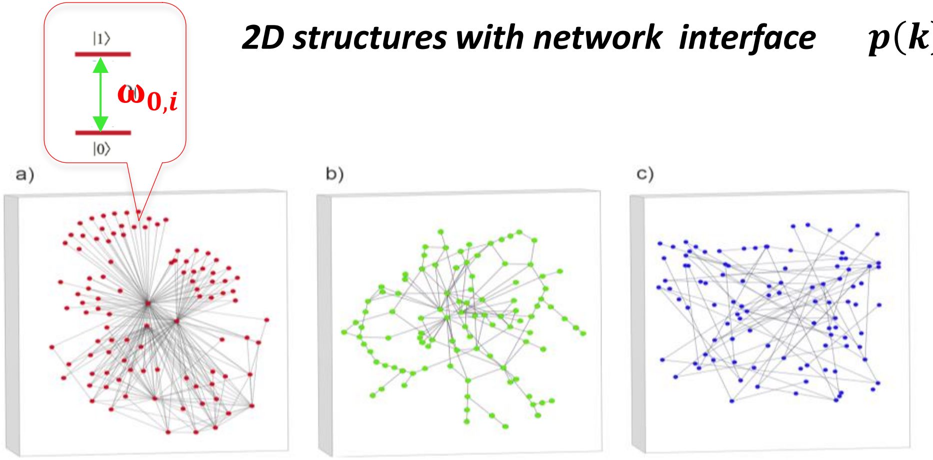
Qubits | Photons | Photon-qubit interaction | Injection field

$$H = \frac{1}{2}\sum_{i=1}^{N}\omega_{0,i}\sigma_i^z + \omega_{ph}\frac{1}{N}\sum_{i=1}^{N}\sum_{v=1}^{k_i} a_v^+ a_v + \frac{g}{\sqrt{N}}\sum_{i=1}^{N}\sum_{v=1}^{k_i}\left(a_v\sigma_i^+ + a_v^+\sigma_i^-\right) + \hbar P\left(a_v^+ - a_v\right)$$

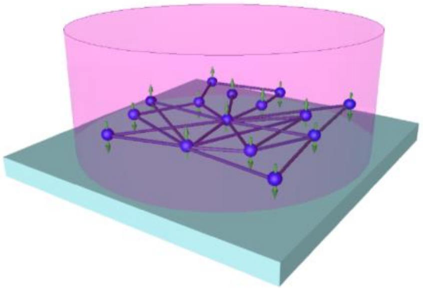
$\omega_{0,i}$ - qubit frequency transition

$a_v^+(a_v)$ – photonic creation (annihilation) operator for v mode with frequency ω_{ph}

2D structures with network interface $p(k) \propto \frac{1}{k^{\gamma}}$



$\gamma = 1.5$ $\gamma = 2.5$ $\gamma = 5$



V. DeGiorgio and Marlan O. Scully, Analogy between the Laser Threshold Region and a Second-Order Phase Transition Phys. Rev. A 2, 1170 (1970)

Basic Approach

Maxwell-Bloch Equation

$$\dot{\alpha} = (-i\omega_{ph} - \kappa)\alpha - ig\sum_{j=1}^{N} p_j + P$$

$$\dot{p_j} = (-i\omega_{0,j} - \Gamma_j)p_j + igk_j\alpha\sigma_j^z$$

$$\dot{\sigma_j^z} = \frac{1}{\tau_j}(\sigma_{j,0}^z - \sigma_j^z) + 2igk_j(p_j\alpha^* - p_j{}^*\alpha)$$

$\tau_j -$ spontaneous emission time;

$\Gamma_j -$ dephasing rate;

$\kappa -$ photon losing rate;

$p_j = \langle a_j^\dagger b_j \rangle$ - average polarization

$\alpha(t) = \langle a_v \rangle$ *is average photonic field*

$\sigma_j^z = \langle b_j^\dagger b_j - a_j^\dagger a_j \rangle$ - average inversion

Photon-field (transport) diffusion

$$\dot{E} = -\left(\kappa - \sum_{j=1}^{N} \frac{(g^2 k_j \sigma_{j,0}^z(\Gamma_j - i\Delta_j)}{\Delta_j^2 + \Gamma_j^2}\right)E - \sum_{j=1}^{N} \frac{4g^4 k_j^3 \tau_j \Gamma_j (\Gamma_j - i\Delta_j)\sigma_{j,0}^z}{(\Delta_j^2 + \Gamma_j^2)^2}|E|^2 E + P,$$

H. Haken, Light: Laser light dynamics, 1981

Non-Equilibrium Phase Transition

ITMO UNIVERSITY

Approximation

$$\dot{p}_j(t) = 0; \;\; \sigma_j^z(t) = 0$$

$$\Gamma_j \gg \kappa, \frac{1}{\tau_j}, \Delta_j$$

Order parameter is field amplitude

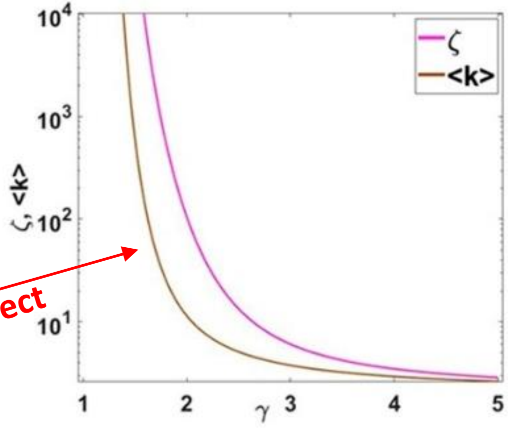
$$\alpha \simeq \sqrt{\frac{\sigma_z}{\sigma_{z,thr}} - 1}$$

Threshold to lasing for normalized population imbalance

$$\boldsymbol{\sigma_{z,thr}} = \frac{\boldsymbol{\kappa\Gamma}}{\boldsymbol{g^2 N}} \frac{\mathbf{1}}{\boldsymbol{\langle k \rangle}}$$

Convenient Laser term

Network effect

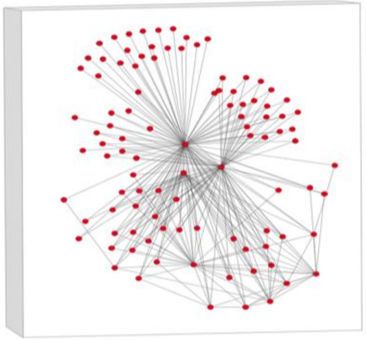


Above threshold

$$\alpha \simeq P^{1/3}$$

Threshold free

$$\langle k \rangle \to \infty \; (\gamma \to 1)$$



$\gamma = 1.5$

The rate of photon transport enhanced $\langle k \rangle$ times

Rabi splitting scales as $g\sqrt{N\langle k \rangle}$

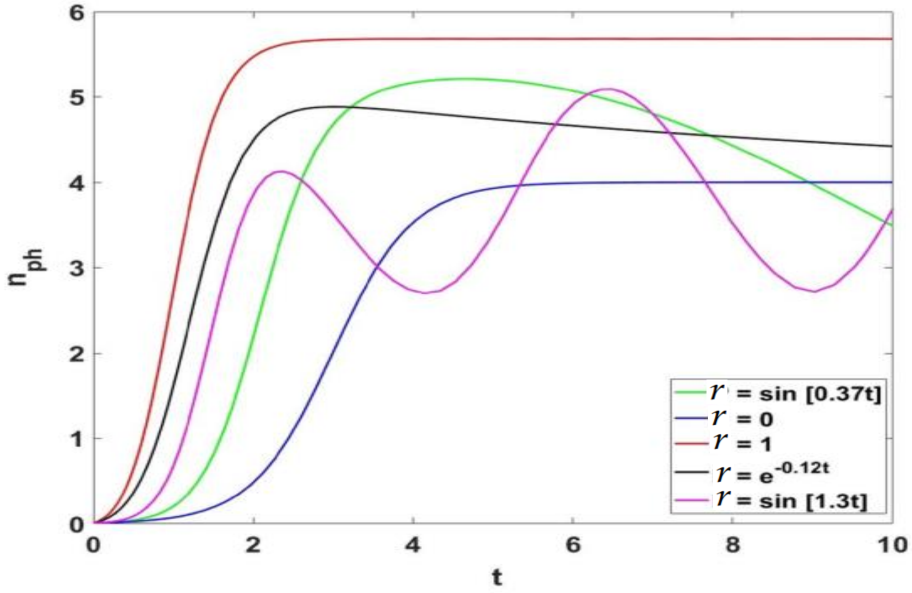
Diffusion

Basic equation

$$n_{ph} \equiv |\Psi|^2$$

$$\dot{n}_{ph} = 2An_{ph} - 2Bn_{ph}^2 + 2r\sqrt{n_{ph}},$$

$$A = \frac{\kappa}{2}\left(C_\Gamma D_0 - 1\right), \qquad B = \frac{C_\Gamma^2\kappa^2 D_0}{(\gamma_P + \gamma_D)},$$



r=0

$$n_{ph} = \frac{Ave^{2At}}{1 + Bve^{2At}};$$

$v = \bar{n}/(A - B\bar{n});$ $\bar{n}$ - начальное значение

$r \neq 0$

$$n_{ph} = \frac{(e^{At}(A\sqrt{\bar{n}} + r) - r)^2}{A^2}$$

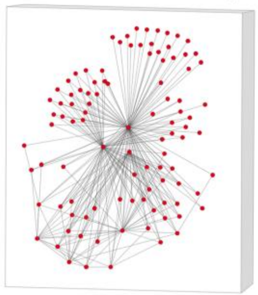
$r_c = A^2\bar{n}$ критическое значение контролирующего поля, при котором наступает усиление

Outline

ITMO UNIVERSITY

In NISQ era quantum inspired (heuristic) algorithms realized by means of photon involved simulators may be more successful for solving some of NP-hard problems for moderate (up to hundred of thousands) number of qubits.

We can minimize computational overheads and improve speedup by means of direct arrangement of hardware circuits for a given NP-hard problem.



The rate of photonic transport may be enhanced in complex networks $\langle k \rangle \gg 1$ times. Such a regime occurs due to simultaneous interaction of two-level systems with a quantized field through numerous waveguide channels (graph edges) responsible for the hubs formation.

A. Bazhenov, M. M. Nikitina, D. V. Tsarev, and A. P. Alodjants, *Random Laser Based on Materials in the Form of Complex Network Structures* **JETP Letters, Vol. 117, No. 11, pp. 814–820 (2023)**

A. Melnikov, M. Kordzanganeh, Alexander Alodjants & Ray-Kuang Lee, *Quantum machine learning: from physics to software engineering,* **Advances in Physics: X, 8:1 (2023)**

Alodjants, A.; Zacharenko, P.; Tsarev, D.; Avdyushina, A.; Nikitina, M.; Khrennikov, A.; Boukhanovsky, *A. Random Lasers as Social Processes Simulators*. ***Entropy*** **2023, *25*, 1601.**

A. Alodjants, A. Bazhenov, A. Khrennikov, A.V. Bukhanovsky, *Mean-field theory of social laser* **Scientific Reports, 12. 1-17 (2022)**

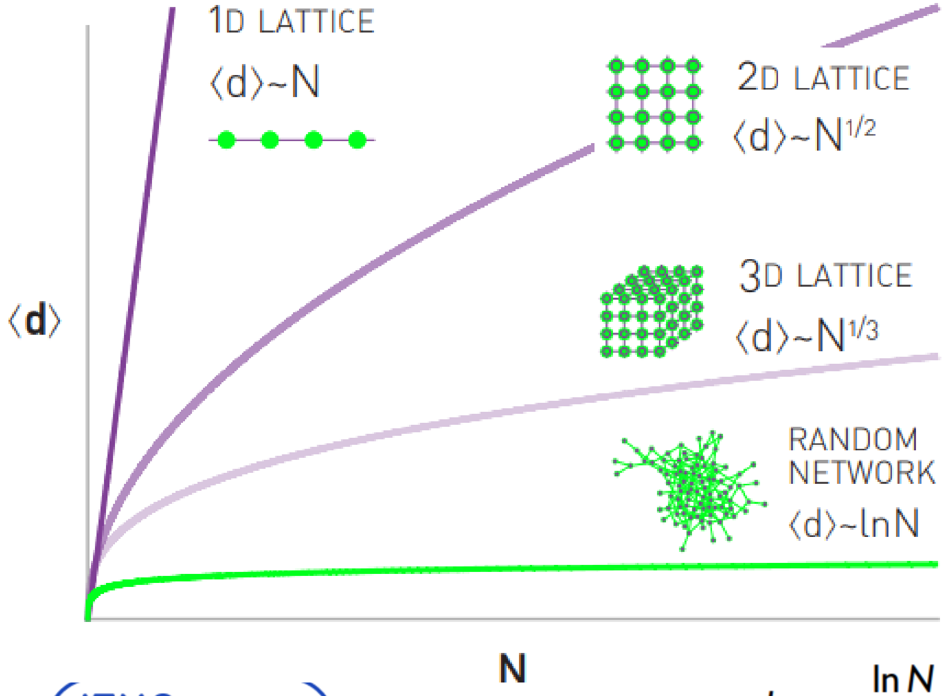
Thank you for your attention!

ITsMOre than a UNIVERSITY

Alexander_ap@list.ru

Small world phenomenon

Diameter of a random network



$$d_{max} \approx \frac{\ln N}{\ln \langle k \rangle},$$

Distance between two randomly chosen nodes in a network is short - six degrees of separation effect

$\langle k \rangle$ nodes at distance one (d=1).

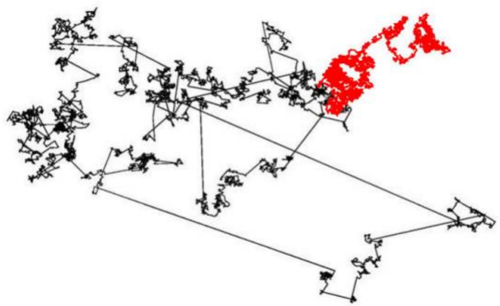
$\langle k \rangle^2$ nodes at distance two (d=2).

$\langle k \rangle^3$ nodes at distance three (d =3).

...

$\langle k \rangle^d$ nodes at distance d.

Random vs Levy Walks



Information spreading in complex structures

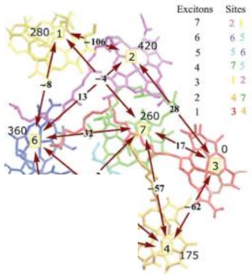
ITMO UNIVERSITY

For simulation of QUBO (or some other) NP-hard computational problem we need to use some mapping procedure to the graph

- ***Can we avoid minor embedding (or, some similar) procedure?***
- ***How we can use graph topology for speedup information processing?***

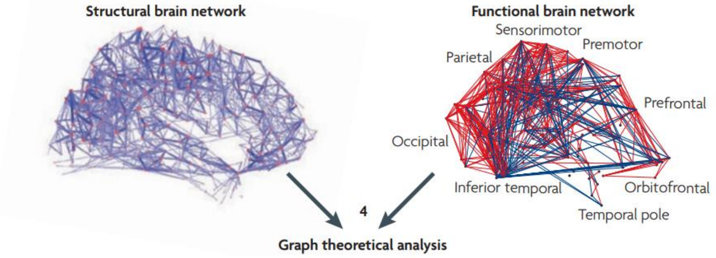
Biological networks

Fenna–Matthews–Olsen (FMO) antenna complexes involved I n natural photosynthesis



Social Networks

Our Brain



We should use complex network advantages !

Bullmore, E., Sporns, O.
Nat Rev. Neurosci 10, 186 (2009)