HIGHER SCHOOL OF ECONOMICS
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Anisotropic model of qubit resonator interaction: reading, quantum correlations and weak chaos

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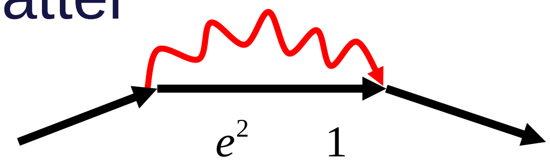
Outline

- Introduction
- Rabi model
- Coupling qubits with resonator
- An anisotropic model
- Symmetry and coherent states
- Random tree-type matrices
- Summary

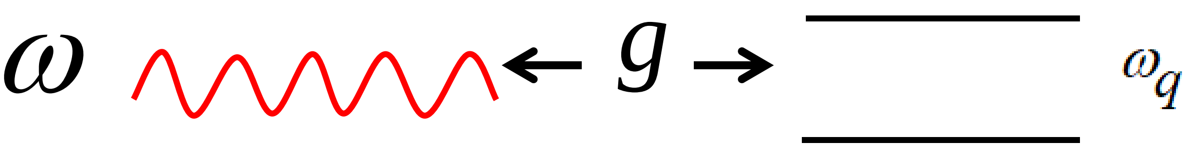
Coupling between light and matter



In QED there is the small value of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

Purcell discovered that spontaneous emission is enhanced when atoms are coupled to a resonant cavity. But the interaction between atoms and cavities can also have the opposite effect; if the coupling between atom and cavity is strong enough — strong coupling — then spontaneous emission can be suppressed. In this case, atom and cavity rapidly exchange quanta of energy, forming hybridized excited states — a molecule of matter and light.



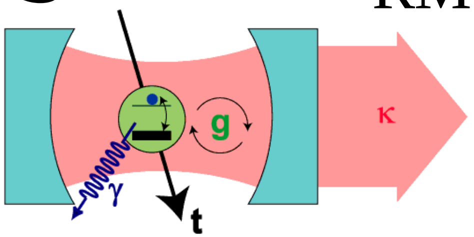
$$\omega \sim \omega_q$$

Weak, strong and ultrastrong c coupling strength comparable to the transition frequencies

A Circuit Analog for Cavity QED

$$g = \vec{d} \bullet \vec{E}_{\mathrm{RMS}} / \hbar$$

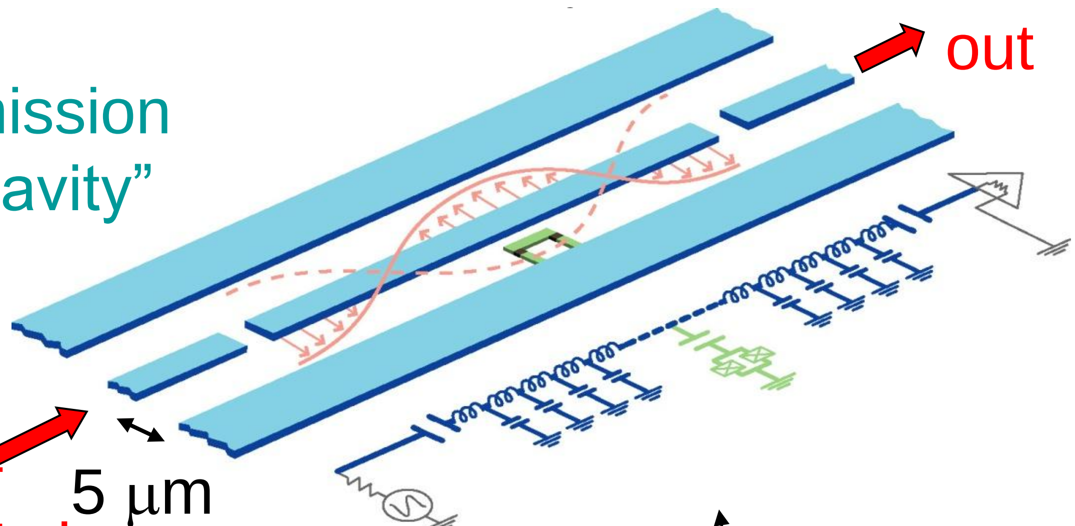
$$d \sim 40{,}000\, e a_0$$



$$g = d E_{\mathrm{RMS}}$$

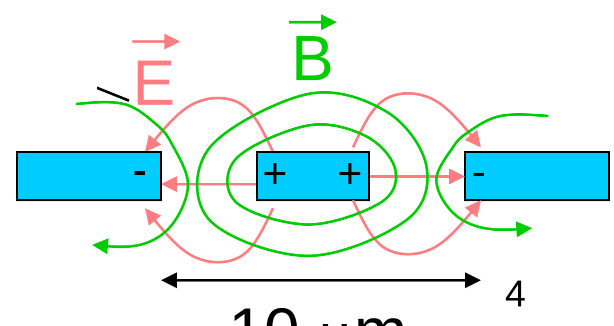
$$E = E_{\mathrm{RMS}}(a + a^{\dagger})$$

transmission line "cavity"



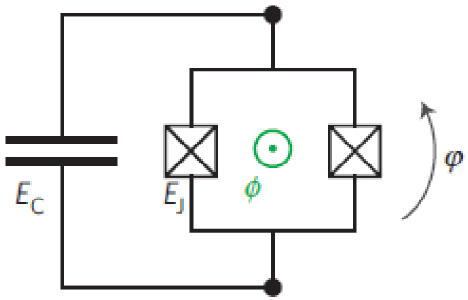
Lumped element equivalent circuit

Cross-section of mode:



Blais, Huang, Wallraff, SMG & RS, PRA 2004

Circuit Quantum Electrodynamics (QED)



C=10 *p*F $L_J \approx 1\ nH$

$$\frac{\omega_J}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2eI_c}{\hbar C}} = \frac{1}{2\pi\sqrt{L_J C}} \approx 1 GHz$$

$$T \approx 10\ mK$$

T=24 mK=500 MHz

$100 \times 100\ \ nm^2$

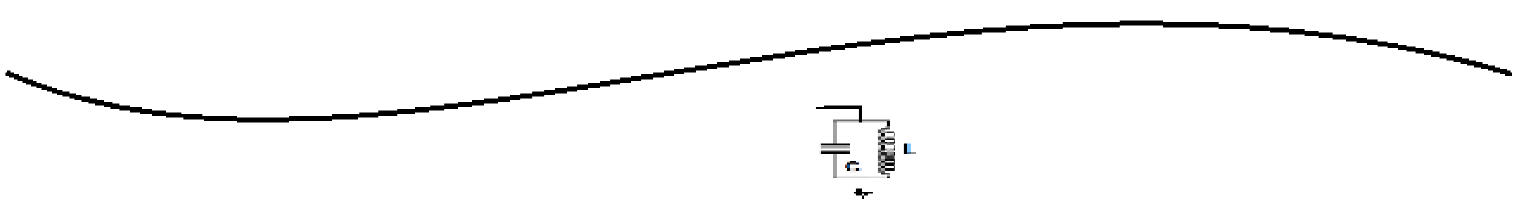
$$k_B T << \hbar\omega_J$$

$$Q >> 1$$

$L_J = \Phi_0 / I_c$

Typically, the wavelength of microwave photons in superconducting circuits is much larger than the characteristic size of the embedded elements, which makes it possible to use equivalent circuits of quantum circuits containing local elements for their description: capacitances, inductors, Josephson junctions, etc.

$$\lambda >> L$$



Models for light–matter coupling

Rabi model $\hat{H}_{\mathrm{Rabi}} = \omega_{\mathrm{c}} \hat{a}^{\dagger} \hat{a} + \frac{1}{2} \omega_{\mathrm{q}} \hat{\sigma}_z + \hat{H}_{\mathrm{int}}, \quad \hat{H}_{\mathrm{int}} = g \hat{X} \hat{\sigma}_x = g_1 (\hat{a} \hat{\sigma}_+ + \hat{a}^{\dagger} \hat{\sigma}_-) + g_2 (\hat{a} \hat{\sigma}_- + \hat{a}^{\dagger} \hat{\sigma}_+),$

Rabi, I. I. Space quantization in a gyrating magnetic field. Phys. Rev. 51, 652 (1937).

Jaynes-Cummings model $H = \hbar \omega a^{\dagger} a + \frac{1}{2} \hbar \omega_{\mathrm{q}} \sigma_z + \hbar g \left(a^{\dagger} \sigma_- + a \sigma_+ \right)$

Jaynes, E. T. & Cummings, F. W. Comparison of quantum and semiclassical radiation theories with application to the beam maser. Proc. IEEE 51, 89 (1963)

	1 atom	*N* atoms
Without RWA	Quantum Rabi model[220]	Dicke model[8], Hopfield model[222]
With RWA	Jaynes–Cummings model[39]	Tavis–Cummings model[48]

RWA, rotating-wave approximation.

A.F. Kockum *et al.* Nature Physics Reviews v. 1 2019 |

experiment from 1992 with a ratio $g/\omega = 10^{-8}$

Thompson, R.J.; Rempe, G.; Kimble, H.J. Observation of normal-mode splitting for an atom in an optical cavity. Phys. Rev. Lett. 1992, 68, 1132–1135

strong coupling regime $g \sim \omega$

Casanova, J.; Romero, G.; Lizuain, I.; García-Ripoll, J.J.; Solano, E. Deep Strong Coupling Regime of the Jaynes-Cummings Model. Phys. Rev. Lett. 2010, 105, 263603

ultra-strong coupling regime $0.1 \lesssim g/\omega \lesssim 0.3$,

Forn-Díaz, P.; Lamata, L.; Rico, E.; Kono, J.; Solano, E. Ultrastrong coupling regimes of light-matter interaction. Rev. Mod. Phys. 2019, 91, 025005

The Jaynes-Cummings model and its descendants

by Jonas Larson &Themistoklis Mavrogordatos

[1] L. Allen and J. L. Eberly, Optical resonance and two-level atoms (Dover Publications, 1975).
[2] W. Paul, “Nobel lecture,” (1989). [3] H. G. Dehmelt, “Nobel lecture,” (1989).
[4] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. 70, 101 (1998).
[5] D. J. Wineland, “Nobel lecture,” (2012). [6] S. Haroche, “Nobel lecture,” (2012).
[7] C. Cohen-Tannoudji, “Nobel lecture,” (1997).
[8] S. Chu, “Nobel lecture,” (1997).
[9] W. D. Philips, “Nobel lecture,” (1997).
[10] T. W. Hänsch, “Nobel lecture,” (2005).
[11] J. L. Hall, “Nobel lecture,” (2005).
[12] R. J. Glauber, “Nobel lecture,” (2005).
[13] E. A. Cornell, “Nobel lecture,” (2001).
[14] C. E. Wieman, “Nobel lecture,” (2001).
[15] W. Ketterle, “Nobel lecture,” (2001)
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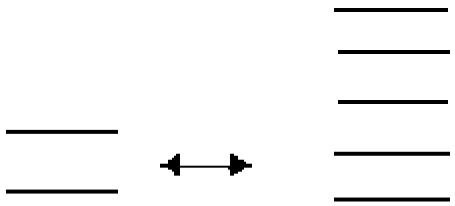
[1976] W. Heisenberg, The physical principles of the quantum theory (University of Chicago Press, Dover Publications, 1930).

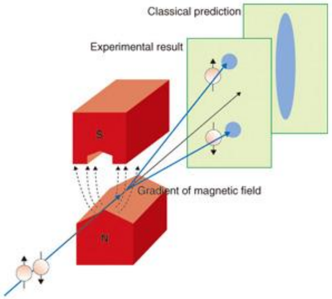
Jaynes-Cummings model: dispersive regime

$$H=\frac{\hbar\omega_q}{2}\sigma_z+\hbar a^+a+\hbar g\left(a\sigma_+ + a^+\sigma_-\right)$$



$$H_{eff}=\hbar\chi\left(\sigma_+\sigma_- + a^+a\sigma_z\right)$$

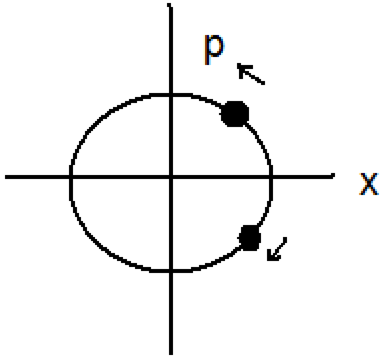
$$\chi=\frac{g^2}{\omega_q-\omega}$$



$$\left|\psi(t)\right\rangle=\hat{U}(t)\left|\psi(0)\right\rangle,$$

$$\left|\psi(0)\right\rangle=\left|g\right\rangle\otimes\left|n\right\rangle,\ \ \left|\psi(t)\right\rangle=e^{-iH_{eff}t/\hbar}\left|\psi(0)\right\rangle=e^{-i\chi nt}\left|g\right\rangle\otimes\left|n\right\rangle$$

$$\left|\psi(0)\right\rangle=\left|e\right\rangle\otimes\left|n\right\rangle,\ \ \left|\psi(t)\right\rangle=e^{i(n+1)\chi t}\left|e\right\rangle\otimes\left|n\right\rangle$$



$$\left|\psi(0)\right\rangle=\left|g\right\rangle\otimes\left|\alpha\right\rangle,\ \ \left|\psi(t)\right\rangle=\left|g\right\rangle\otimes\left|\alpha e^{i\chi t}\right\rangle$$

$$\left|\psi(0)\right\rangle=\left|e\right\rangle\otimes\left|\alpha\right\rangle,\ \ \left|\psi(t)\right\rangle=\left|e\right\rangle\otimes\left|\alpha e^{-i\chi t}\right\rangle e^{-i\chi t}$$

$$\left|\psi(0)\right\rangle=\frac{1}{\sqrt{2}}\left(\left|g\right\rangle+e^{i\phi}\left|e\right\rangle\right)\otimes\left|\alpha\right\rangle,\ \ \left|\psi(t)\right\rangle=\frac{1}{\sqrt{2}}\left(\left|g\right\rangle\otimes\left|\alpha e^{i\chi t}\right\rangle+e^{i\phi}e^{-i\chi t}\left|e\right\rangle\otimes\left|\alpha e^{-i\chi t}\right\rangle\right)$$

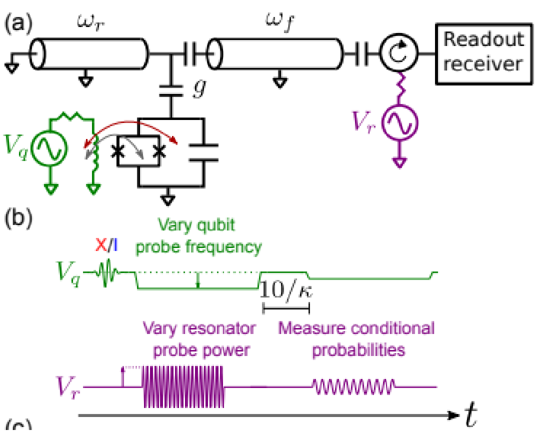
Measurement-Induced State Transitions in a Superconducting Qubit: Within the Rotating Wave Approximation,
Google Quantum AI, Goleta, CA , Phys. Rev. Applied 20, 054008 (2023)

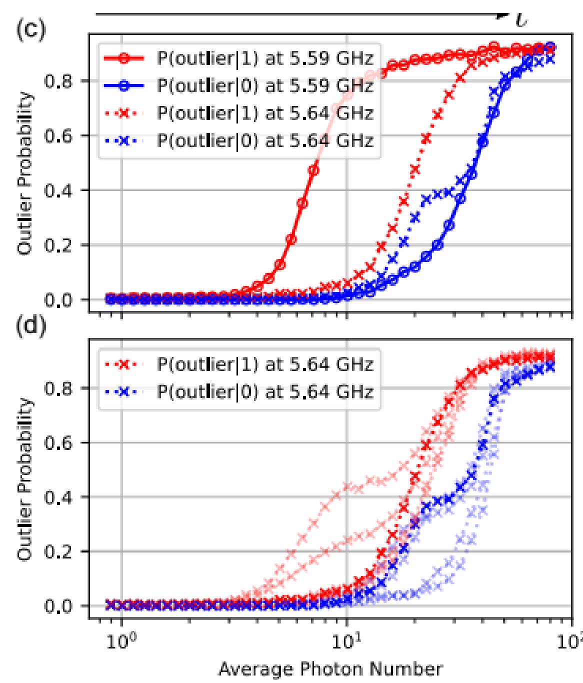
$\omega_q/(2\pi) = 6.340$ GHz

nonlinearity $\eta \equiv \omega_q - \omega_{21} \approx 2\pi \times 195$ MHz

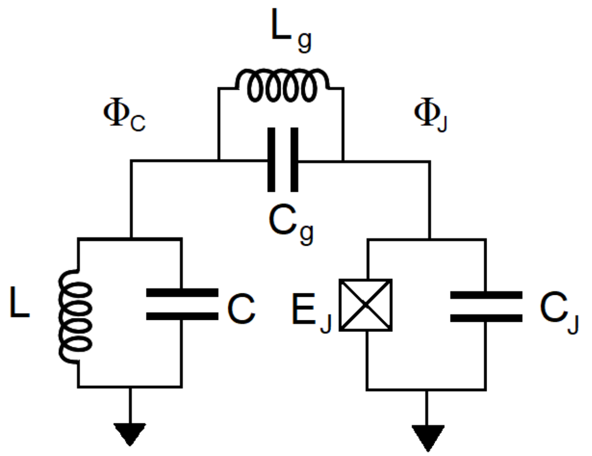
$\omega_r/(2\pi) = 4.750$ GHz

$g/(2\pi) \approx 130$ MHz

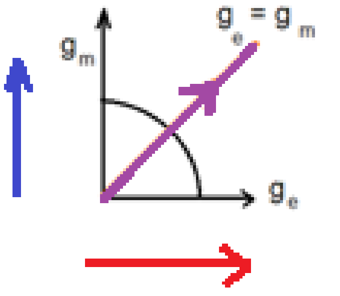




Mixed coupling between a qubit and a resonator



(a)



(b)

An equivalent scheme of the qubit-resonator system with electro-dipole and magneto-dipole coupling. The system consists of a contour and a Josephson oscillator formed by one or a pair of parallel weak connections.

(a) The interaction between the subsystems is carried out using an oscillatory circuit .

(b) The plane of coupling constants (the line in this model exactly corresponds to the RWA approximation or the Jaynes-Cummings model).

Lagrangian and Hamiltonian

$$\mathcal{L} = \frac{C\dot{\Phi}_C^2}{2} - \frac{\Phi_C^2}{2L} + \frac{C_J\dot{\Phi}_J^2}{2} - E_J(\Phi_J) + \frac{C_g}{2}(\dot{\Phi}_C - \dot{\Phi}_J)^2 - \frac{1}{2L_g}(\Phi_C - \Phi_J)^2.$$

$$Q_C = C\dot{\Phi}_C - C_g\dot{\Phi}_J,$$

$$Q_J = C_J\dot{\Phi}_J - C_g\dot{\Phi}_C,$$

$$H = \frac{C_L Q_C^2}{2D} + \frac{\Phi_C^2}{2L_G} + \frac{C_\Sigma Q_J^2}{2D} + V_J(\Phi_J) + \frac{C_g}{D}Q_C Q_J - \frac{1}{L_g}\Phi_C\Phi_J, \qquad \omega = 1/\sqrt{\tilde{L}\tilde{C}}$$

$$C_\Sigma = C + C_g \qquad C_\Xi = C_J + C_g \qquad 1/L_G = 1/L + 1/L_g \qquad D = C_\Sigma C_\Xi - C_g^2 \qquad V_J(\Phi_J) = E_J(\Phi_J) + \frac{\Phi_J^2}{2L_g}$$

$$Q_C = \left(\frac{\hbar\omega\tilde{C}}{2}\right)^{1/2}(-i)(a + a^\dagger), \quad \Phi_C = \left(\frac{\hbar}{2\omega\tilde{C}}\right)^{1/2}(a + a^\dagger),$$

$$H=\frac{C_L Q_c^2}{2D}+\frac{\Phi_C^2}{2L_G}+\frac{C_\Sigma Q_J^2}{2D}+V_J(\Phi_J)+\frac{C_g}{D}Q_C Q_J-\frac{1}{L_g}\Phi_C\Phi_J,$$

$$Q_C=\left(\frac{\hbar\omega\tilde{C}}{2}\right)^{1/2}(-i)(a+a^\dagger),\ \ \Phi_C=\left(\frac{\hbar}{2\omega\tilde{C}}\right)^{1/2}(a+a^\dagger),$$

$$\tilde{E}_C=e^2/2\tilde{C}$$

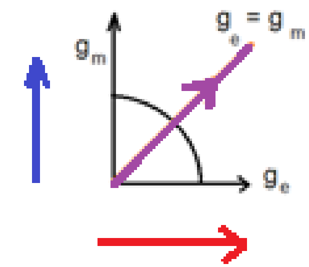
$$\tilde{E}_J=E_J+\Phi_0^2/L_g$$

$$Q_J=2e\left(\frac{\tilde{E}_C}{2\tilde{E}_J}\right)^{1/4}\sigma_y,\ \ \Phi_J=\frac{\Phi_0}{2\pi}\left(\frac{2\tilde{E}_C}{\tilde{E}_J}\right)^{1/4}\sigma_x,$$

$$\sim\hat{\mu}\cdot\hat{h}\qquad \frac{C_g}{D}Q_C Q_J\qquad ig_m\left(a-a^\dagger\right)\sigma_y\qquad g_m=\frac{eC_g}{D}\sqrt{\frac{\hbar\omega\tilde{C}}{2}}\left(\frac{\tilde{E}_J}{2\tilde{E}_C}\right)^{1/4}$$

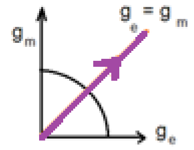
$$\sim\hat{d}\cdot\hat{e}\qquad -\frac{1}{L_g}\Phi_C\Phi_J\qquad g_e\left(a^\dagger+a\right)\sigma_x\qquad g_e=-\frac{1}{L_g}\sqrt{\frac{\hbar}{2\omega\tilde{C}}}\frac{\hbar}{2e}\left(\frac{2\tilde{E}_C}{\tilde{E}_J}\right)^{1/4}$$

$$\boxed{H_{AR}=\frac{1}{2}\omega_{\mathrm{q}}\sigma_z+\omega a^\dagger a+g_e\left(a^\dagger+a\right)\sigma_x+ig_m\left(a-a^\dagger\right)\sigma_y}$$



Anisotropic Rabi model properties and limiting Jaynes &Cummings Hamiltonian

The JCM provides very good agreement with experiments in atom optics where the dipole coupling strength is many orders of magnitude smaller than the mode frequency



$$g_e = g_m = g/2$$

$$H_{AR} \to H_{JC} = \frac{1}{2}\omega_q \sigma_z + \omega a^\dagger a + g\left(a^\dagger \sigma_- + a\sigma_+\right),$$

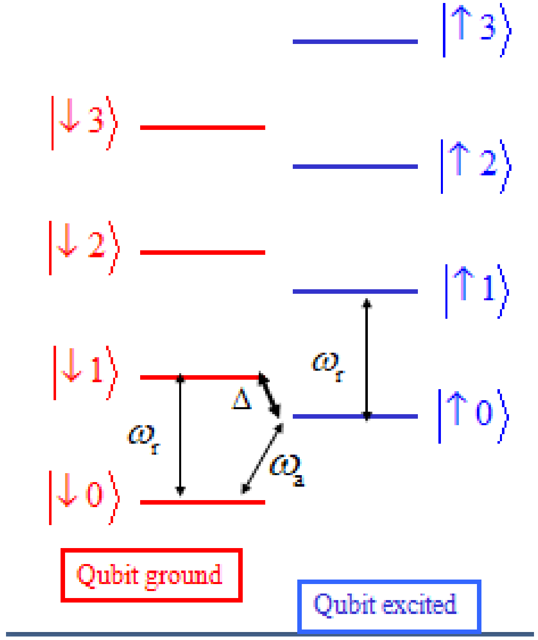
$$C = \omega\left(\frac{1}{2}\sigma_z + a^\dagger a\right),$$

$$|+1,n\rangle = \cos\theta_n |n\rangle|+1\rangle + \sin\theta_n |n+1\rangle|-1\rangle,$$
$$|-1,n\rangle = -\sin\theta_n |n\rangle|+1\rangle + \cos\theta_n |n+1\rangle|-1\rangle,$$

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\omega_q - \omega}\right)$$

$$E_{\pm,n} = \omega\left(n + \frac{1}{2}\right) \pm \tfrac{1}{2}\sqrt{4g^2(n+1) + (\omega_q - \omega)^2}$$

$$E_{+,0} = \frac{\omega_q}{2}$$



Symmetry and splitting of the Hilbert space

$$P(\pi)=\exp\left(i\pi\left(\frac{1}{2}\sigma_z+a^\dagger a\right)\right)=\exp\left(\frac{i\pi}{2}\sigma_z\right)R,$$

$$R=(-1)^{a^\dagger a}=\exp(i\pi a^\dagger a)=\cos(\pi a^\dagger a)$$

$$S=\frac{1}{2}\left((1-R)\sigma_z-i(1+R)\sigma_y\right),\quad S^{-1}=\frac{1}{2}\left((1-R)\sigma_z+i(1+R)\sigma_y\right).$$

$$S^{-1}\left(a+a^\dagger\right)\sigma_x S=\left(a+a^\dagger\right)I,$$

$$S^{-1}\left(a-a^\dagger\right)\sigma_y S=iR\left(a-a^\dagger\right)\sigma_z,$$

$$S^{-1}\sigma_z S=-R\sigma_z,$$

$$Ra+aR=0,$$

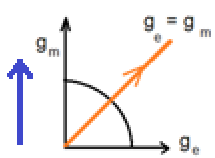
$$\bar{H}_{AR}=S^{-1}H_{AR}S=\omega a^\dagger a+g_e\left(a^\dagger+a\right)I-\sigma_z\left\{\frac{1}{2}\omega_{\mathrm{q}}+g_m\left(a^\dagger-a\right)\right\}R$$

$$\sigma=\pm 1$$

$$\bar{H}_{AR}(\sigma)\equiv\bar{H}_\sigma=\omega a^\dagger a+g_e\left(a^\dagger+a\right)-\sigma\left(\frac{1}{2}\omega_{\mathrm{q}}+g_m\left(a^\dagger-a\right)\right)R$$

$$\tilde{H}_{AR}=S^{-1}\Omega^{-1}H_{AR}\Omega S=\omega a^\dagger a-ig_m\left(a-a^\dagger\right)I-\sigma_z\left\{\frac{1}{2}\omega_{\mathrm{q}}+ig_e\left(a^\dagger+a\right)\right\}R,$$

$$\bar{C}_e=S^{-1}CS=\omega\left(-\frac{1}{2}R\sigma_z+a^\dagger a\right)\quad ,$$



$$\langle m|\bar{H}_{AR}|n\rangle = \sigma\left(-\frac{1}{2}\omega_{\mathrm{q}}(-1)^n + \omega n\right)\delta_{m,n} + g_e\left(\sqrt{n}\delta_{m,n-1} + \sqrt{n+1}\delta_{m,n+1}\right) + \\ +\sigma g_m\left(\sqrt{n}\delta_{m,n-1} - \sqrt{n+1}\delta_{m,n+1}\right)(-1)^n$$

$$\langle m|\bar{C}_e|n\rangle = \omega\left(-\frac{1}{2}(-1)^n + n\right)\delta_{m,n}$$

$$l(n) = \left(\sum_j |\psi_n(j)|^4\right)^{-1}$$

$$P(s) \sim \exp(-\mu s)$$

$$s = (E_{j+1} - E_j)/\langle E_l\rangle$$

The Poisson distribution

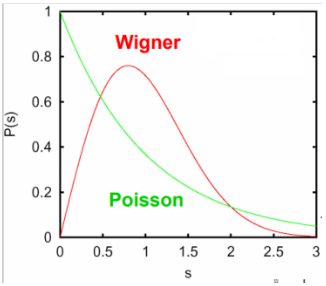
$$P(s)$$

$$P(s) \sim s\exp(-\mu s)$$

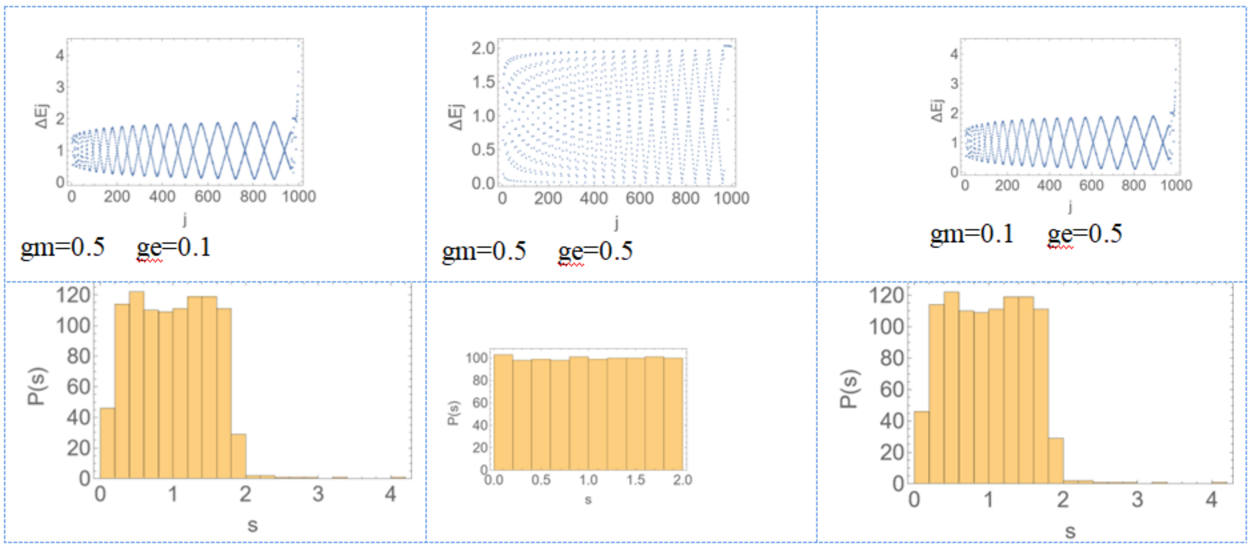
$$H_{m,n} = E_n\delta_{m,n} + V_{m,n}$$

The Wigner distribution

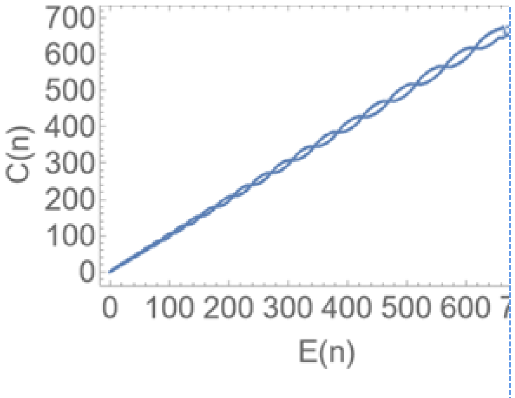
$$\Delta E_n = H_{n+1,n+1} - H_{n,n} = \sigma\omega_{\mathrm{q}}(-1)^n + \omega$$

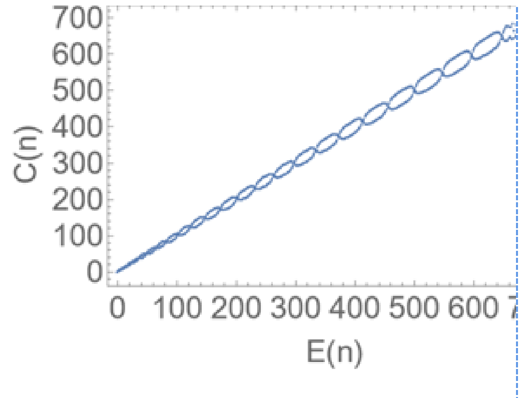


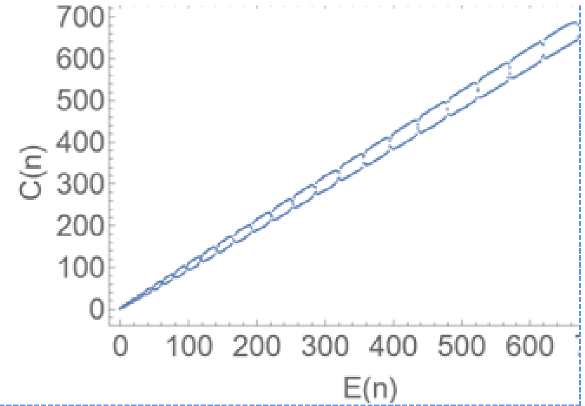
N=1000

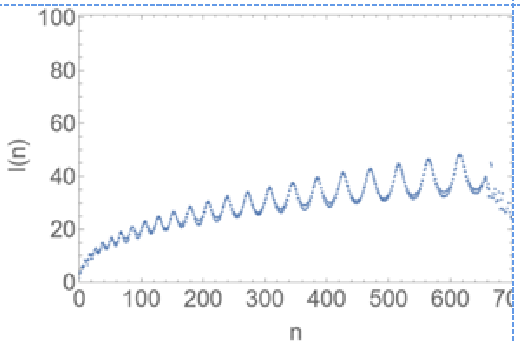


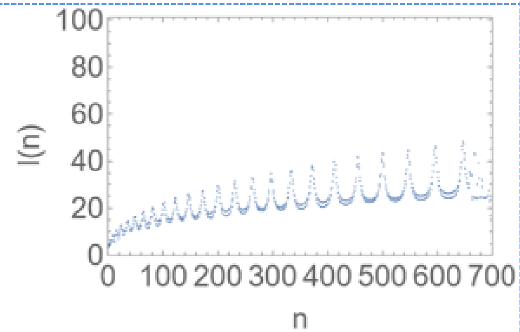
Quantum web

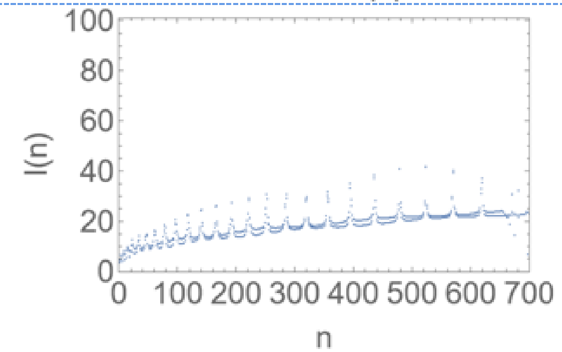




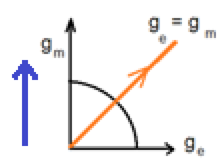


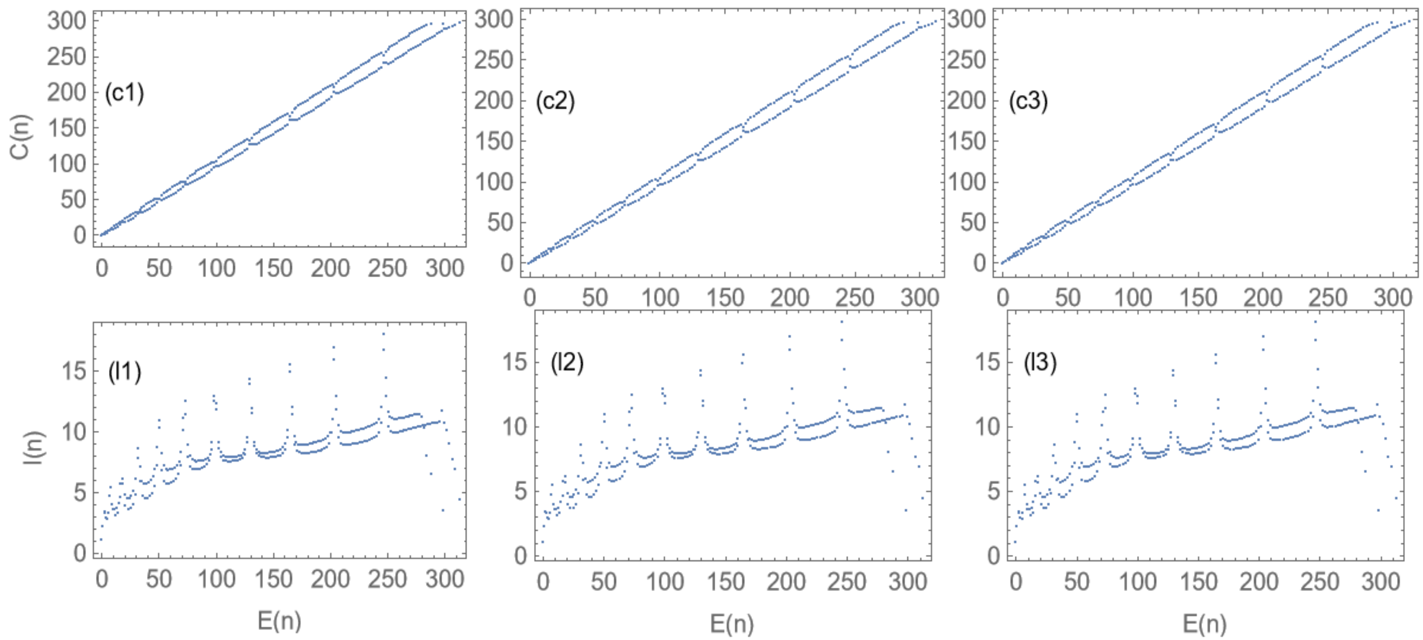






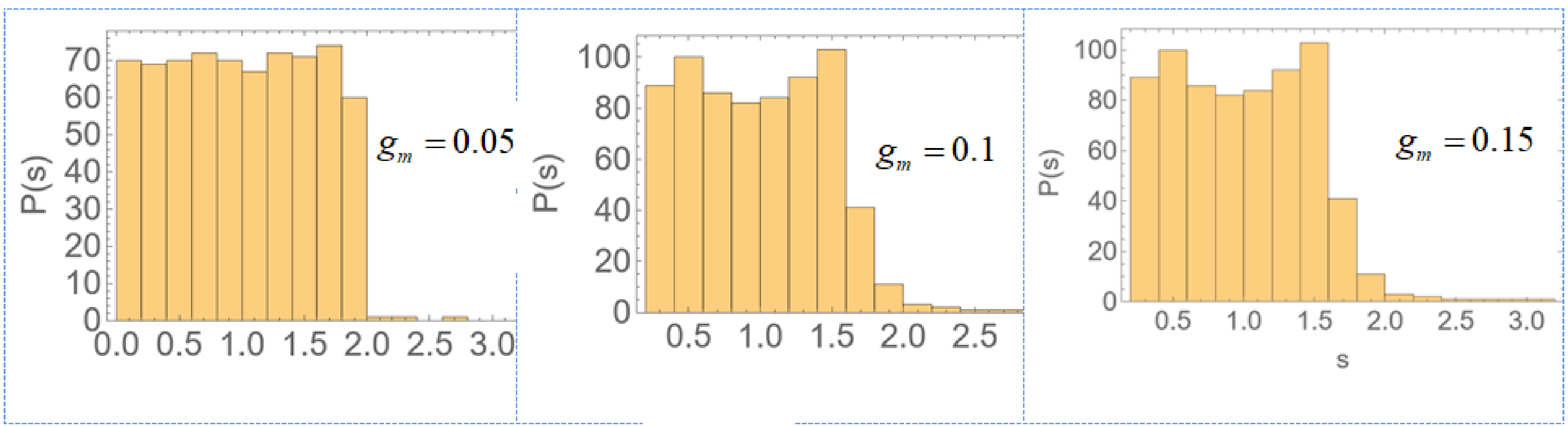
`ge:=0.75;` `gm:=0.05;`	`ge:=0.75;` `gm:=0.1;`	`ge:=0.75;` `gm:=0.15;`

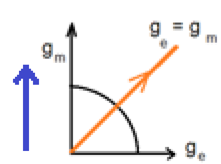




`ge:=0.75 gm:=0.1;` `ge:=1.0; gm:=0.1;` `ge:=2; gm:=0.1;`

$g_e = 0.75$





$$D(\alpha)=\exp\left(-\alpha\left(a^{\dagger}-a\right)\right), D^{-1}(\alpha)aD(\alpha)=a+\alpha, \qquad \alpha=-\frac{g_e}{\omega},$$

$$H_D^{\varepsilon}\equiv D^{-1}(\alpha)\bar{H}_{\sigma}D(\alpha)=\omega a^{\dagger}a-\frac{g_e^2}{\omega} \text{ -}\sigma\left(\frac{1}{2}\omega_{\mathrm{q}}+g_m\left(a^{\dagger}-a\right)\right)R^{\varepsilon}(\alpha),$$

$$R^{\varepsilon}(\alpha)=D^{-1}(\alpha)RD(\alpha)=\cos\left(\pi\left(a^{\dagger}+\alpha\right)\left(a+\alpha\right)\right)$$

$$\langle m|H_D^{\varepsilon}|n\rangle=\left(\omega n-\frac{g_e^2}{\omega}\right)\delta_{m,n} \text{ -}\sigma\left(\frac{1}{2}\omega_{\mathrm{q}}R^{\varepsilon}{}_{m,n}(\alpha)+g_m\left(\sqrt{m}R^{\varepsilon}{}_{m-1,n}(\alpha)-\sqrt{m+1}R^{\varepsilon}{}_{m+1,n}(\alpha)\right)\right)$$

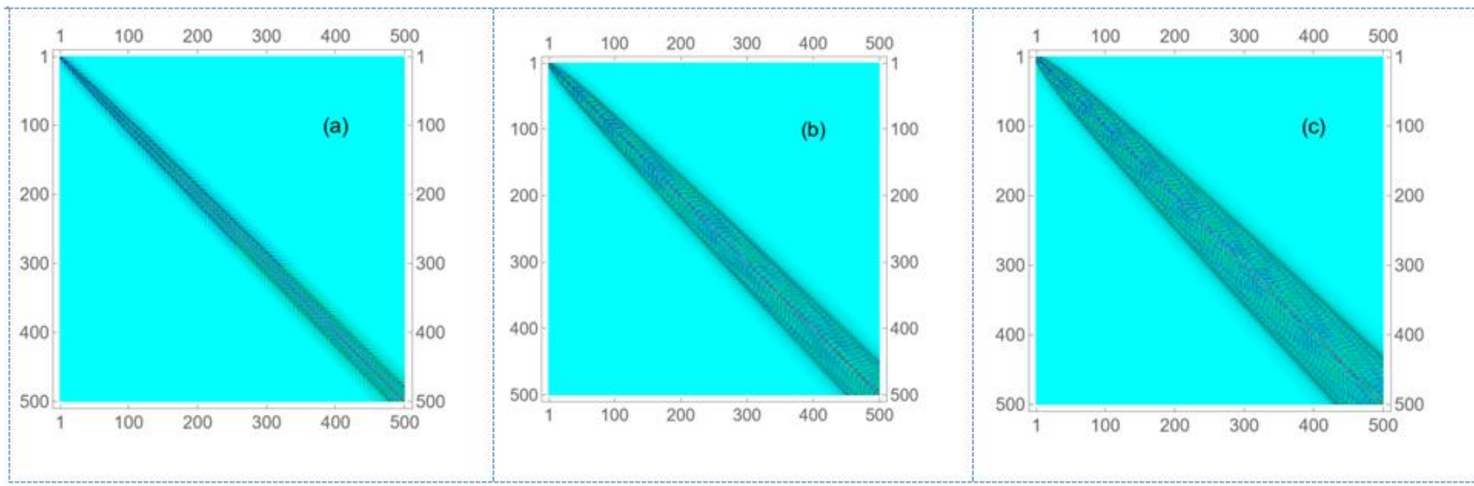
$$R^{\varepsilon}{}_{m,n}(\alpha)=\sum_{l=0}^{\infty}D_{m,l}(-\alpha)(-1)^{l}D_{l,n}(\alpha), \qquad D_{m,n}(\alpha)=\begin{cases}\sqrt{\frac{n!}{m!}}e^{-|\alpha|^2/2}\alpha^{m-n}L_n^{m-n}(|\alpha|^2), m>n,\\ \sqrt{\frac{m!}{n!}}e^{-|\alpha|^2/2}\alpha^{n-m}L_n^{n-m}(|\alpha|^2), n>m,\end{cases}$$

$$\Delta H_{n,n}=-\sigma\frac{\omega_{\mathrm{q}}}{2}(R^{\varepsilon}{}_{n+1,n+1}(\alpha)-R^{\varepsilon}{}_{n,n}(\alpha))+\omega$$

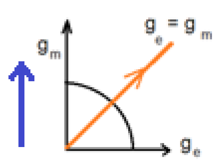
:

$$H_{em} = D^{-1}(\alpha_m)D^{-1}(\alpha_e)\bar{H}_{AR}D^{-1}(\alpha_e)D^{-1}(\alpha_m) = D^{-1}(\alpha_e+\alpha_m)H_{AR}D^{-1}(\alpha_e+\alpha_m)$$

$$g_e = 0.25$$

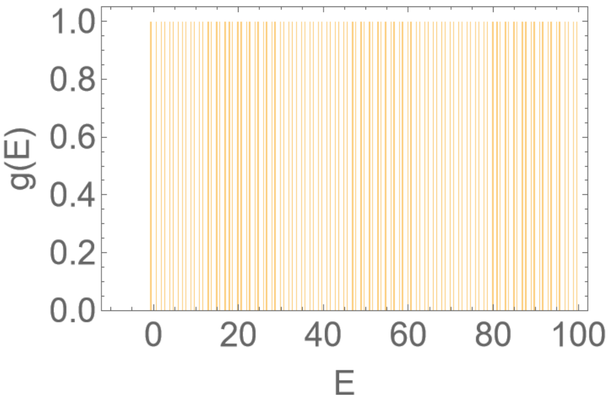


$g_m = 0$ $g_m = 0.5$ $g_m = 0.75$

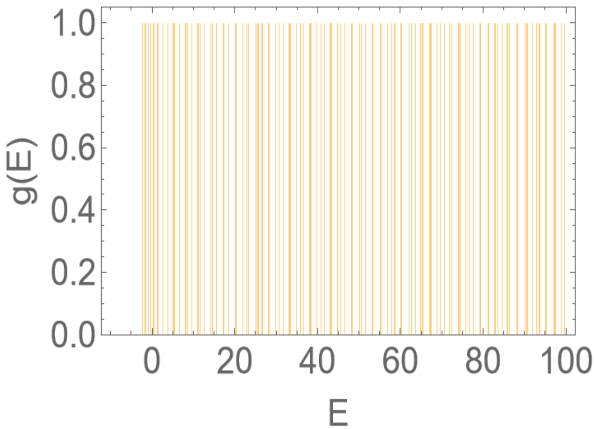


Old fence

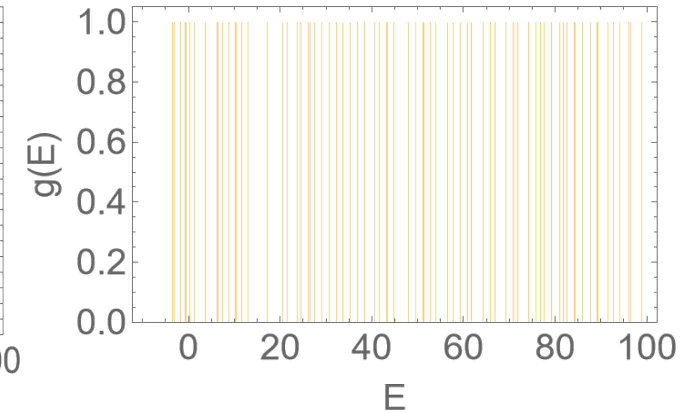
latches



ge:=1.5;
gm:=0;



ge:=1.5;
gm:=1;



ge:=1.5;gm:=2;

Statistical properties of structured random matrices

The best-known examples of structured matrices are Toeplitz matrices in 1911
O. Toeplitz, Zur Theorie der Quadratischen und Bilinearen Formen von Wendlichvielen Veranderlichen,

Teil: Theorie der L-Formen, Math. Ann. 70, 351 (1911).
Toeplitz (Tmn), Tmn = tm−n H. Hankel, in Über eine Besondere Classe der Symmetrischen Determinanten, edited by W. F. Kaestner (Göttingen, 1861). [11]
Hankel matrices were introduced in 1861 [10] and [11].
Hankel (Hmn), Hmn = hm+n
Toeplitz-plus-Hankel [(T + H)mn] (T + H)mn = tm−n + hm+n matrices,
whose matrix elements have the following form, , , (1) with m, n = 1,...,N and ti, hj arbitrary real or complex numbers.

Matrices are omnipresent structures in extremely varied branches of physics and mathematics, and there exist many different types of matrices tailored for specific problems. E.Bogomolny

Summary

- The generalized scheme of the qubit-resonator system with the electro-dipole and the magneto-dipole coupling is proposed

- As an example, an anisotropic one-qubit system + photon is considered, which is characterized by weak chaos

- Results show that the qubit-resonator system with an anisotropic coupling can be used for quantum control devices