Quantum Information Scrambling and Entanglement: A Mathematical Connection

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$$|\psi\rangle\langle\psi|$$

Outlines

1. What is Quantum Information Scrambling .?
2. Wooter’s Concurrence.
3. A Mathematical Connection .

Quantum information scrambling and entanglement in bipartite quantum states

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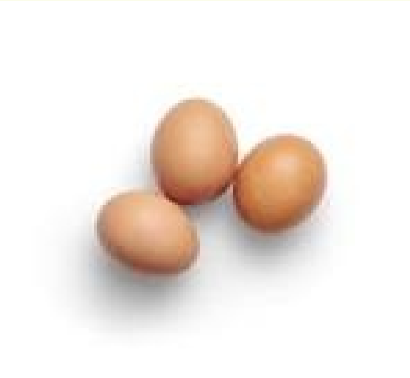
Abstract
Investigating the influence of quantum information (QI) scrambling on quantum correlations in a physical system is an interesting problem. In this article, we establish the mathematical connections among the quantifiers known as quantum information scrambling, Uhlmann fidelity, Bures metric and bipartite concurrence. We study these connections via four-point out-of-time-order correlation function used for quantum information scrambling. Further, we study the dynamics of all the quantifiers and investigate the influence of QI scrambling on entanglement in two qubits prepared in Bell states. We also investigate the QI scrambling and entanglement balancing points in Bell states under Ising Hamiltonian.

Keywords Quantum information scrambling · Uhlmann fidelity · Bures metric · Concurrence · OTOC · Balancing points

Scrambling



From omelette, one can not recover eggs (irreversible process)

Quantum Information Scrambling

- A measure of quantumm chaos:
- Quantified through out of order time correlator (OTOC): Dicvovered in 1968 in Fermi Gas.

A. Larkin and Yu. N. Ovchinnikov, Quasiclassical Method in the Theoryof Superconductivity, Sov. Phys. JhETP 28, 6, 1200 (1969)

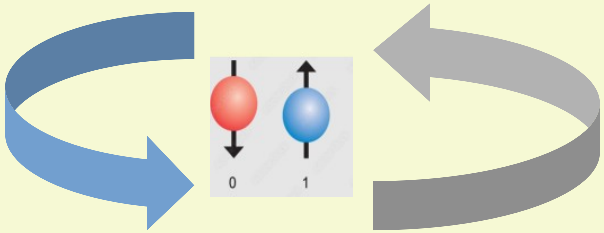
Quantum Information Scrambling (QIS)

$$\langle C(t)\rangle = \langle [W(t),V]^{\dagger}.[W(t),V]\rangle$$

$$f = |\langle Y|X\rangle|^2$$

$|\psi\rangle$ $\quad$ $|Y\rangle$

$W(t).V$



$V.W(t)$

$|X\rangle$ $\quad$ $|\psi\rangle$

Conditions for QIS

$$[W(t),V] \neq 0$$

$$[H,W(0)] \neq 0.$$

$$W(t) = e^{iHt}W(0)e^{-iHt} = W(0) + it[H,W(0)] + \frac{t^2}{2!}[H,[H,W(0)]] + \frac{it^3}{3!}[H,[H,[H,W(0)]]] + \ldots\ldots\ldots$$

Bounded Operators $\Longrightarrow$

$$||H|| \leq \epsilon$$
$$||W(0)|| \leq \epsilon$$

QIS Simplified

$$\langle C(t)\rangle = \langle [W(t), V]^\dagger . [W(t), V]\rangle$$

$$W(0)^\dagger = W(0) \qquad W(t)^\dagger = W(t) \qquad V^\dagger = V$$

$$\mathrm{C(t){=}[W(t),V]^\dagger}.[W(t), V] = 2.I - \{W(t).V.W(t).V\}^\dagger - W(t).V.W(t).V$$

$$\langle C(t)\rangle_\rho = 2\left[1 - \Re\{Z\}\right] \qquad (1),$$

Where $\mathrm{Z{=}Tr[M]}$ $\mathrm{M{=}W(t).V.W(t).V}.\rho$

Uhlmann Fidelity and QIS: Mathematical Connection

Following Eq. 1 and using cyclic property of Trace operation we can rewrite the factor Re(Z)

$$\Re[\langle\psi|W(t).V.W(t).V|\psi\rangle] = \Re[\langle y|x\rangle]$$

Where $\||x\rangle = W(t).V|\psi\rangle$ Forward evolution of pure state $\||\psi\rangle$

$\||y\rangle = V.W(t)|\psi\rangle\,.$ Backward evolution of pure state $\||\psi\rangle$

Uhlmann Fidelity for Pure States

$$f = |\langle y|x\rangle|^2$$

$$\langle C(t)\rangle_\rho = 2\left[1 - \sqrt{f - \left(\Im\{Z\}\right)^2}\right]. \qquad (2)$$

If $\Im\{Z\} = 0$

$$\langle C(t)\rangle_\rho = 2\left[1 - \sqrt{f}\right]. \qquad (3)$$

⇒ $$D = \sqrt{2(1 - \sqrt{f})} \qquad (4)$$

⇒ W. K. Wootters, Statistical distance and Hilbert space, Phys. Rev. D., 23, 357 (1981).

QIS and Bures Metric: Mathematical Connection

From Eq (2) and (4)

$$\langle C(t)\rangle_\rho = 2\left[1 - \sqrt{\left(1 - \frac{D^2}{2}\right)^2 - \left(\Im\{Z\}\right)^2}\right] \qquad (5)$$

If $\Im\{Z\} = 0$

$$\langle C(t)\rangle_\rho = D^2\,. \qquad (6)$$

QIS and Two Qubits Concurrence: Mathematical Connection

Concurrence

$$C_r(|\psi\rangle) = |\langle\psi|\sigma_y \otimes \sigma_y|\psi^\star\rangle| \qquad (7)$$

$$C_r(|\psi\rangle) = \left|Tr[(\sigma_y \otimes \sigma_y).(|\psi^\star\rangle\langle\psi|)]\right|. \qquad (8)$$

If we assume

$(|\psi^\star\rangle = |\psi\rangle)$

$$C_r(\rho) = \left|Tr[(\sigma_y \otimes \sigma_y).\rho]\right| \qquad (9)$$

Hence the concurrence in chaotic matrix M can be calculated as

$$C_r(M) = \left|Tr[(\sigma_y \otimes \sigma_y).M\right| \qquad (10)$$

Since we now $\mathrm{M{=}W(t).V.W(t).V.}\rho$

Hence $C_r(M) = \left|Tr[(\sigma_y \otimes \sigma_y).W(t).V.W(t).V.\rho]\right|. \qquad (11)$

$$\mathrm{M{=}W(t).V.W(t).V.}\rho \qquad (M^\dagger \neq M)$$

In general M is non-Hermitian

Analytical structure of M

$$M = \begin{pmatrix} a & b & c & -a \\ d & e & e & f \\ g & h & h & i \\ j & k & l & -j \end{pmatrix}, \quad M^T = \begin{pmatrix} a & d & g & j \\ b & e & h & k \\ c & e & h & l \\ -a & f & i & -j \end{pmatrix}$$

All the elements of matrix are complex numbers

$$|Tr[W(t).V.W(t).V.\rho]| = |Tr[(\sigma_y \otimes \sigma_y).W(t).V.W(t).V.\rho]|$$

Eq. (11) becomes

$$C_r(M) = |Tr[W(t).V.W(t).V.\rho]| \qquad (12)$$

$$C_r(M) = \sqrt{f} \qquad (13)$$

Adjusting Eq. (2) and (13)

$$C_r(M) = \sqrt{\left[1 - \frac{\langle C(t)\rangle_\rho}{2}\right]^2 + \left[\Im\{Z\}\right]^2}. \qquad (14)$$

If $\Im\{Z\} = 0$

$$C_r(M) = 1 - \left[\frac{\langle C(t)\rangle_\rho}{2}\right]. \qquad (15)$$

The relation is Linear

Properties of QIS

1. Positivity, $\langle C(t) \rangle_\rho \geq 0\,.$
2. Bounded limits, $0 \leq \langle C(t) \rangle_\rho \leq 2.$
3. Unitary invariant, $U C(t)\, U^\dagger = C(t)\,.$

Thanks