Steklov Mathematical Institute

Russian Quantum Center

Quantum evolution through the prism of operator growth

Igor Ermakov, JINR Dubna, 28 of May 2024

Papers and People

Presented results are mostly based on:
arXiv:2401.08187 (Unified framework for efficiently computable quantum circuits)
Authors:

Igor Ermakov
Steklov Institute of Mathematics
RQC

Oleg Lychkovskiy
Skoltech
RQC
Steklov Institute of Mathematics

Tim Byrnes
East China Normal University
NYU Shanghai

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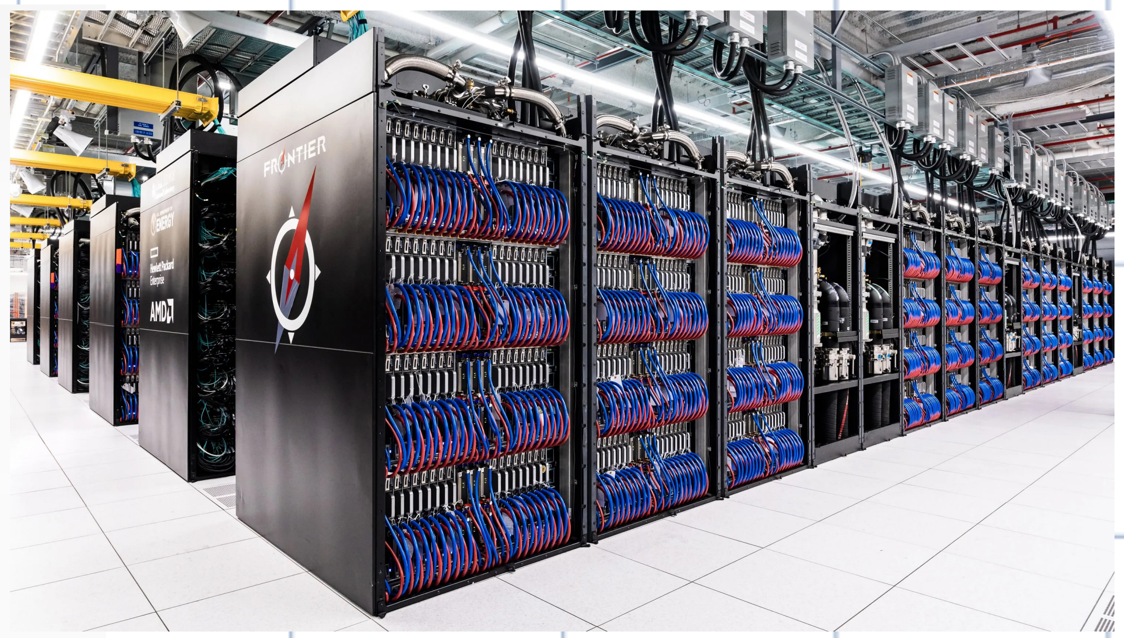

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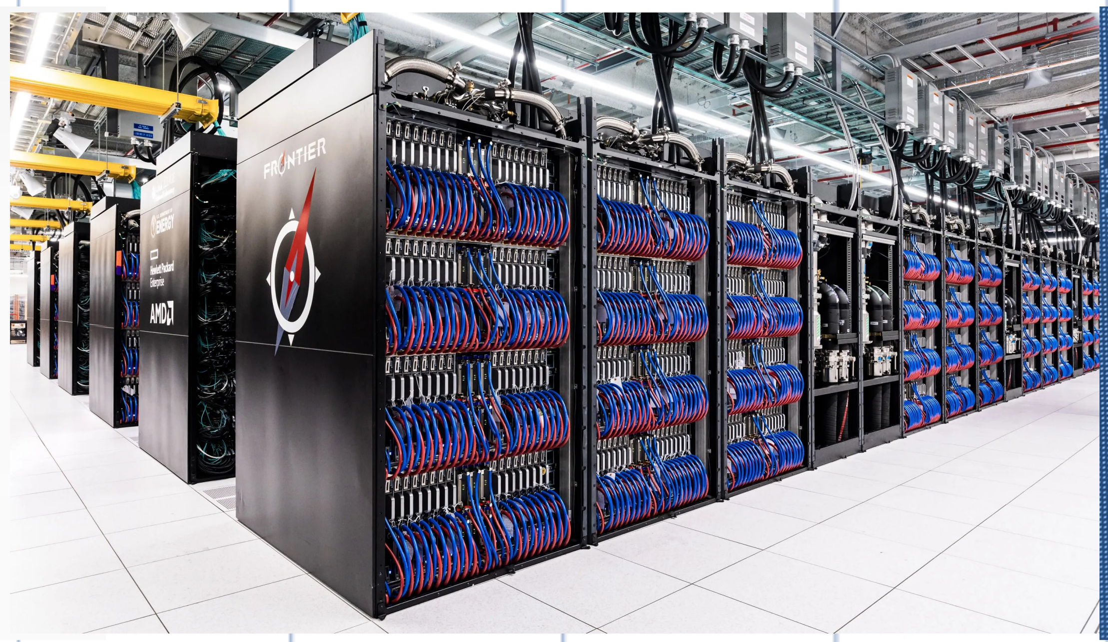
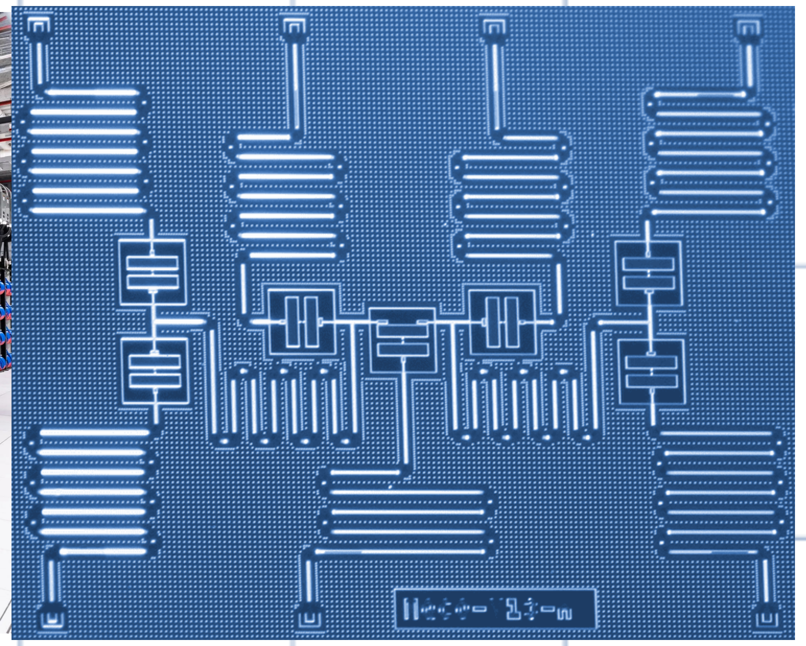

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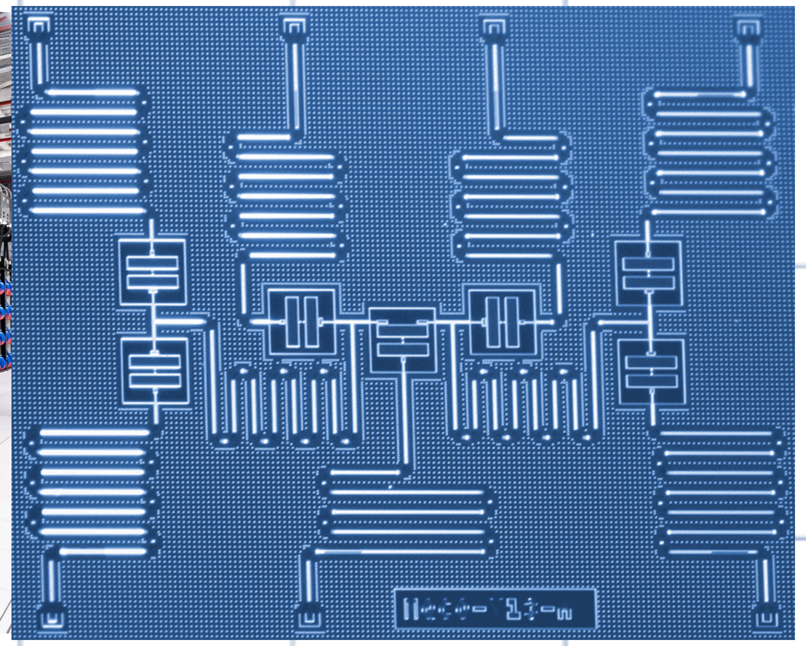

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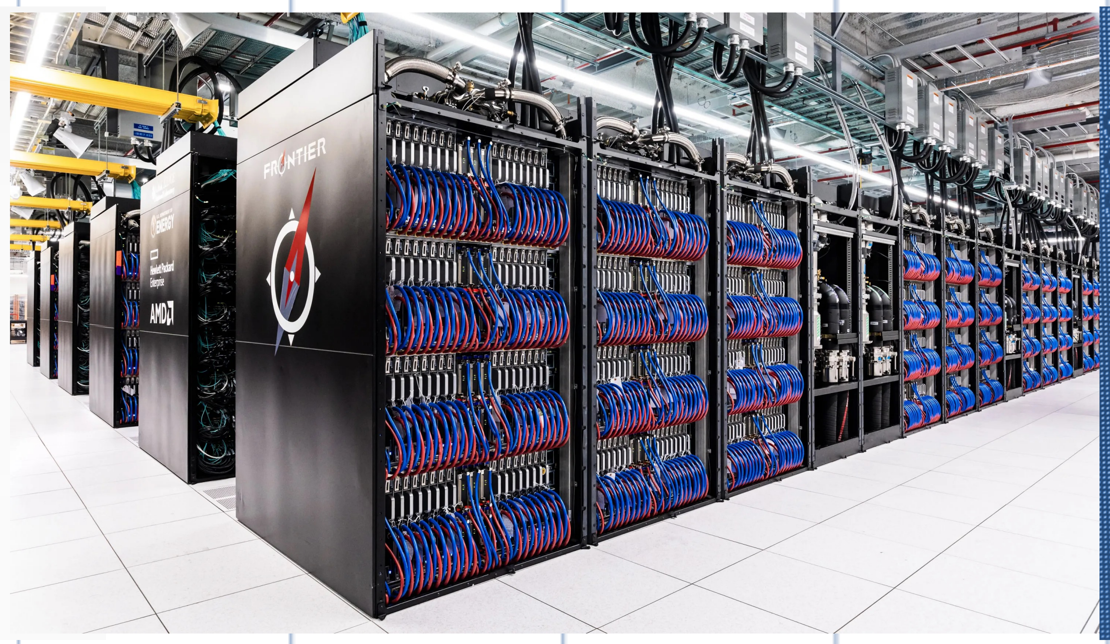
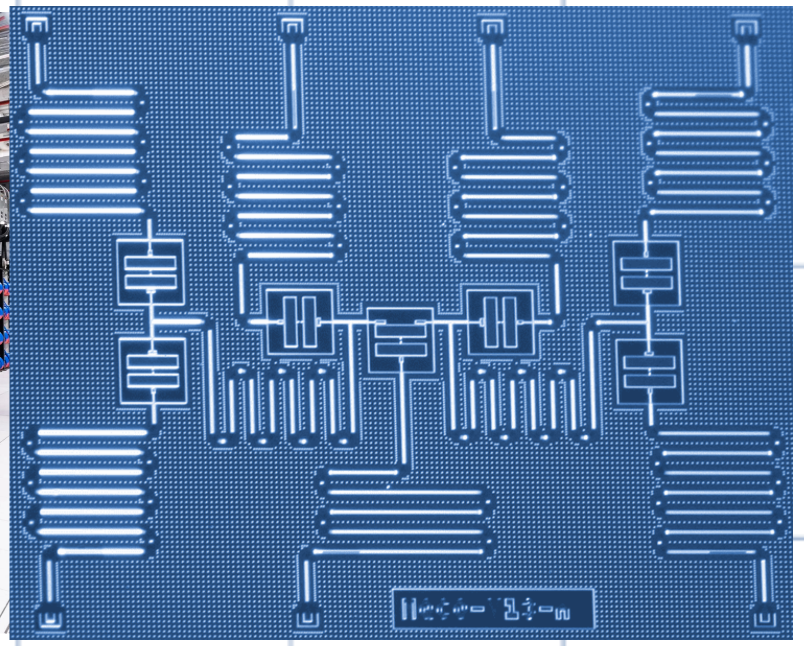

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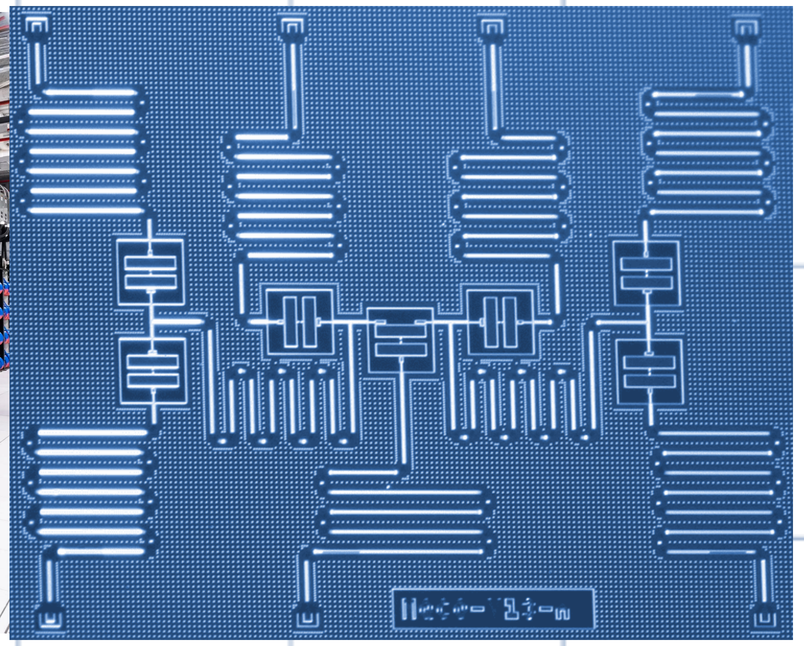

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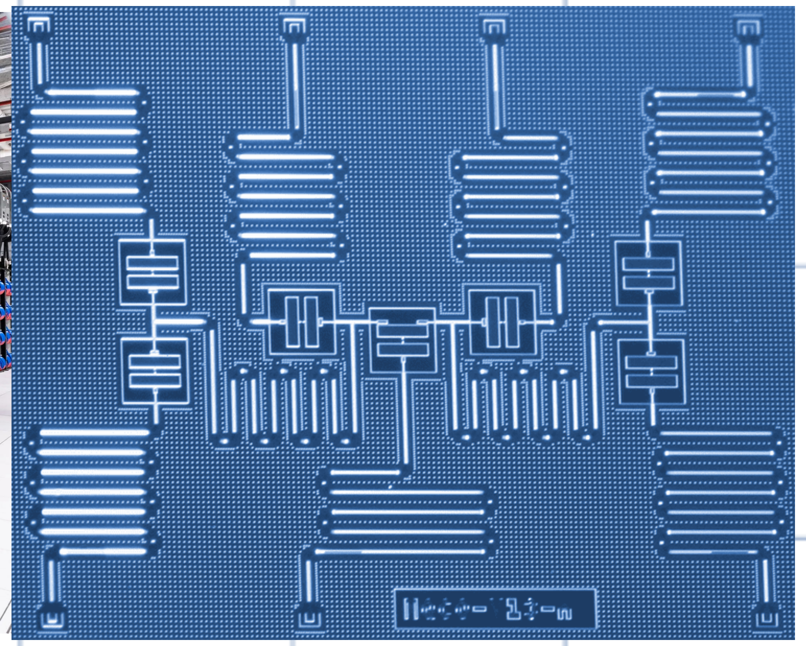

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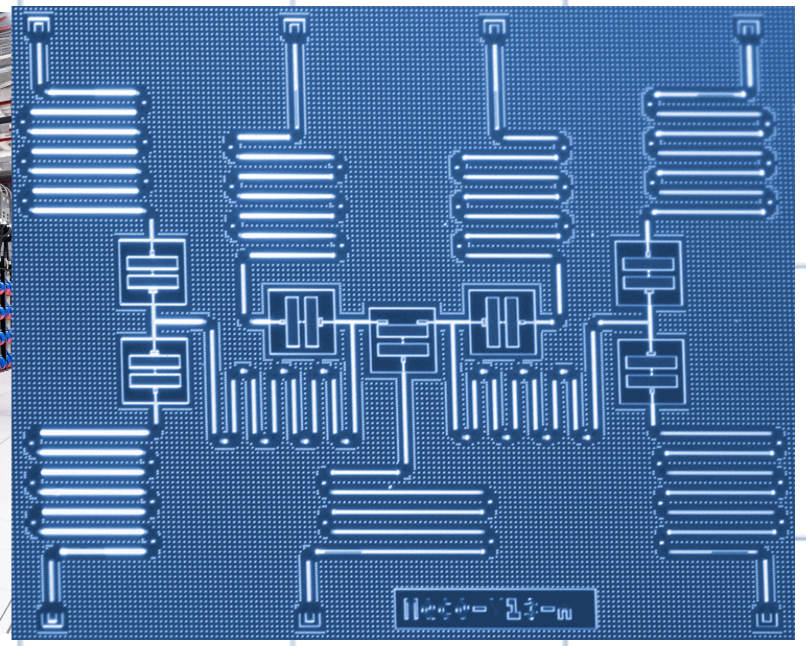

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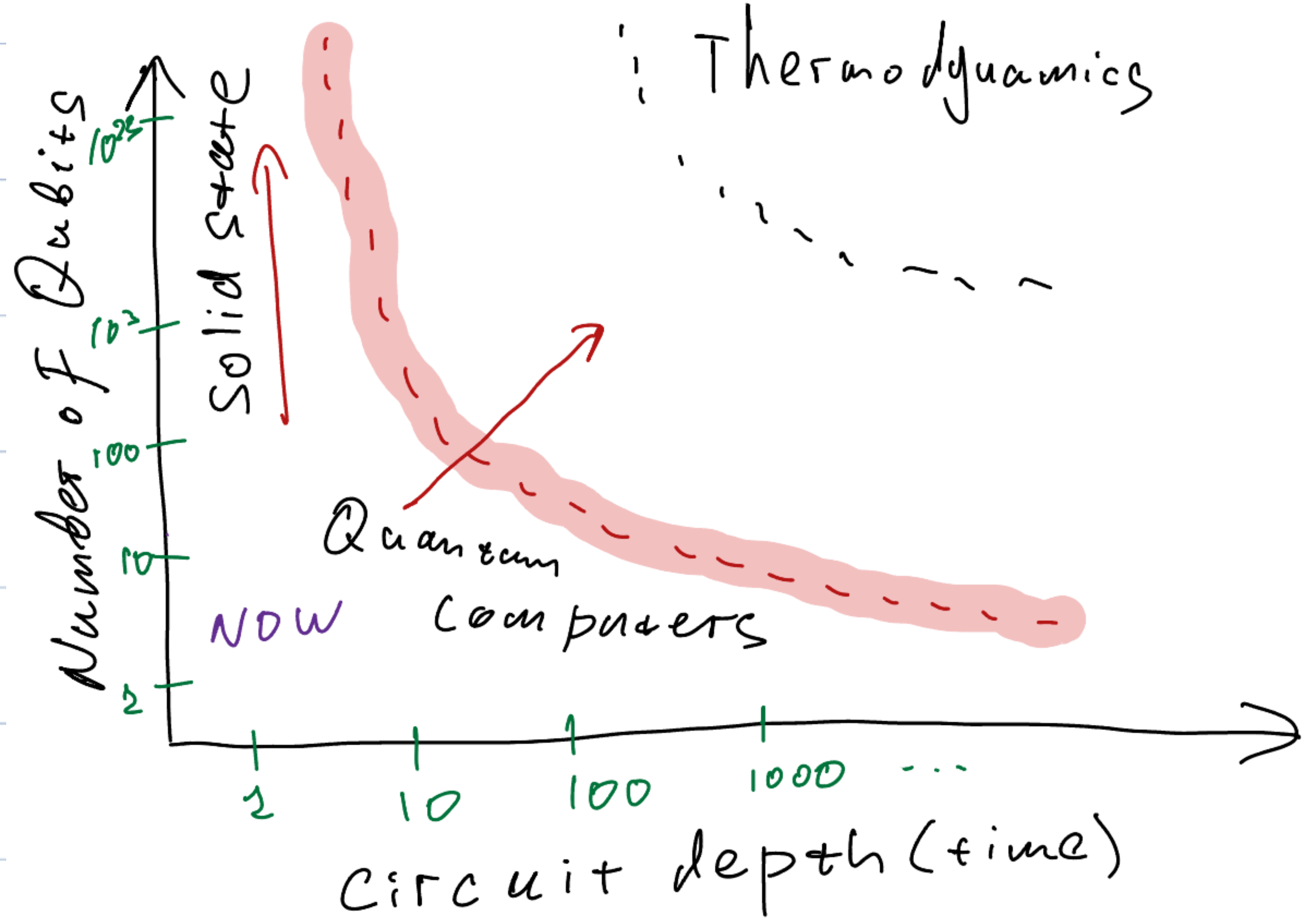

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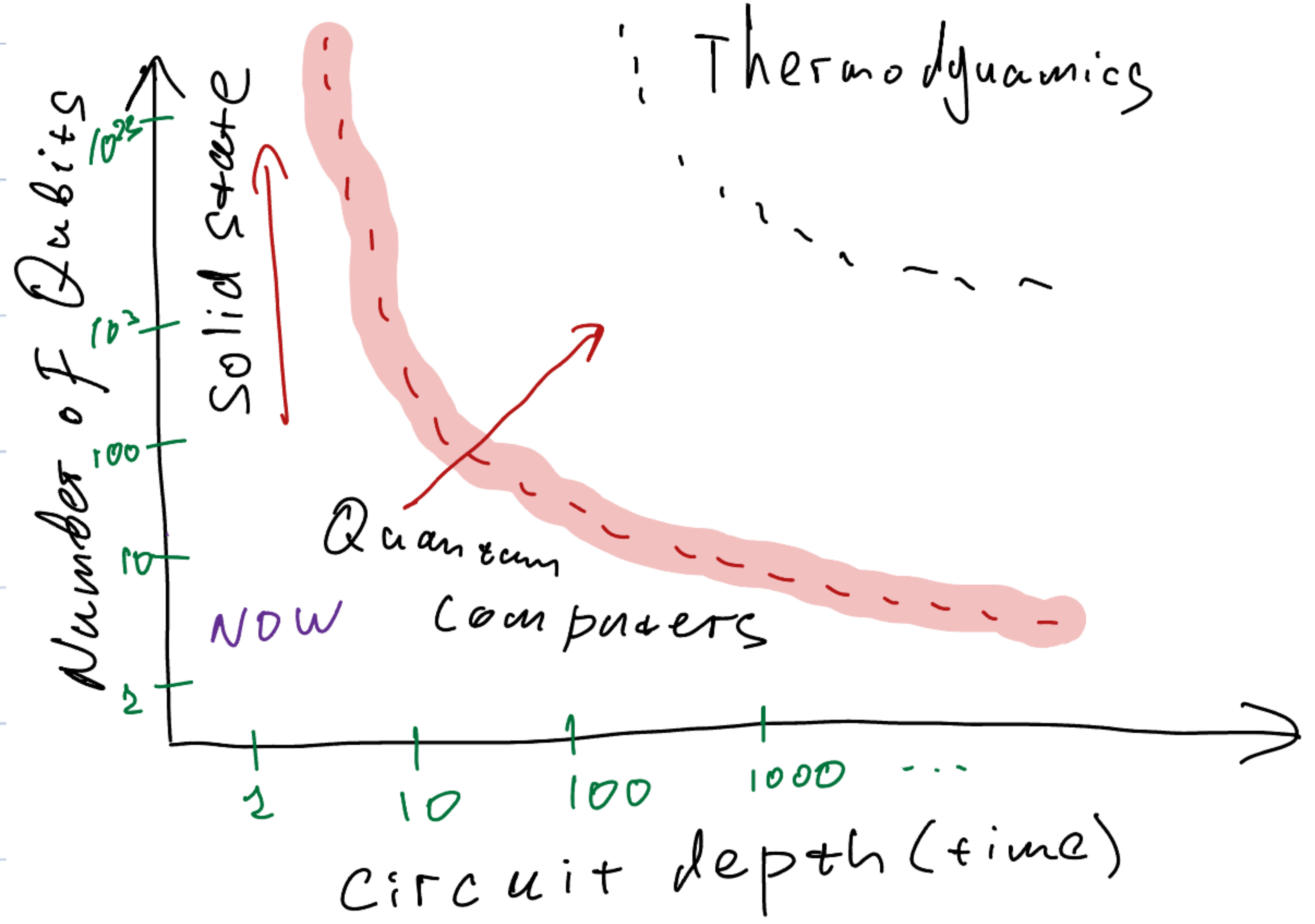

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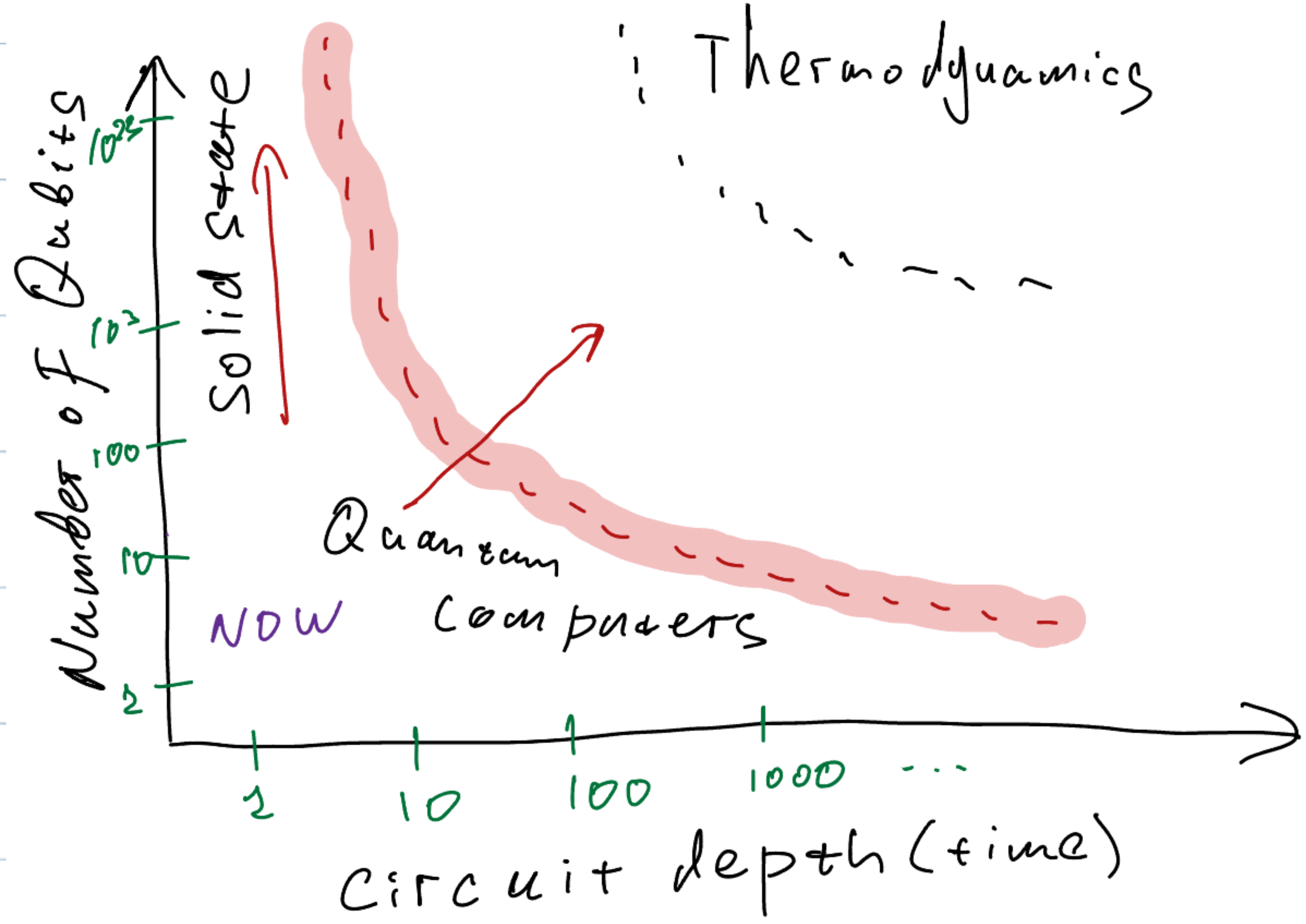
Look for different examples of difficult yet solvable problems to push this boundary
(More easily accessible results, yet not so strict)



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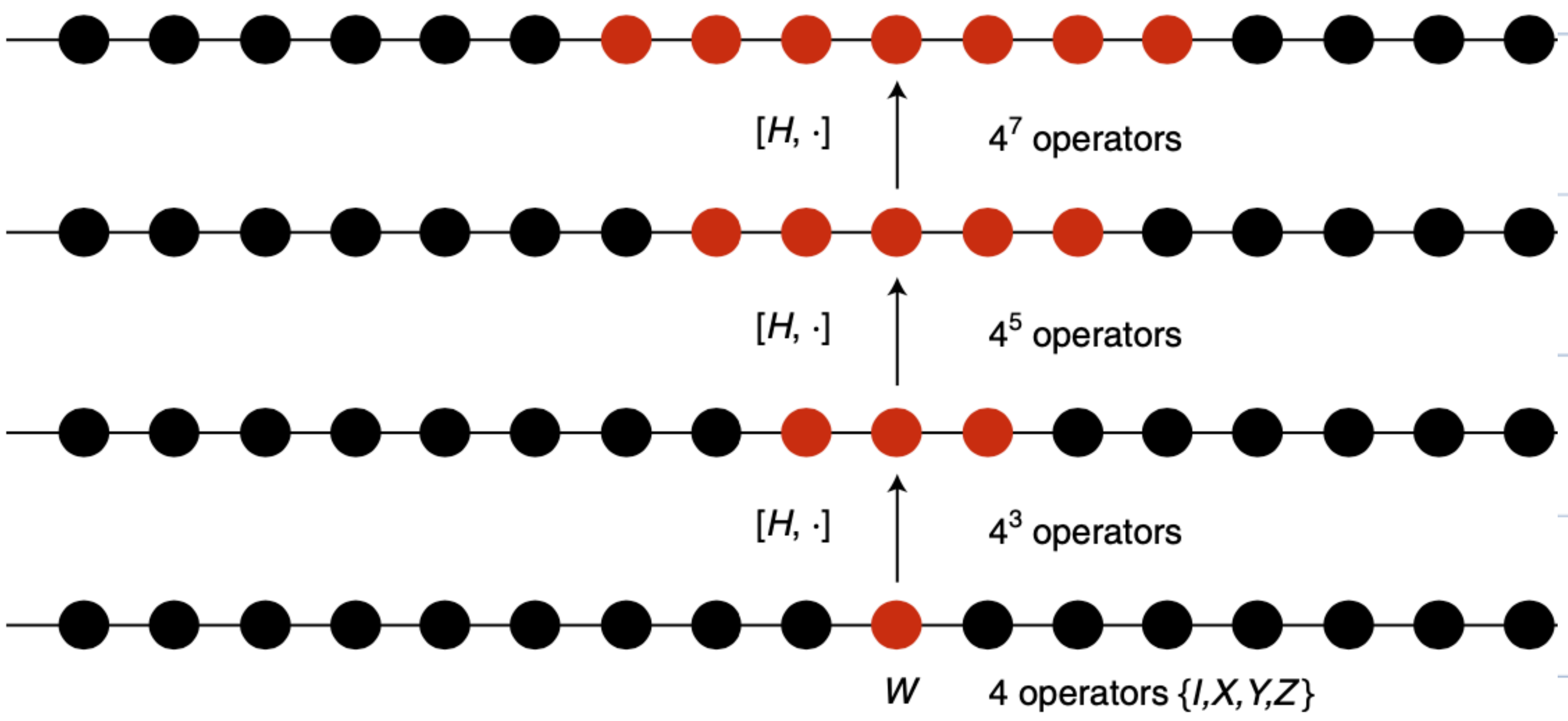

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Swingle, Brian. "Unscrambling the physics of out-of-time-order correlators." *Nature Physics* 14.10 (2018): 988-990.

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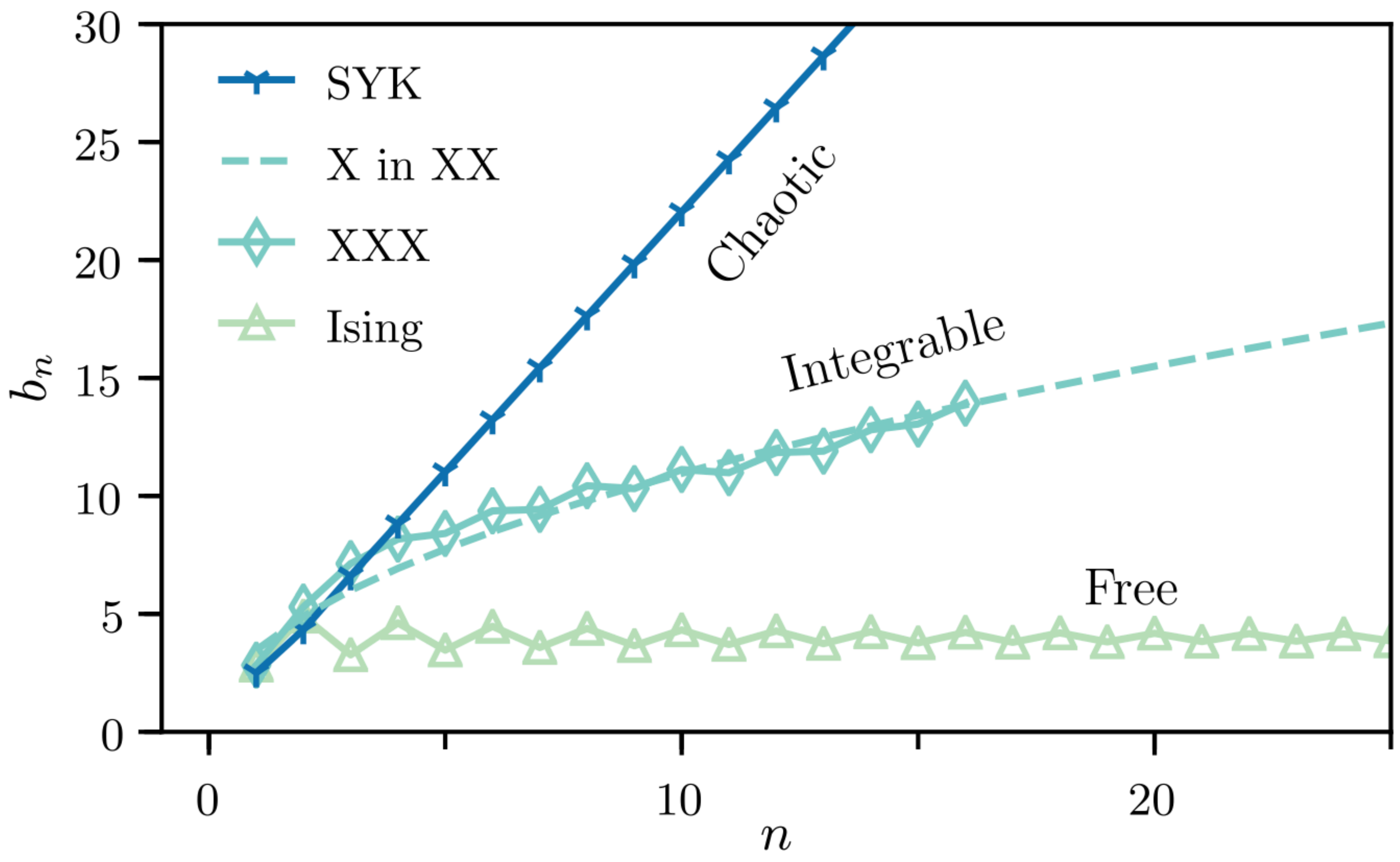

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Pauli strings form orthogonal basis in the 4^L dimensional operator space

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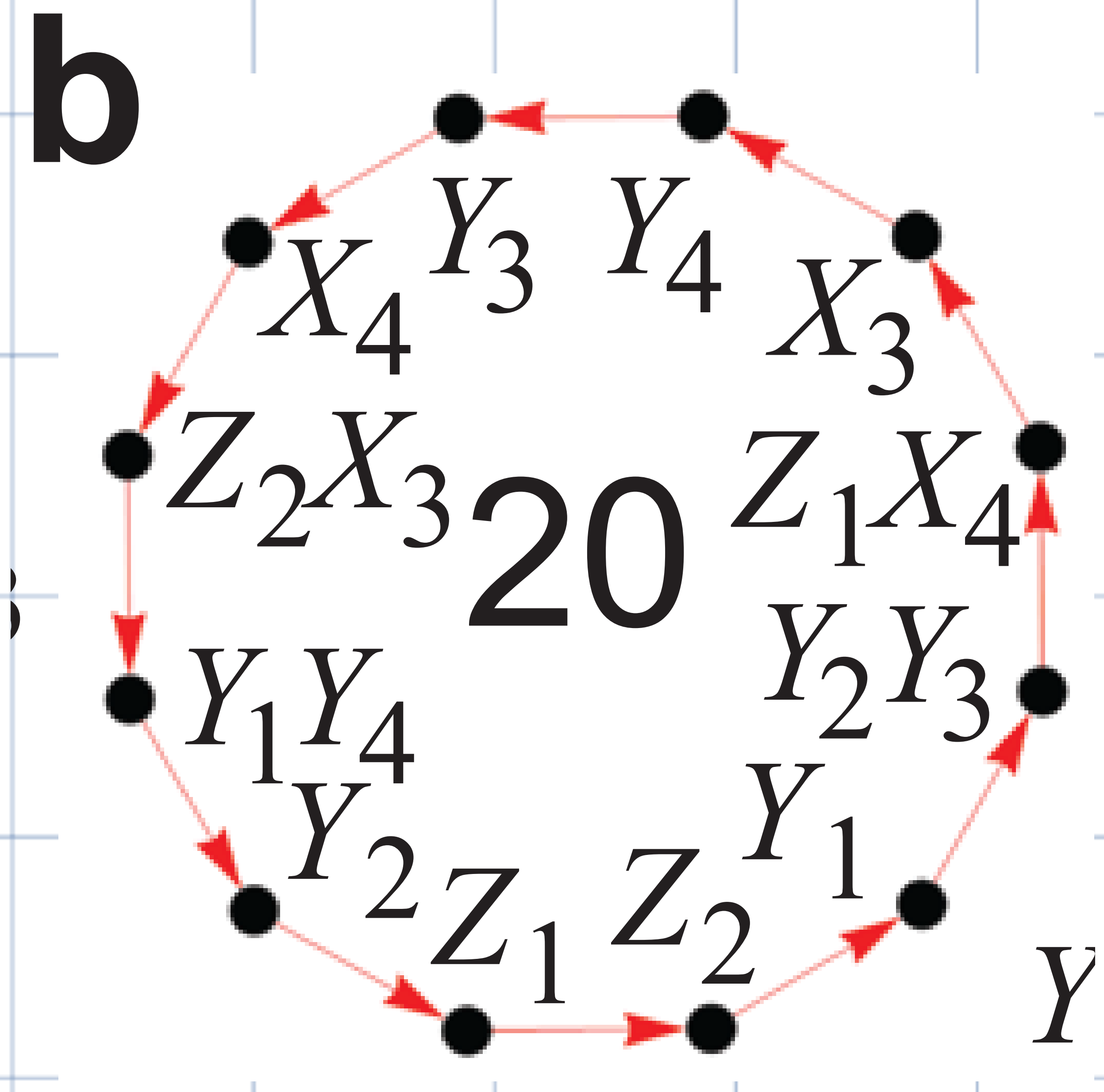

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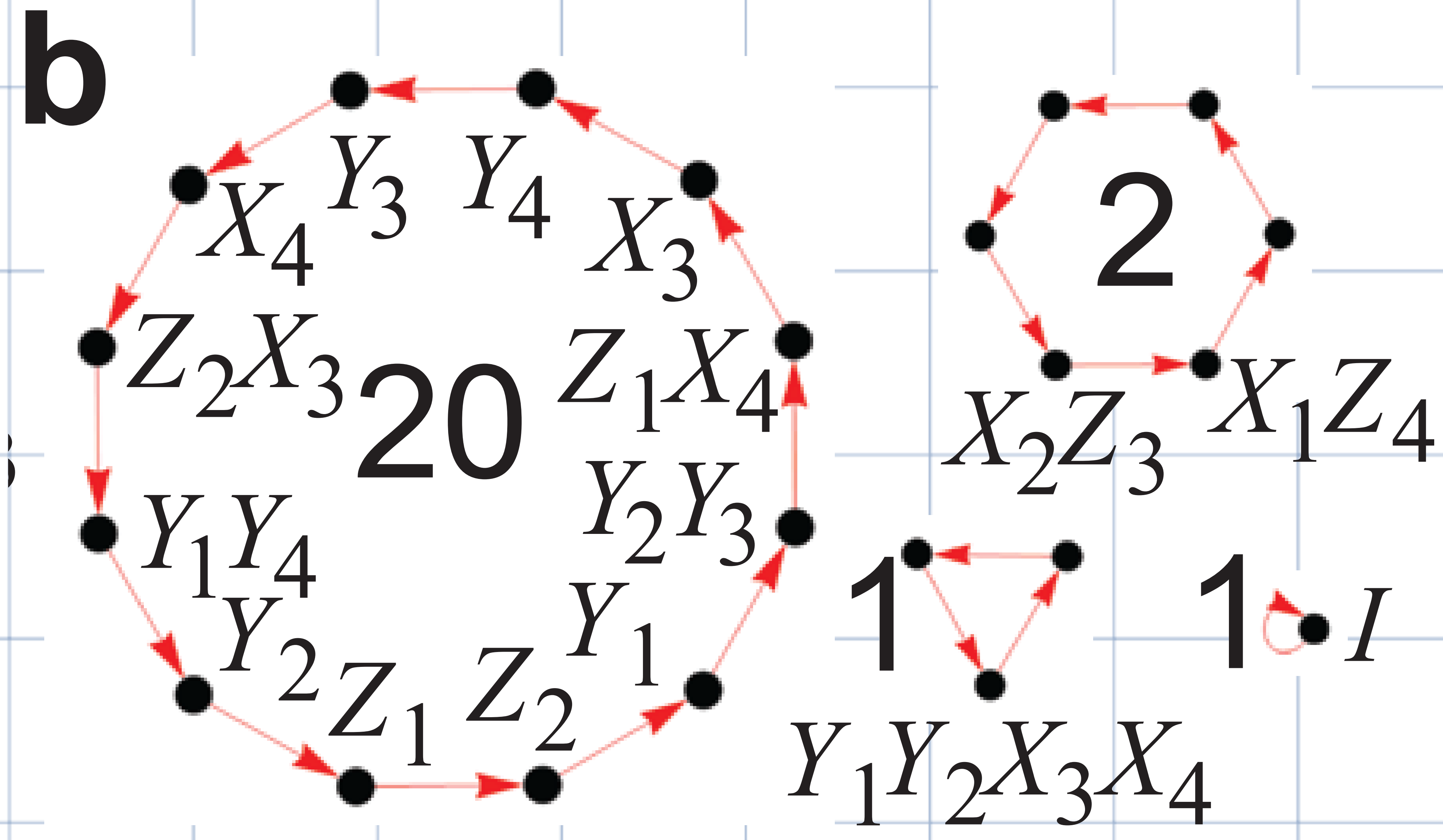

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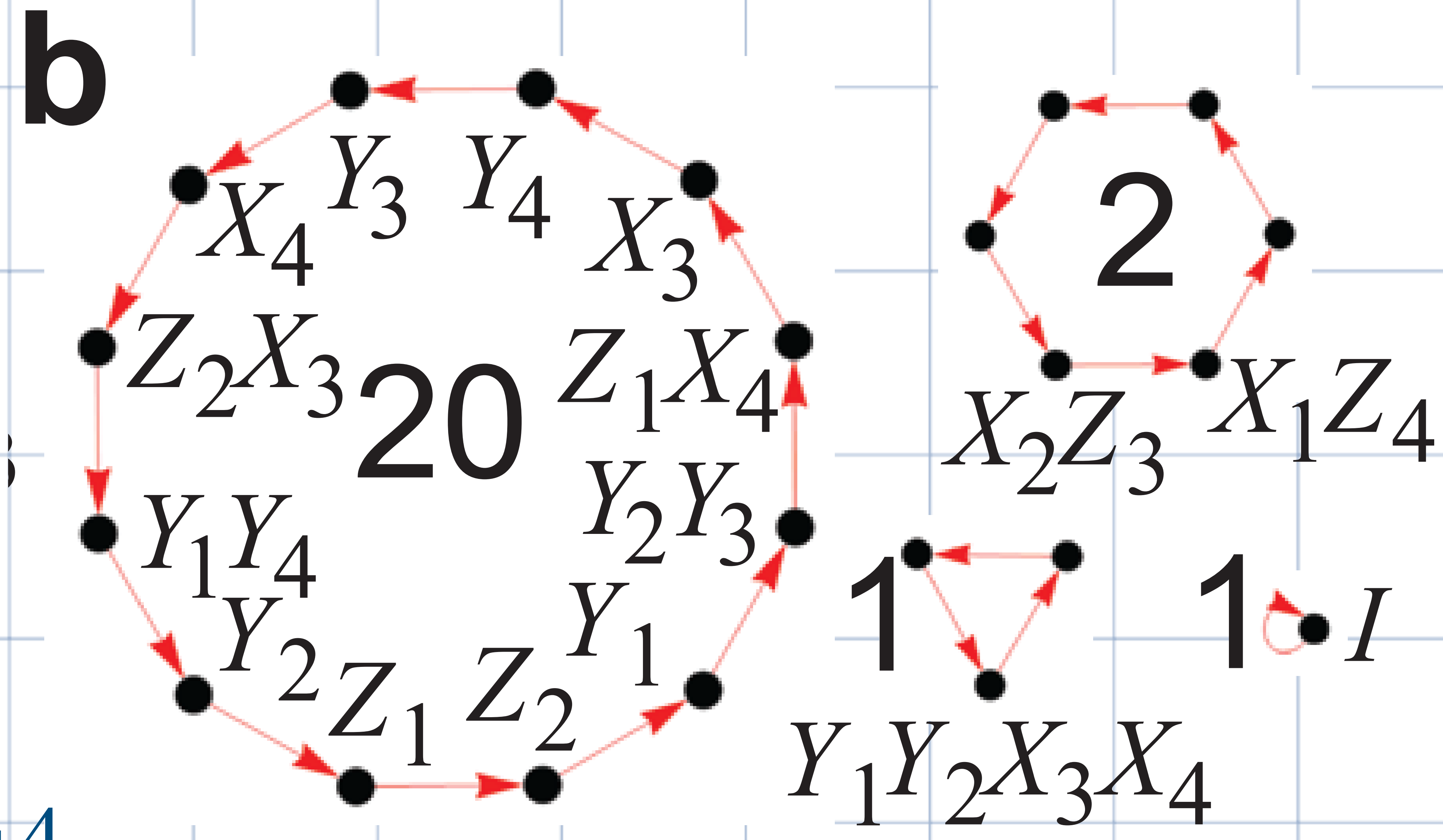

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$$20 \cdot 12 + 2 \cdot 6 + 1 + 1 = 4^4$$

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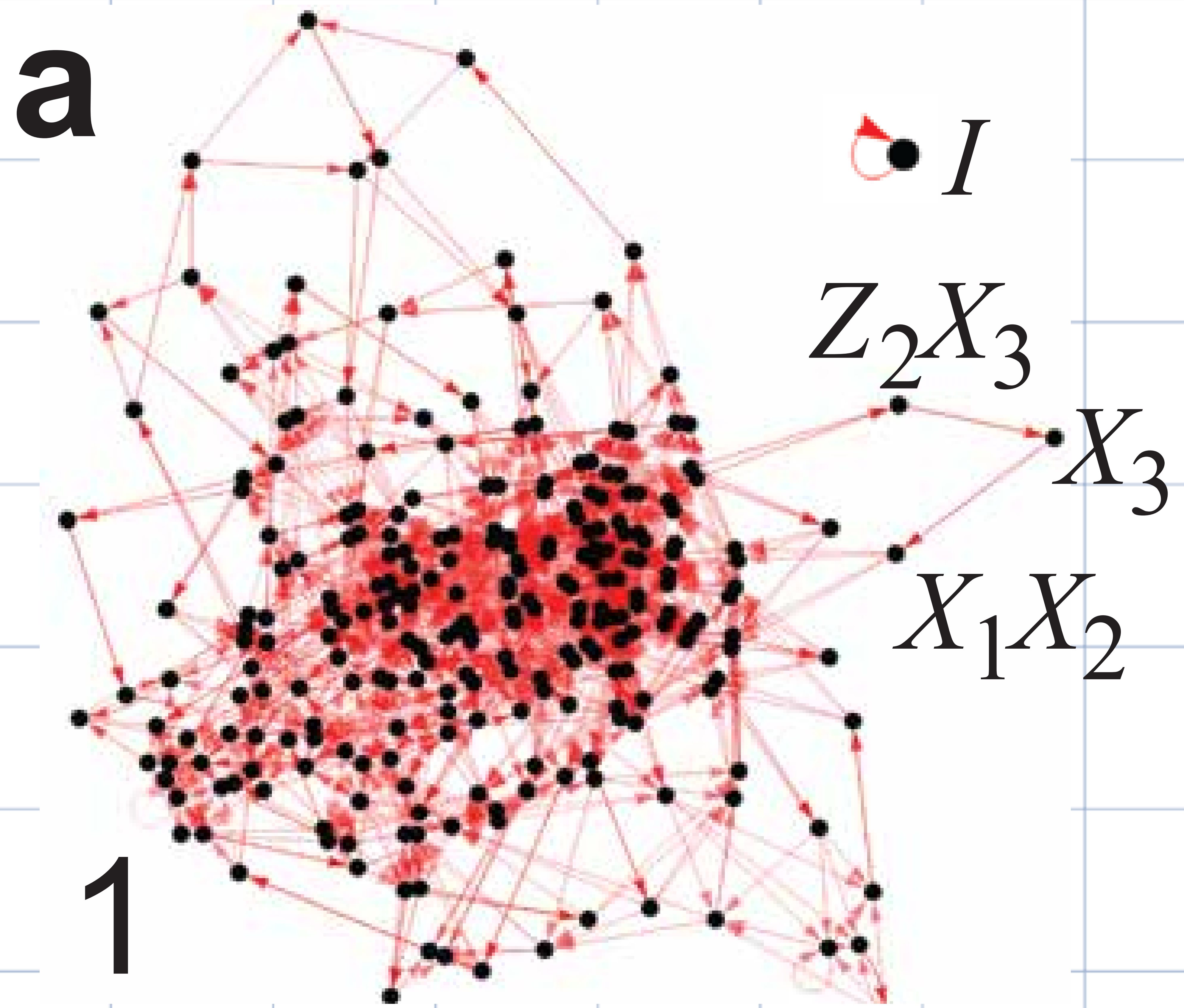

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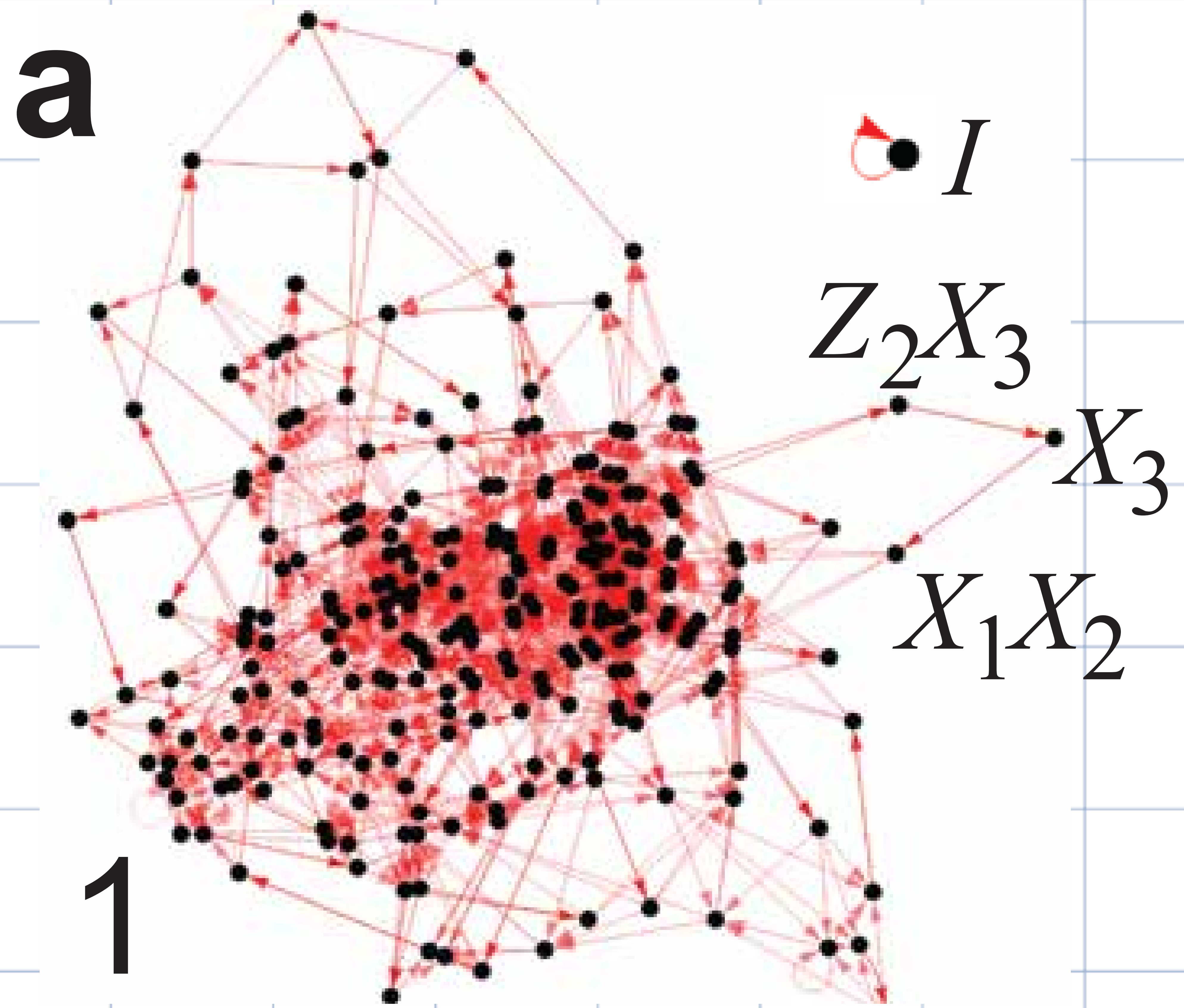

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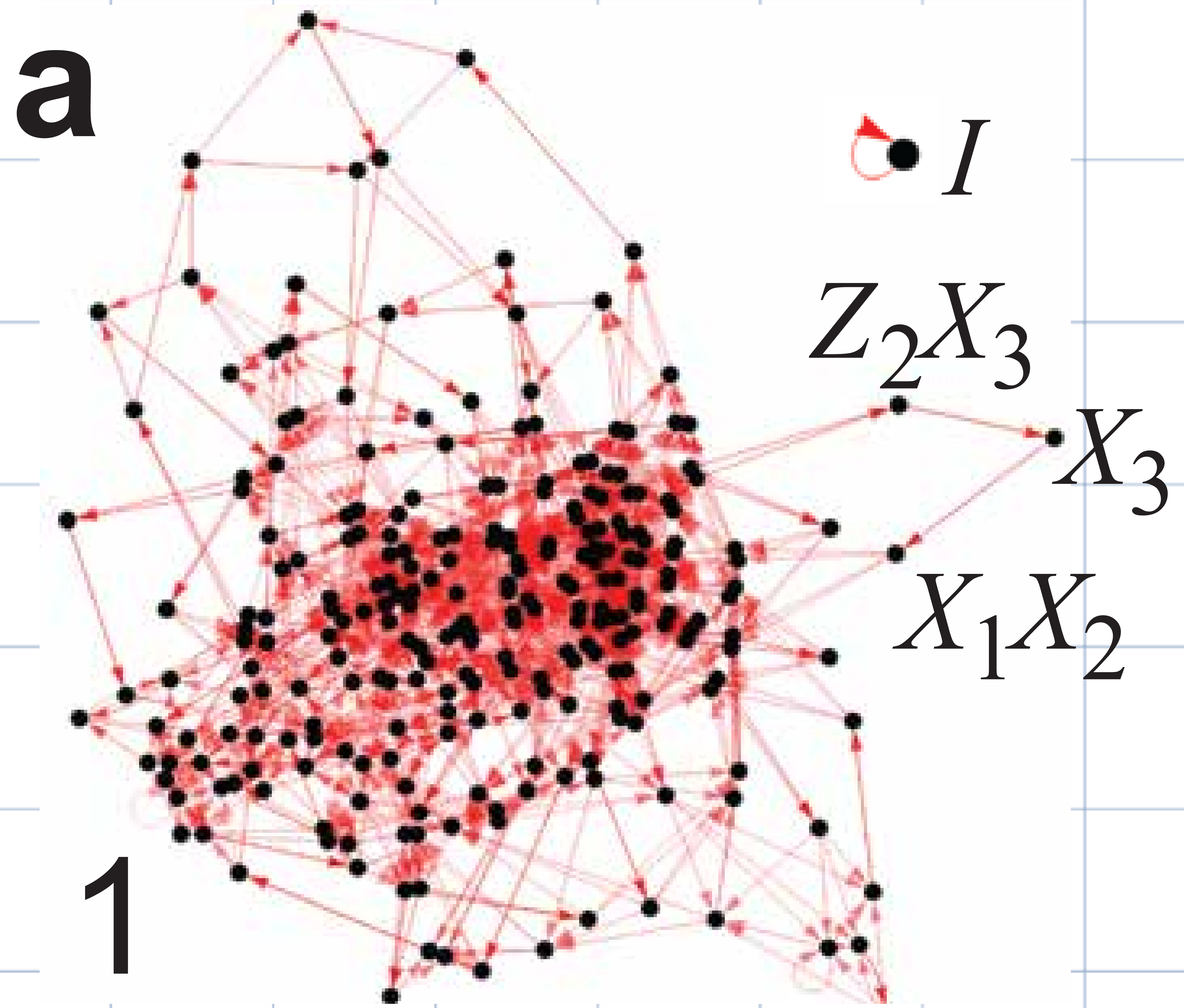

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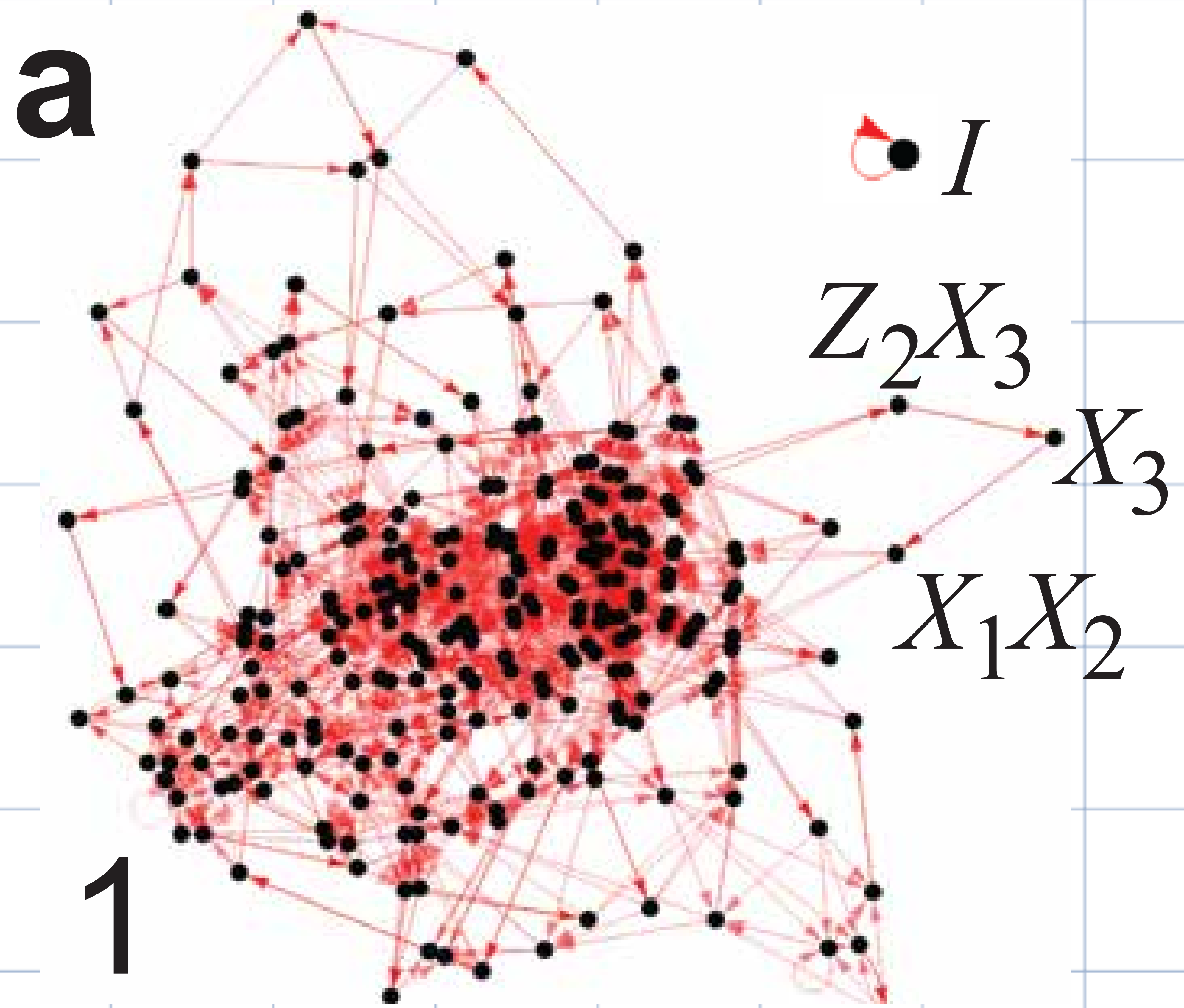
$\{B_n\}_{n=1}^{4^L}$ Basis of Pauli strings



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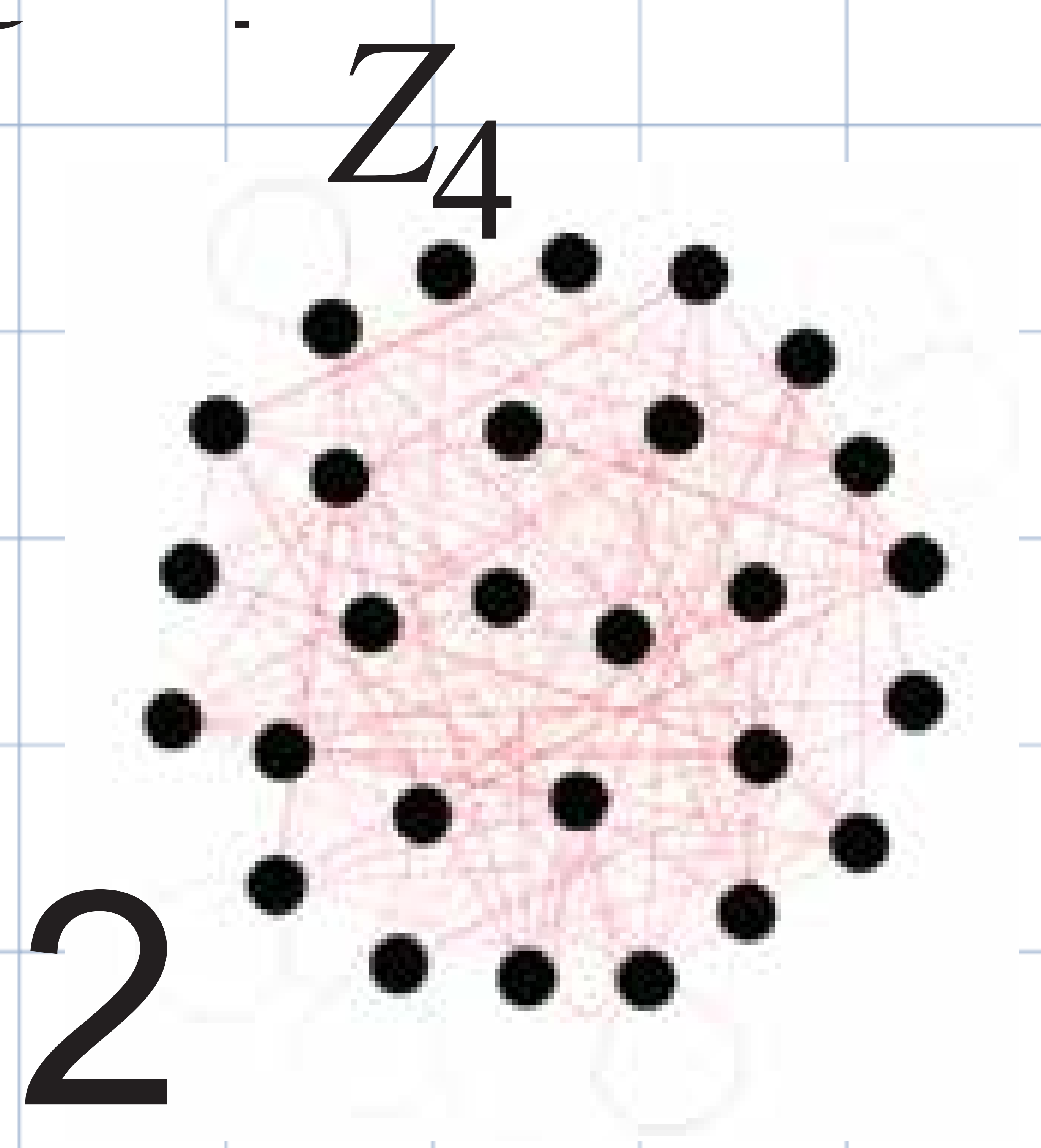

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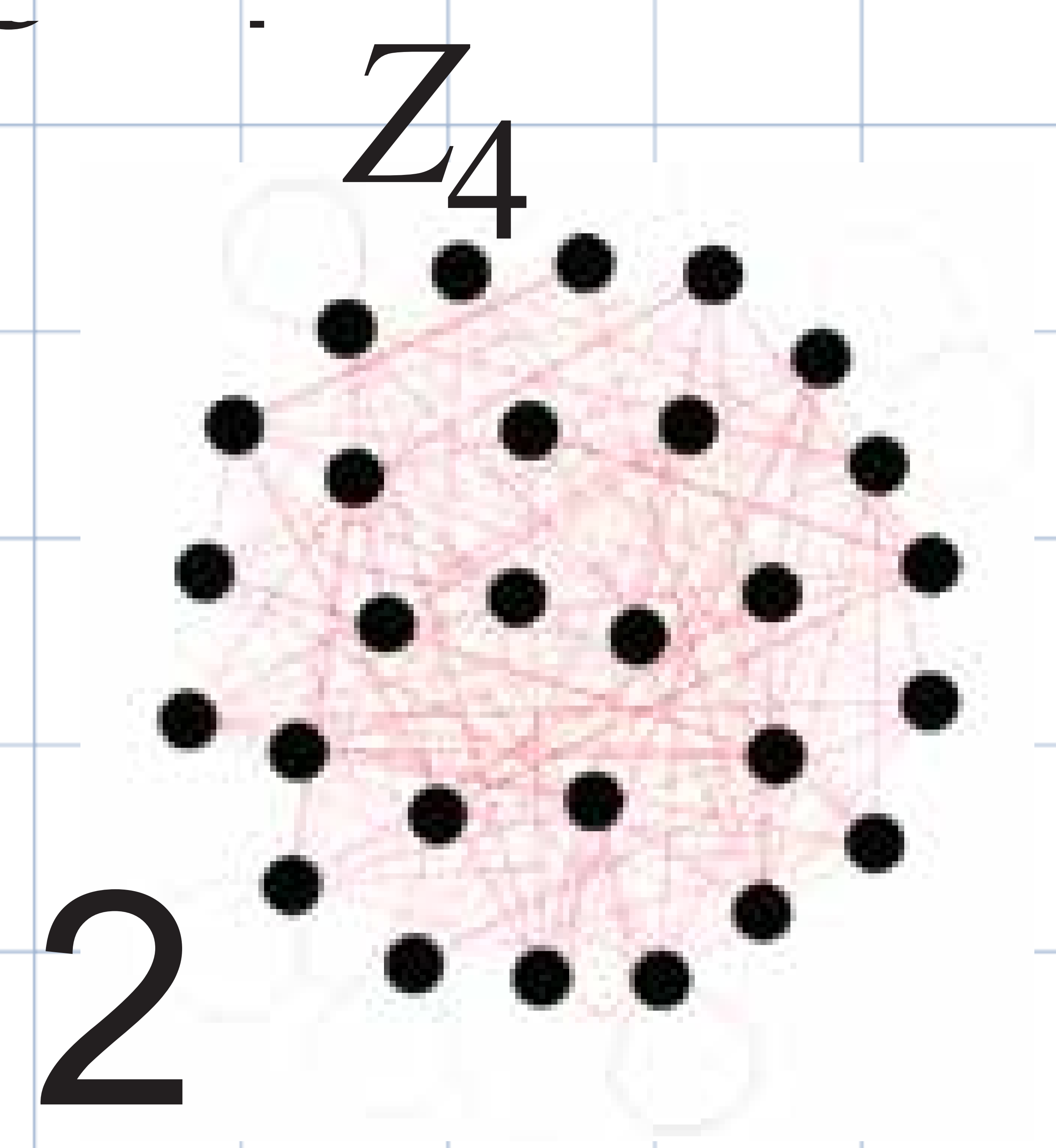

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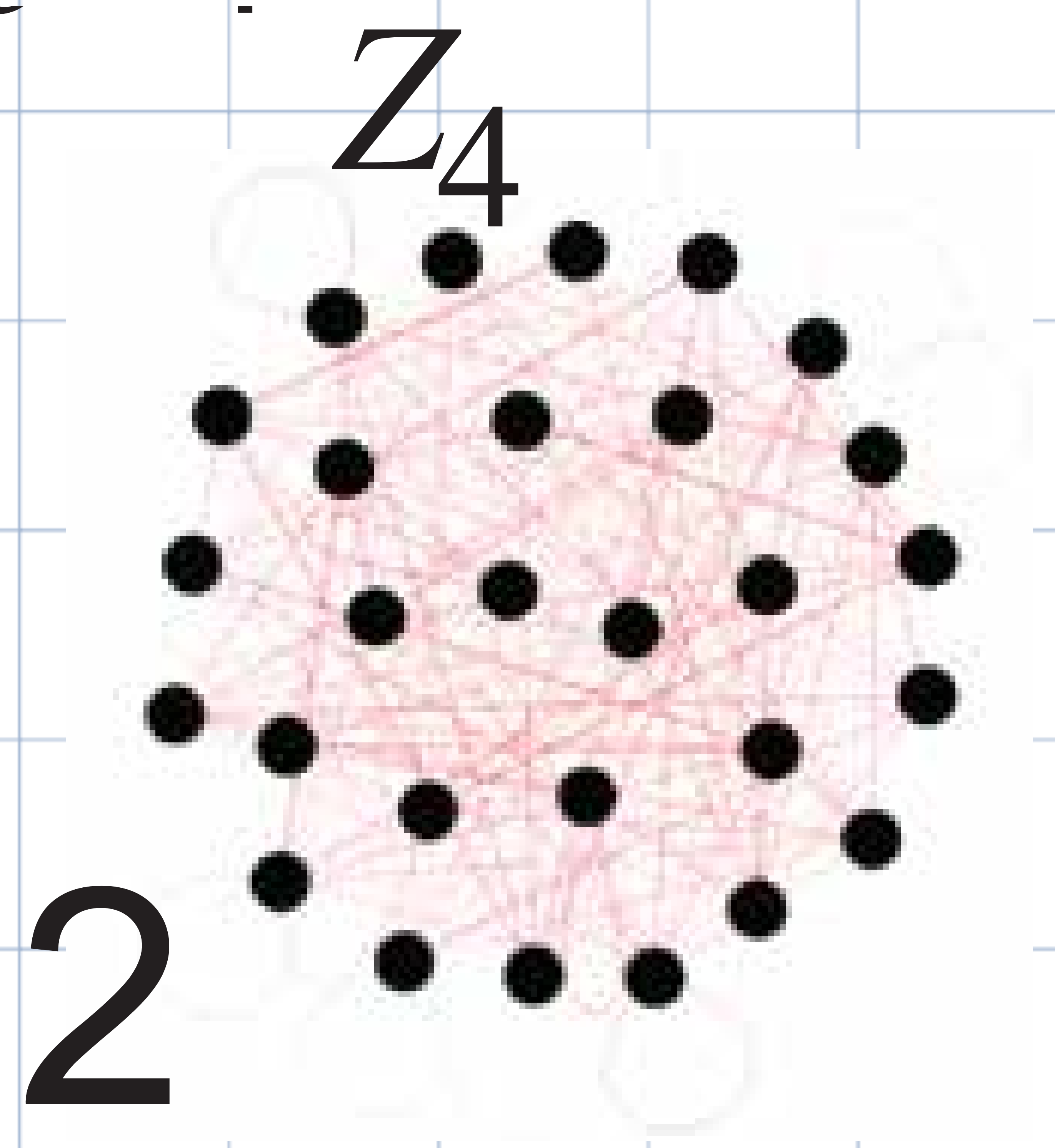
Matchgate circuits are equivalent to free-fermionic spin chains



Who is matchgate?

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Let us introduce the matchgate operator $G(A,B)$ acting on qubits A and B

$$G(A,B) = \begin{pmatrix} A_{11} & 0 & 0 & A_{12} \\ 0 & B_{11} & B_{12} & 0 \\ 0 & B_{21} & B_{22} & 0 \\ A_{21} & 0 & 0 & A_{22} \end{pmatrix}$$

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$$H = \sum_{i=1}^{L} J_i^{xx}(t) X_i X_{i+1} + J_i^{yy}(t) Y_i Y_{i+1} + J_i^{xy}(t) X_i Y_{i+1} + J_i^{yx}(t) Y_i X_{i+1} + h_i^z(t) Z_i$$

where X_i, Y_i, Z_i are Pauli matrices acting on ith site, L in the number of qubits, J_i^α, h_i^z are, in general, time dependent coefficients

Classical simulability of matchgates (PI-SO)

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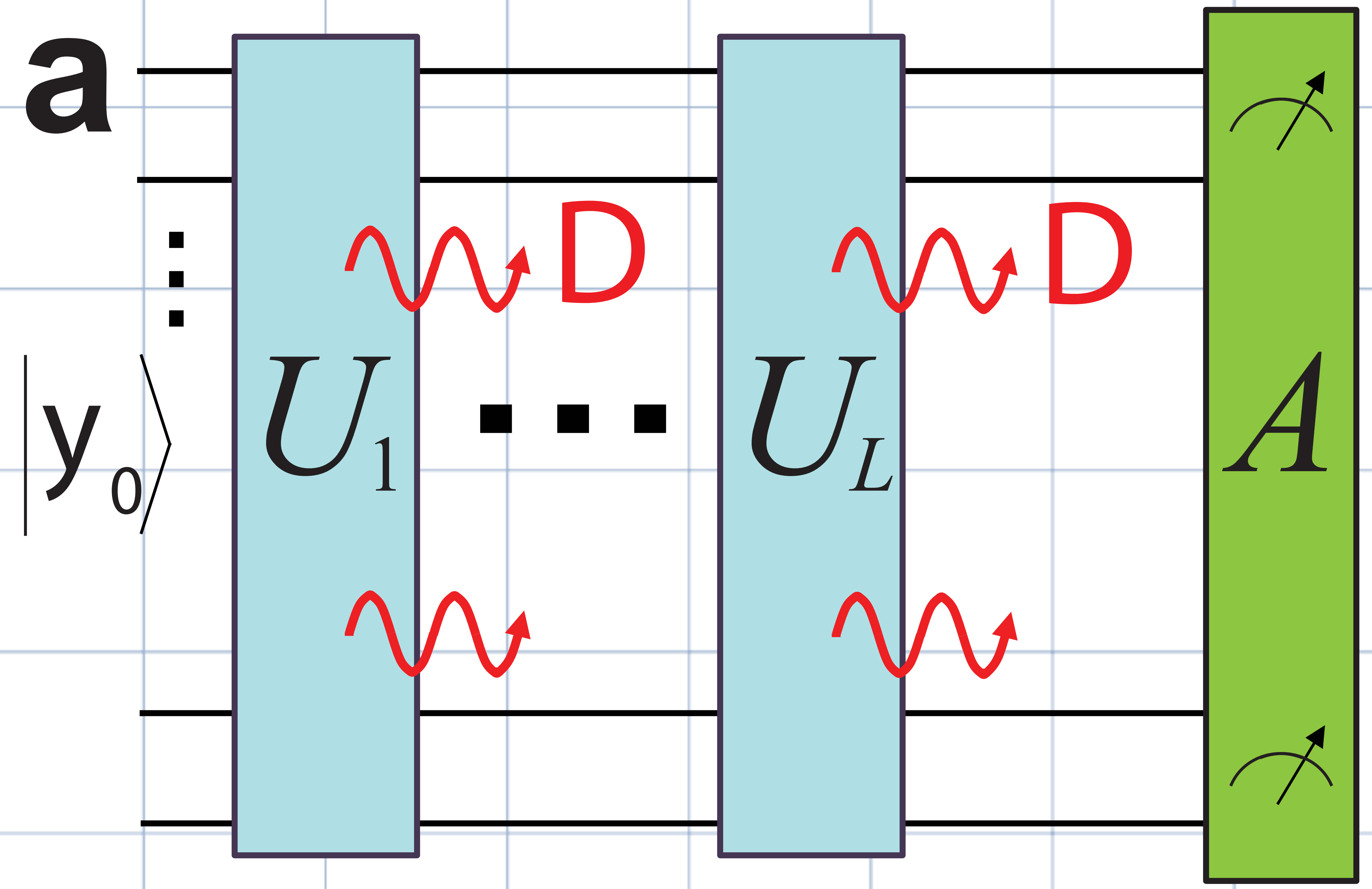

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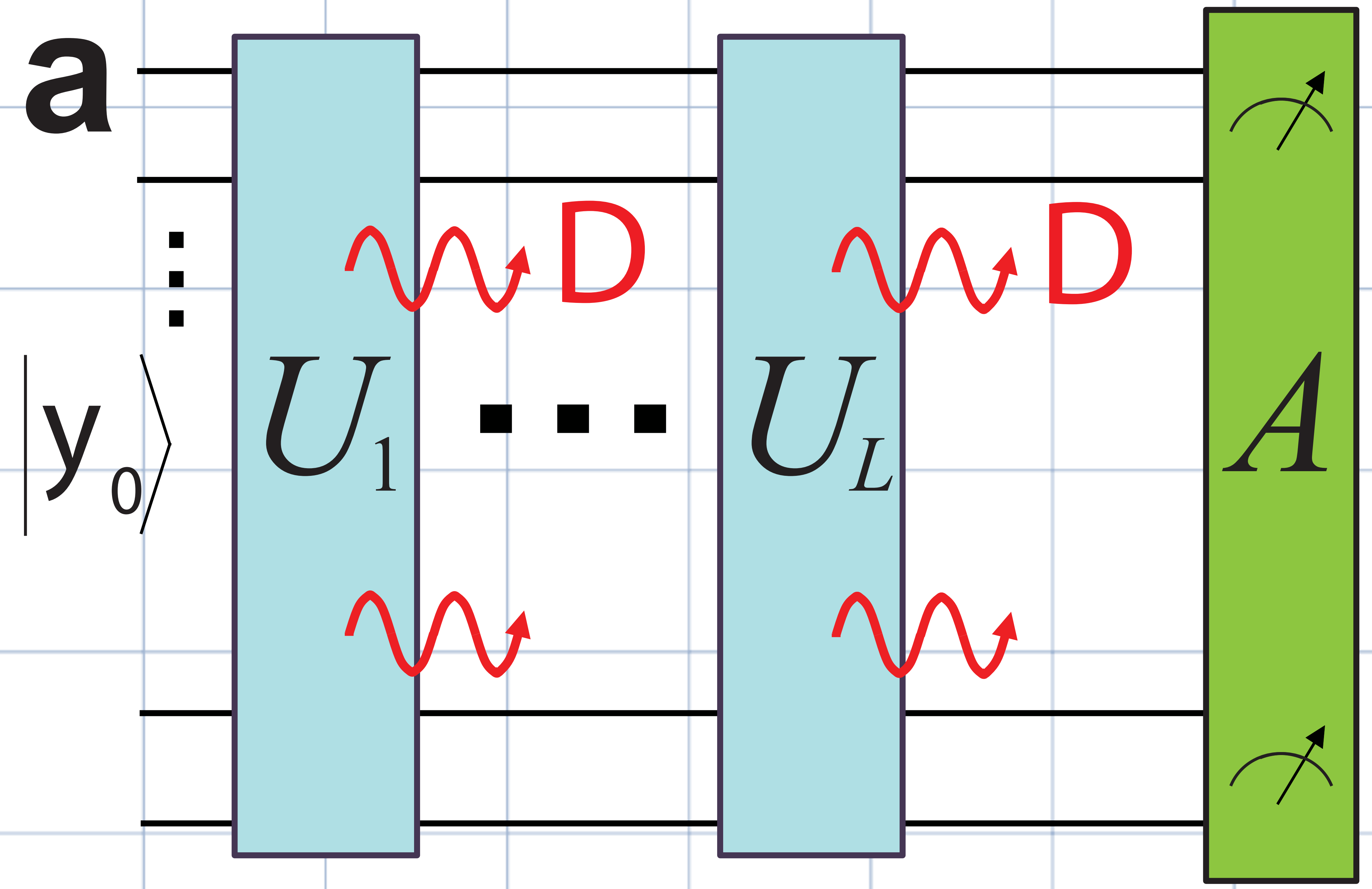

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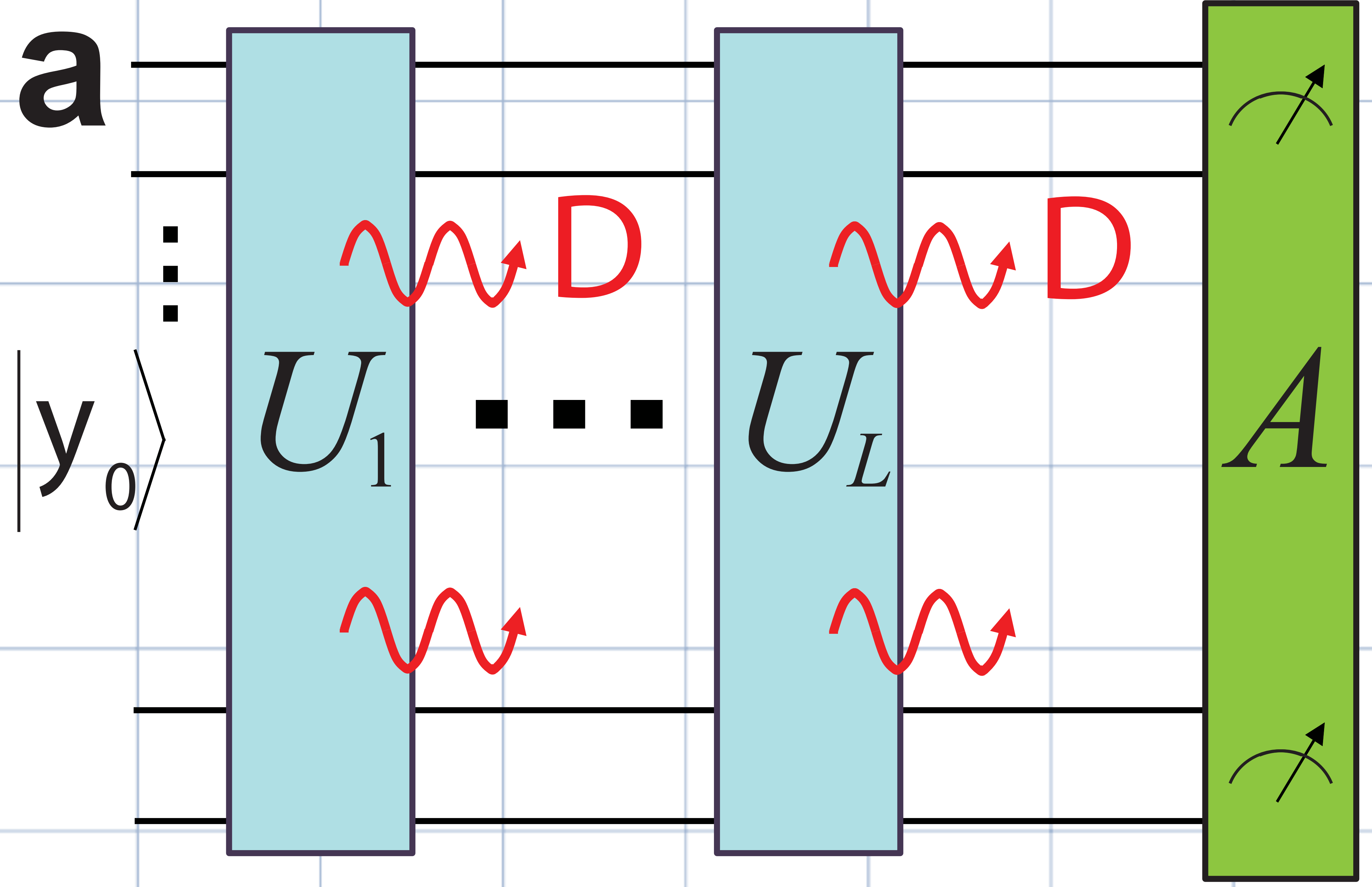

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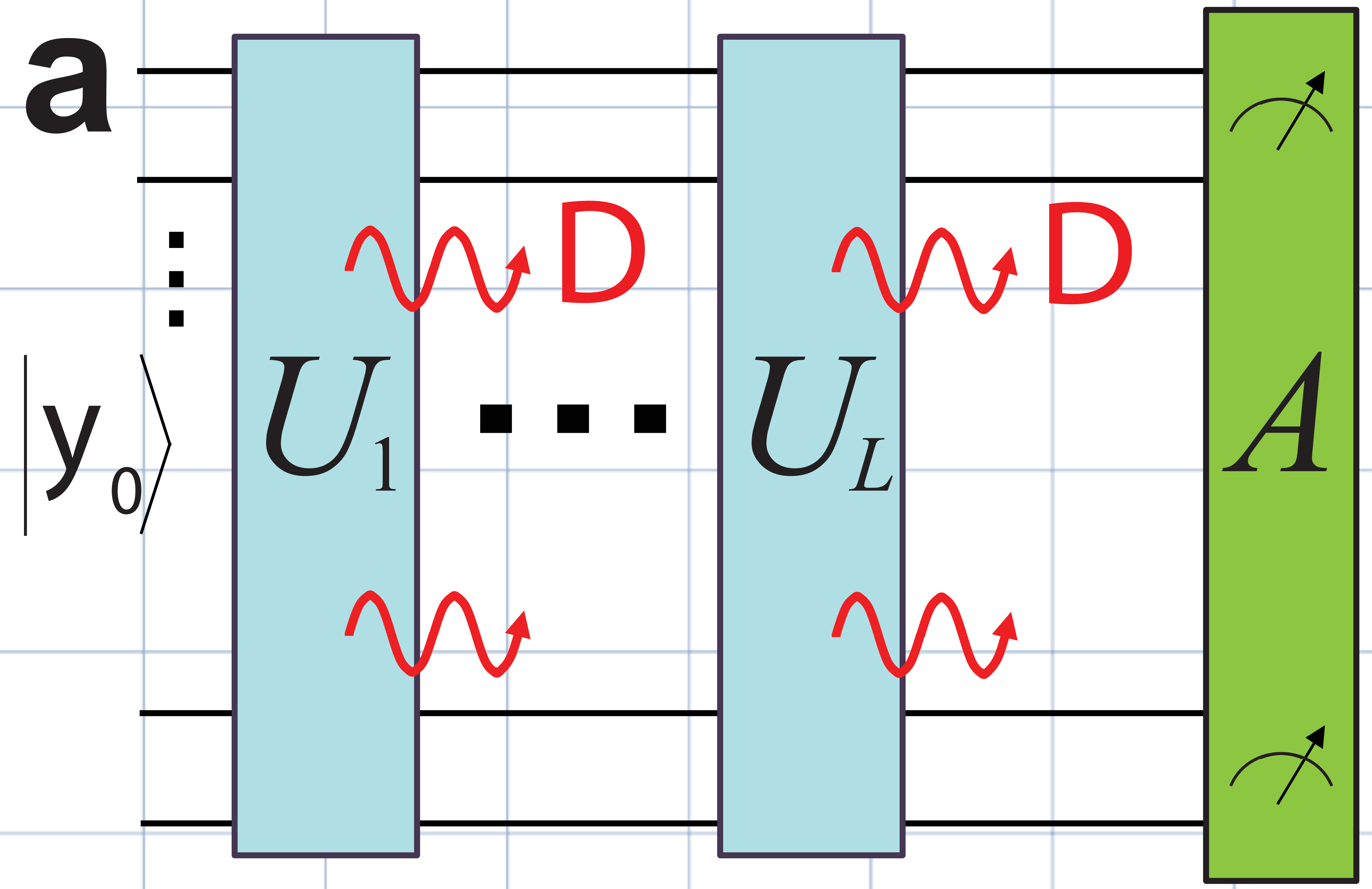

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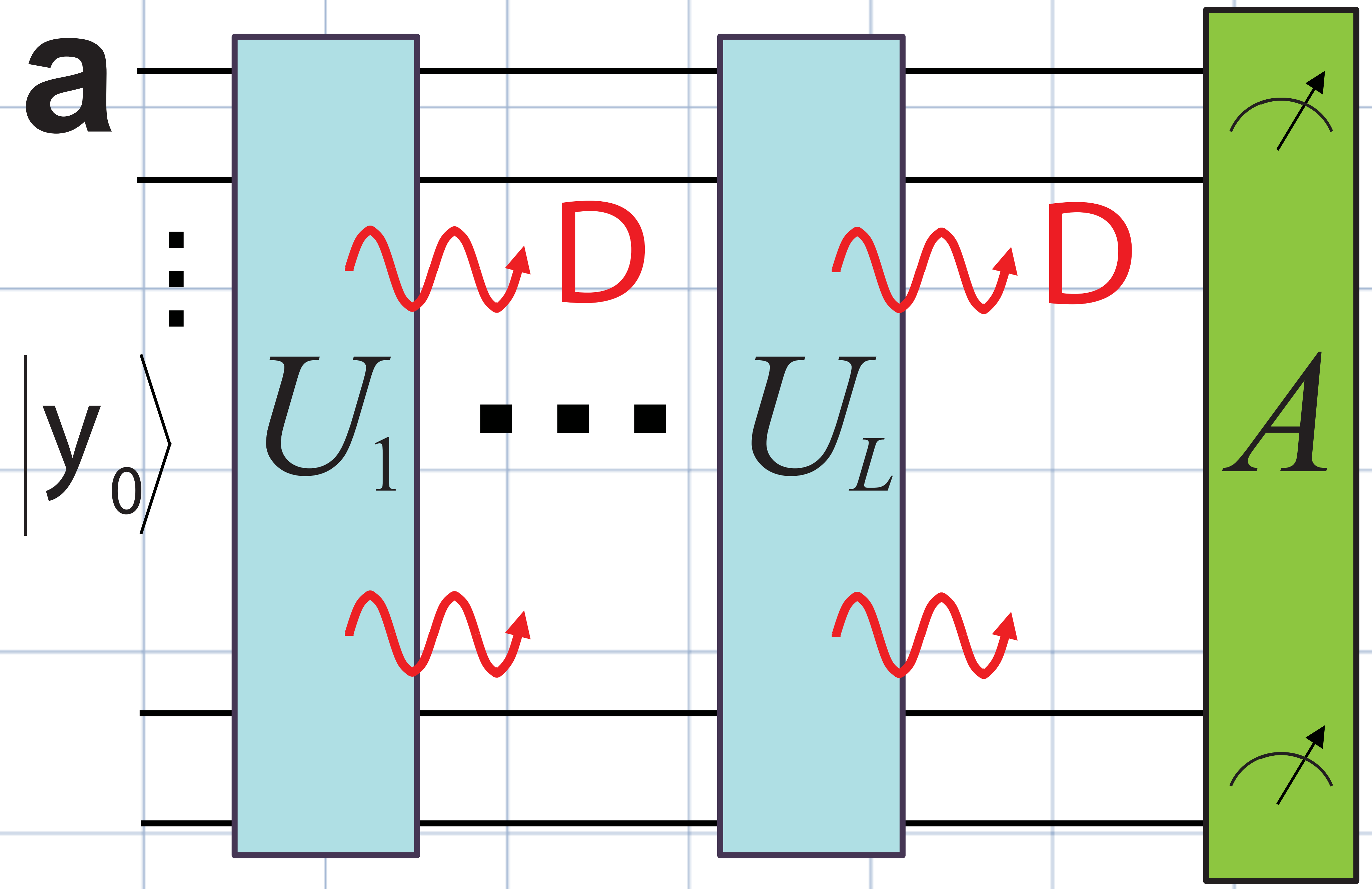

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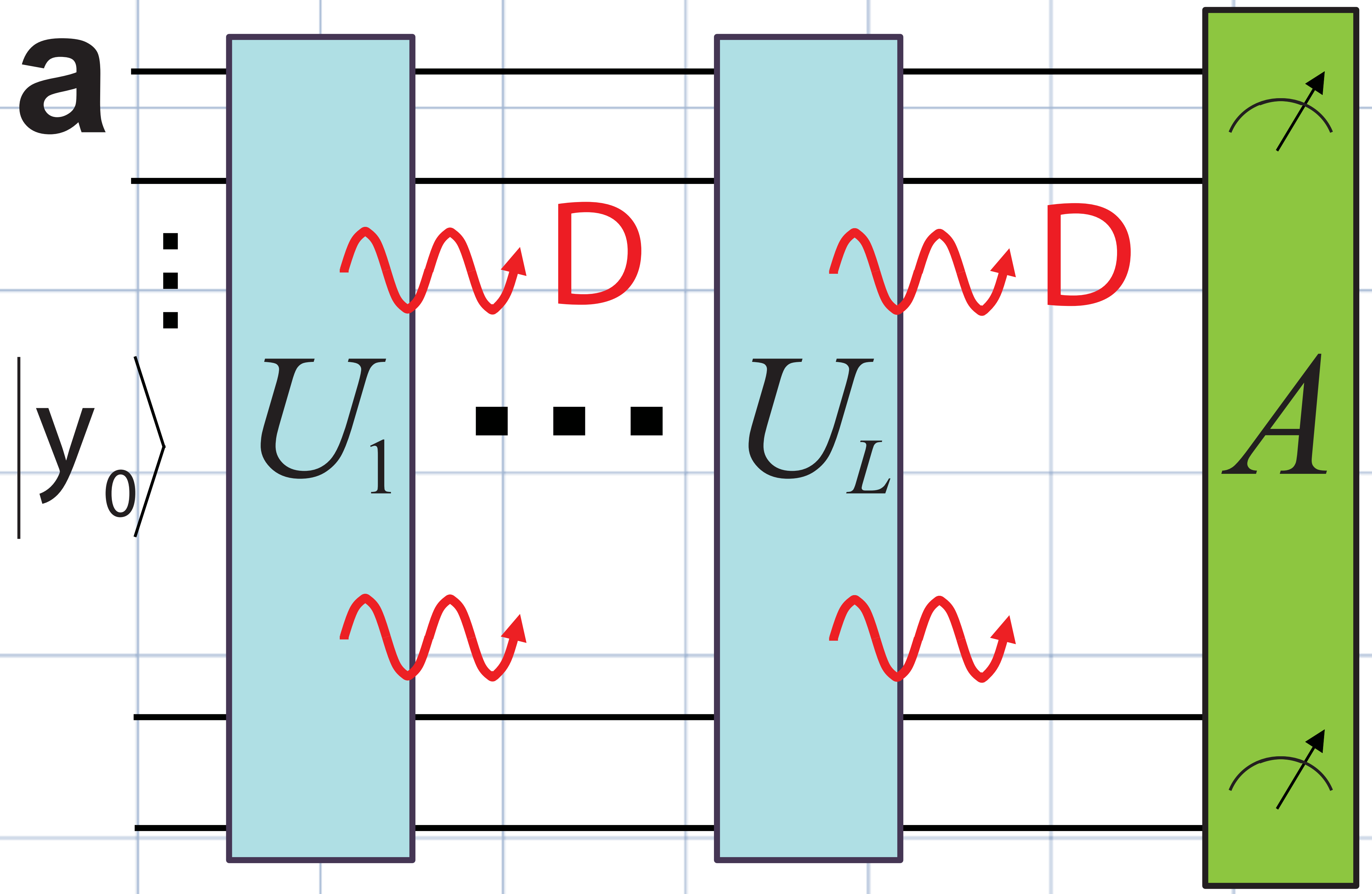

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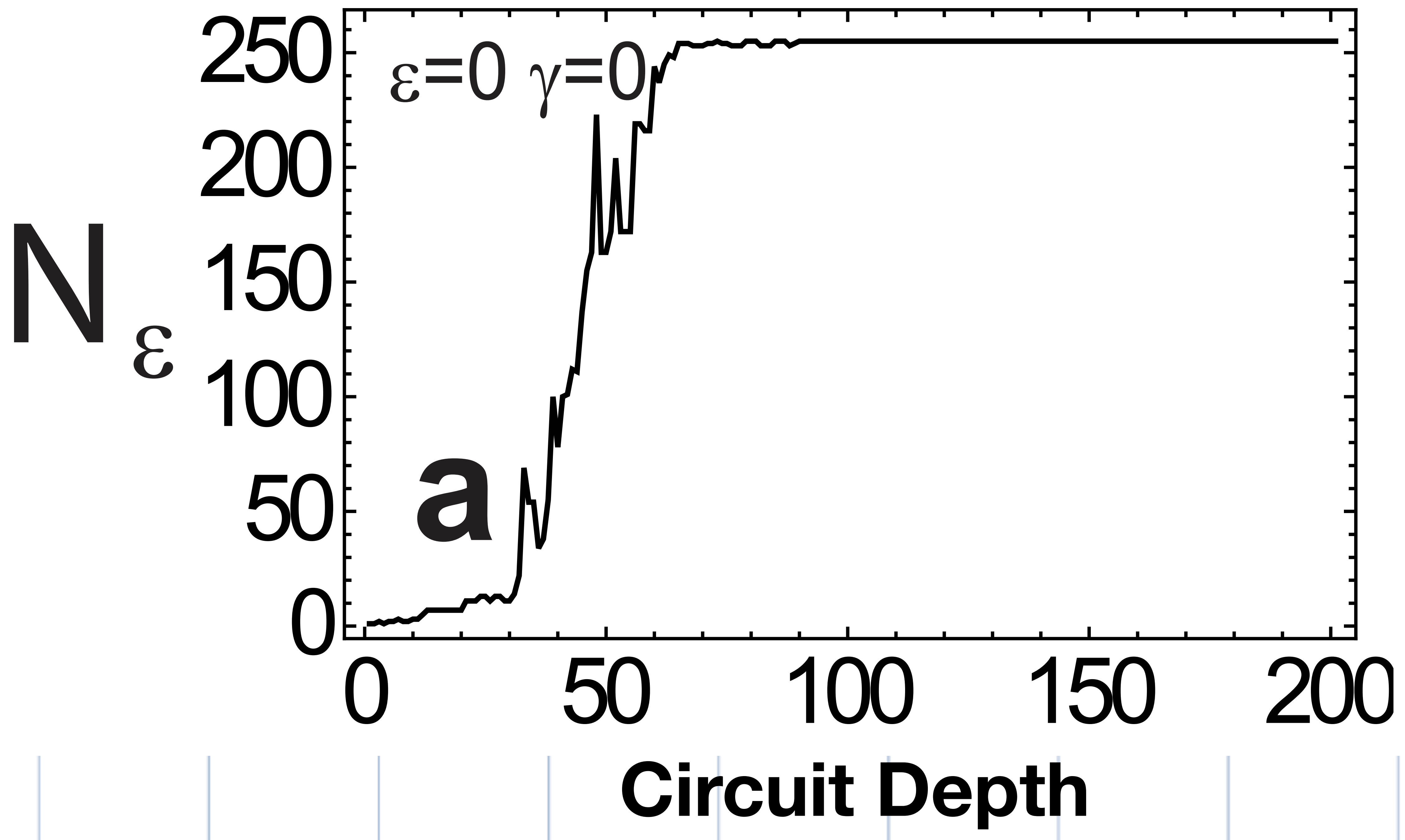

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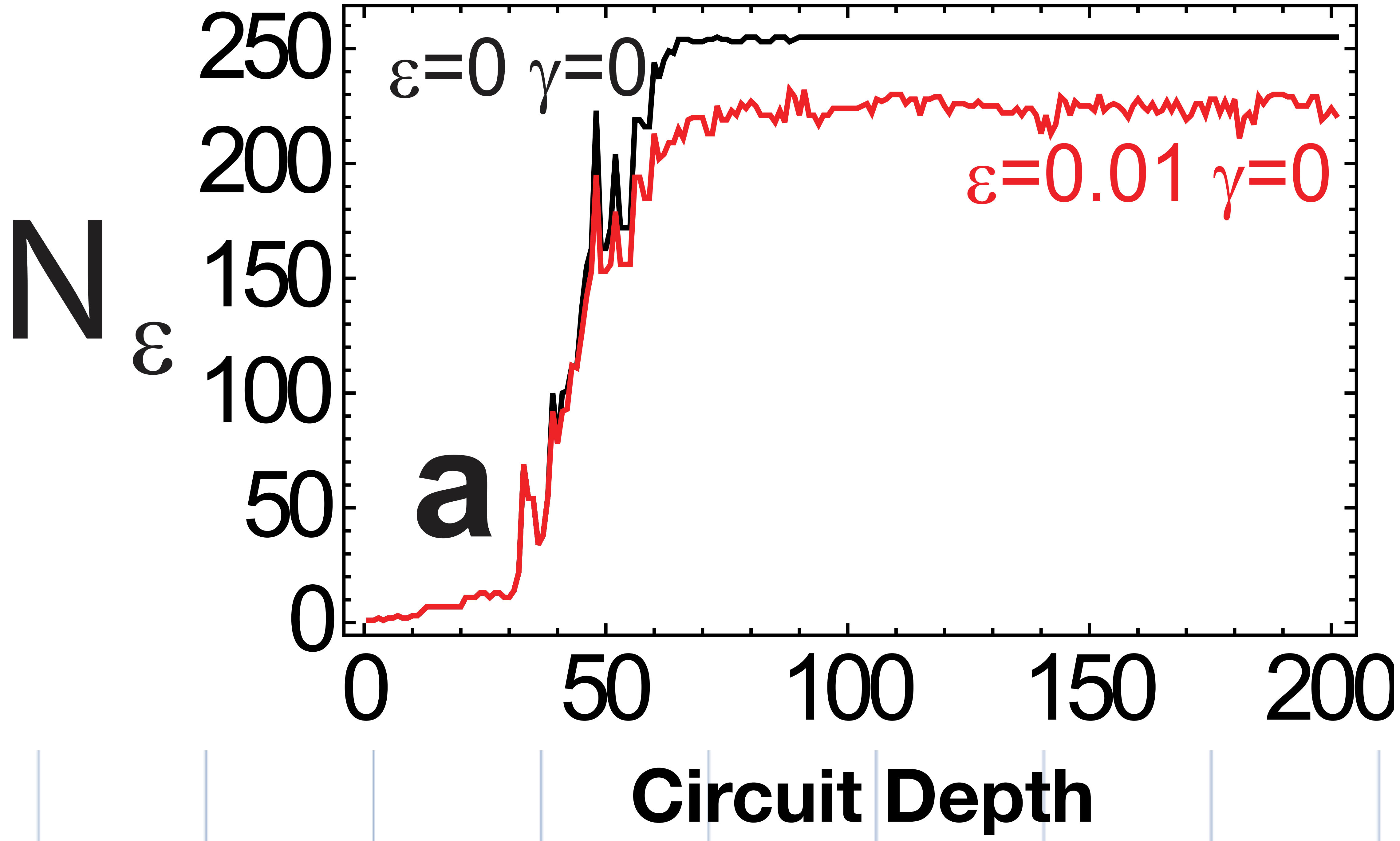

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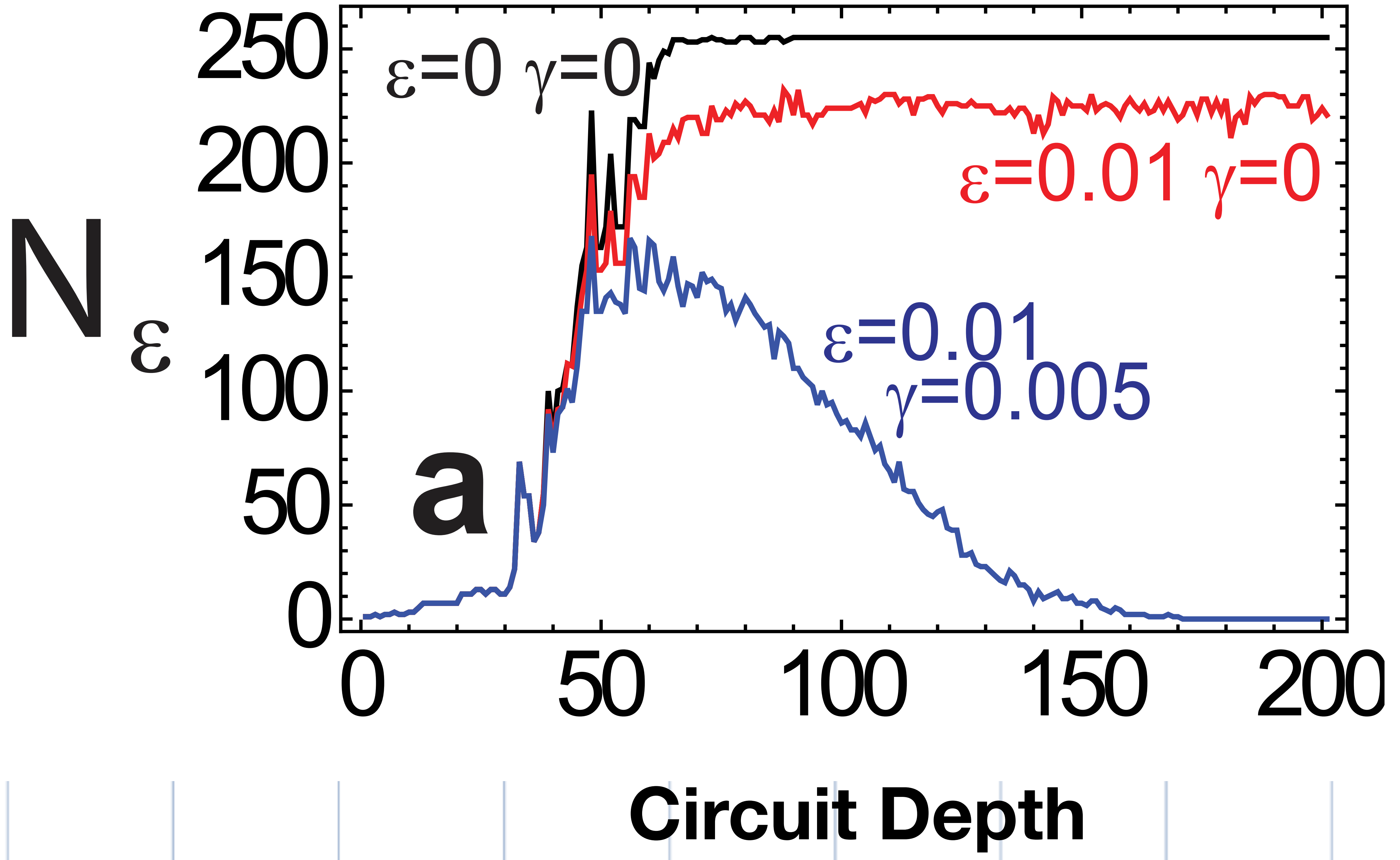

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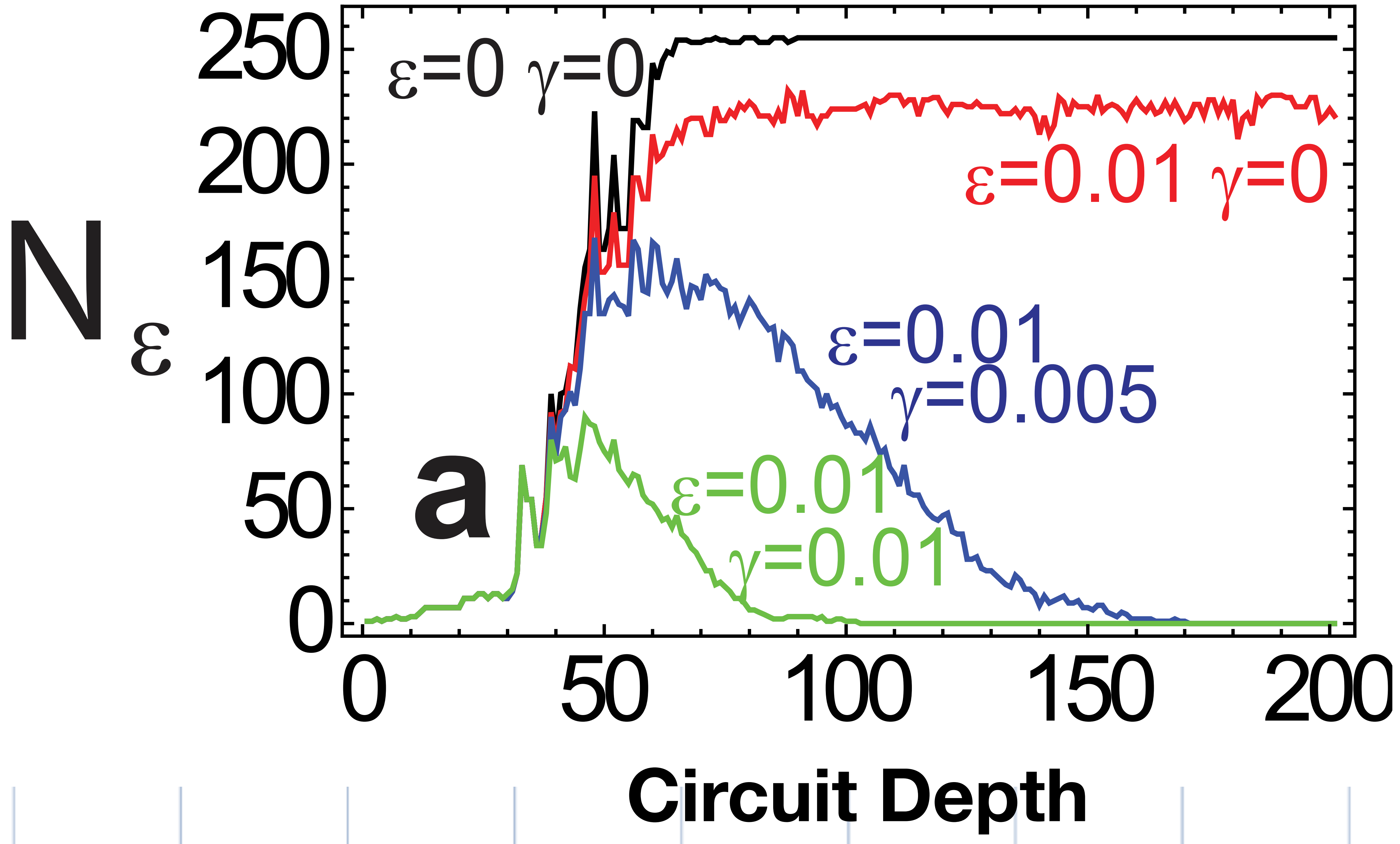

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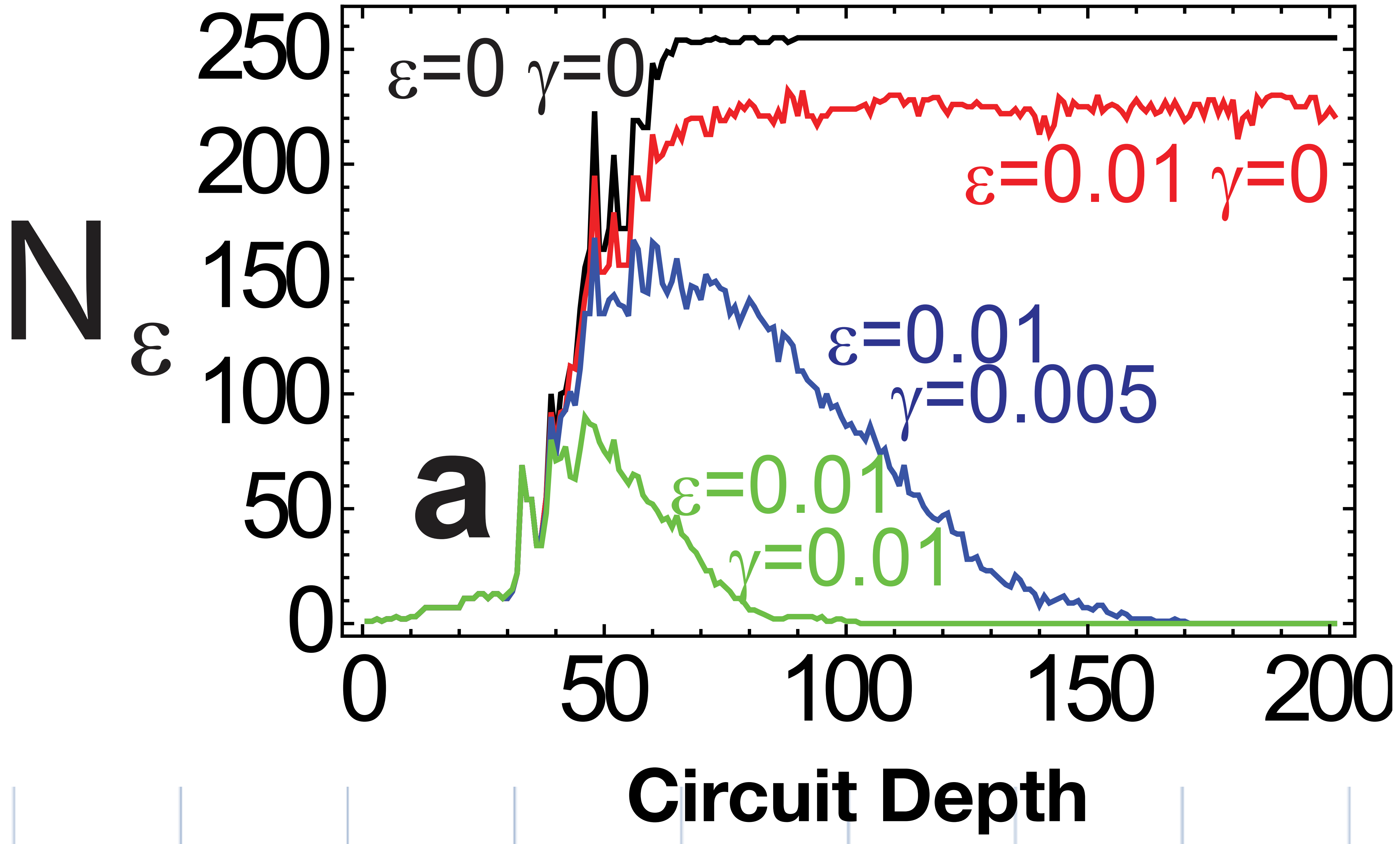


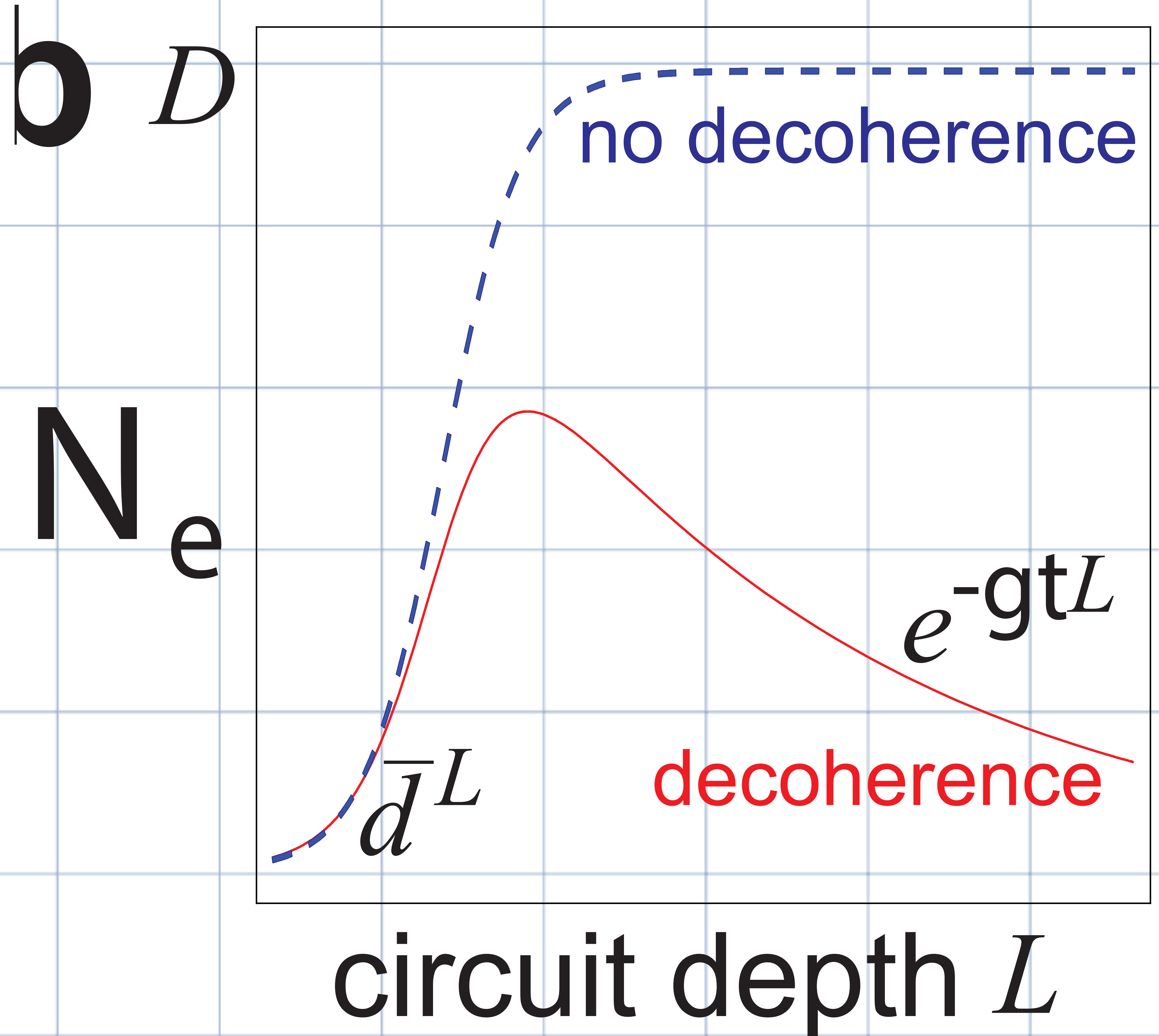
This peak remains suppressed in thermodynamic limit

General picture

In general, the number of significant operators can be approximated as:

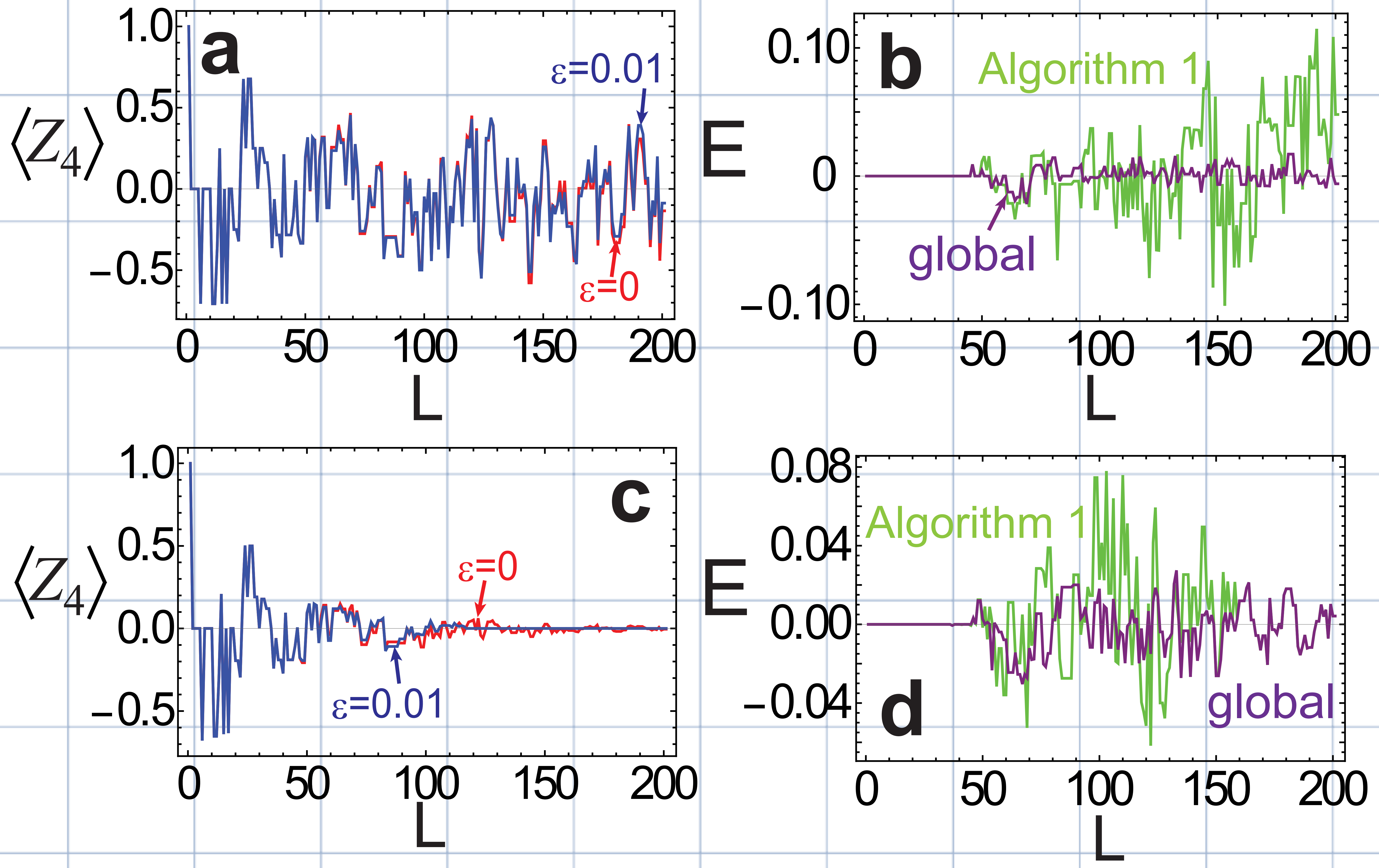
$$\mathcal{N}_\epsilon \propto \frac{De^{-\epsilon\gamma\tau L}\bar{d}^L}{D - 1 + a\bar{d}^L}$$

Here D order of the graph and
$\bar{d}$ — mean vertex out degree per layer
τ, α — fit parameters



Truncation algorithm for simulation of observables

We developed a numerical algorithm that utilizes the fact that operator evolution is restricted



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Dissipative quantum dynamics seems to be more easily simulatable than unitary

Thanks for attention!