Quantum Computing with QuDits

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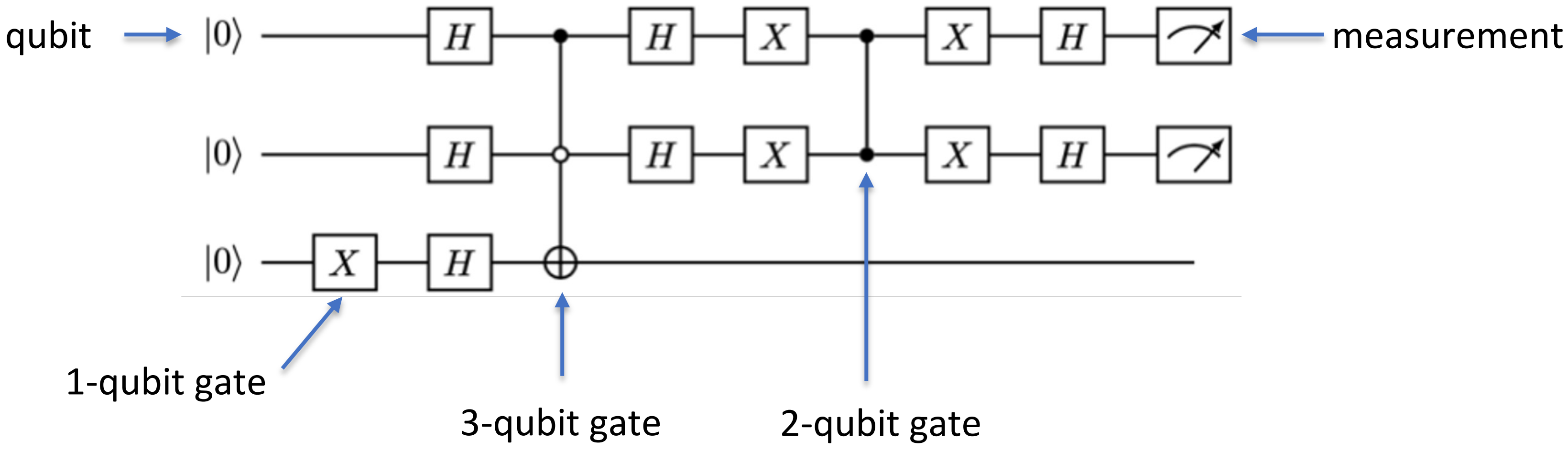
Gate-based quantum computing

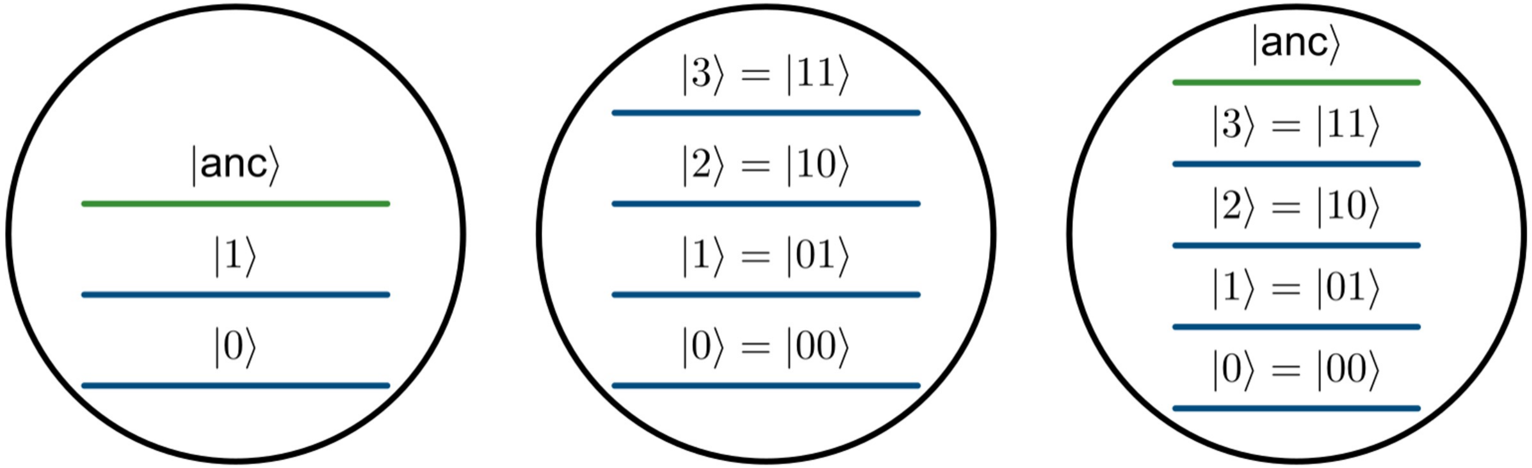
- Quantum circuit is a sequence of quantum gates



- Multi-qubit gates → single particle + two-particle gates
- Two-particle gates introduce more noise then single-particle gates

QuDits – d-level quantum systems

- Physical systems used as qubits usually have more than 2 levels
- Qubit(s) can be embedded in quDit's space



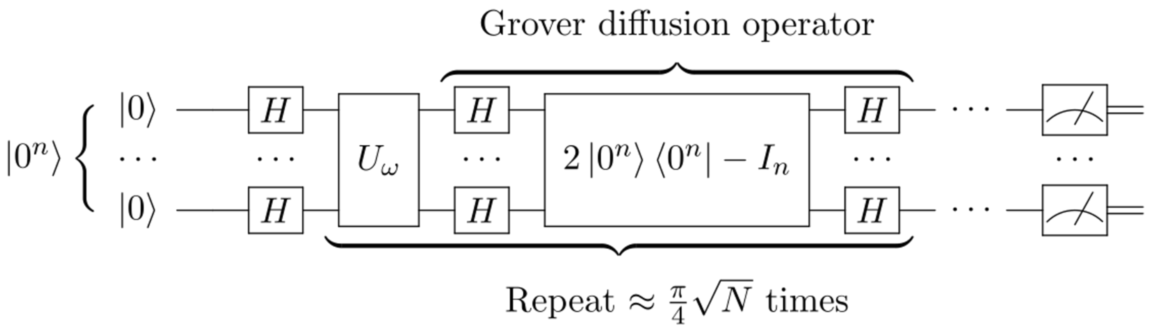
d = 3 qutrit
qubit + ancillary state

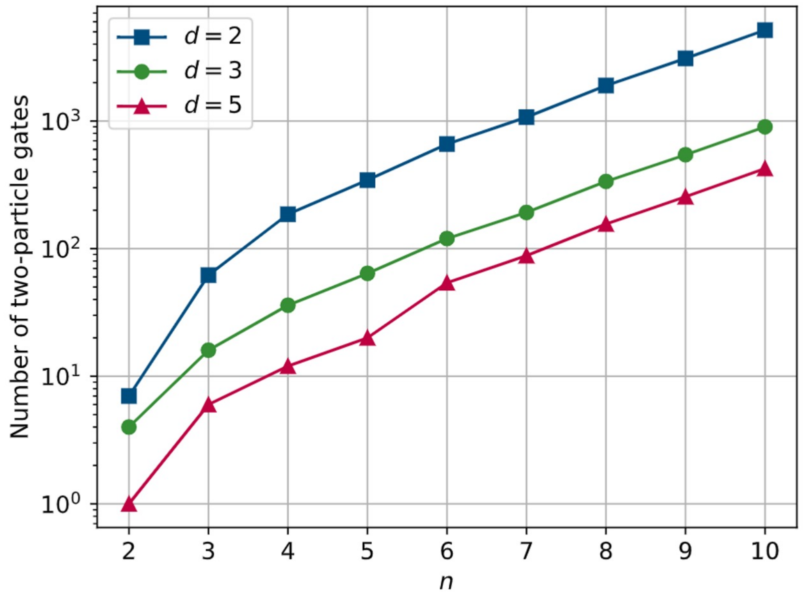
d = 4 ququart
2 qubits

d = 5 ququint
2 qubits + ancillary level

QuDits for qubit circuit implementation

- Simplified multi-qubit gate decomposition
- Decrease in the number of information carries
- Reduction in the number of two-particle gates in the circuit





Entropy 2023, 25(2), 387

Trapped ions as quDits

A universal qudit quantum processor with trapped ions

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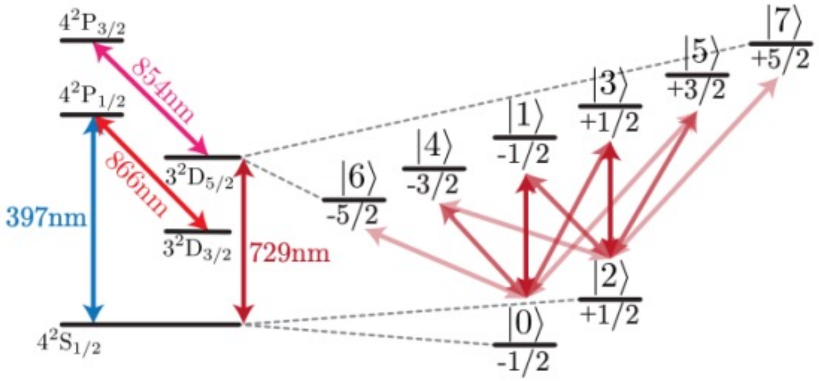

Nature Physics **18**, 1053-1057 (2022)

Realizing quantum gates with optically-addressable $^{171}Yb^+$ ion qudits

M.A. Aksenov,[1,2] I.V. Zalivako,[1,2] I.A. Semerikov,[1,2] A.S. Borisenko,[1,2] N.V. Semenin,[1,2] P.L. Sidorov,[1,2] A.K. Fedorov,[2] K.Yu. Khabarova,[1,2] and N.N. Kolachevsky[1,2]
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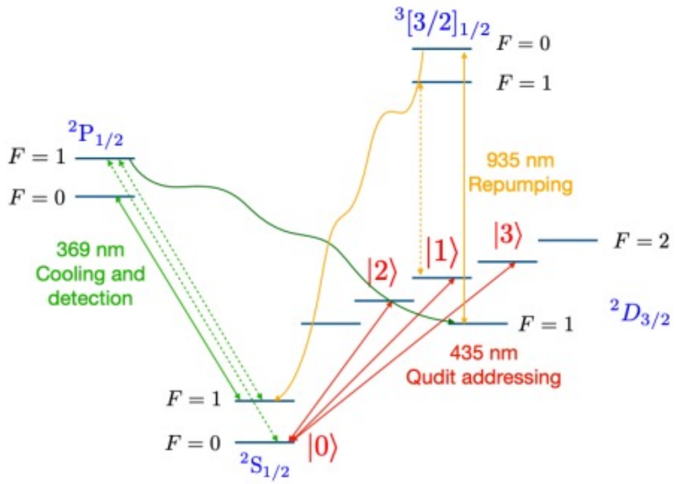
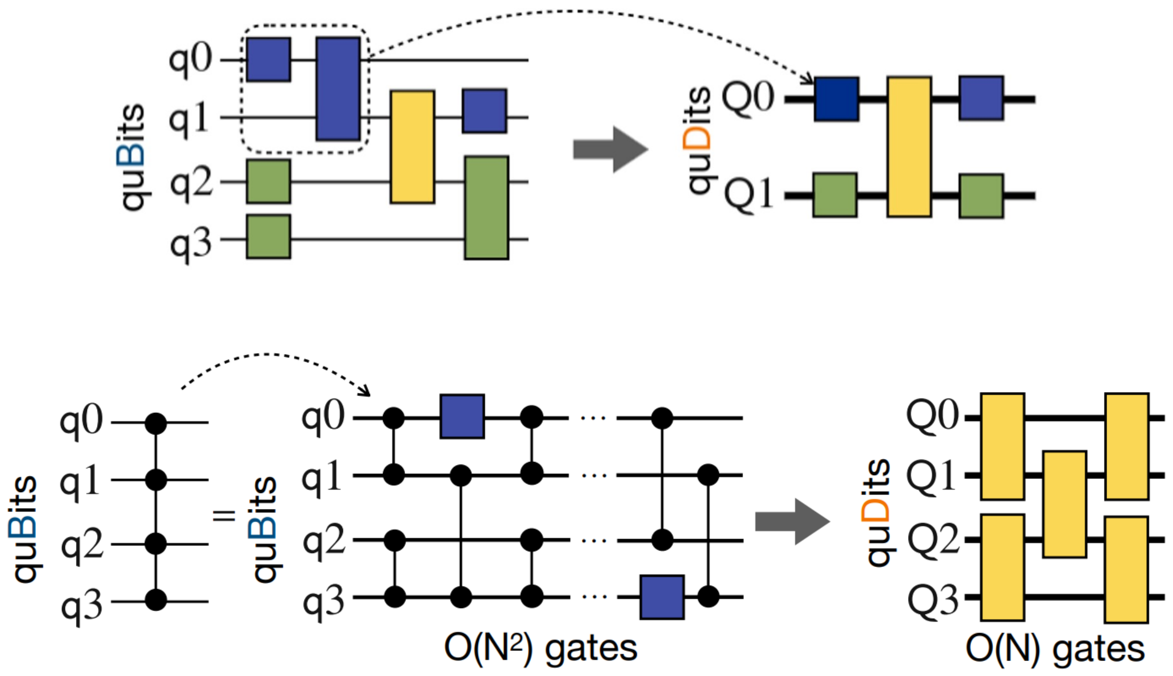


Phys. Rev. A **107**, 052612 (2023)

Qubit quantum circuits with quDits

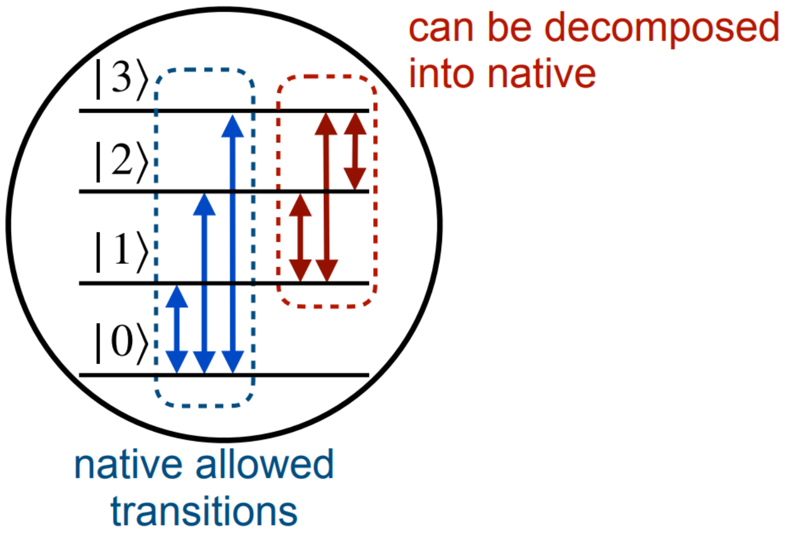
- To implement qubit circuit with quDits we need to define how to realize:
    - Single-qubit gates inside qudit
    - Two-qubit gates with qudits
    - Multi-qubit gates with qudits
- Basic gates for physical platform are important!



arXiv:2311.12003

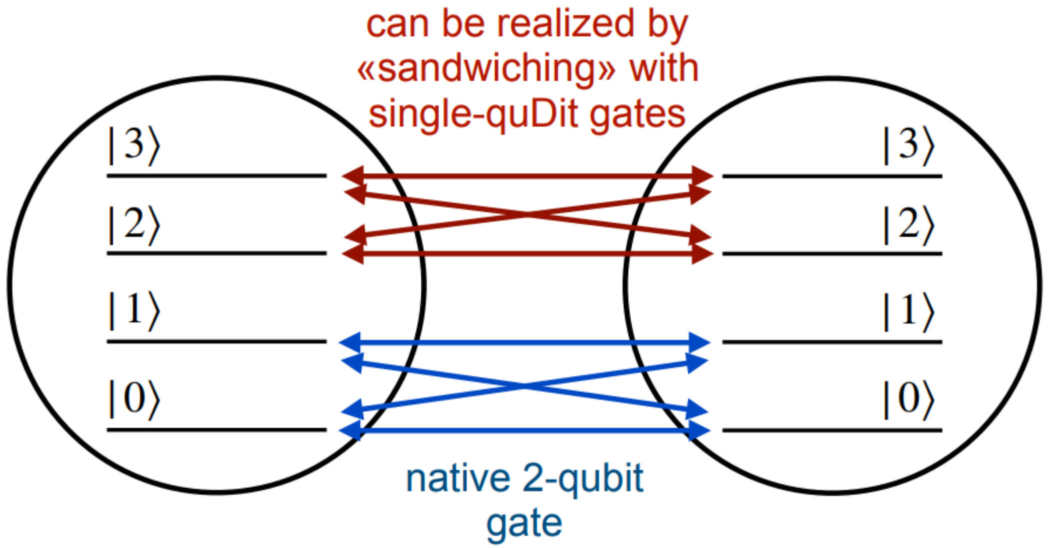
Basic gates for trapped-ion quDits

- Single-qudit gates



$$R^{\alpha,\beta}(\phi,\theta) = \exp\left(-\imath\left[\sigma_x^{\alpha,\beta}\cos\phi + \sigma_y^{\alpha,\beta}\sin\phi\right]\frac{\theta}{2}\right)$$

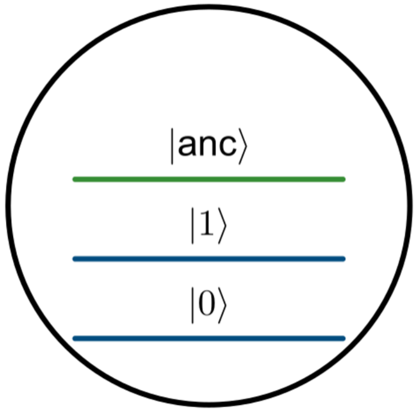
- Two-qudit Mølmer–Sørensen gate



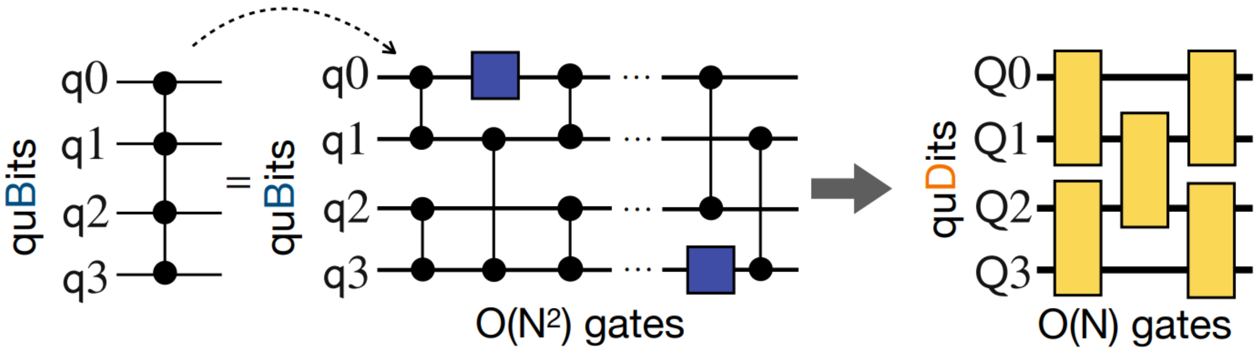
$$\mathrm{MS}(\chi) = \exp\left(-\imath\left[\sigma_x^{0,1}\otimes\sigma_x^{0,1}\right]\chi\right)$$

d = 3: qubit circuit with qutrits

- Single-qubit gate $R(\phi, \theta)$ → Single-qutrit $R^{01}(\phi, \theta)$ in qubit subspace
- Two-qubit gate $MS(\chi)$ → Two-qutrit $MS^{0101}(\chi)$ in qubit subspace
- Multi-qubit gate → Developed qutrit-based decomposition

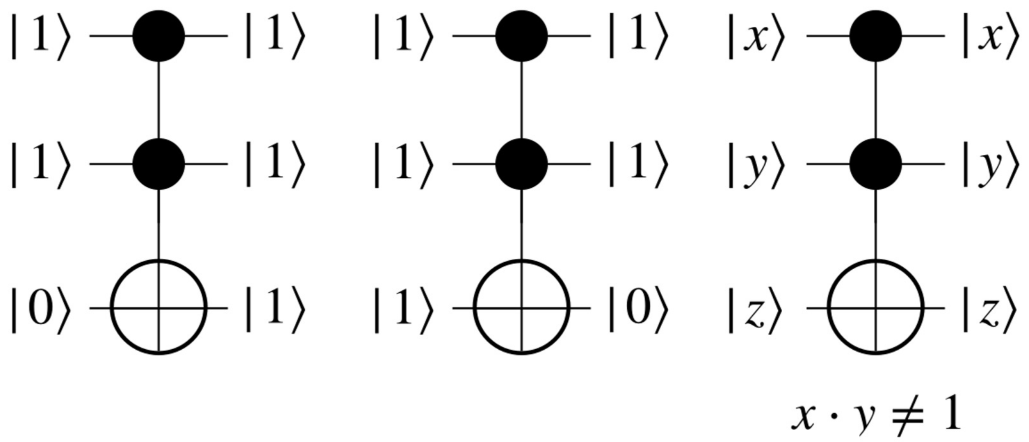


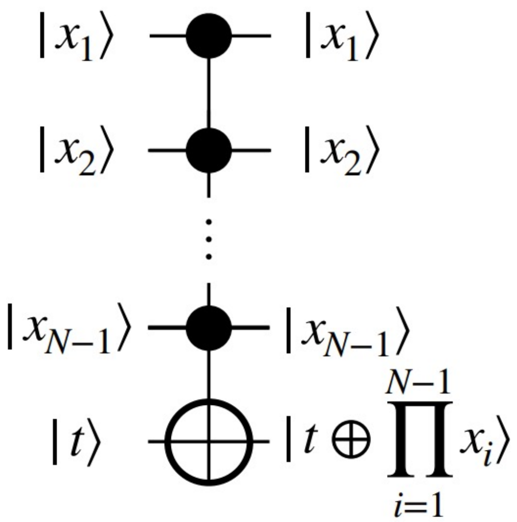
d = 3 qutrit
qubit + ancillary state



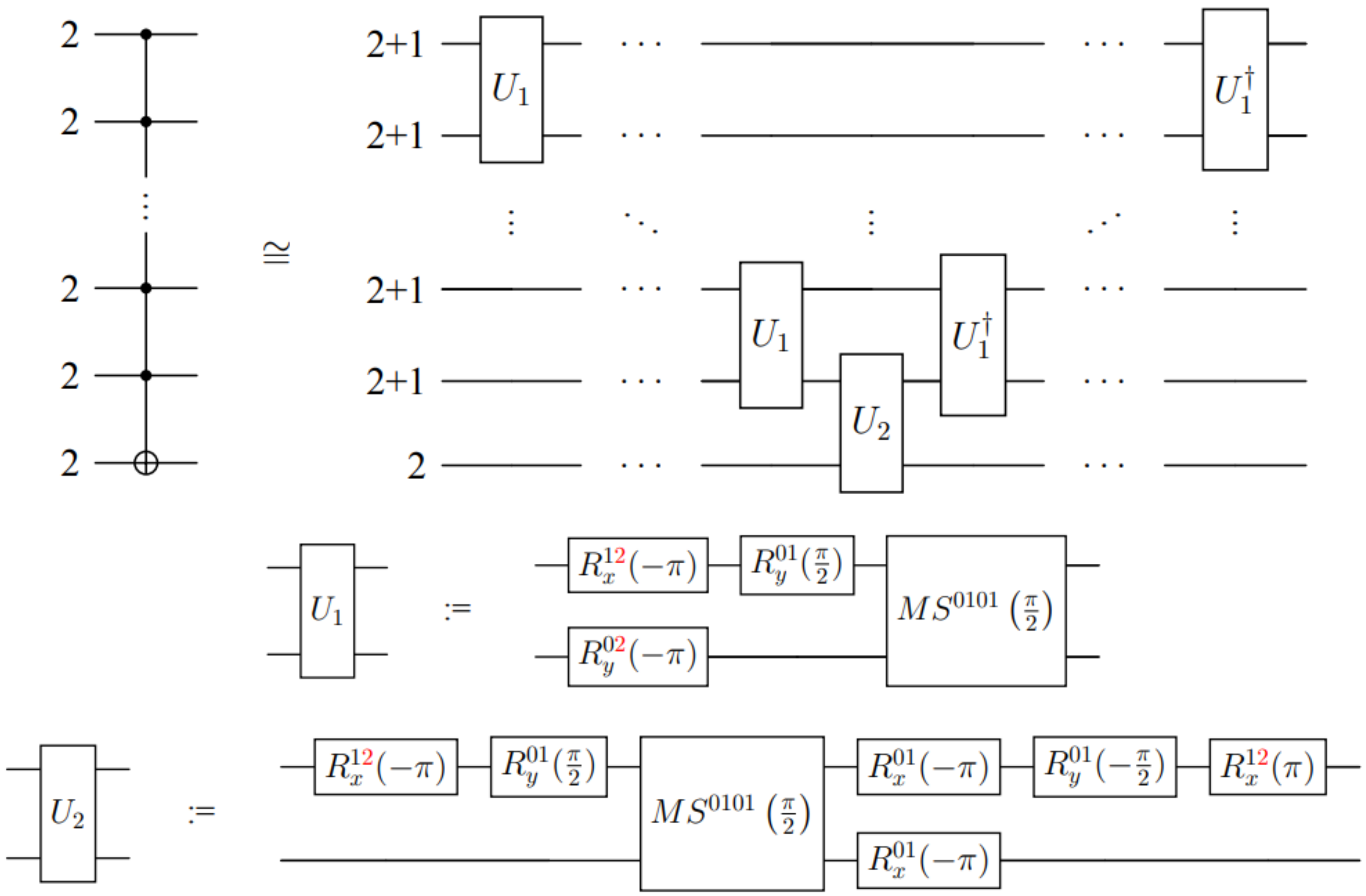
Multi-qubit generalized Toffoli gate

- Used in Grover's search algorithm, factorization problem and in error correction
- From $C^{N-1}X$ other multi-controlled unitaries can be constructed
- N qubits → $O(N^2)$ 2-qubit gates
- N qutrits → $2N-3$ 2-qutrit gates (d = 3)





d = 3: Multi-qubit $C^{N-1}X$ gate with ion qutrits



Resulting decomposition: **2N-3** MS(π/2)

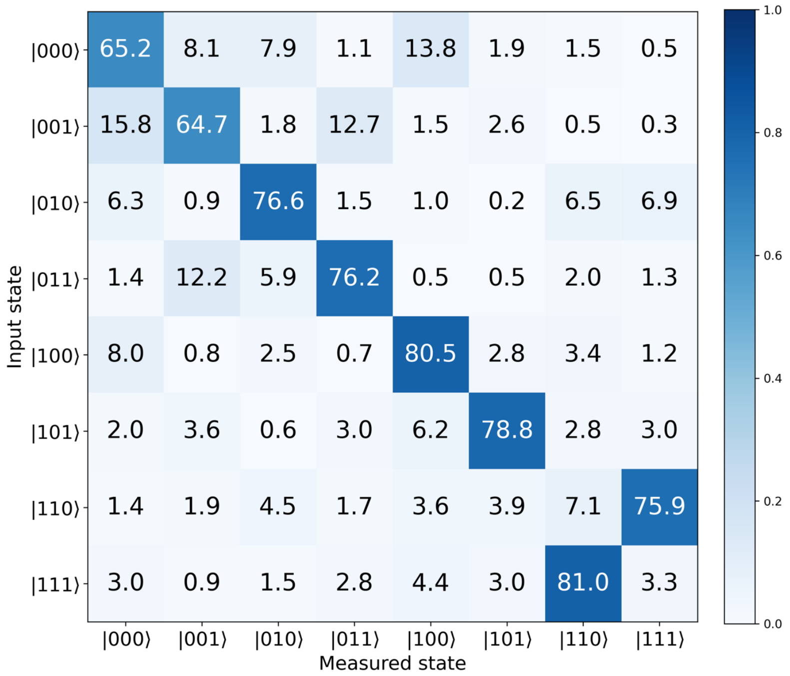
Standard approach:
12N - 30 CNOTs + N-3 ancillas /
O(N²) CNOTs with no ancillas

Phys. Rev. A 109, 022615 (2024)

3-qubit Toffoli CCX gate with qutrits (d = 3)

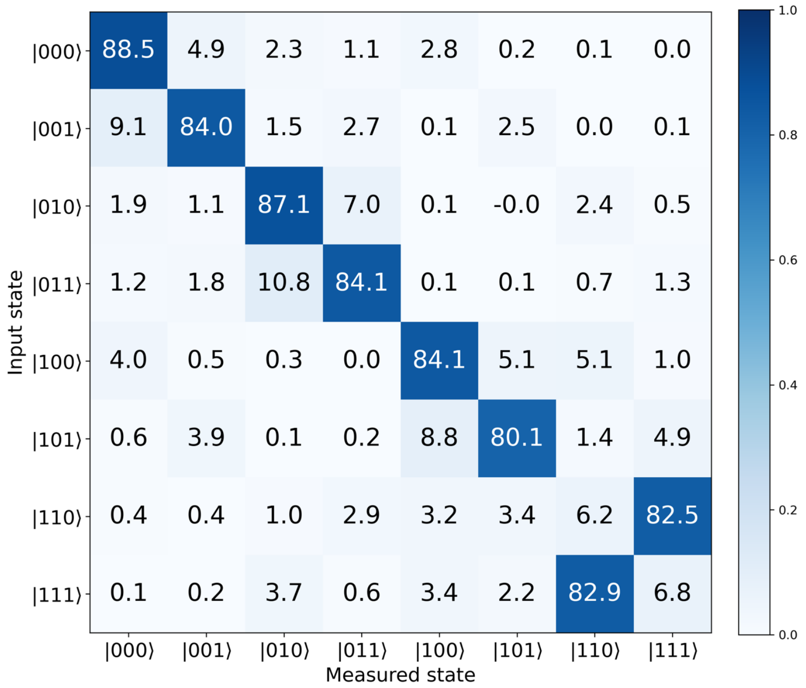
- Experimental fidelity of the computational basis truth table

quBits, "F" ~ 73%



6 2q gates

quDits, "F" ~ 82%

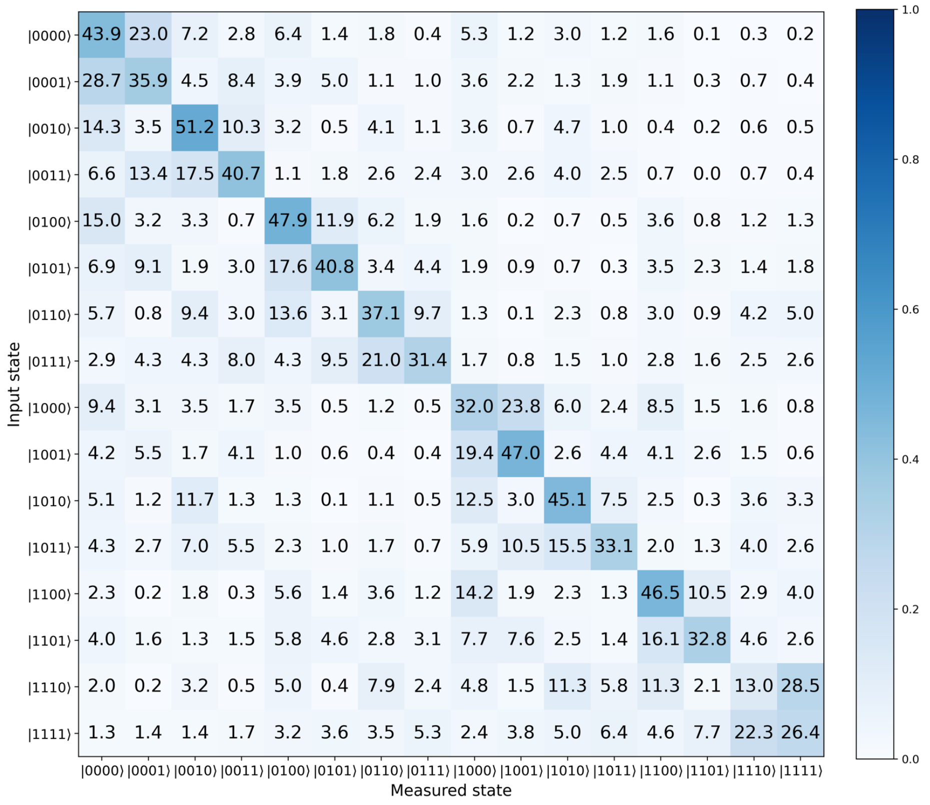


3 2q gates

4-qubit CCCX gate with qutrits (d = 3)

- Experimental fidelity of the computational basis truth table

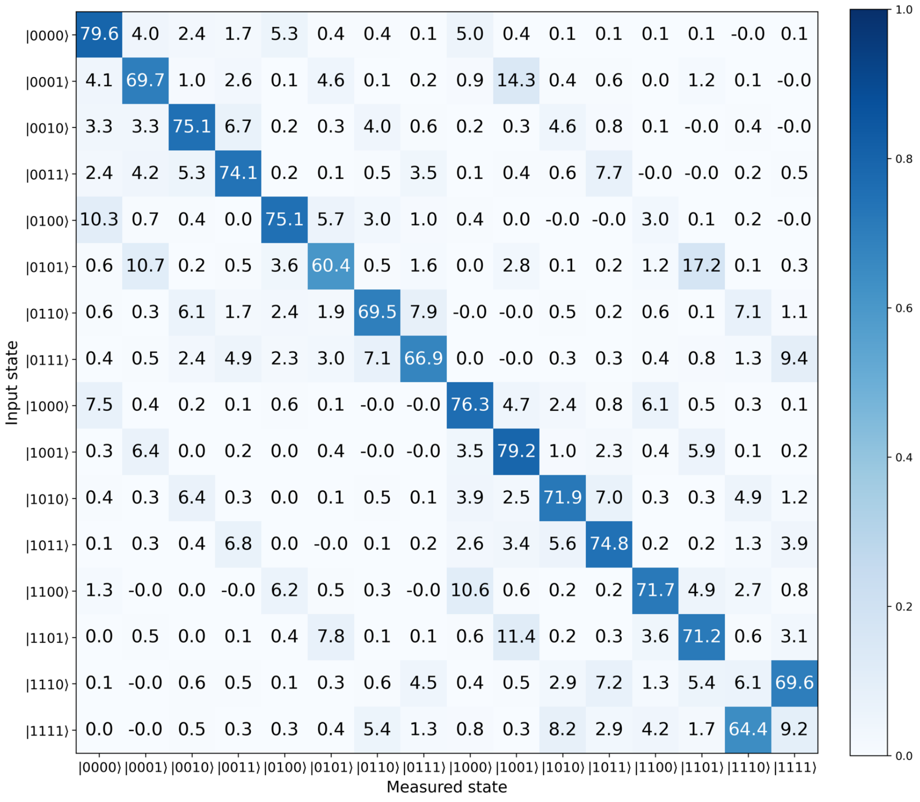
quBits, "F" ~ 39%



Input \ Measured	|0000⟩	|0001⟩	|0010⟩	|0011⟩	|0100⟩	|0101⟩	|0110⟩	|0111⟩	|1000⟩	|1001⟩	|1010⟩	|1011⟩	|1100⟩	|1101⟩	|1110⟩	|1111⟩
|0000⟩	43.9	23.0	7.2	2.8	6.4	1.4	1.8	0.4	5.3	1.2	3.0	1.2	1.6	0.1	0.3	0.2
|0001⟩	28.7	35.9	4.5	8.4	3.9	5.0	1.1	1.0	3.6	2.2	1.3	1.9	1.1	0.3	0.7	0.4
|0010⟩	14.3	3.5	51.2	10.3	3.2	0.5	4.1	1.1	3.6	0.7	4.7	1.0	0.4	0.2	0.6	0.5
|0011⟩	6.6	13.4	17.5	40.7	1.1	1.8	2.6	2.4	3.0	2.6	4.0	2.5	0.7	0.0	0.7	0.4
|0100⟩	15.0	3.2	3.3	0.7	47.9	11.9	6.2	1.9	1.6	0.2	0.7	0.5	3.6	0.8	1.2	1.3
|0101⟩	6.9	9.1	1.9	3.0	17.6	40.8	3.4	4.4	1.9	0.9	0.7	0.3	3.5	2.3	1.4	1.8
|0110⟩	5.7	0.8	9.4	3.0	13.6	3.1	37.1	9.7	1.3	0.1	2.3	0.8	3.0	0.9	4.2	5.0
|0111⟩	2.9	4.3	4.3	8.0	4.3	9.5	21.0	31.4	1.7	0.8	1.5	1.0	2.8	1.6	2.5	2.6
|1000⟩	9.4	3.1	3.5	1.7	3.5	0.5	1.2	0.5	32.0	23.8	6.0	2.4	8.5	1.5	1.6	0.8
|1001⟩	4.2	5.5	1.7	4.1	1.0	0.6	0.4	0.4	19.4	47.0	2.6	4.4	4.1	2.6	1.5	0.6
|1010⟩	5.1	1.2	11.7	1.3	1.3	0.1	1.1	0.5	12.5	3.0	45.1	7.5	2.5	0.3	3.6	3.3
|1011⟩	4.3	2.7	7.0	5.5	2.3	1.0	1.7	0.7	5.9	10.5	15.5	33.1	2.0	1.3	4.0	2.6
|1100⟩	2.3	0.2	1.8	0.3	5.6	1.4	3.6	1.2	14.2	1.9	2.3	1.3	46.5	10.5	2.9	4.0
|1101⟩	4.0	1.6	1.3	1.5	5.8	4.6	2.8	3.1	7.7	7.6	2.5	1.4	16.1	32.8	4.6	2.6
|1110⟩	2.0	0.2	3.2	0.5	5.0	0.4	7.9	2.4	4.8	1.5	11.3	5.8	11.3	2.1	13.0	28.5
|1111⟩	1.3	1.4	1.4	1.7	3.2	3.6	3.5	5.3	2.4	3.8	5.0	6.4	4.6	7.7	22.3	26.4

14 2q gates

quDits, "F" ~ 72%



Input \ Measured	|0000⟩	|0001⟩	|0010⟩	|0011⟩	|0100⟩	|0101⟩	|0110⟩	|0111⟩	|1000⟩	|1001⟩	|1010⟩	|1011⟩	|1100⟩	|1101⟩	|1110⟩	|1111⟩
|0000⟩	79.6	4.0	2.4	1.7	5.3	0.4	0.4	0.1	5.0	0.4	0.1	0.1	0.1	0.1	-0.0	0.1
|0001⟩	4.1	69.7	1.0	2.6	0.1	4.6	0.1	0.2	0.9	14.3	0.4	0.6	0.0	1.2	0.1	-0.0
|0010⟩	3.3	3.3	75.1	6.7	0.2	0.3	4.0	0.6	0.2	0.3	4.6	0.8	0.1	-0.0	0.4	-0.0
|0011⟩	2.4	4.2	5.3	74.1	0.2	0.1	0.5	3.5	0.1	0.4	0.6	7.7	-0.0	-0.0	0.2	0.5
|0100⟩	10.3	0.7	0.4	0.0	75.1	5.7	3.0	1.0	0.4	0.0	-0.0	-0.0	3.0	0.1	0.2	-0.0
|0101⟩	0.6	10.7	0.2	0.5	3.6	60.4	0.5	1.6	0.0	2.8	0.1	0.2	1.2	17.2	0.1	0.3
|0110⟩	0.6	0.3	6.1	1.7	2.4	1.9	69.5	7.9	-0.0	-0.0	0.5	0.2	0.6	0.1	7.1	1.1
|0111⟩	0.4	0.5	2.4	4.9	2.3	3.0	7.1	66.9	0.0	-0.0	0.3	0.3	0.4	0.8	1.3	9.4
|1000⟩	7.5	0.4	0.2	0.1	0.6	0.1	-0.0	-0.0	76.3	4.7	2.4	0.8	6.1	0.5	0.3	0.1
|1001⟩	0.3	6.4	0.0	0.2	0.0	0.4	-0.0	-0.0	3.5	79.2	1.0	2.3	0.4	5.9	0.1	0.2
|1010⟩	0.4	0.3	6.4	0.3	0.0	0.1	0.5	0.1	3.9	2.5	71.9	7.0	0.3	0.3	4.9	1.2
|1011⟩	0.1	0.3	0.4	6.8	0.0	-0.0	0.1	0.2	2.6	3.4	5.6	74.8	0.2	0.2	1.3	3.9
|1100⟩	1.3	-0.0	0.0	-0.0	6.2	0.5	0.3	-0.0	10.6	0.6	0.2	0.2	71.7	4.9	2.7	0.8
|1101⟩	0.0	0.5	0.0	0.1	0.4	7.8	0.1	0.1	0.6	11.4	0.2	0.3	3.6	71.2	0.6	3.1
|1110⟩	0.1	-0.0	0.6	0.5	0.1	0.3	0.6	4.5	0.4	0.5	2.9	7.2	1.3	5.4	6.1	69.6
|1111⟩	0.0	-0.0	0.5	0.3	0.3	0.4	5.4	1.3	0.8	0.3	8.2	2.9	4.2	1.7	64.4	9.2

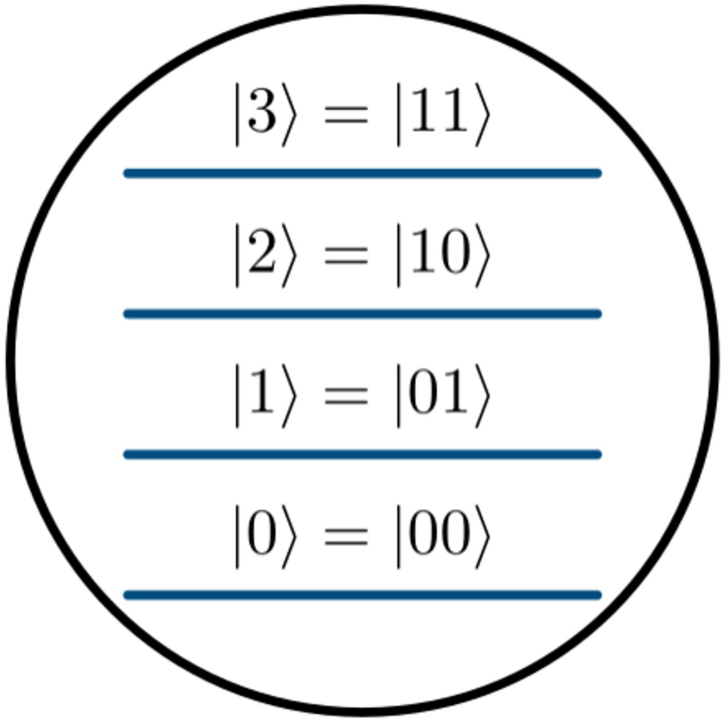
5 2q gates

d = 4: ququart ↔ 2 qubits

- Ququart space → tensor product of the spaces of two qubits

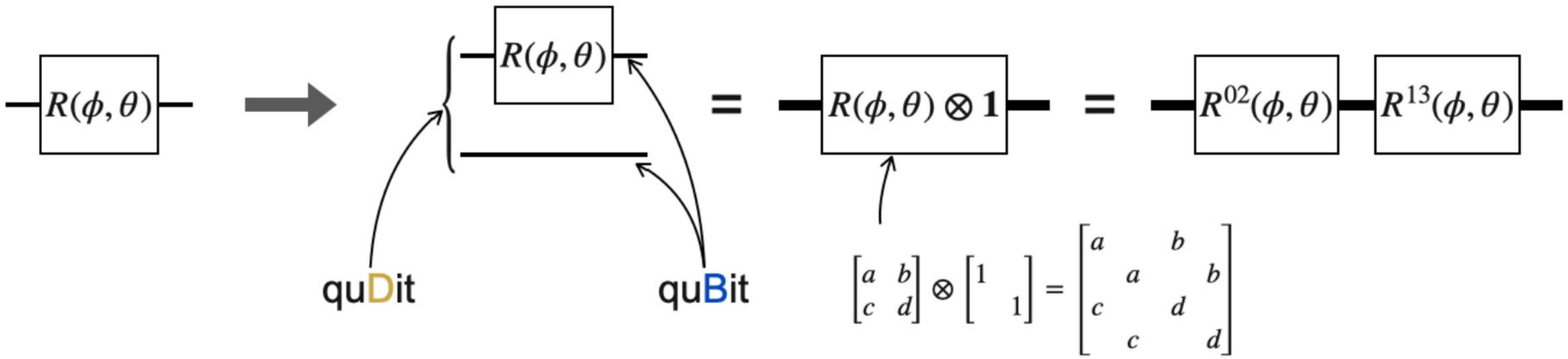
$$|0\rangle_Q \leftrightarrow |0\rangle_q \otimes |0\rangle_{q'}$$
$$|1\rangle_Q \leftrightarrow |0\rangle_q \otimes |1\rangle_{q'}$$
$$|2\rangle_Q \leftrightarrow |1\rangle_q \otimes |0\rangle_{q'}$$
$$|3\rangle_Q \leftrightarrow |1\rangle_q \otimes |1\rangle_{q'}$$

- Single- and two-qubit gates are defined by this level correspondence

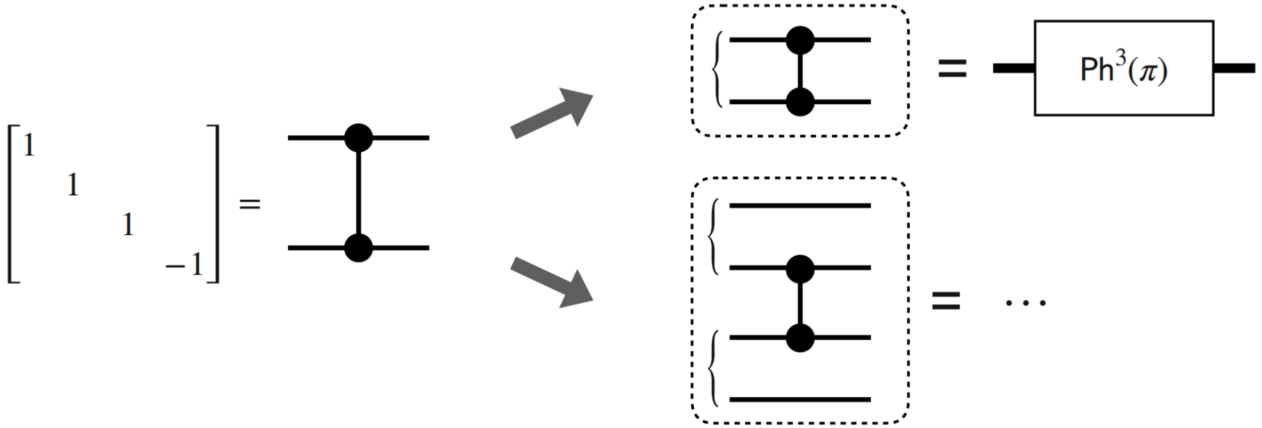


d = 4: Embedding 2 qubits in 1 ququart (d = 4)

- Single-qubit gate → 2 single-ququart gates



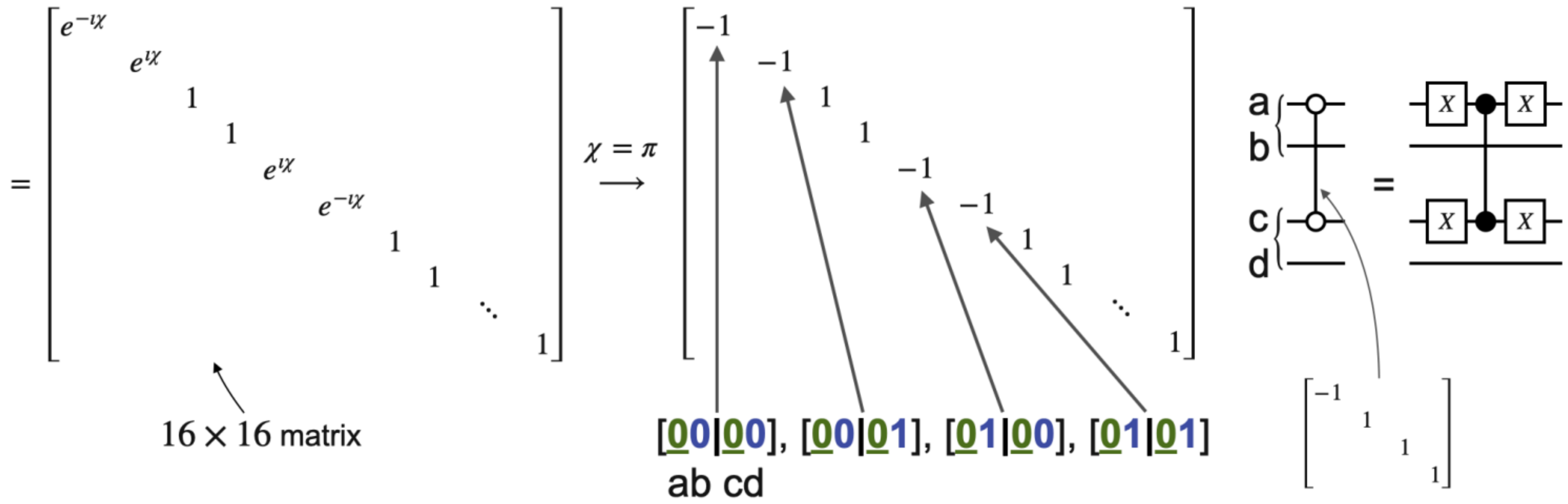
- Two-qubit gate →
  - Single-ququart
  - Two-ququart*



2-qubit gate between qubits in different ququarts

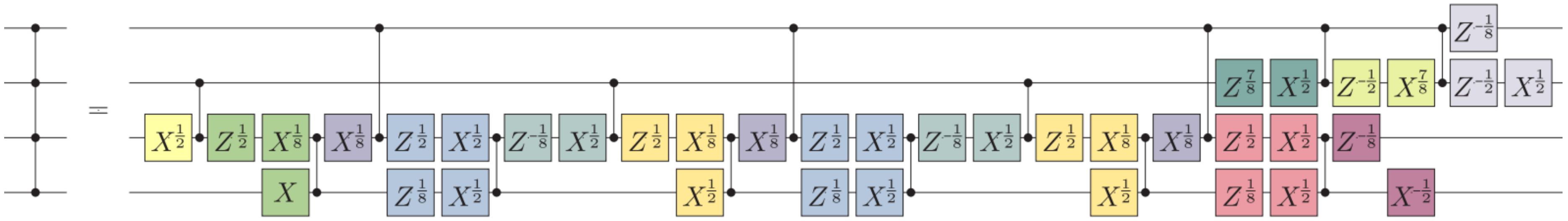
- $MS(\pi) \sim CZ$ between two qubits embedded in different ququarts

$$\mathrm{MS}(\chi) = \exp\left(-\imath\left[\sigma_x^{0,1} \otimes \sigma_x^{0,1}\right]\chi\right) \rightarrow \exp\left(-\imath\left[\sigma_z^{0,1} \otimes \sigma_z^{0,1}\right]\chi\right) = \exp\left(-\imath \mathrm{diag}[1,-1,0,0] \otimes \mathrm{diag}[1,-1,0,0]\chi\right)$$

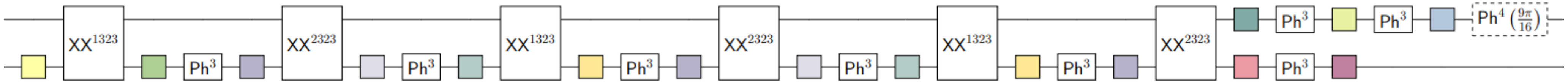


Template decomposition of multi-qubit gates with ququarts (d = 4)

- CCCZ decompositions with 4 qubits — 14 CZ gates



- CCCZ decompositions with with 2 ququarts — 6 $MS(\pi)$ gates



arXiv:2109.13223

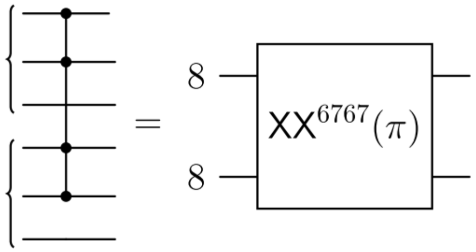
Phys. Rev. A 109, 022615 (2024)

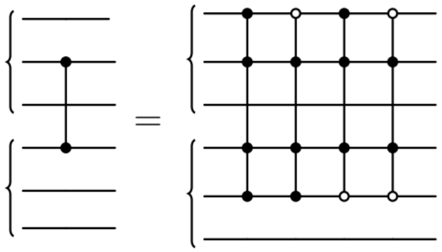
High-dimensional ion quDits

- Improved multi-qubit gate decompositions for *d* = 5, 6, 7
- Gate set for qubits in quocts (*d* = 8):
  - Single-qubit rotations
  - 4-qubit gate in different quocts with MS gate
  - 2-qubit gate in different quocts with 4 MS gates

TABLE I. Properties of $\mathsf{C}^{N-1}\mathsf{X}$ gate decompositions for N qubits embedded in $N/2$ qudits (N is even).

Qudit dimension	Number of $\mathsf{XX}^{ijk\ell}(\chi)$ gates	Depth
$d=5$	$12N-18$	$\mathcal{O}(N)$
$d=6$	$N-3$	$\mathcal{O}(N)$
$d=7$	$N-3$	$\mathcal{O}(\log N)$



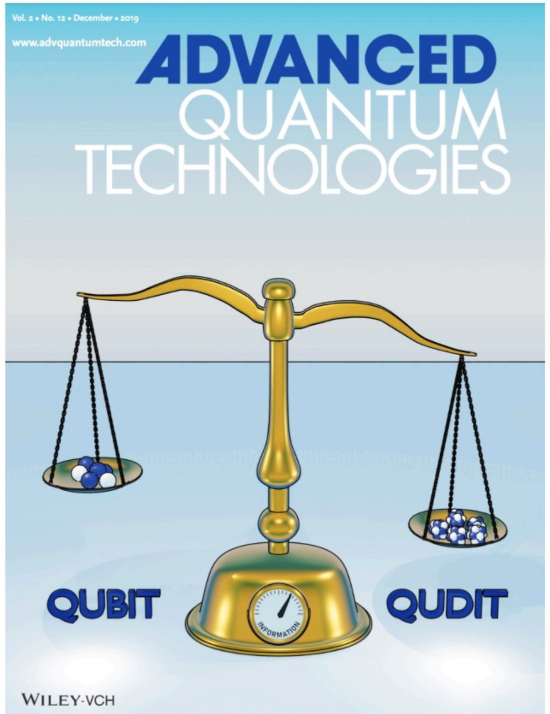


Phys. Rev. A 109, 022615 (2024)

Conclusion

- Multi-level structure of quantum systems, including ions, can be used as an additional resource for quantum computing
- When realizing qubit circuits with qudits, reduction in the number of two-particle gates can be achieved
- Single-qubit + Two-qubit + Multi-qubit gates with qudits with $d = 3,\ldots,8 \rightarrow$ arbitrary qubit circuits with trapped-ion qudits
- Improvement in CCX and CCCX gate realization with trapped-ion qutrits was demonstrated
- Simplifying multiqubit gate realization with iSWAP native operation is also possible: see Phys. Rev. A 105, 032621 (2022)
- The review on qudit-based decompositions: arXiv:2311.12003

Thank you for your attention!



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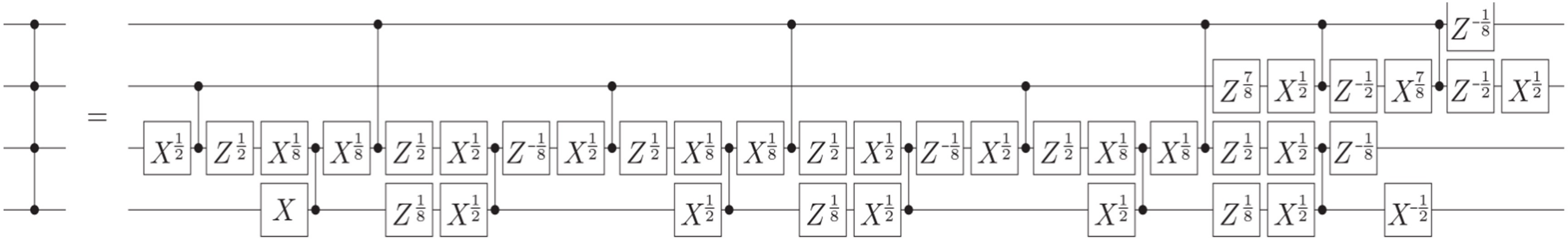
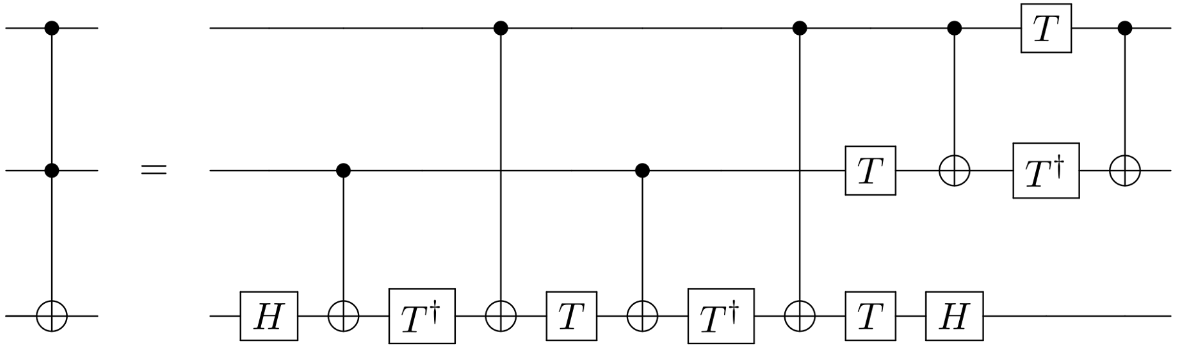
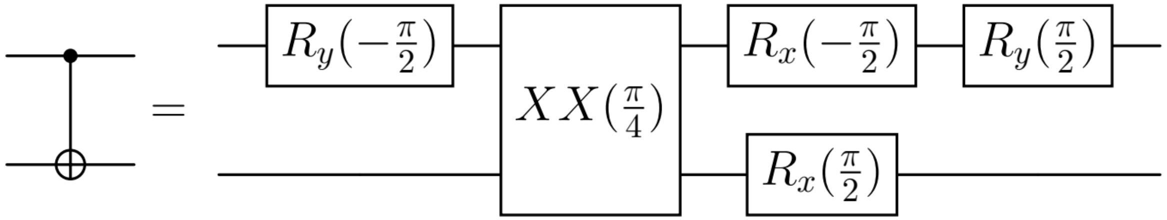
CCZ and CCCZ qubit-based decomposition

We compare with these qubit-based decomposition (6 and 14 two-qubit gates)







Grover’s algorithm with qudits: trapped-ions

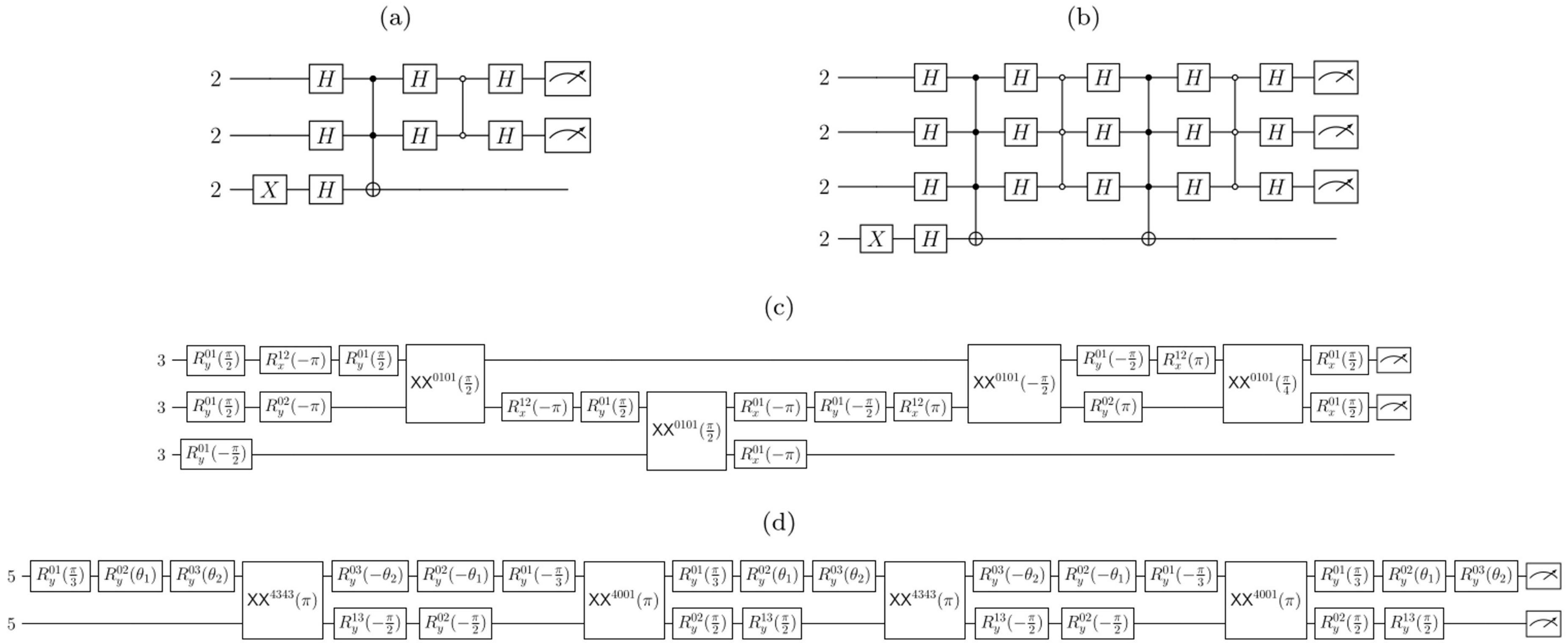


Figure 5. Circuits of Grover’s algorithm implementation. In (a) and (b) hardware-agnostic circuits for searching “11” and “111” hidden strings with $2+1$ and $3+1$ qubits are presented correspondingly. Empty circles correspond to controls on state $|0\rangle$. In (c) circuit from (a) is transpiled to the single-qutrit and two-qutrit ion gates. Multiqubit decomposition from Fig. 1 is used. The circuit depicted in (d) corresponds to the ququint-based realization of circuit (b). Straightforward qubit-to-ququint mapping is used: the first (second) qubit pairs are embedded in the first (second) ququint. The highest levels of ququints are used to simplify the implementation of multi-qubit gates, as presented in Fig. 3 (a,b). Here, $\theta_1 := 2\arcsin(3^{-1/2})$, $\theta_2 := \pi/2$.

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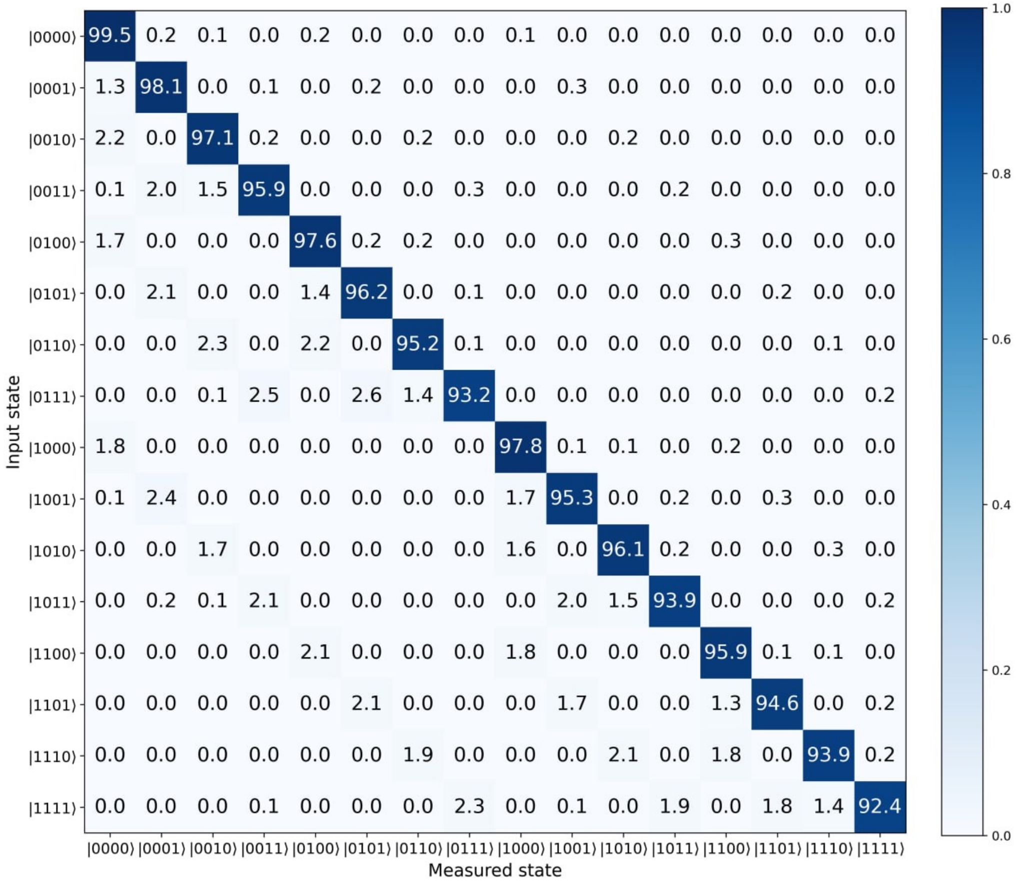
Ion qudit processor parameters

- 10 ions
- d = 4
- 1Q: 99.95%
- 2Q: 92-95%
- 10 ions
- All-to-all coupling map



Readout