The problems of long-wave neutron optics

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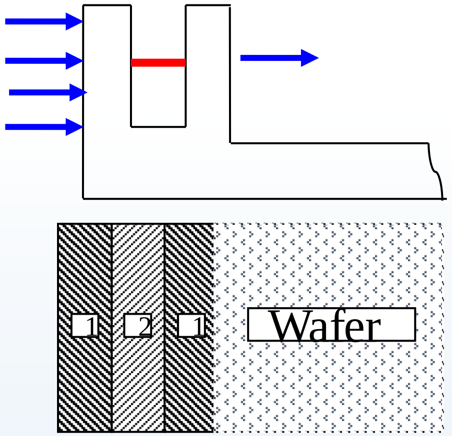
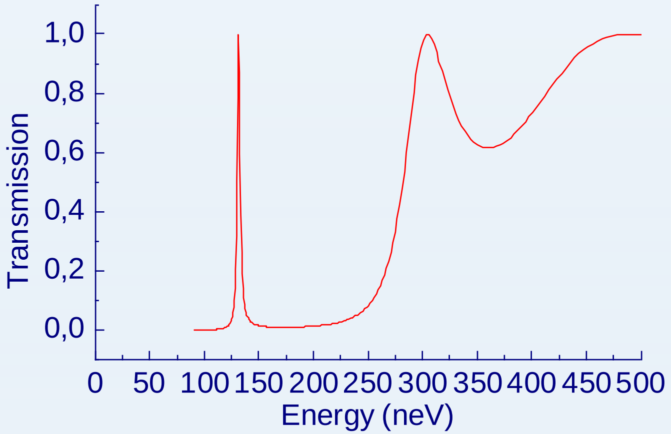
Dubna
15 May 2024

UCN spectroscopy.

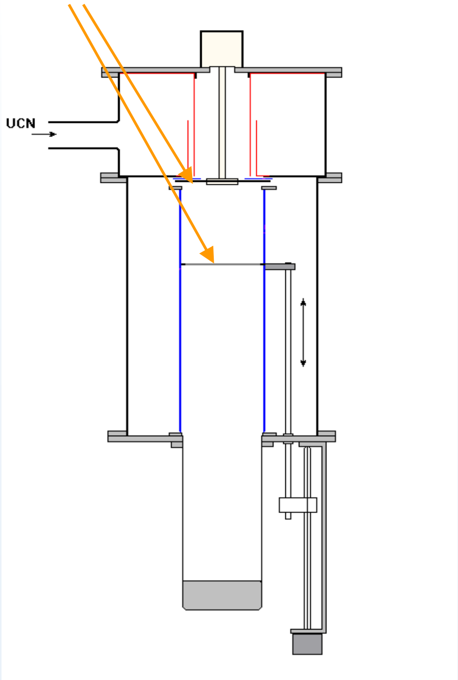
Gravity spectrometer with Fabry-Perot interferometers

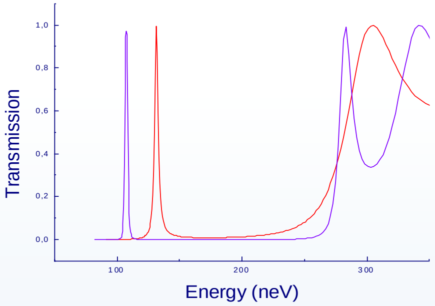


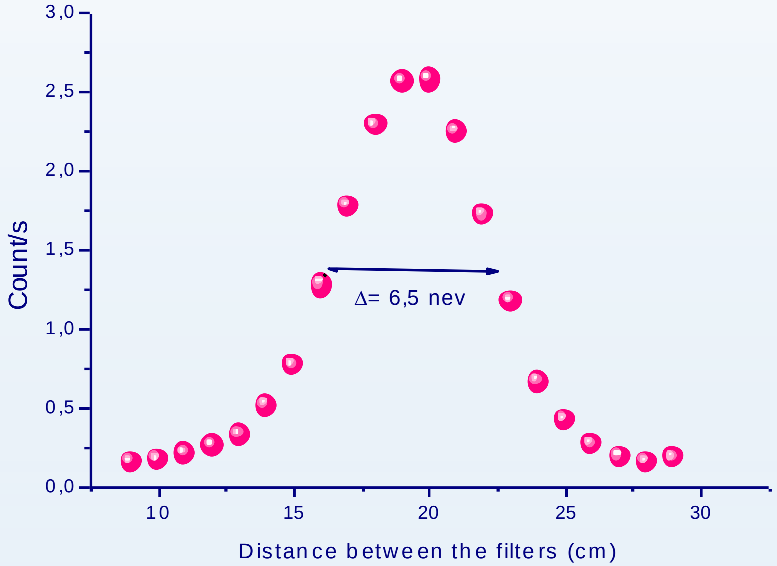
$$U_{1,2} = \frac{2\pi\hbar^2}{m}(\rho b)_{1,2}$$



Two NIFs with variable distance between them

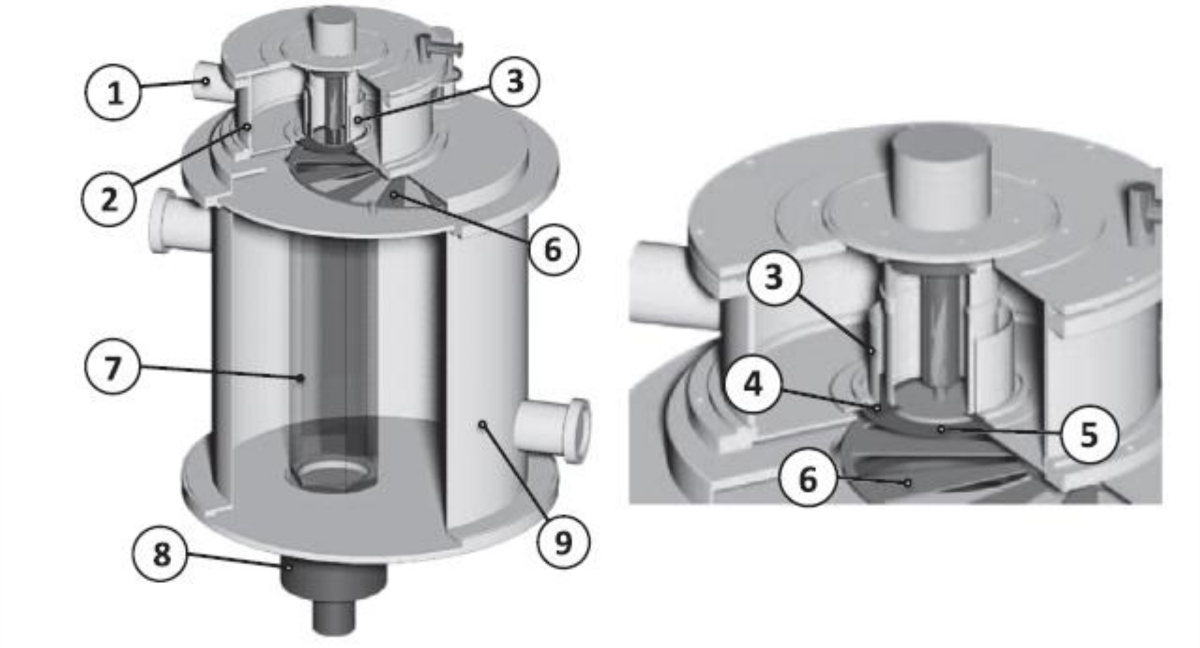




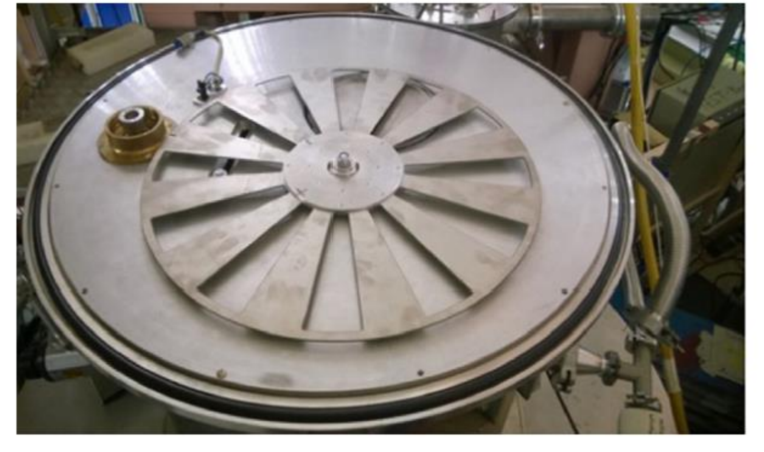


I.V.Bondarenko, S.N.Balashov, A.Cimmino, et al. NIM A, 440 (2000), 591-596

Time-of-flight Fourier spectrometer



1 – feeding guide,2 – entrance chamber,3 – annular channel, 4 – filter- monochromator, 5 – grating, 6 – rotor of the Fourier modulator, 7 – vertical glass guide, 8 – detector, 9 – vacuum vessel

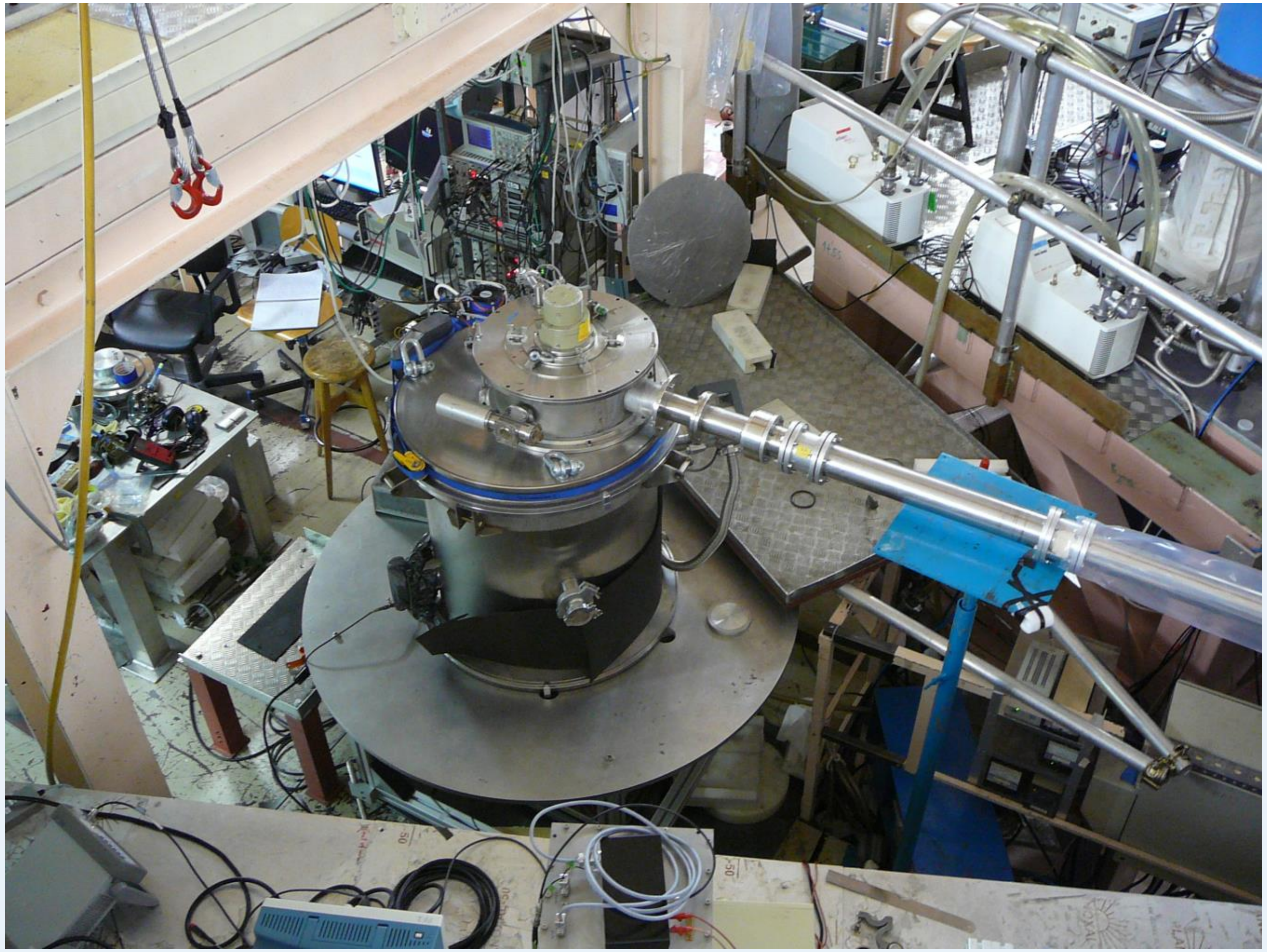


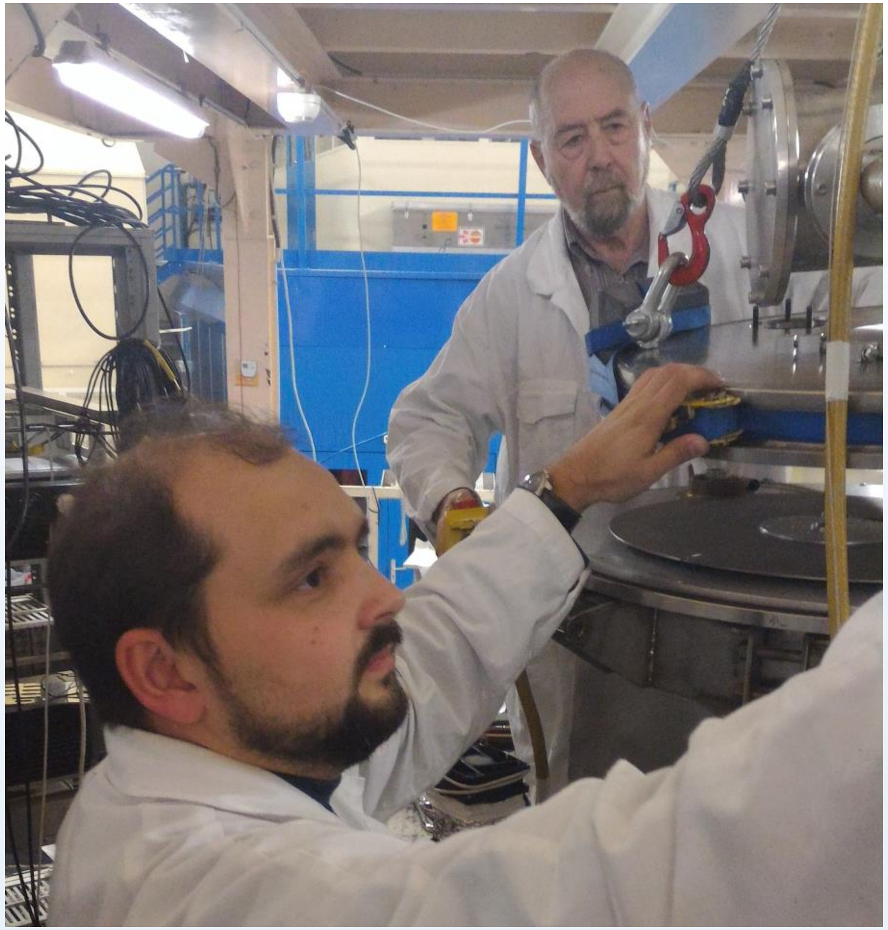
The spectrometer may be used for obtaining UCN energy spectra in the energy range of 60–200 neV with a resolution of about 5neV. The accuracy of determination of the line position was estimated to be several units of 10^{-10} eV.

Modulation frequency 6 – 360 Hz

G.V. Kulin, A.I. Frank, S.V. Goryunov et al. NIM A, **819** (2016) 67-72

Time-of-flight Fourier spectrometer

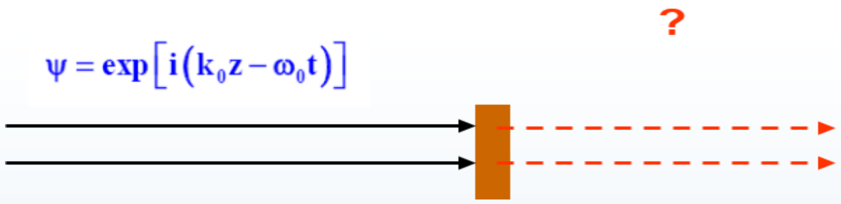


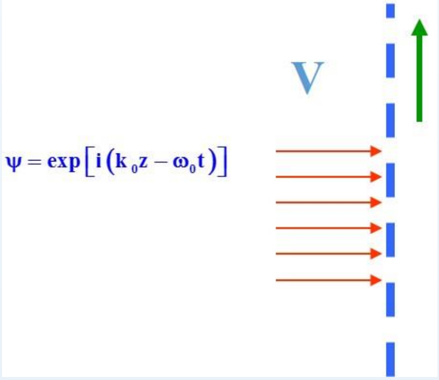


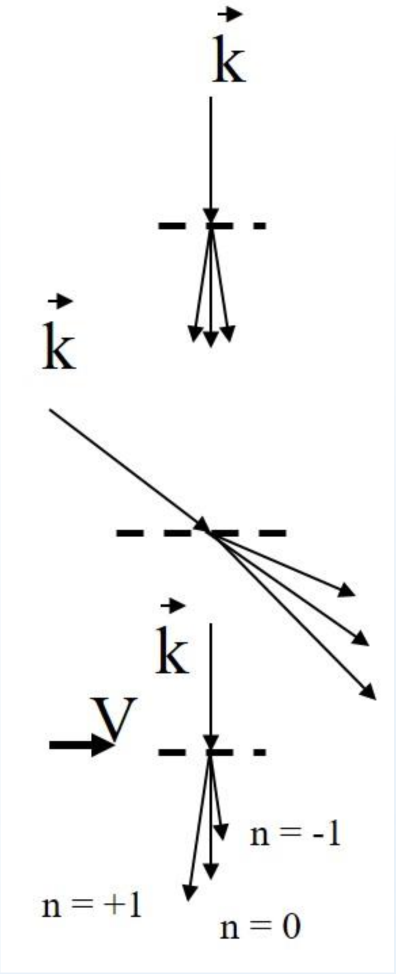
Nonstationary diffraction

a) Moving grating

Diffraction by a moving grating as a phase modulation of the neutron wave







$$\Psi(z,y,t) = \sum_j a_j \exp[i(k_j z + q_j y - \omega_j t)]$$

$(k_0 L << 1)$

$$\boxed{a_j = \frac{1}{d}\int_0^L T(x)\exp(-iq_j x)\,dx}$$

$q_j = j \cdot \left(2\pi/d\right) = jq_0$

$\omega_j = \omega_0 + j\Omega$

$k_j \cong k_0\left(1 + j\frac{\Omega}{\omega_0}\right)^{\frac{1}{2}}$

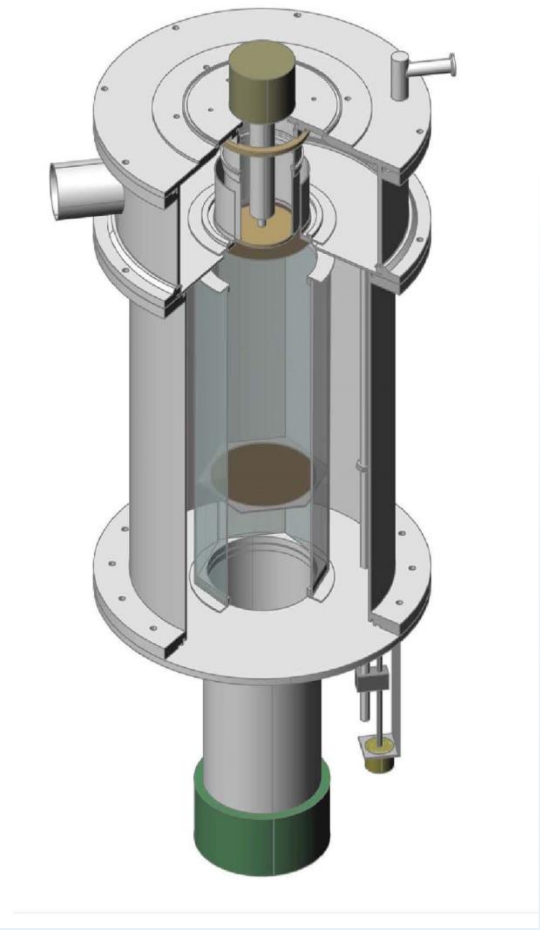
$j = 0, \pm 1, \pm 2....$

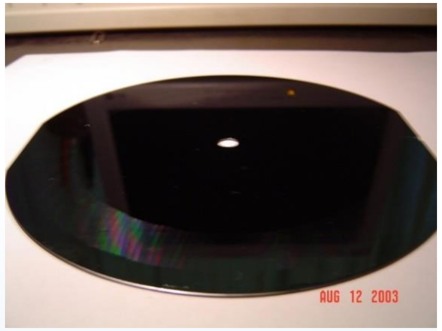
$\Omega = \frac{2\pi}{T} = 2\pi f = 2\pi\left(\frac{V}{d}\right)$

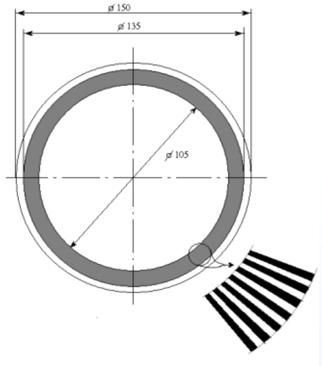
d – space period of a grating

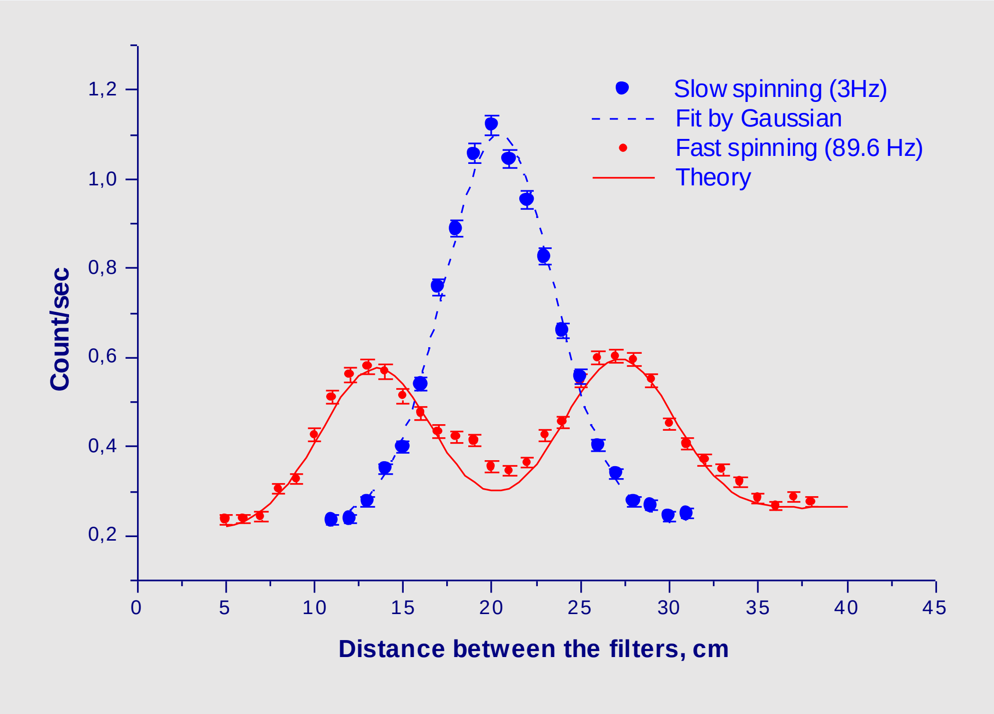
V.G.Nosov, A.I.Frank. J. Mosc. Phys. Society, 1, 1 (1991).
A.I.Frank, V.G.Nosov. Phys. Lett. A. 188, 120 (1994).

Demonstration of quantum spectrum splitting

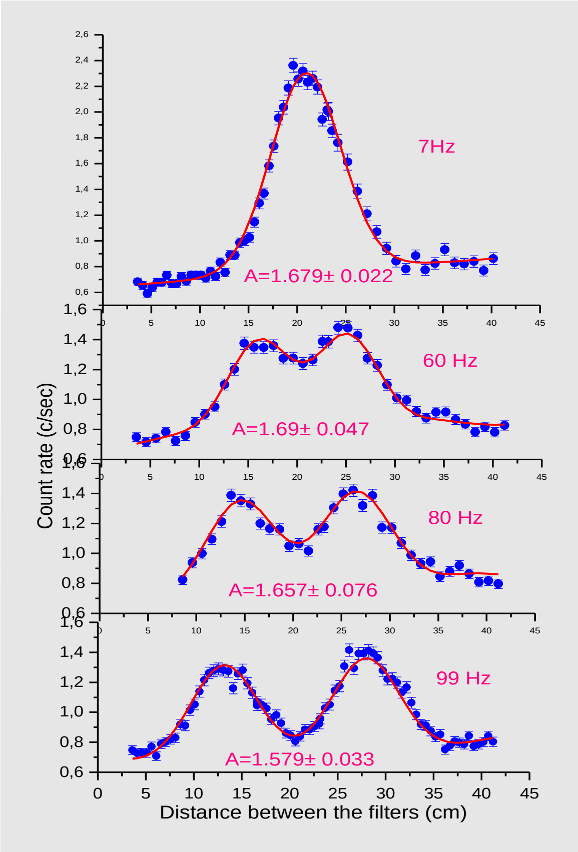








***A*.I. Frank, . Geltenbort, G. V. Kulin et al. Phys. Lett. A 311 (2003) 6**

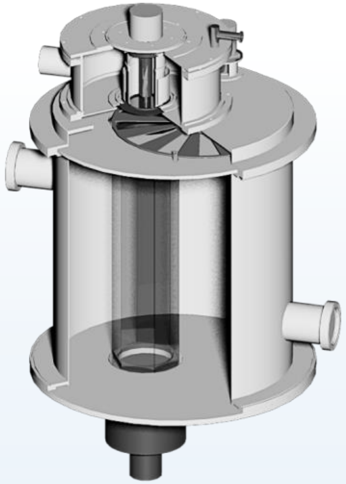


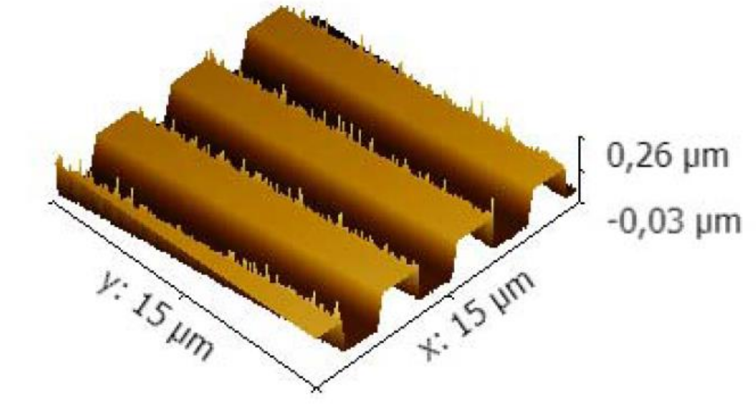
$$|\mathbf{a}_1|^2_{\mathbf{th}} = 0.405$$

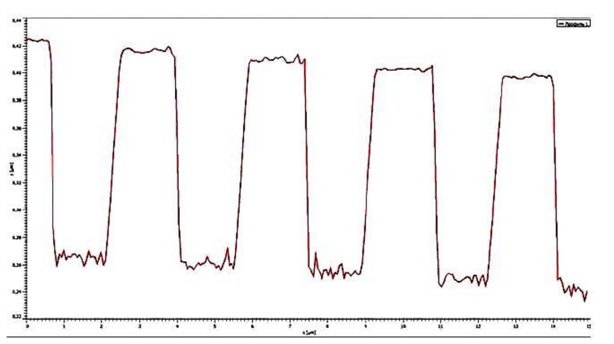
$$|\mathbf{a}_1|^2_{\mathbf{exp}} = 0.383(8)$$

A.I. Frank, P. Geltenbort, G. V. Kulin et al. JETP Letters, 81 (2005) 427.

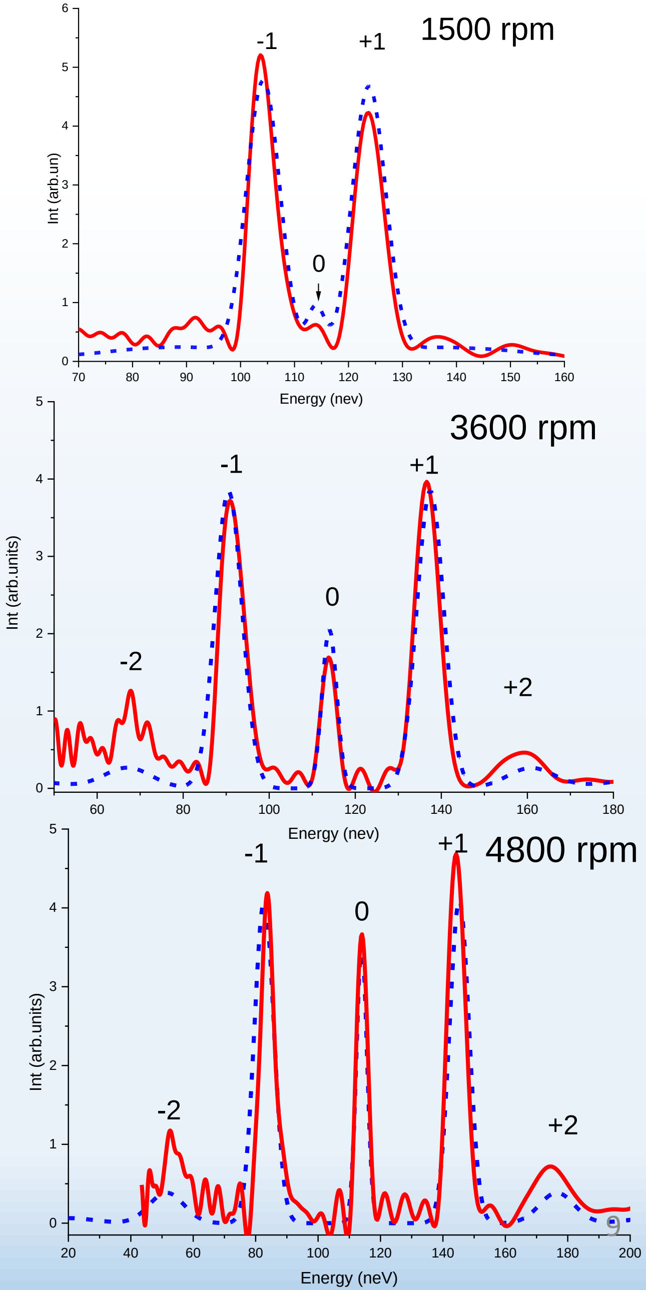
TOF Fourier spectrometry and comparing obtained spectra with dynamic theory of neutron diffraction





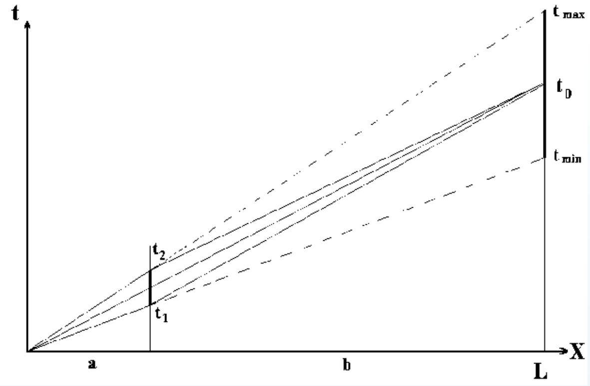


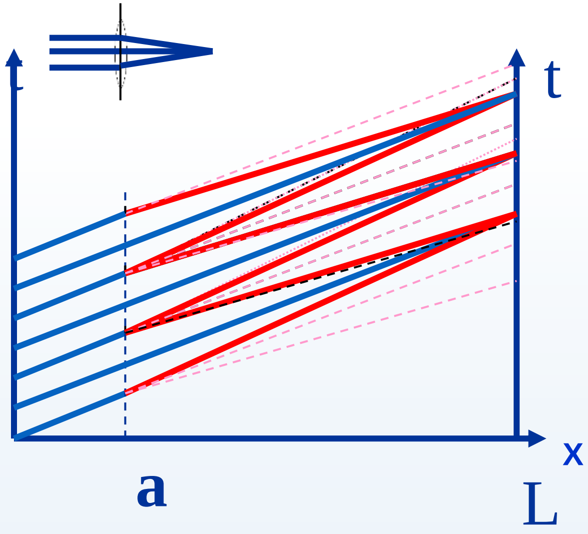
Angular period of grating
0.0665mrad
(4µ at the middle diameter)

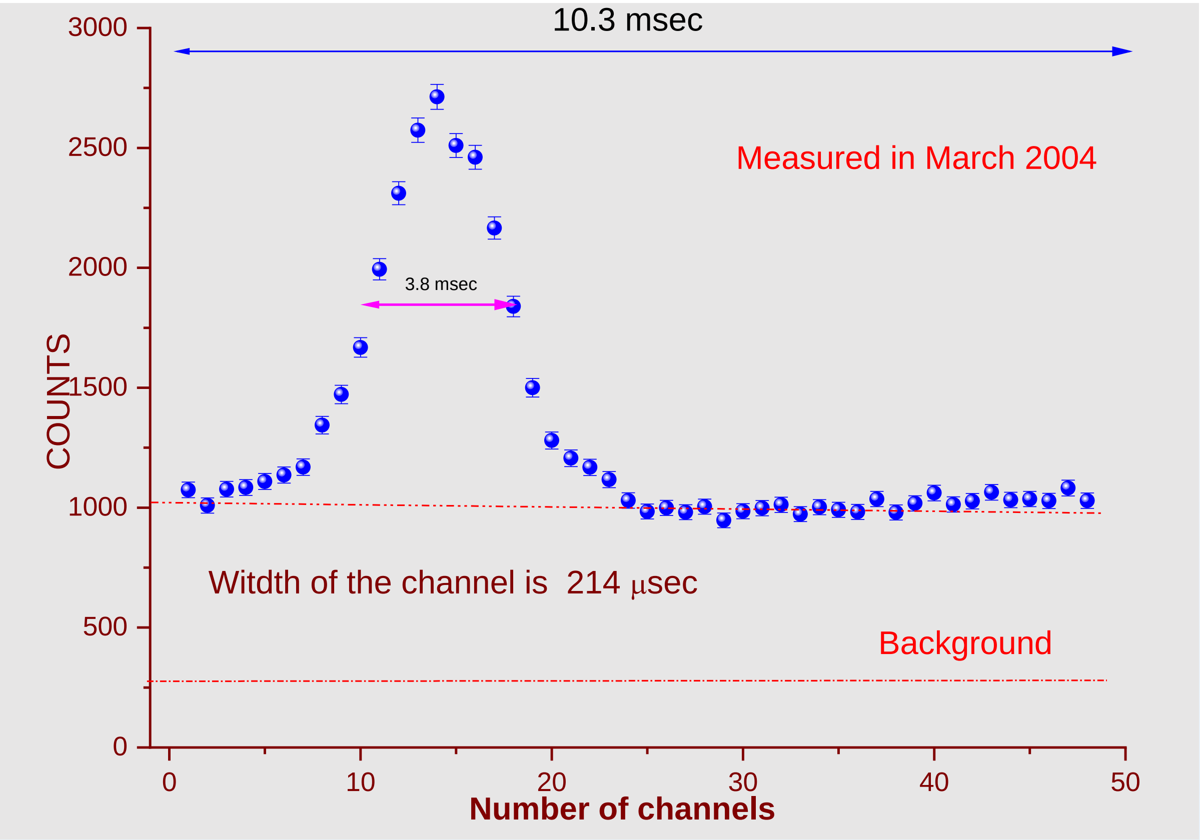


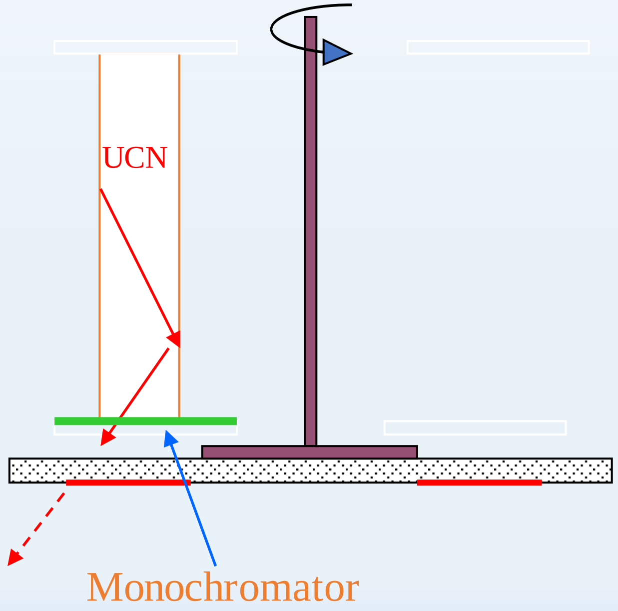
G.V. Kulin, A.I. Frank, S.V. Goryunov et al. Phys. Rev. A 93033606, (2016)

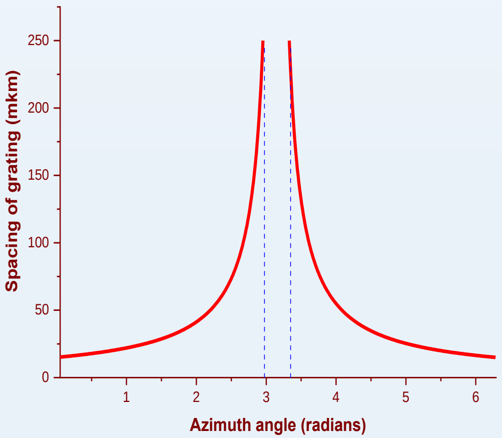
Neutron focusing in time





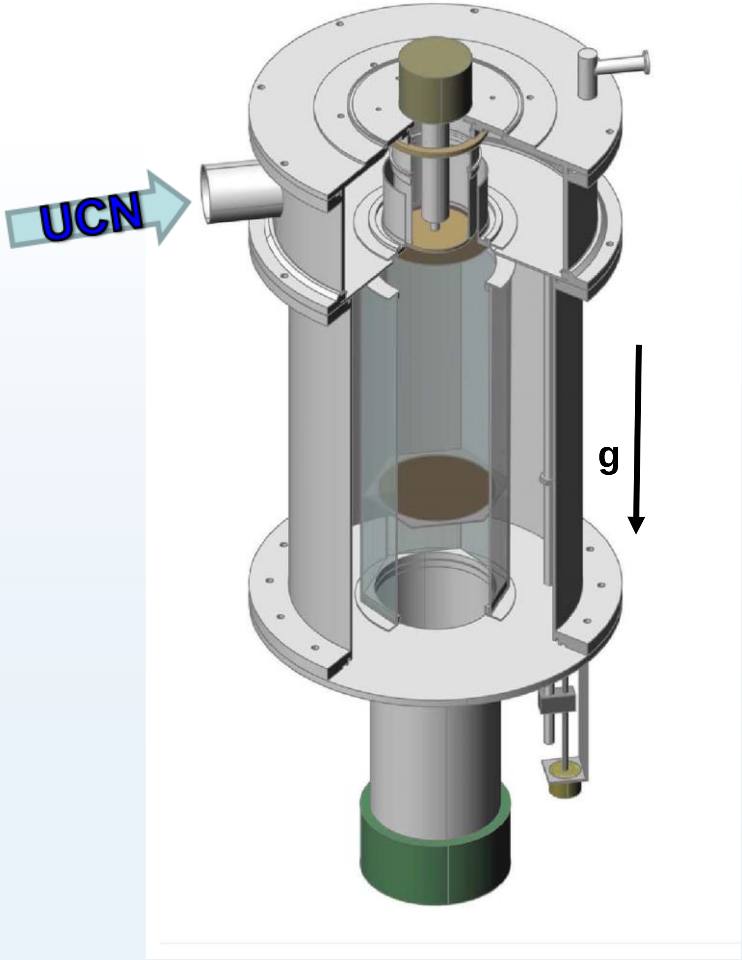


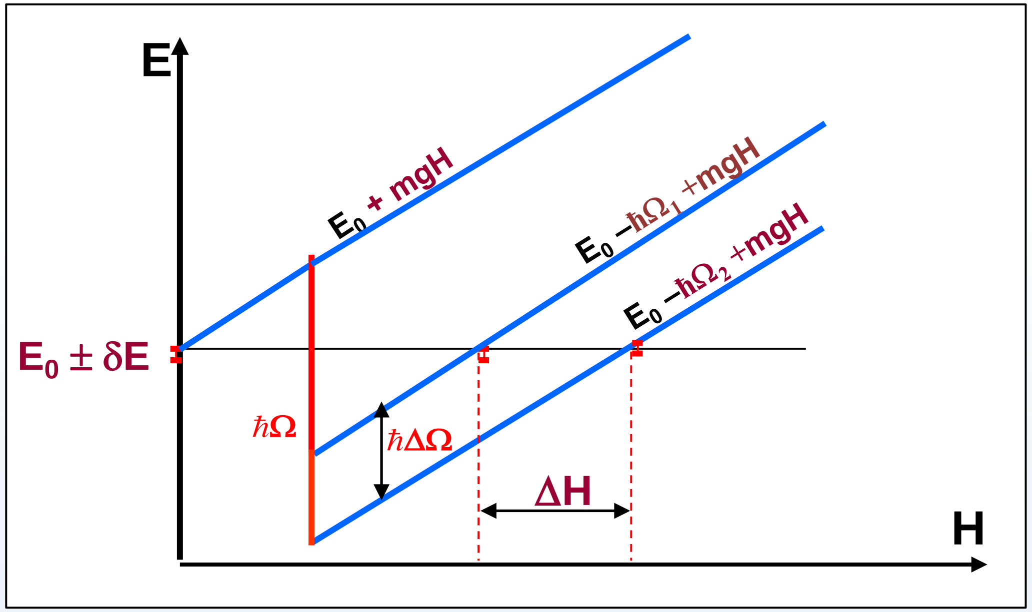




A. I. Frank, P. Geltenbort, G. V. Kulin et al. JETP Lett. 78, (2003) 188

Test of the weak equivalence principle for neutrons





The idea was to compare the change of energy **mgH** with energy **ħΩ** transferred to neutron by a moving grating

Frank A.I., Masalovich S.V., Nosov V.G. (ISINN-12). E3-2004-169, 215, Dubna, (2004)

$$\boxed{m_g a_n = \hbar \frac{\Delta\Omega}{\Delta H}}$$

$$\boxed{1 - \frac{m_g a_n}{m_n g} = (1.8 \pm 2.1) \cdot 10^{-3}}$$

A.I. Frank, P. Geltenbort, M. Jentschel, et al. JETP Letters, 86, 225 (2007)

International symposium “New trends of developing fundamental and applied physics: problems, achievements and prospects”. Tashkent, November 10-11, 2016

Nonstationary diffraction

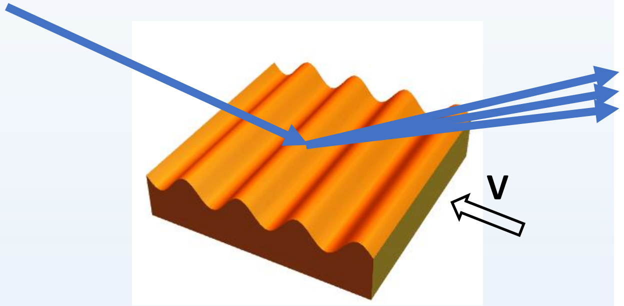
a) Surface Acoustic Waves (SAW)

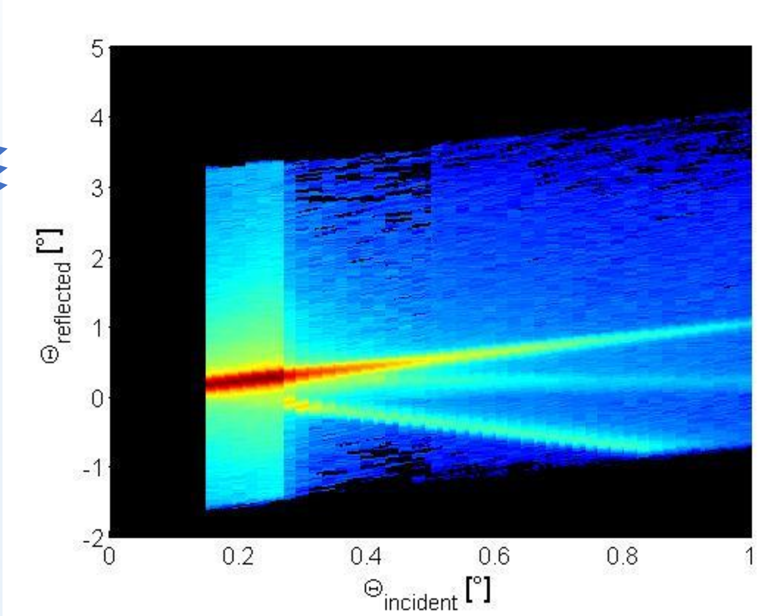
Nonstationary diffraction by SAW

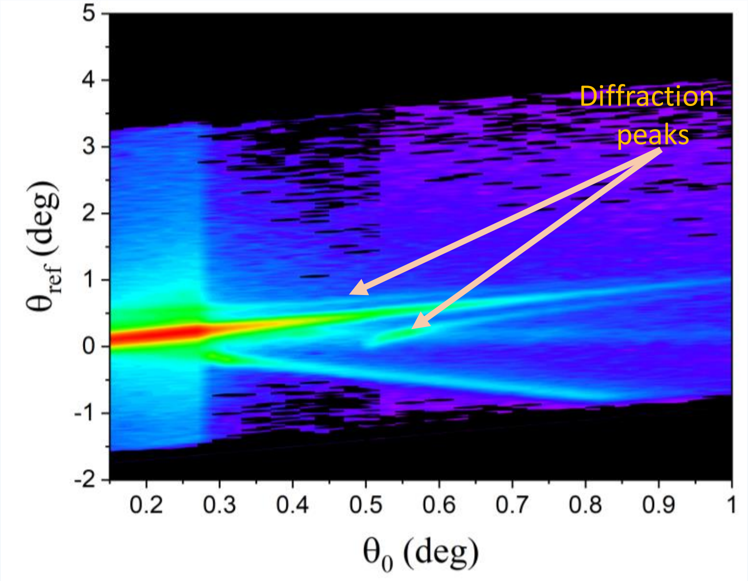
Motivation:

a) This nonstationary quantum phenomenon was observed only in one experiment and poorly researched

b) The matter from which the neutrons are reflected moves with a gigantic acceleration



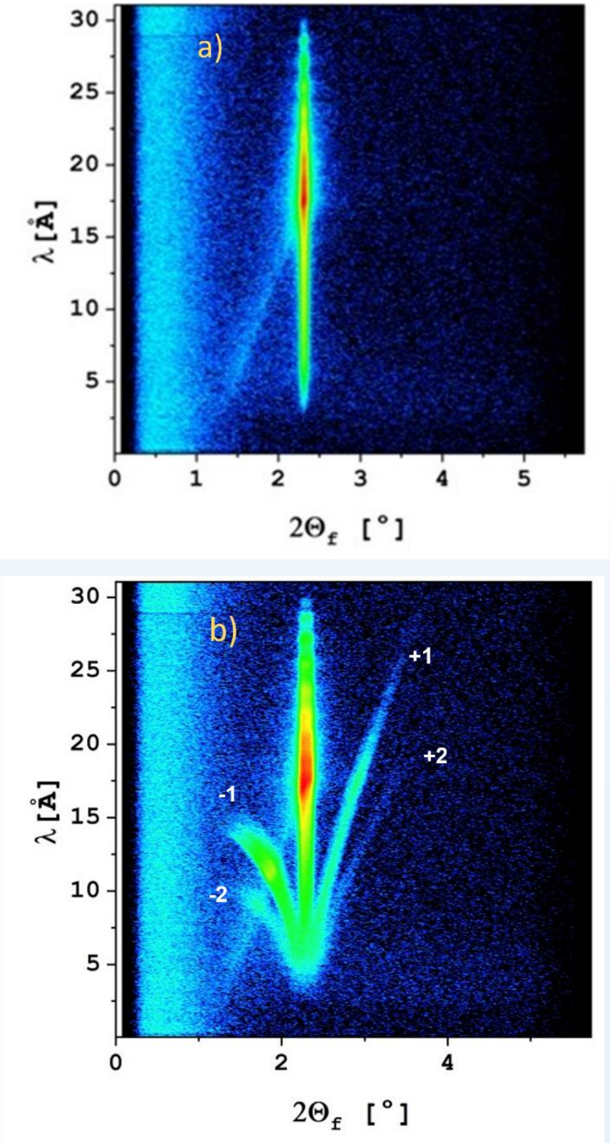




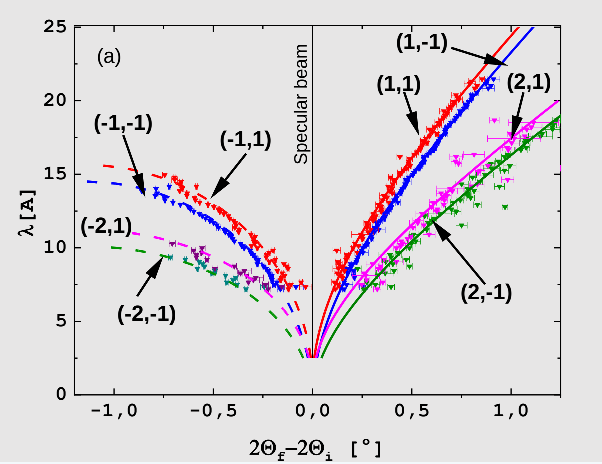
$\lambda = 4Å$ $\theta - 2\theta$ geometry

G. V. Kulin, A. I. Frank, V. A. Bushuev et al. Phys. Rev. B, 101, 165419 (2020).

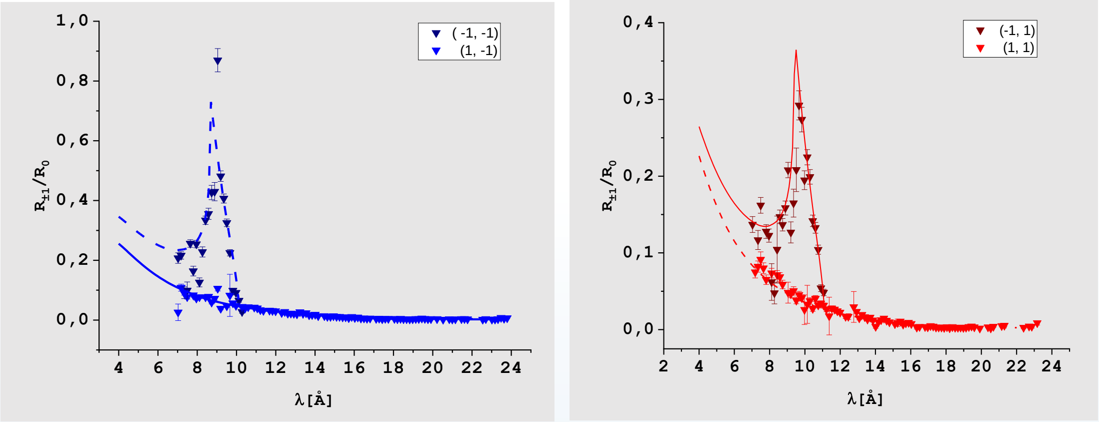
TOF mode. Cold neutrons



The paper in preparation



Angular distributions of diffracted beams as a function of wavelength



The intensity of ± 1 orders of diffraction. Theory and experiment.

	Type of experiment	E, eV	Acceleration, m/sec^2	Critical accel., m/sec^2	Reference
1.	Reflection from surface acoustic waves	2.8×10^{-4} 6.7×10^{-4}	**6.3×10^{7}**	**2×10^{9}** **4.8×10^{9}**	W.A.Hamilton, A.G. Klein, G.I. Opat, P.A. Timmins, 1987
2.	Reflection from vibrating mirror	1.4×10^{-4}	**10^{6}**	**1.1×10^{9}**	J. Felber, R. Gahler, C. Rauch and R. Golub, 1996
3.	Reflection from surface acoustic waves	4.4×10^{-3}	**4×10^{8}**	**3.4×10^{10}**	**NREX, FRM II**
3.	Reflection from surface acoustic waves	3.3×10^{-3} (5Å) — 1.3×10^{-4} (25Å)	**1×10^{7} (35MHz)** **—** **1×10^{9} (117MHz)**	**1×10^{9} (25Å)** **—** **4.5×10^{10} (5Å)**	**D17, ILL**

Neutron waves in matter

a) Dispersion law

Refractive index and potential-like dispersion law

$$k^2 = k_0^2 - 4\pi\rho b$$

L.Foldy, 1945

Potential –like dispersion law (PDL)

$$\boxed{n = \frac{k}{k_0}} \qquad \boxed{n^2 = 1 - \frac{4\pi}{k_0^2}\rho b}$$

$$\boxed{U = \frac{2\pi\hbar^2}{m}\rho b = \mathrm{const}} \quad \rightleftarrows \quad u = \frac{2\pi\hbar^2}{m} b\delta\left(\vec{r} - \vec{r}_j\right)$$

$$k_{0\perp} \le k_b = \left(4\pi\rho\left|b\right|\right)^{1/2}$$

Total reflection

$$\left|k_0\right| \le k_b \Rightarrow \mathbf{UCN}$$

Dispersion law zoo (multiple scattering)

$$k_1^2 = k_0^2 + 4\pi\rho C f_0 \qquad C = \begin{cases} C = \dfrac{1}{1-(4\pi/3)\rho\alpha} & \text{for light} \\ C = 1 \;\left(f_0 = -b \right) & \text{for neutrons} \end{cases}$$

Lax, 1951

$$n^2 = 1 - \frac{4\pi\rho}{k_0^2}\left(C' - iC''\right)\left(b' - ib''\right)$$

$b''/b' \approx 10^{-4}\text{-}10^{-5}$ $C''b' \cong b''$

I. M.Frank, 1974

Eeffective field corrections

$$n^2 = 1 + \frac{4\pi}{k^2}\rho f c, \qquad f = -b + ikb^2 \qquad c = \left(1 - J\right)^{-1} \approx 1 + J' + J''$$

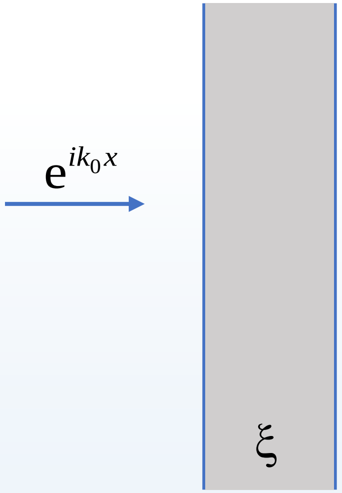
$$J = Nb\int \exp\left(i\mathbf{k}\cdot\mathbf{r}\right)G\left(\mathbf{r}\right)\left[1 - g\left(r\right)\right]d\mathbf{r} \qquad G\left(\mathrm{r}\right) = \exp\left(ikr\right)/r$$

V.F.Sears,1982

$$\boxed{n^2 = 1 - \frac{4\pi\rho b/k_0^2}{1 + \left(4\pi\rho b/nk_0^2\right)\int e^{ix}\sin\left(nx\right)\left[g\left(x/k_0\right) - 1\right]dx}}$$

M. Warner & J.E.Gubernatis,1985

UCN and hypotheses of super-slow neutrons



$$\psi_{scat} = 2\pi\rho b \frac{e^{ik_0x}}{ik_0}\xi \qquad |\psi_{scat}| << 1, \quad \xi \gg a \approx \rho^{-1/3}$$

$$\frac{2\pi\rho b}{k_0} a << 1$$

If $k_0 \le 4\pi\rho b a$ the rescattering at the distance $\hat{\rho} = \left(k_b^2 a\right)^{-1}$ take place, what leads to the uncertainty $\Delta k \approx \hat{\rho}^{-1} \approx k_0$

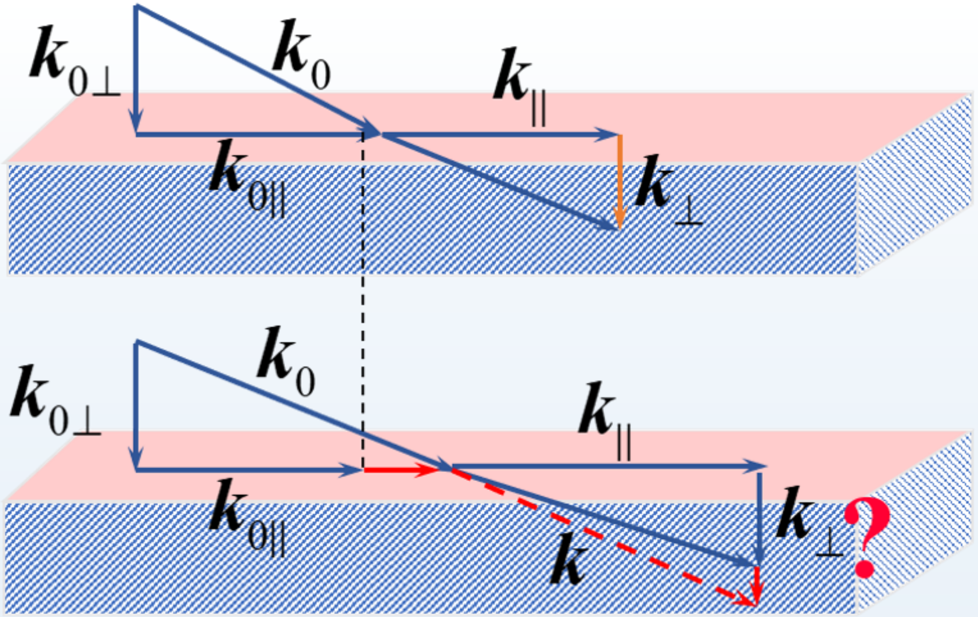
Region of applicability of PDL $k_0 >> 4\pi\rho b a = \chi^2 a$ $\chi^2 = 4\pi\rho b$

Unknown dispersion law for super slow neutron $(v \le 10\text{cm/s})$

Small correction for the dispersion law to UCN are possible

V.G.Nosov & A.I.Frank Phys. Rev.A. **55** (1997) 1129

Specific feature of the potential dispersion law



$$k_{II}^2 = k_{0II}^2$$

$$k^2 = k_0^2 - \chi^2; \qquad \chi^2 = 4\pi N b$$

$$k_\perp^2 = k_{0\perp}^2 - \chi^2; \qquad b \quad \chi^2 = 4\pi\rho b$$

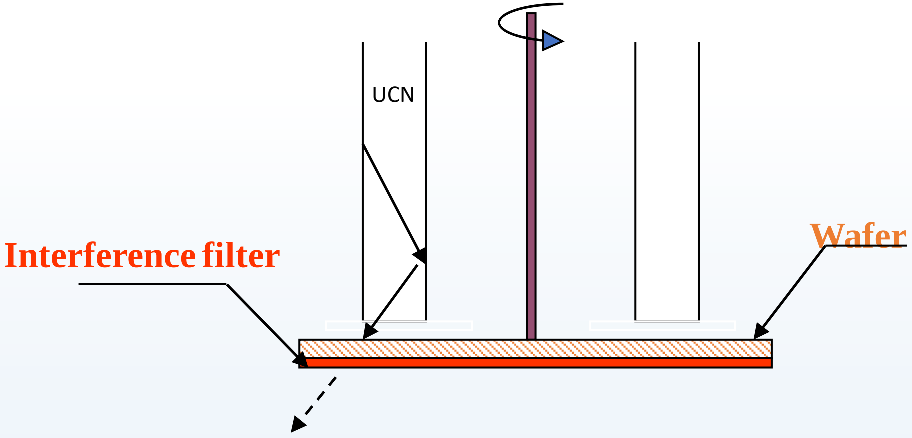
I.M.Frank, 1974,
A,G.Klein, S.A.Werner, 1983

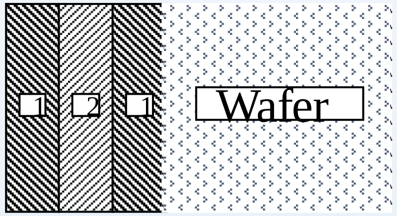
$$k^2 = k_0^2 - \chi^2 + \varepsilon\left(k_0^2\right); \qquad \chi^2 = 4\pi\rho b$$

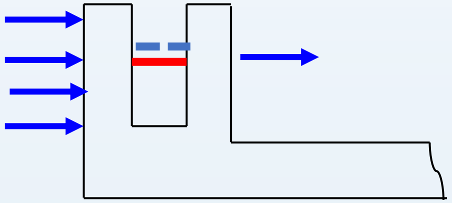
$$k_\perp^2 = k_{0\perp}^2 - \chi^2 + \varepsilon\left(k_0^2\right);$$

Strategy of the test experiment – looking for $k_\perp$ with variation of $k_{\|}$ at $k_{0\perp} = \mathrm{const}$

First attempt of the testing dispersion law for UCN using rotating interference filter

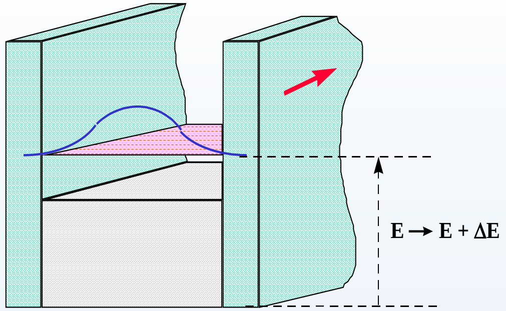


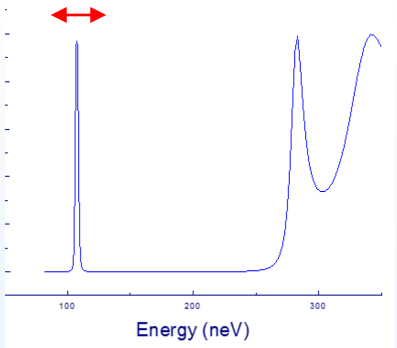




$$U_{1,2} = \frac{2\pi\hbar^2}{m}(\rho b)_{1,2}$$

V.G.Nosov and A.I.Frank. Phys. At.Nucl. 58, 402 (1995) ISINN 3 (1995)



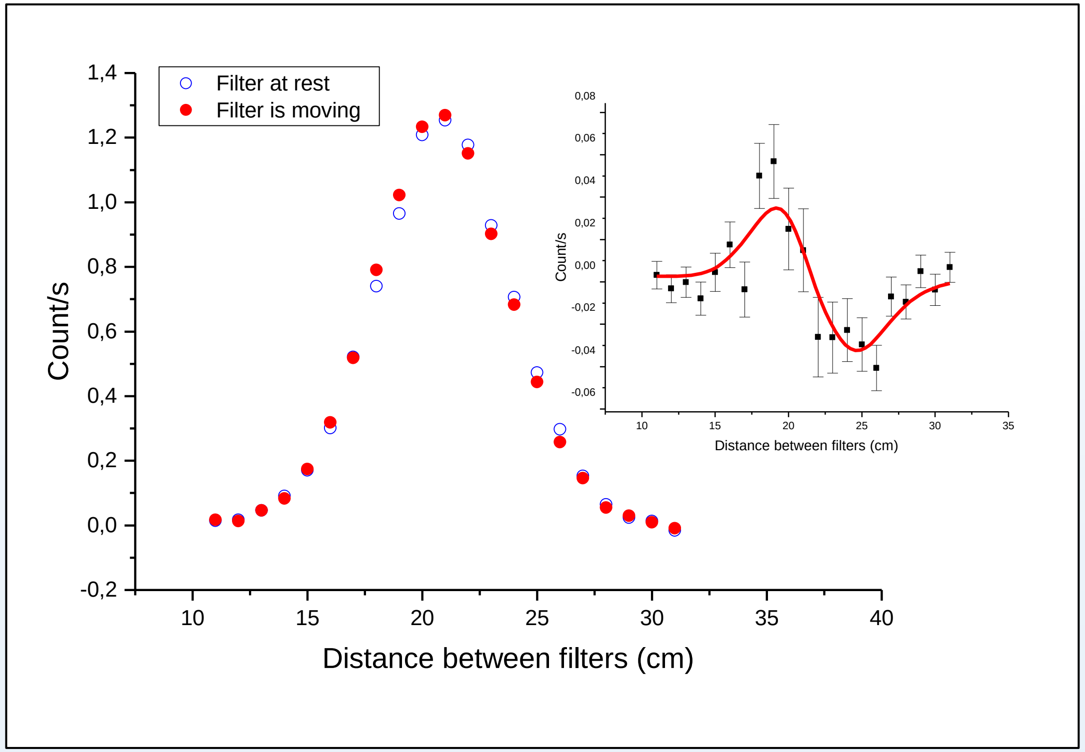


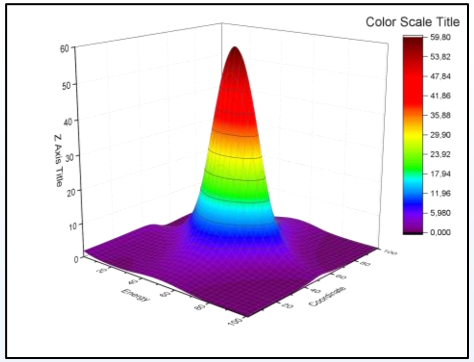
In case of deviation $(\varepsilon \neq 0)$ *from the potential-like dispersion law the position of resonant should shift when the filter is spinning*

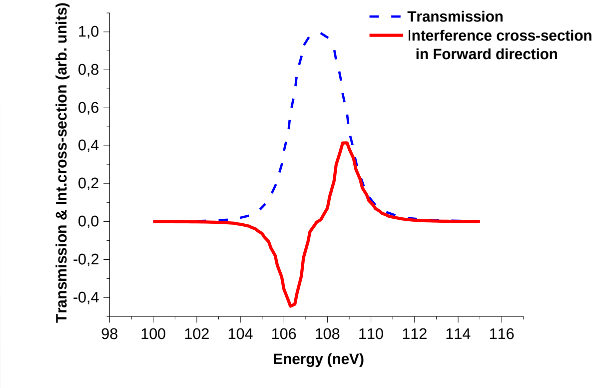
$$k^2 = k_0^2 - 4\pi N b + \varepsilon\left(k_0^2\right)$$

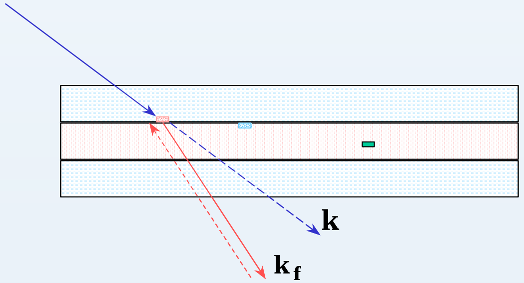
$$k_\perp^2 = k_{0\perp}^2 - 4\pi N b + \varepsilon\left(k_0^2\right)$$

Unexpected result and possible explanation









$$f\left(k',k\right) = -\frac{m}{2\pi\hbar^2}\int \tilde{\Psi}_f\left(\vec{r}\right)V_1\left(\vec{r}\right)\Psi\left(\vec{r}\right)d\vec{r}$$

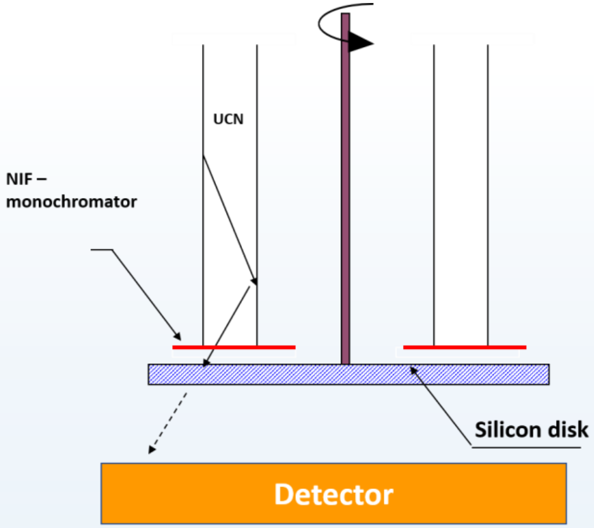
$$\sigma_{int} = -\frac{4\pi}{k}Im\left\{T^* f\left(k_t,k_t\right)\right\}$$

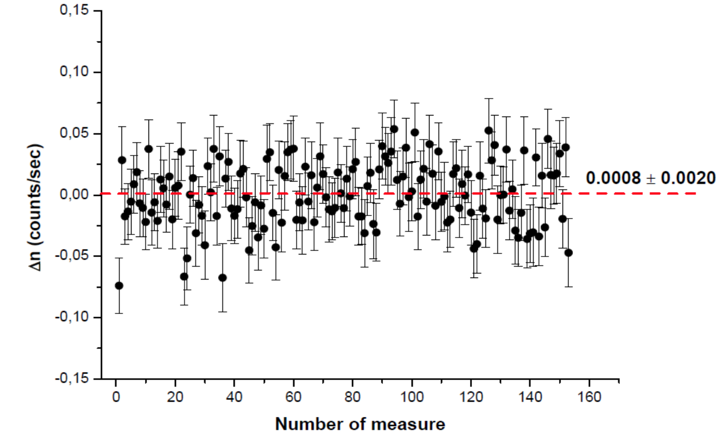
I.V. Bondarenko, A. V. Krasnoperov, A. I. Frank at al.
JETP Letters, 67 (1998) 786.

1. *In the resonant conditions the amplitude of neutron scattering at roughness's rises by some orders of magnitude*
2. *Transmitted wave and forward scattered wave are interfered.*
3. *The phase of the transmitted wave changes its sign in the resonance*

A.I.Frank, S.N. Balashov, I.V. Bondarenko et al. 2001

Test the UCN dispersion law with rotating sample





$$v_x = 6 \Leftrightarrow 36 \text{ m/s}$$

$$\Delta n / n = (0.6 \pm 1.4) \times 10^{-3}$$

Transmittivity depends on two parameters: real and imaginary parts of “potential”

$$U = V - iW = \frac{2\pi\hbar^2}{m} N\left(1 + J' + iJ''\right)\left(b' - ib''\right)$$

$$\delta J' = \ \leq 3 \times 10^{-3} \ \mathit{if} \ \ \delta W = 0$$

$$\delta J'' = \ \leq 3 \times 10^{-8} \ \text{if} \ \ \delta \mathrm{V} = 0$$

G.V.Kulin, A.N.Strepetov, A.I.Frank et al. Phys. Lett. A, 378, (2014) 2553

1. The theory predicts that dispersion law must differs from the potential form both for the cold neutrons and for the super slow neutrons.
2. This prediction is not yet verified by experiment.
3. For the case of UCN only one experiment was performed to confirm the model of effective potential. The result cannot be interpreted unambiguously
4. Any data concerning the testing the hypothesis of Super Slow Neutron still absent.

Neutron velocity in a matter

$$V = nV_0 \quad (??)$$

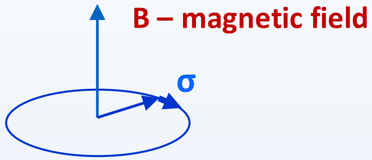
Direct measure of the neutron velocity in matter

Delay time in a refractive sample

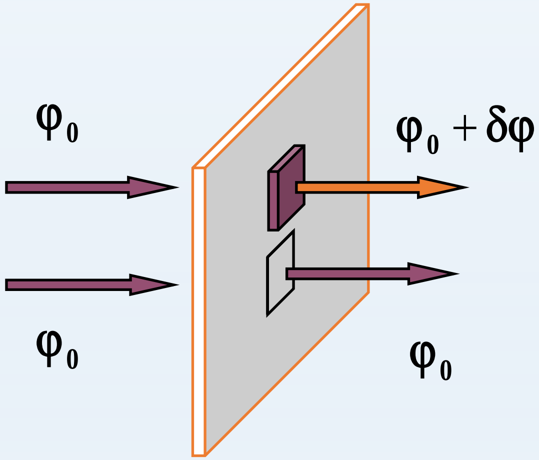
$$v = nv_0 \ ?$$

$$\boxed{\Delta t = \frac{L}{nv_0} - \frac{L}{v_0} = \frac{L}{v_0}\left(\frac{1-n}{n}\right)}$$

$$\delta\varphi = \Delta t \cdot \omega_L \qquad \omega_L = \frac{2\mu B}{\hbar}$$

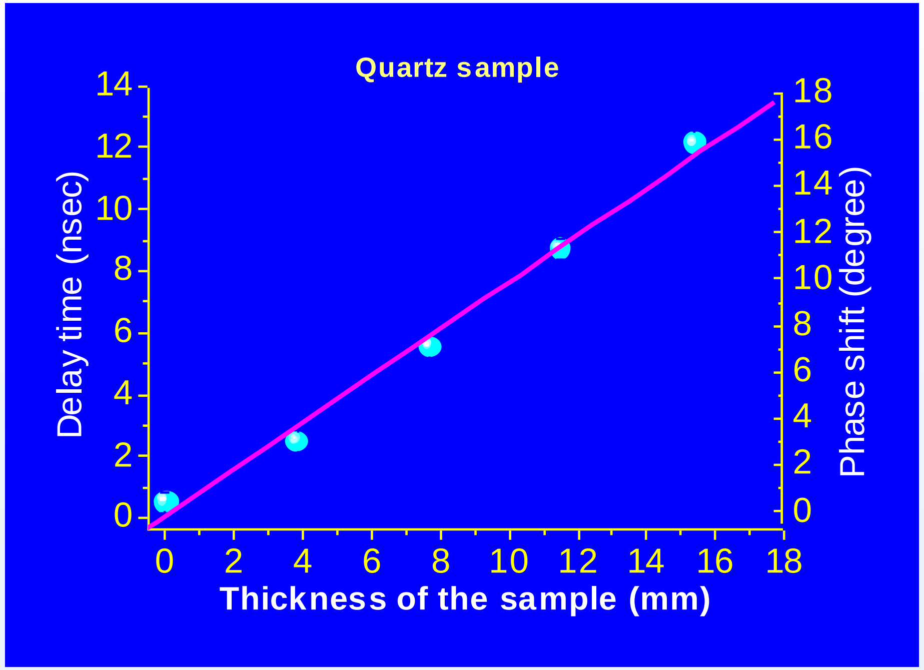


Precession (Larmor) frequency



$$\lambda = 18.85 \text{ Å}$$

$$V \simeq 210\,\text{m/s}$$



$$\delta t_{min} = 3.7\times10^{-10}\ \textbf{sec} \qquad t \approx 1.7\times10^{-2}\ \textbf{sec}$$

A.I. Frank, I.V. Bondarenko, A.V. Kozlov et al. . Physica B: Cond. Matter 297, (2001) 307.

The velocity of neutrons in matter and its relation to the form of dispersion law

Generally speaking, neutron inside a refractive matter is not a particle but a quasi particle with effective mass m^*

$$k = nk_0 \Rightarrow m^* v = nmv_0$$

$\boxed{m^* = m}$ Only in the case of $\boxed{k^2 = k_0^2 + \chi^2}$ $k^2 = k_0^2 - 4\pi\rho b$

$\boxed{v = \hbar \dfrac{nk_0}{m^*}}$ $v = nv_0$ Only in the case of the potential dispersion low validity

A.I. Frank, Physics-Uspekhi, 61 (2018) 900

Concerning the negative neutron effective mass at Bragg diffraction
see Zeilinger A. et al. *Phys. Rev.Lett.* **57,** 3089 (1986)

The velocity of neutrons in matter in the case of non-potential dispersion law

In general case dispersion law may be represent as $k = F\left(k_0^2\right)$

Then the effective mass of neutron in a matter is $m^* = 2mkF'$

If we suppose that $k^2 = k_0^2 - 4\pi\rho b + \varepsilon\left(k_0^2\right) \longrightarrow F = \left(k_0^2 - 4\pi\rho b + \varepsilon\left(k_0^2\right)\right)$ $\quad m^* = m\left(1 + \frac{d\varepsilon}{dk_0^2}\right)$

$$v = \hbar \frac{nk_0}{m^*} \qquad v = nv_0\left(1 + \frac{d\varepsilon}{dk_0^2}\right)^{-1} \qquad n = \left[1 - \frac{4\pi\rho b - \varepsilon\left(k^2\right)}{k_0^2}\right]^{1/2}$$

1. Neutron velocity in a refractive matter differs from the one in vacuum

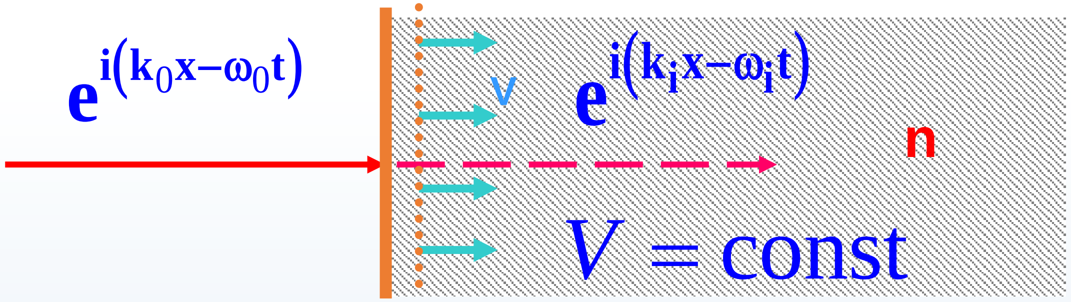
2. The relation $v = nv_0$ is correct only in the case of the potential-like dispersion law.

3. Earlier we reported that we performed experiment for the direct measurement of neutron velocity in a matter. But it was realized recently that this spin-precession experiment is sensitive not to velocity inside the matter v but to the value of nv_0.

4. In fact, neutron velocity inside the matter was measured under the assumption of the validity of the potential dispersion law

Neutron waves in an accelerating matter

Refraction of wave at the border of the moving matter

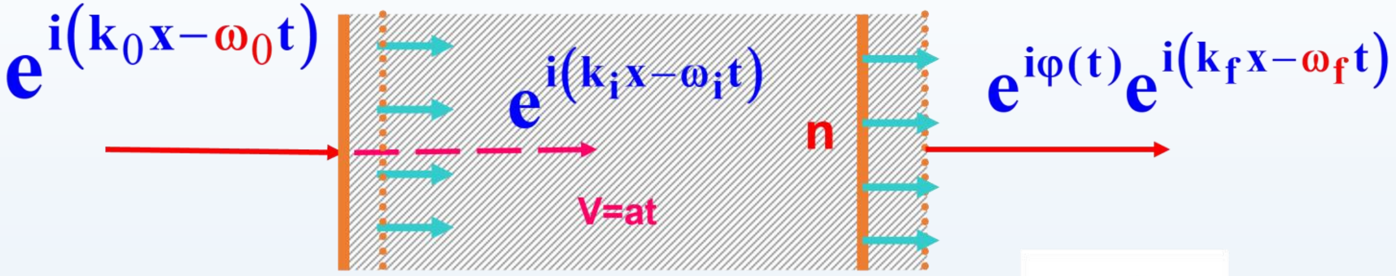


$$\mathbf{k_i = nk_0\left(1+\frac{1-n}{n}\frac{V}{v_0}\right)}$$

$$\boxed{\mathbf{\omega_i = \omega_0 + (n-1)k_0V}}$$

A.I.Frank, V.A. Naumov. Phys. At. Nuc, 76, (2013),1423.

Transmission of wave through the sample moving with acceleration - Accelerating Medium Effect



$$\omega_f \neq \omega_0$$

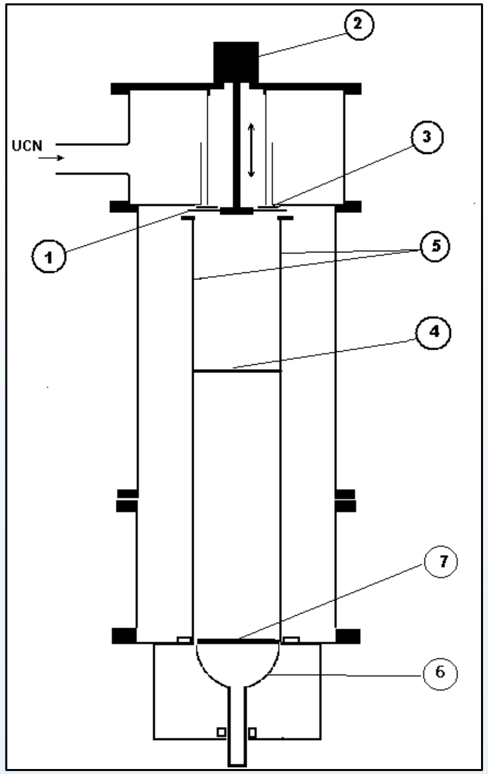
$$\Delta\omega = ka\Delta t \qquad \Delta t = \frac{d}{v_0}\left(\frac{1-n}{n}\right)$$

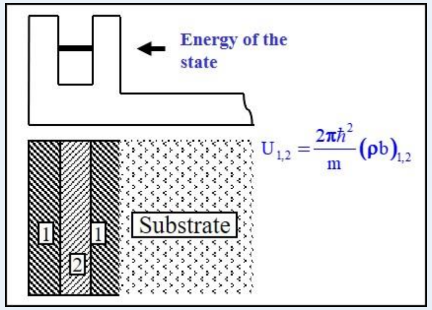
$$\Delta E = \hbar\omega$$

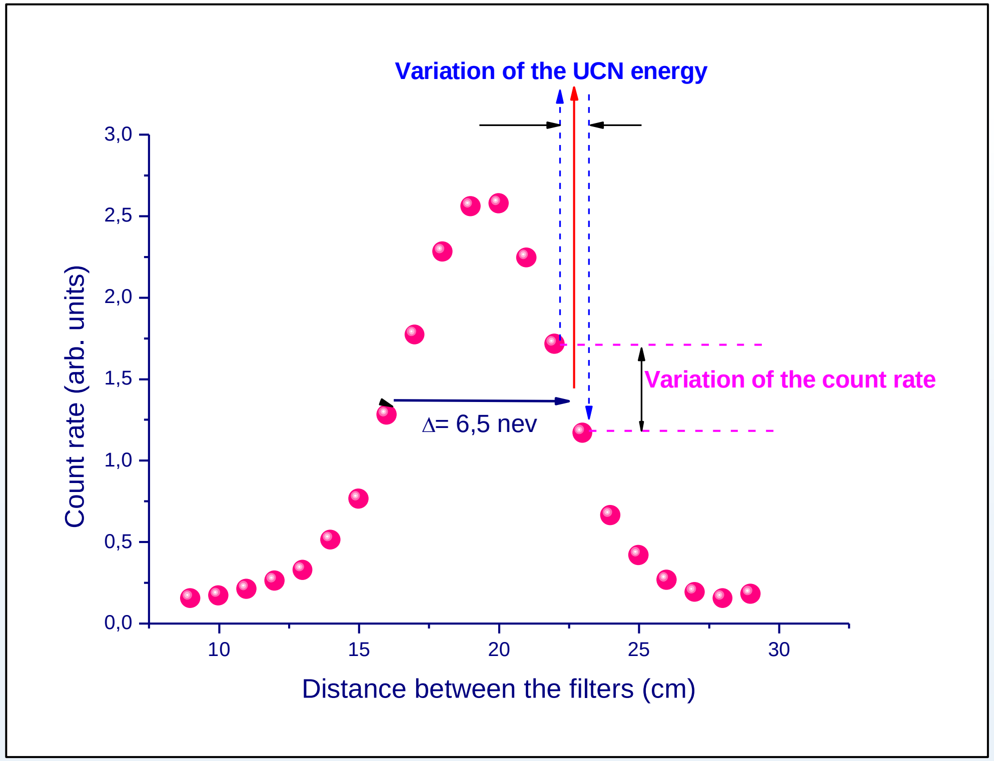
$$\boxed{\Delta E = mad\,\frac{1-n}{n}}$$

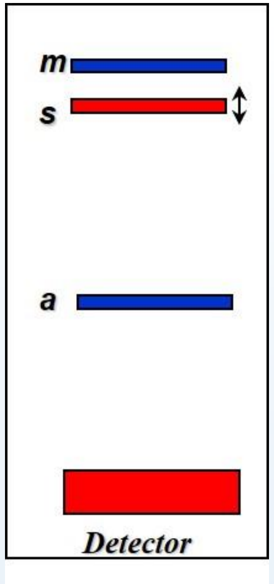
F. V. Kowalski, 1993;
V.G.Nosov, A.I.Frank, 1998

Idea of the experiment





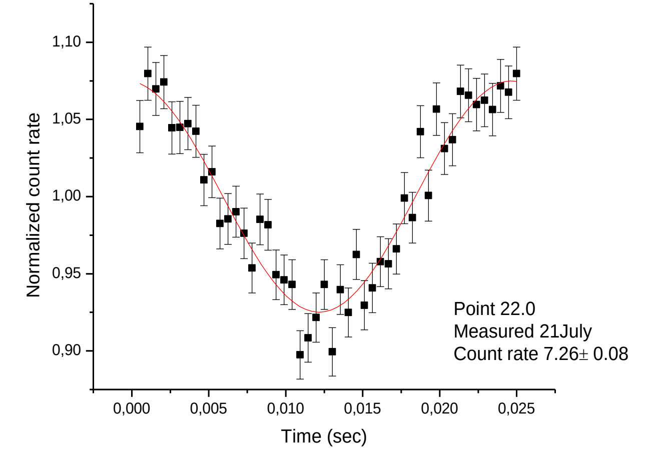


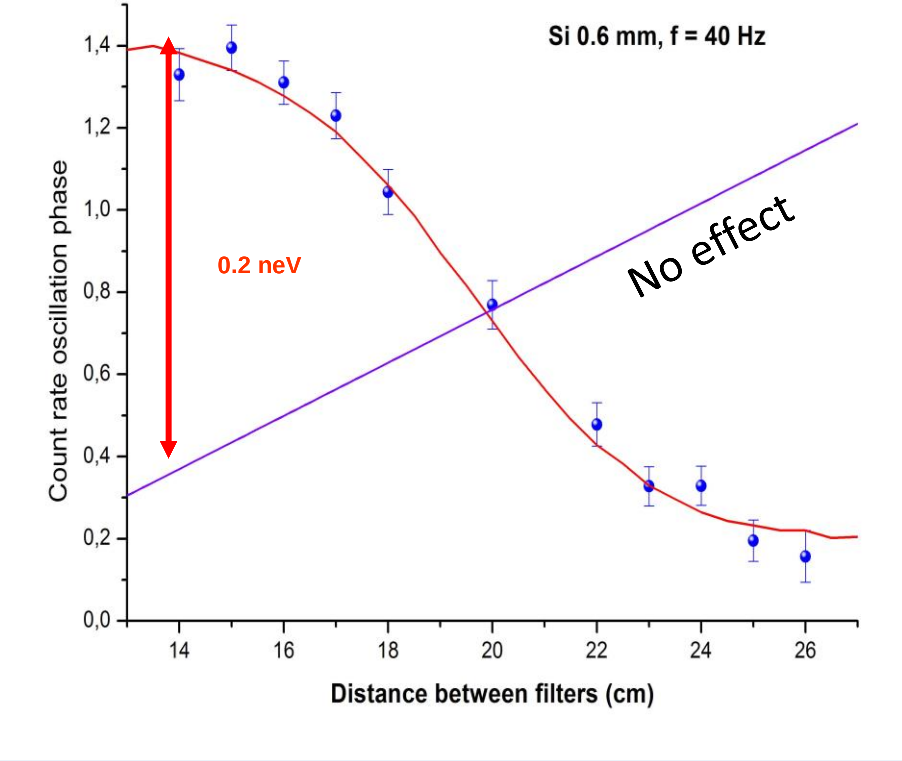


$\Delta E \approx (2\text{-}5)\times10^{-10}$ eV

Periodically variation of the neutron energy, caused by the sample acceleration, leads to the periodical oscillation of the count rate

Oscillation of the count rate and experimental result





$$a \cong -A\Omega^2 \sin\Omega t \qquad V \cong A\Omega \ \cos\Omega t$$

$$f(t) = 1 + B\sin(\Omega t - \varphi)$$

Frequency f = 40, 60 Hz
Oscillation period 0.025, 0.017 sec
Time of flight 0.11 sec

$$w_{max} = A\Omega^2 \approx 60 m/s^2$$

$$\Delta E \cong -K m A\Omega^2 L\left(\frac{1}{n} - 1\right)\sin\Omega t$$

$$K = 0.94 \pm 0.06$$

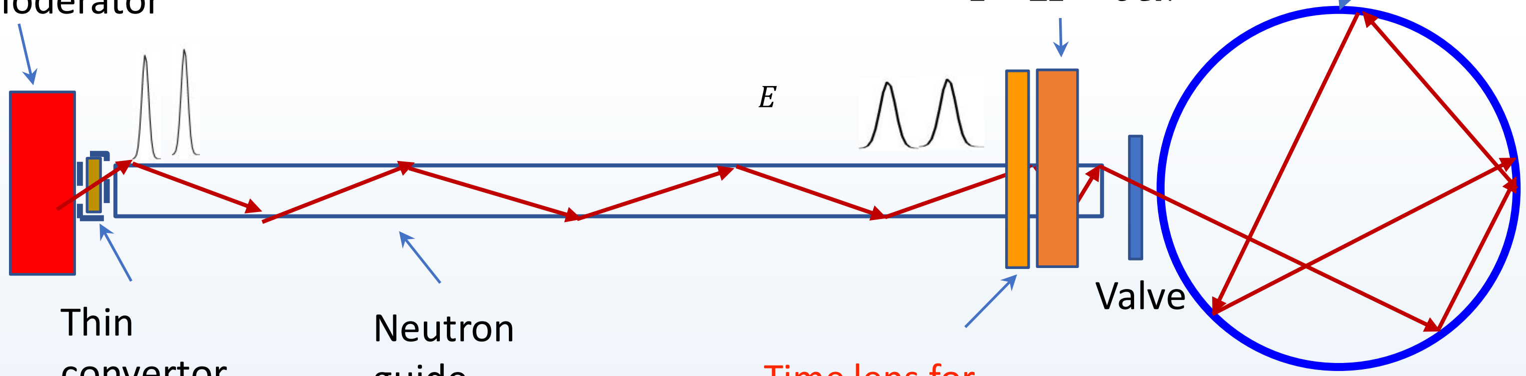
A.I. Frank, P.Geltenbort, G.V.Kulin, et al, Phys. At. Nuclei, 71 1656 (2008) .
A. I. Frank, P.Geltenbort, M. Jentschel, et al. JETP Letters, 93 361, (2011)

1. **The existence of the new optical effect – Acceleration Matter effect was confirmed in two different experiments**
2. **Later the existence of more general Effect of Acceleration, complementing the well-known Doppler effect, was predicted and its reliability was confirmed by a numerical solution of the nonstationary Schrodinger equation.**

1. A.I. Frank, P. Geltenbort, M. Jentschel, D.V. Kustov, G.V. Kulin, V.G. Nosov, A.N. Strepetov. *Effect of Accelerated Matter in Neutron Optics.* Phys. of Atomic Nuc, 71 (2008) 1656-1654.
2. A. I. Frank, P.Geltenbort, M. Jentschel, D.V. Kustov, G.V. Kulin, A.N. Strepetov. *New Experiment on the Observation of the Effect of Accelerating Matter in Neutron Optics*. JETP Letters, **93** (2011) 361–365
3. A.I.Frank. I*nteraction of a wave with an accelerating object and the equivalence principle.* Physics-Uspekhi **53**, 500 (2020).
4. M.A. Zakharov, G.V. Kulin and A.I.Frank. *Interaction of a wave packet with potential structures moving with acceleration.* Eur. Phys. *J. D* **75,** 47 (2021).

Present activity and nearest plans

UCN Source at the IBR2M reactor



UCN trap

Decelerator

$E - \Delta E \rightarrow UCN$

Moderator

E

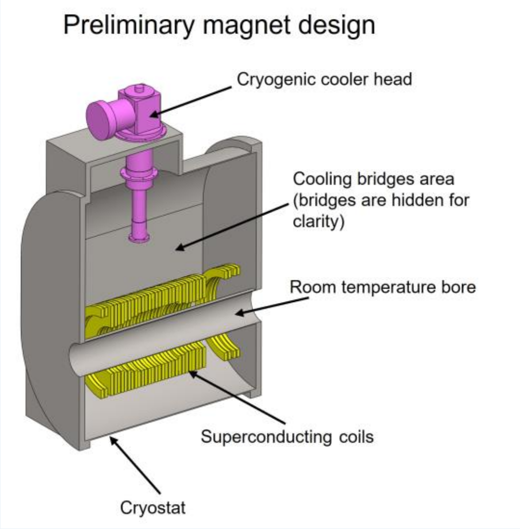
Valve

Thin convertor

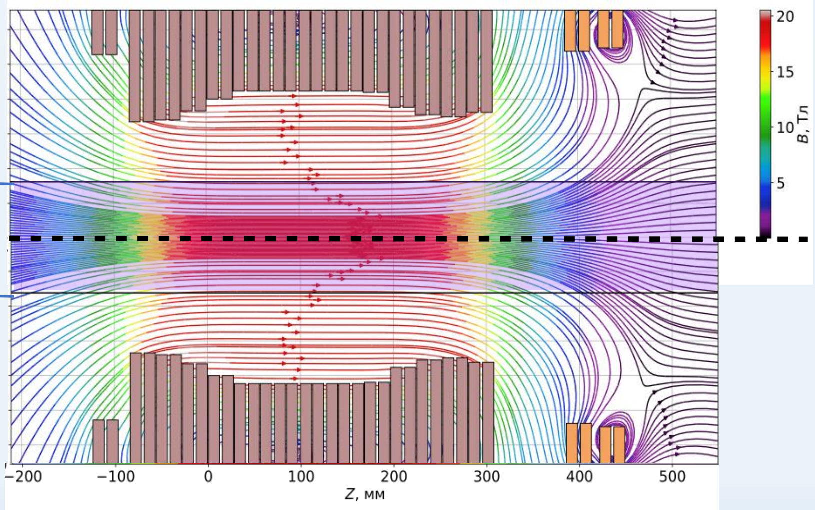
Neutron guide

Time lens for VCN

Talk of G. Kulin



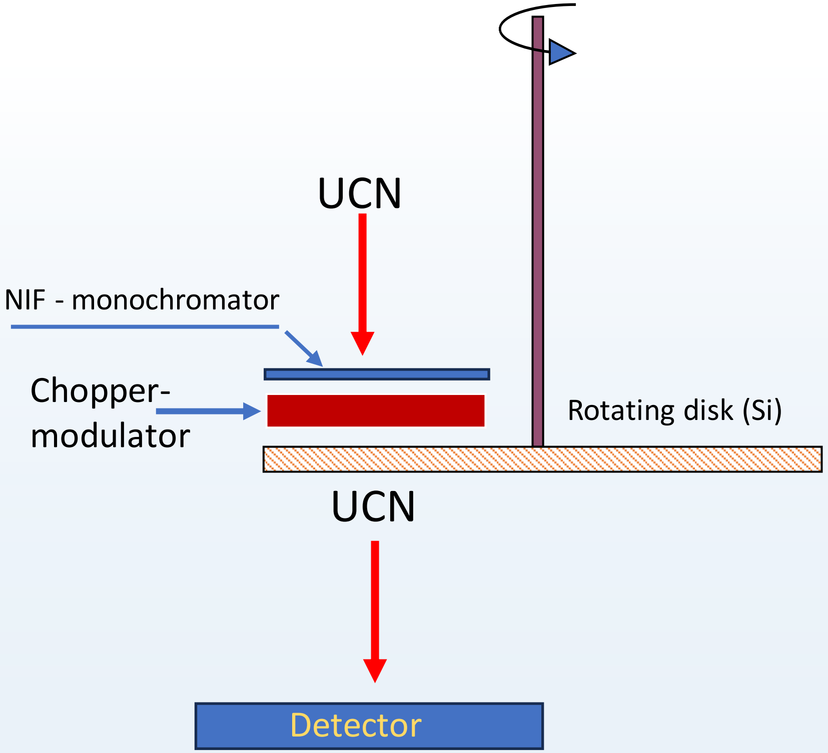
Preliminary magnet design



Neutron guide
Diam. 80mm

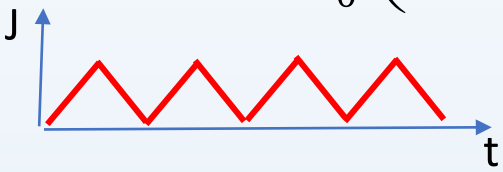
symmetry axis

Preparation to the experiment for the measure of neutron velocity inside the matter

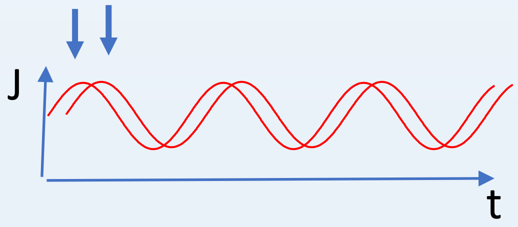


$$k^2 = k_0^2 - 4\pi\rho b + \varepsilon\left(k^2\right) \quad ??$$

$$\tau = \frac{md}{\hbar n k_0}\left(1 + \frac{d\varepsilon}{dk_0^2}\right) \qquad v = d/\tau$$



$$\Delta t = \Delta\Phi/\omega$$



The aim of the experiment is to compare phase shifts of the count rate oscillation for the case when sample is at rest and is spinning

UCN 110 neV. The estimated delay time due to refraction in silicon with thickness of 2mm is about 200 mks

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Yu. Khaydukov

D.V.Roshchupkin

Thank you for your attention!