

The 2nd China-Russia Joint Workshop on NICA Facility

# Deep Learning for HIC: the nuclear EoS

Long-Gang Pang 庞龙刚

Central China Normal University

夸克与轻子物理教育部重点实验室

Sep. 9-12, ShanDong University



# What is deep learning

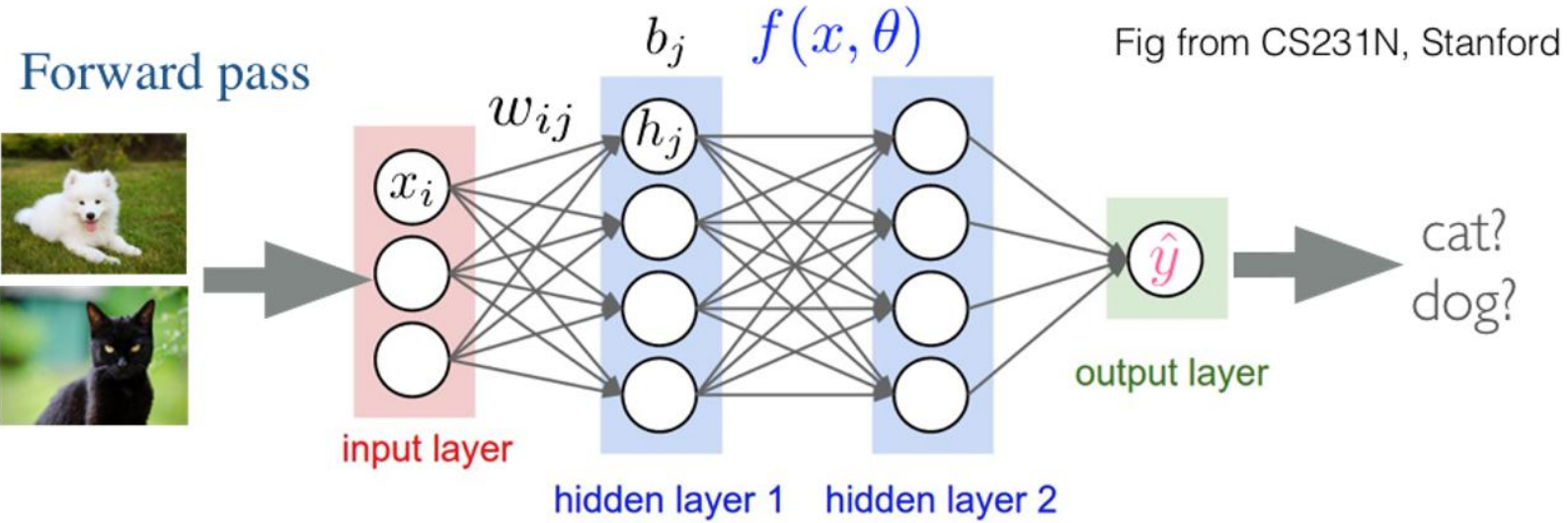


Yann LeCun

**Deep learning** is constructing networks of **parameterized functional modules** & training them from examples using **gradient-based optimization**



# DL: Neural Network with multi hidden layers

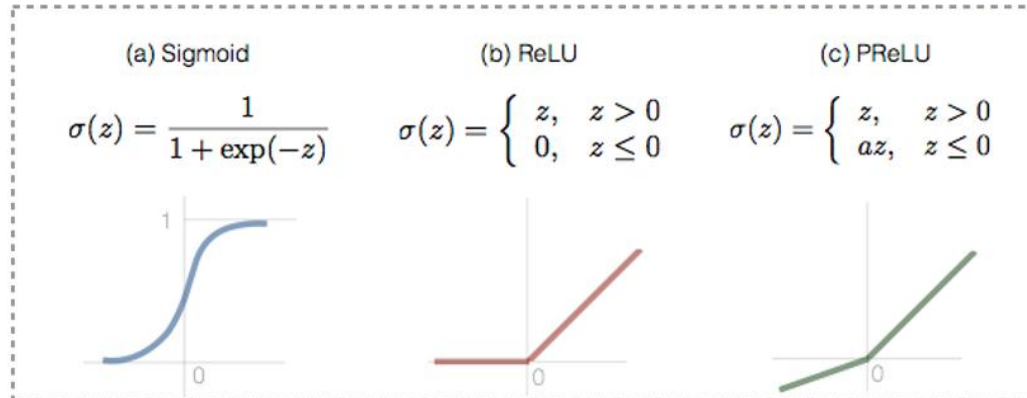


## Linear operation

$$z_j = \sum_{i=1}^N x_i w_{ij} + b_j$$

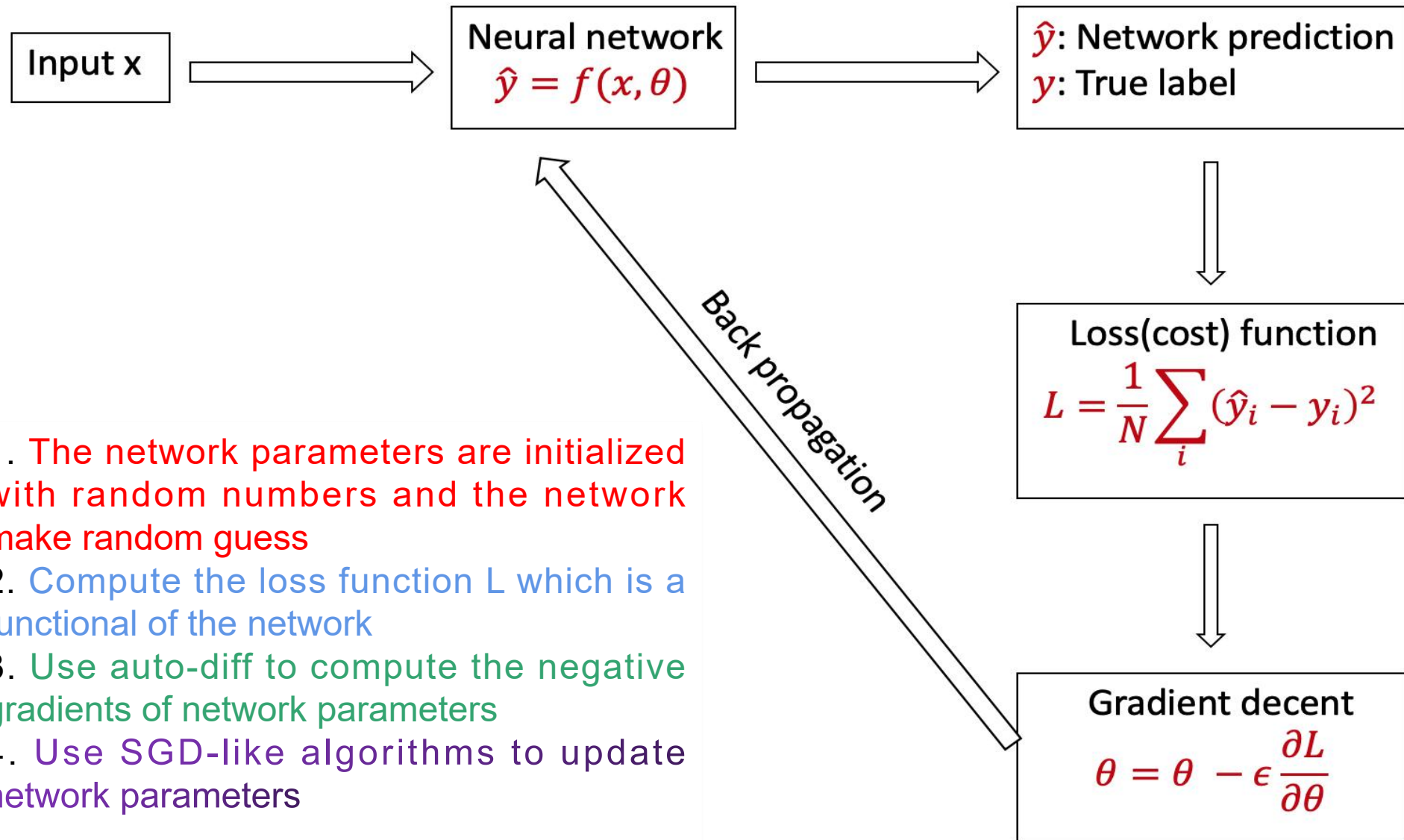
scaling, rotating, boosting,  
changing dimensions

## Non-linear activation function $h_j = \sigma(z_j)$





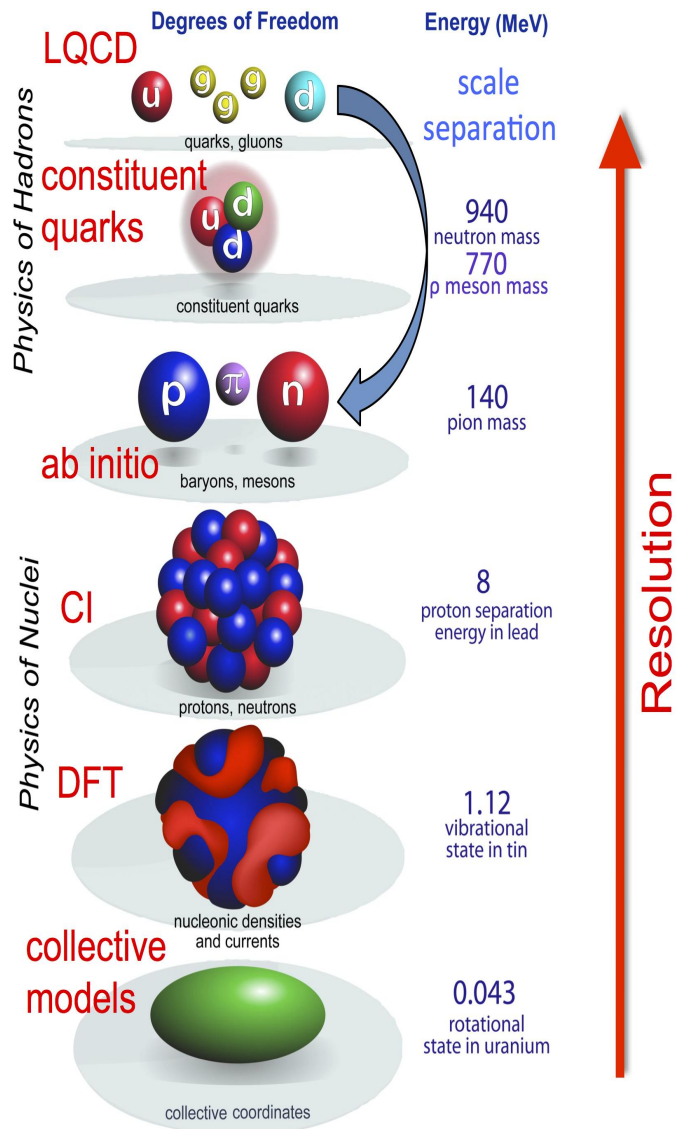
# How does the network learn



1. The network parameters are initialized with random numbers and the network make random guess
2. Compute the loss function  $L$  which is a functional of the network
3. Use auto-diff to compute the negative gradients of network parameters
4. Use SGD-like algorithms to update network parameters

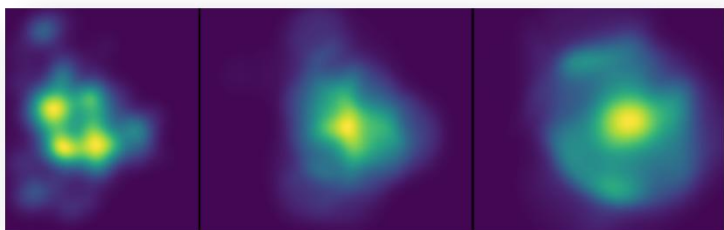
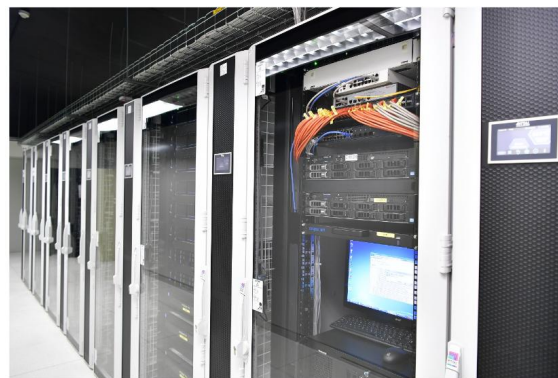
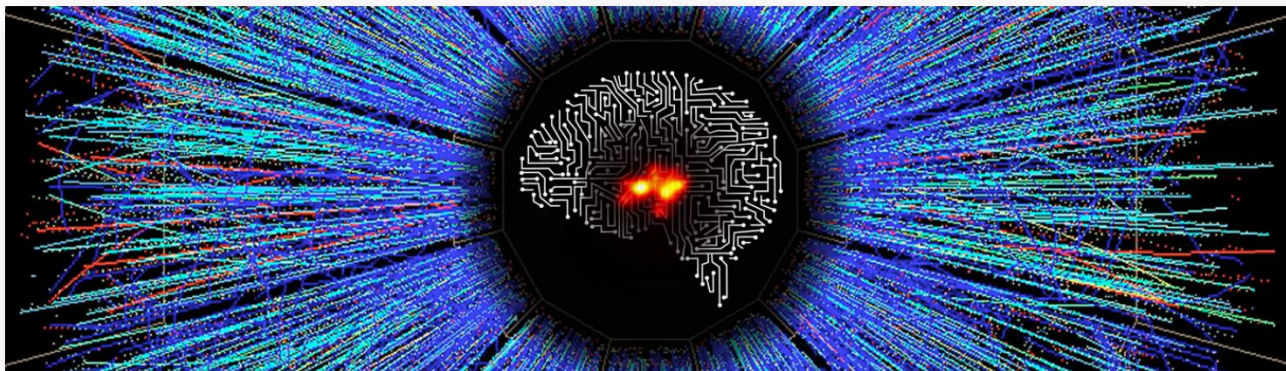


# DL nuclear physics across energy scales

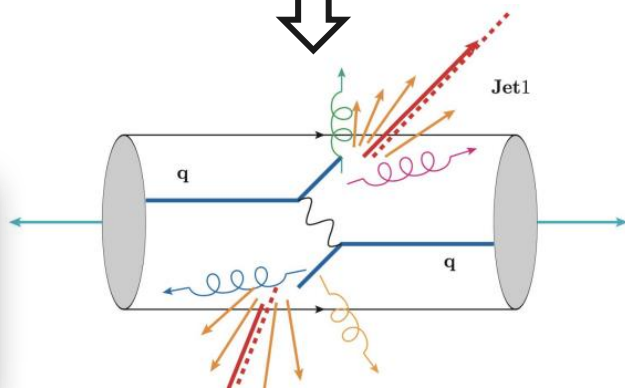


- Deep generative models (such as **normalizing flow** and the **diffusion model**) have been used to **sample Field Configurations in Lattice QCD**
- Deep learning is widely used to **solve inverse problems of HIC** to study the **EoS of hot QCD matter**, the **phase transition**, the transport coefficients  $\eta/s$ , the **impact parameter**, ...
- Deep neural network is used to **represent the many-body wave function of nucleus**, to solve variational problems in ab initio calculations
- Deep learning is used to solve inverse problems of HIC to study the nuclear structure, for instance, the **nuclear deformation**, **neutron skin**, **alpha cluster** and **short range correlation**
- ...

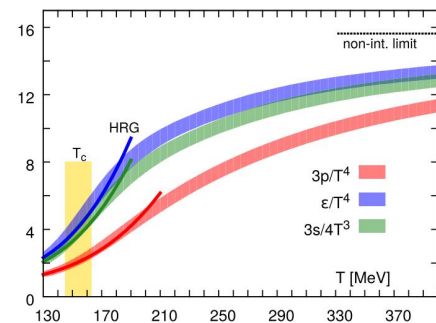
# Different approaches for nuclear EoS



(1) Soft probe: low  $P_t < 3$  GeV/c  
hydrodynamics, transport model, ...

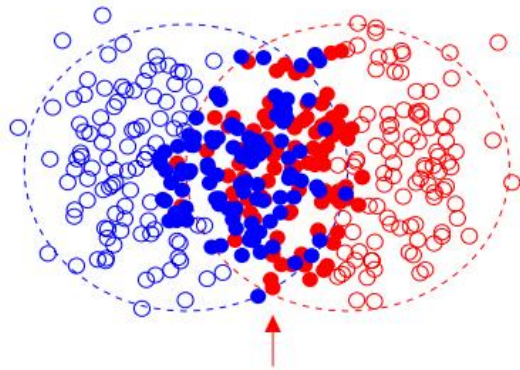


(2) Hard probe:  $P_t > 10$  GeV  
jet eloss, medium response



(3) Theoretical calculations  
Lattice QCD, HRG, fRG, DSE,  
Quasi Parton Model, ...

# Soft probe: relativistic hydro

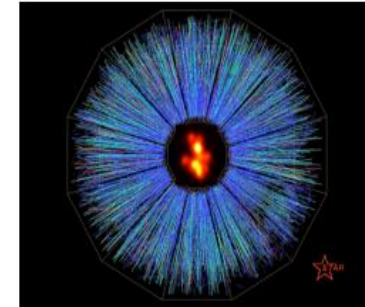


Initial condition

$$\nabla_{\mu} T^{\mu\nu} = 0 \quad \longrightarrow$$
$$T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - P g^{\mu\nu} + \pi^{\mu\nu}$$

EoS

Viscosity



## Name of CLVisc:

1. CCNU-LBNL Viscous Hydro, CCNU = Central China Normal University
2. A 3+1D viscous hydro parallized on GPU using OpenCL

**Purpose:** Describe the **non-equilibrium space-time evolution** of hot QCD matter

**Feature:** **60 times faster** for hydrodynamic evolution, **100 times faster** for hadronization

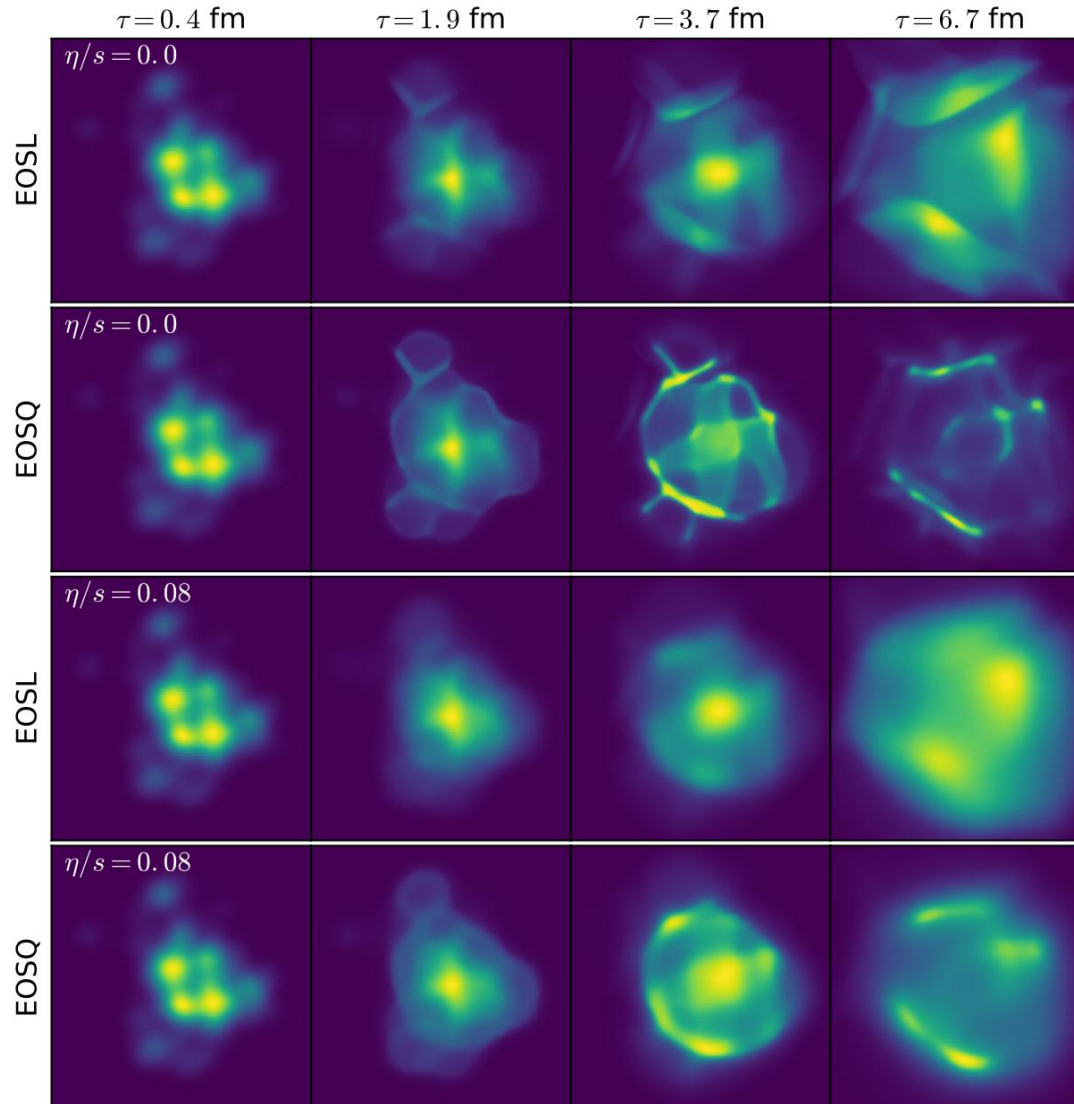
L.G. Pang, Q. Wang and X. N. Wang, PRC 86 (2012) 024911

L.G. Pang, B.W. Xiao, Y. Hatta, X.N.Wang, PRD 2015

L.G. Pang, H.Petersen, XN Wang, PRC97(2018)no.6,064918



# CLVisc for different EoS



**$\eta/s = 0$**   
**Lattice QCD EoS**  
**(smooth cross over)**

**$\eta/s = 0$**   
**First order phase transition**

**$\eta/s = 0.08$**   
**Lattice QCD EoS**

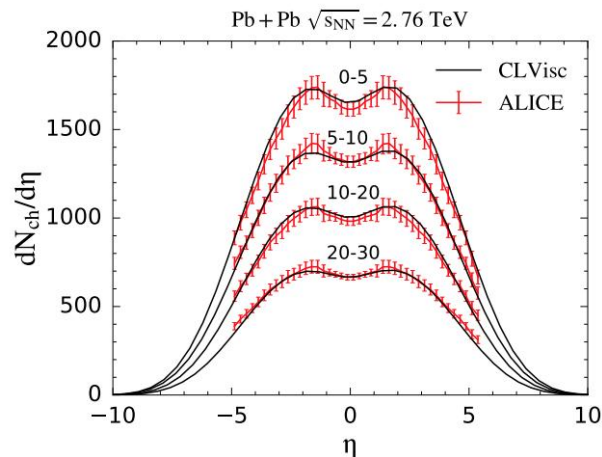
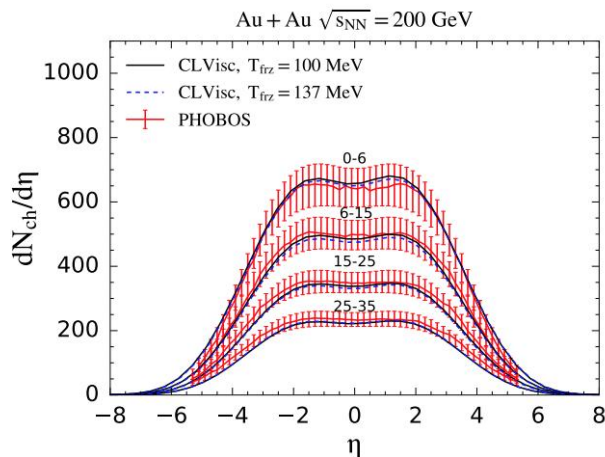
**$\eta/s = 0.08$**   
**First order phase transition**  
 $\eta/s$ : shear viscosity / entropy density

Will the effect of EoS survive the dynamical evolution and exist in the final state hadrons?

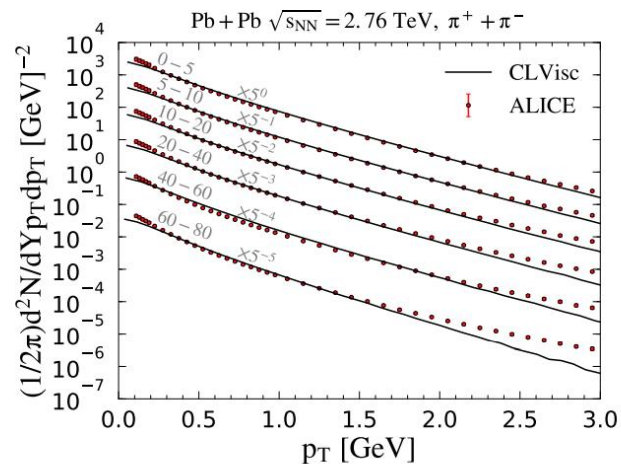
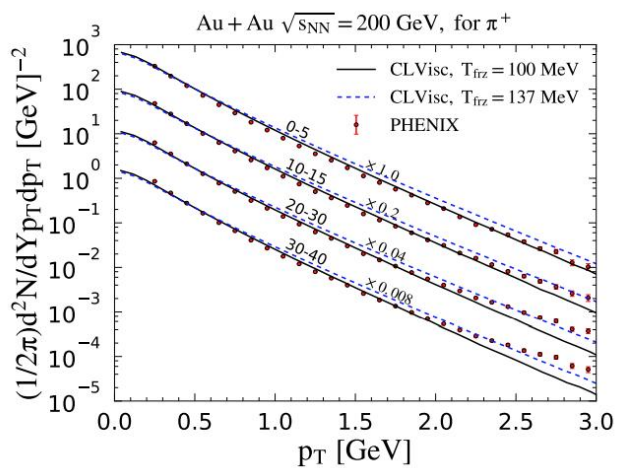


# CLVisc for top RHIC and LHC energies

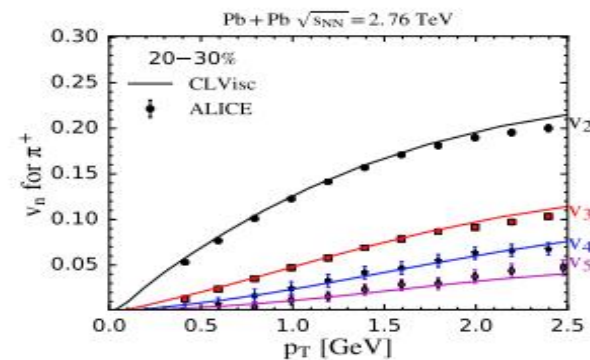
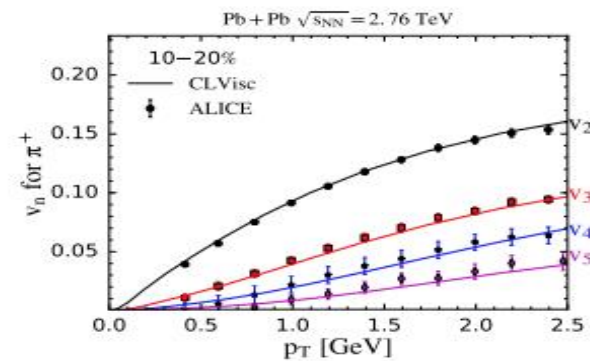
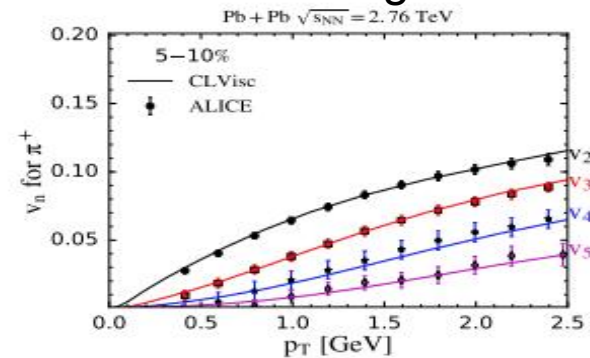
## Longitudinal momentum distribution



## Transverse momentum distribution

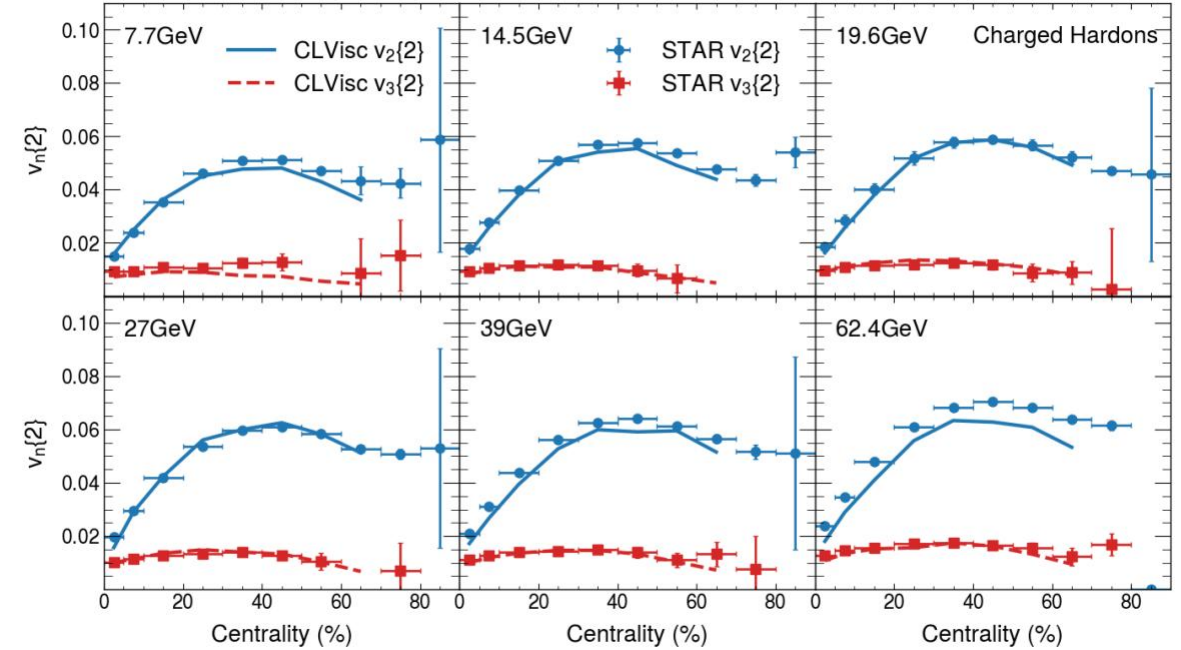
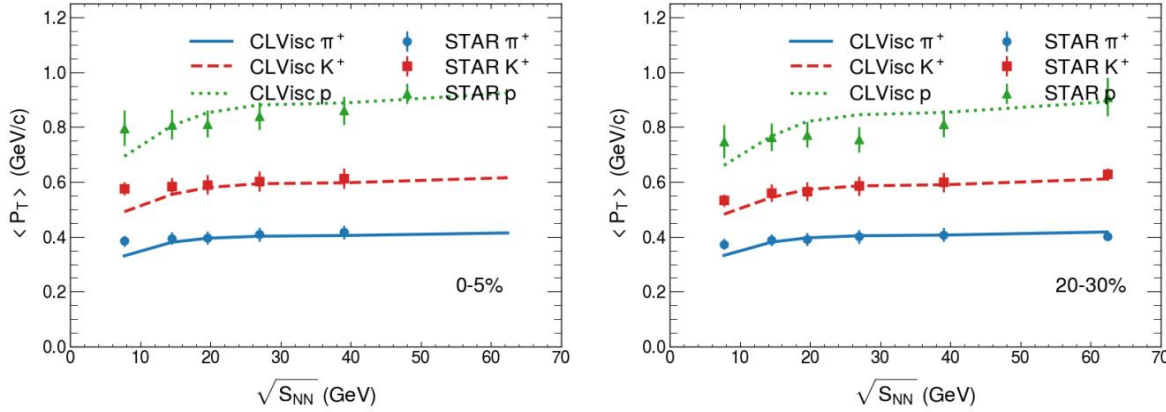


## Fourier decomposition coef. for azimuthal angle





# CLVisc for beam energy scan



$$\begin{aligned} \nabla_{\mu} T^{\mu\nu} &= 0, & T^{\mu\nu} &= eU^{\mu}U^{\nu} - P\Delta^{\mu\nu} + \pi^{\mu\nu}, \\ \nabla_{\mu} J^{\mu} &= 0, & J^{\mu} &= nU^{\mu} + V^{\mu}, \end{aligned}$$

$$\begin{aligned} \Delta_{\alpha\beta}^{\mu\nu} D\pi^{\alpha\beta} &= -\frac{1}{\tau_{\pi}} (\pi^{\mu\nu} - \eta_{\nu}\sigma^{\mu\nu}) & (10) \\ &- \frac{4}{3}\pi^{\mu\nu}\theta - \frac{5}{7}\pi^{\alpha\langle\mu}\sigma_{\alpha}^{\nu\rangle} + \frac{9}{70}\frac{4}{e+P}\pi_{\alpha}^{\langle\mu}\pi^{\nu\rangle\alpha}, \end{aligned}$$

$$\Delta^{\mu\nu} DV_{\nu} = -\frac{1}{\tau_V} \left( V^{\mu} - \kappa_B \nabla^{\mu} \frac{\mu_B}{T} \right) - V^{\mu}\theta - \frac{3}{10}V_{\nu}\sigma^{\mu\nu}, \quad (11)$$

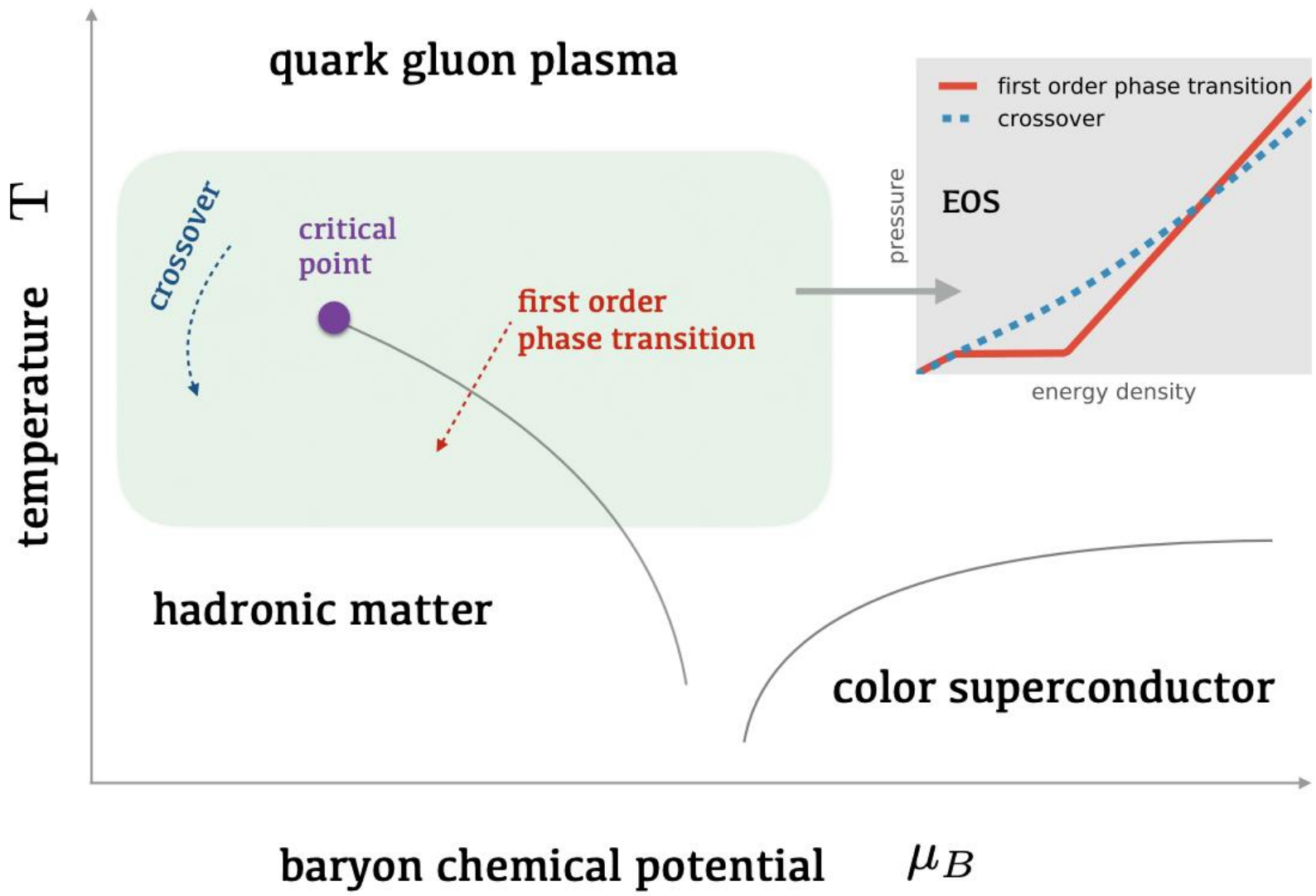
- + net baryon conservation
- + evolution of baryon diffusion current
- + extended to BES regions

XY Wu, GY Qin, LG Pang, XN Wang, PRC 105 (2022) 3, 034909



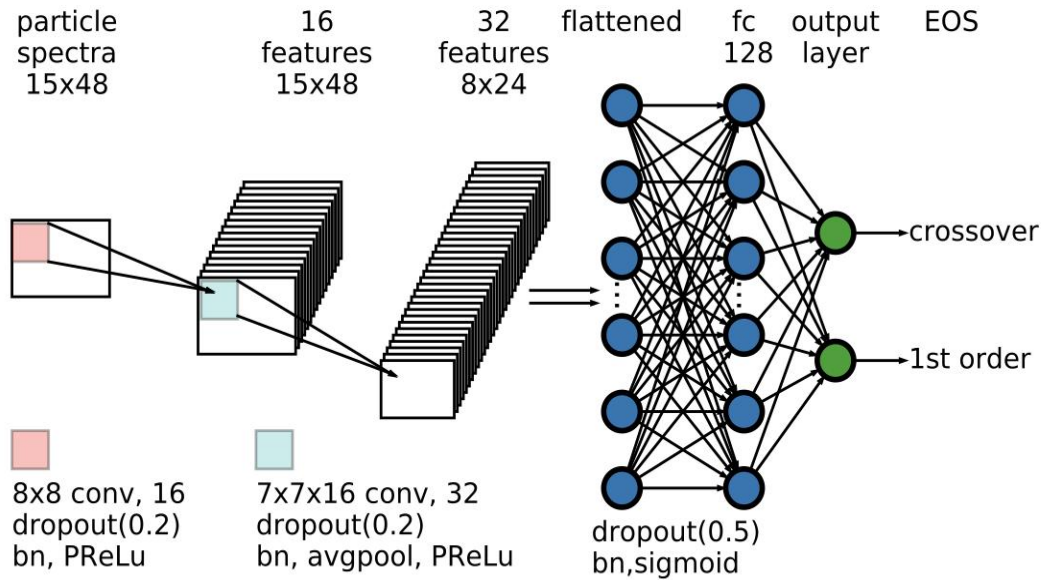


# EoS for different phase transition types



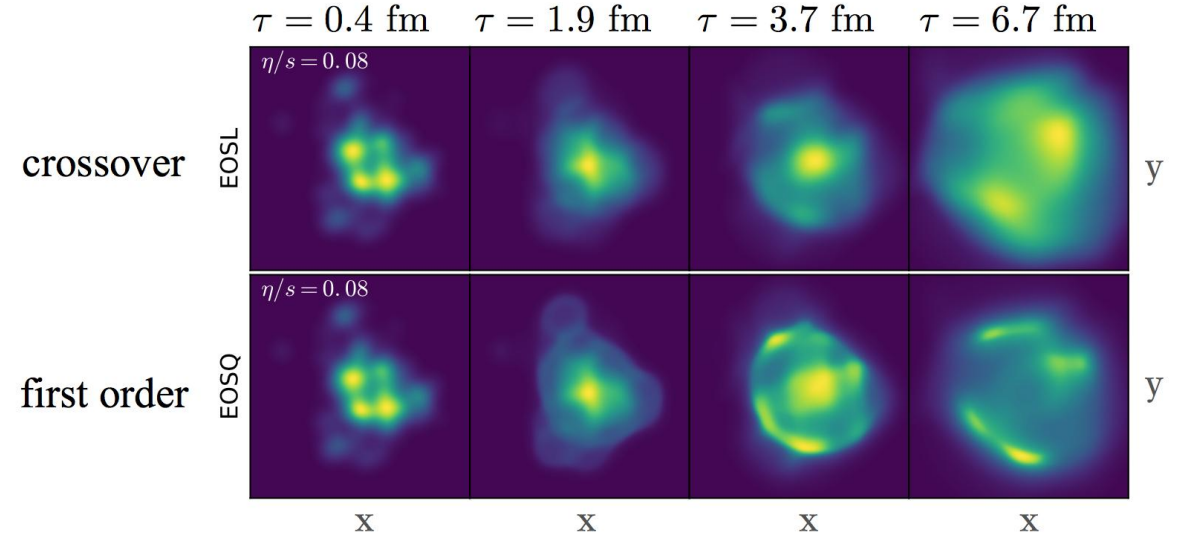
# Determine nuclear phase transitions

$$\nabla_{\mu} T^{\mu\nu} = 0$$



$$l(\theta) = -\frac{1}{N} \sum_{i=1}^N [y_i \log \hat{y}_i + (1 - y_i) \log(1 - \hat{y}_i)] + \lambda \|\theta\|_2^2$$

cross entropy loss      L2 regularization



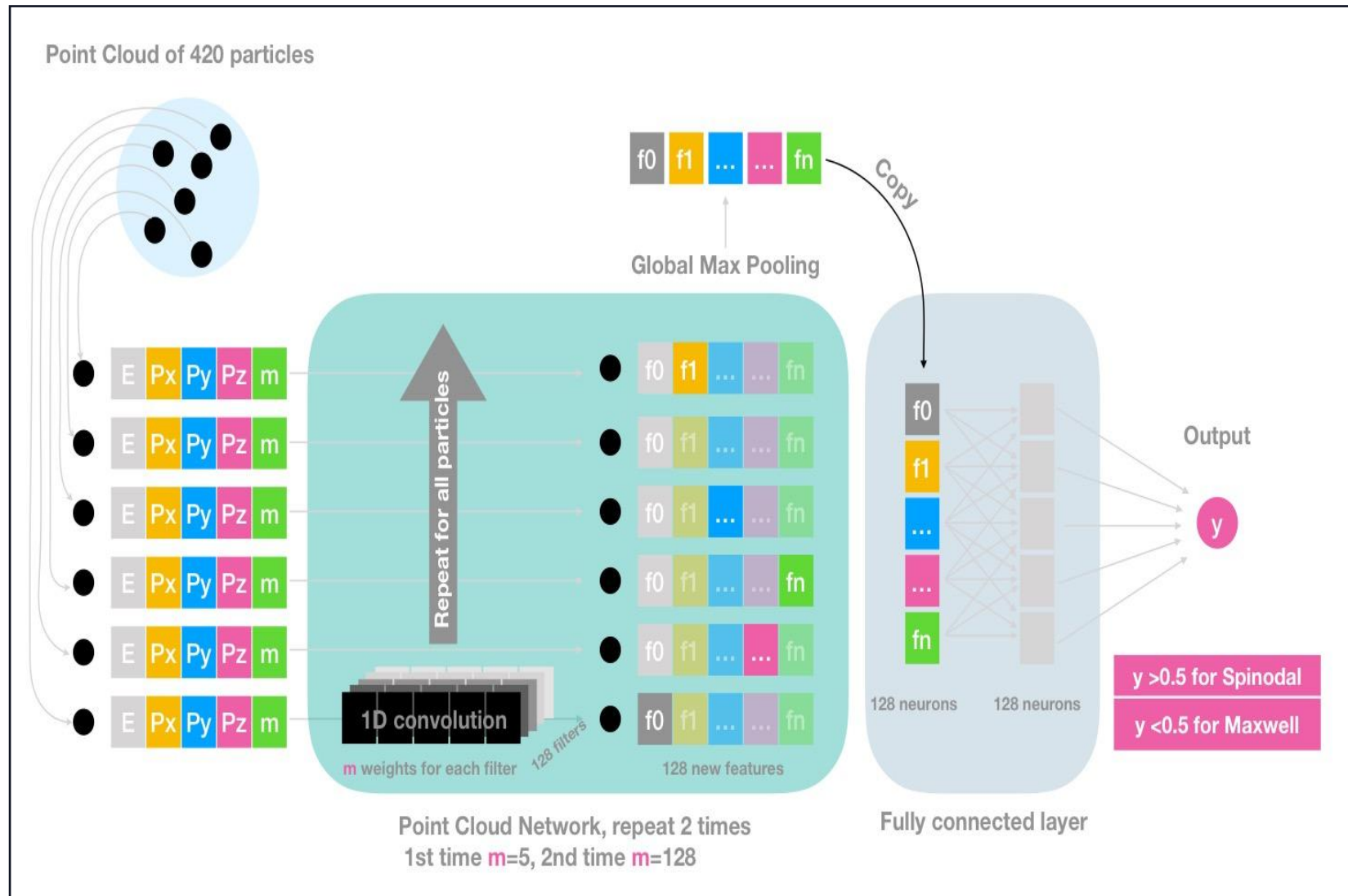
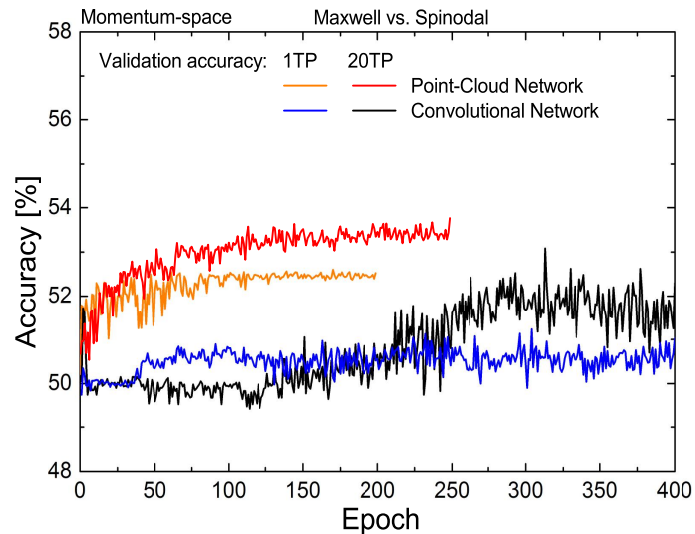
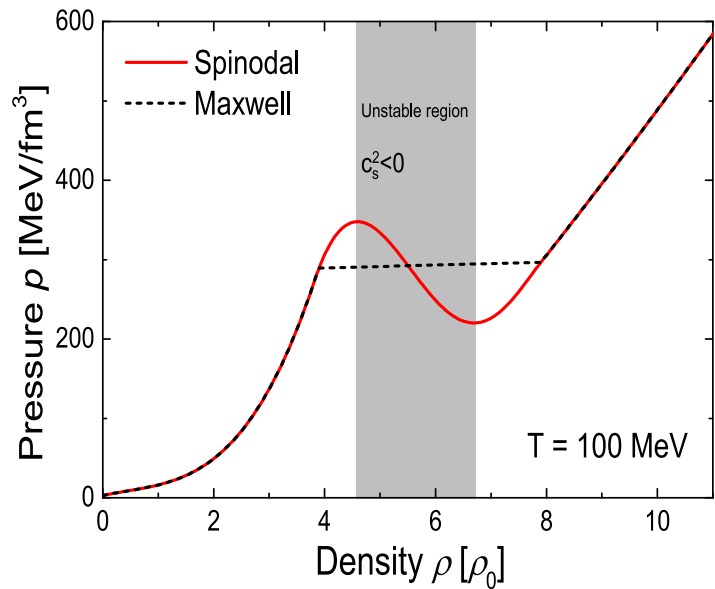
CLVisc 3+1D relativistic hydrodynamics

DL: 93% Classification Accuracy!

Nature Communications 2018, **LG. Pang**, K.Zhou, N.Su, H.Petersen, H. Stoecker, XN. Wang.



# Spinodal vs Maxwell 1<sup>st</sup> order phase transition



J. Steinheimer, L.G. Pang, K. Zhou, V. Koch and J. Randrup, JHEP 12 (2019) 122

# Capture more local correlations

## Dynamical Edge Convolution Network

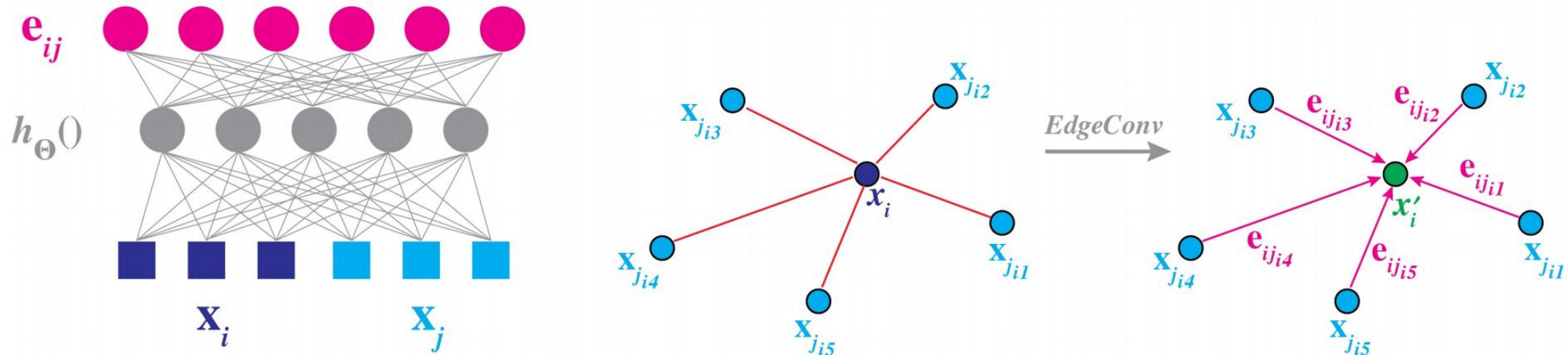
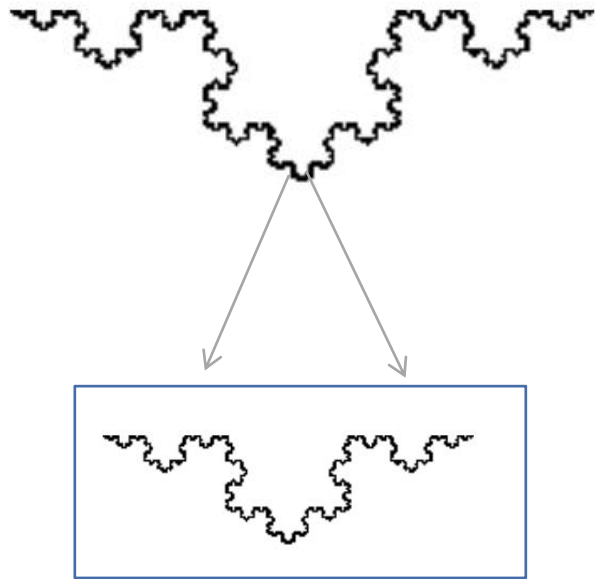


Fig. 2. **Left:** Computing an edge feature,  $e_{ij}$  (top), from a point pair,  $x_i$  and  $x_j$  (bottom). In this example,  $h_{\theta}()$  is instantiated using a fully connected layer, and the learnable parameters are its associated weights. **Right:** The EdgeConv operation. The output of EdgeConv is calculated by aggregating the edge features associated with all the edges emanating from each connected vertex.

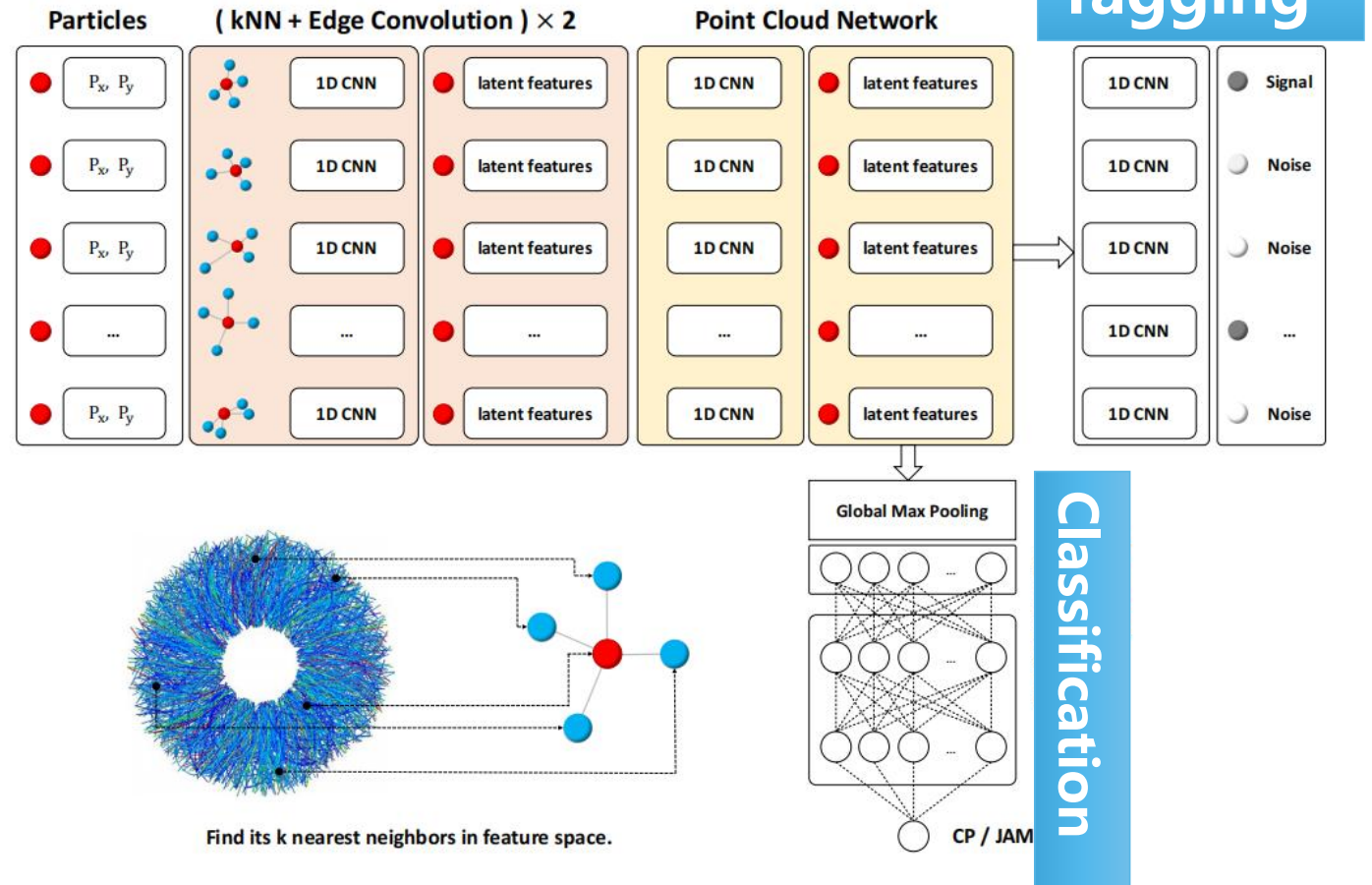


# Looking for self similarity in momentum space



Self similarity, scaling invariance

## Dynamical Edge Convolution Network

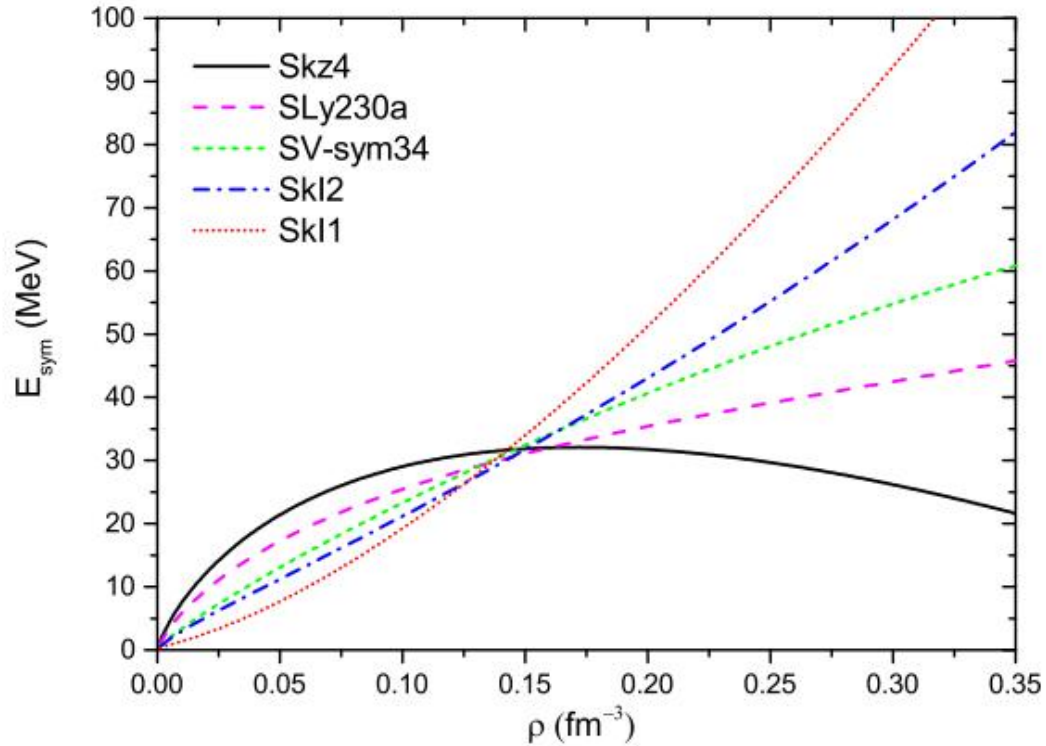


PLB 827(2022) 137001, Y.-G. Huang, L.-G. Pang, X.F. Luo and X.-N. Wang

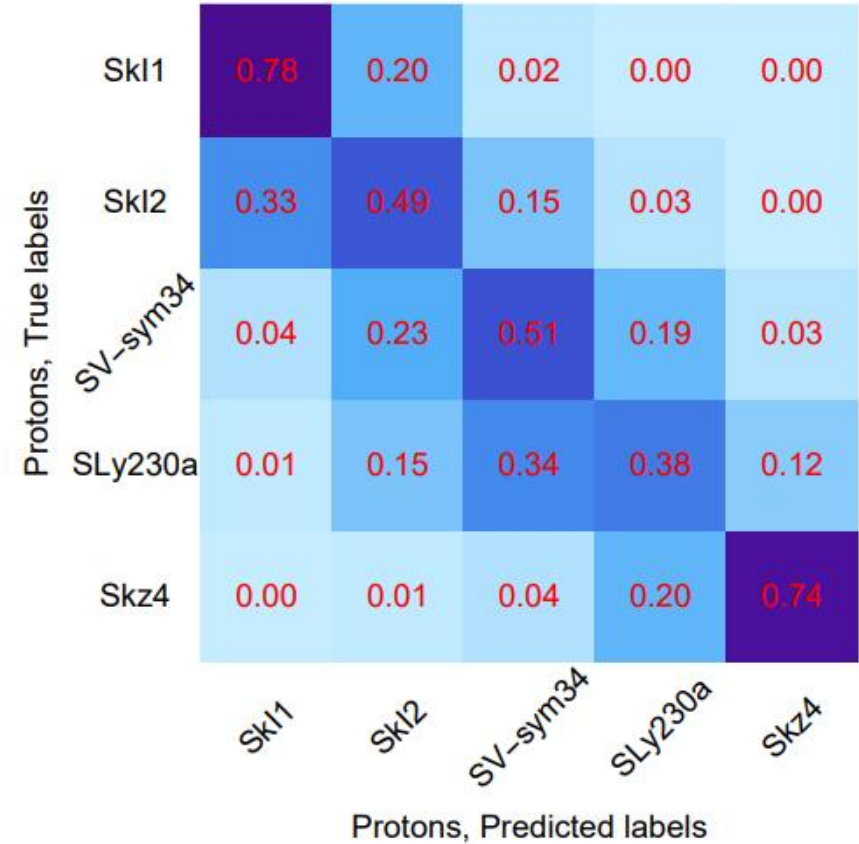


# Nuclear EoS at high density region

Skyrme potential + IMQMD





off-diagonal = misclassified

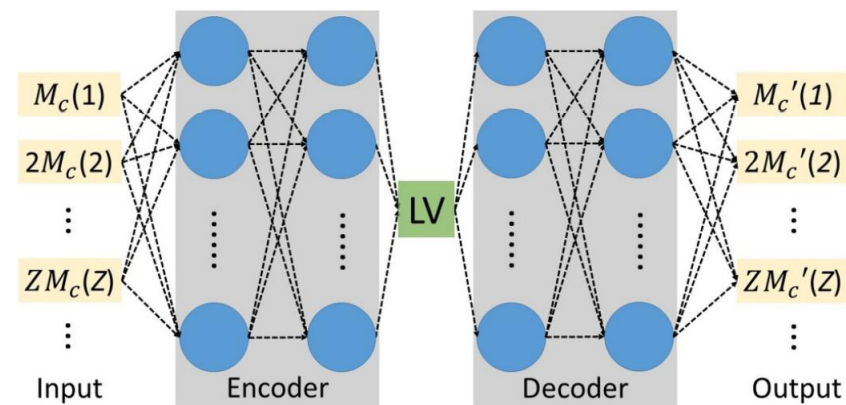
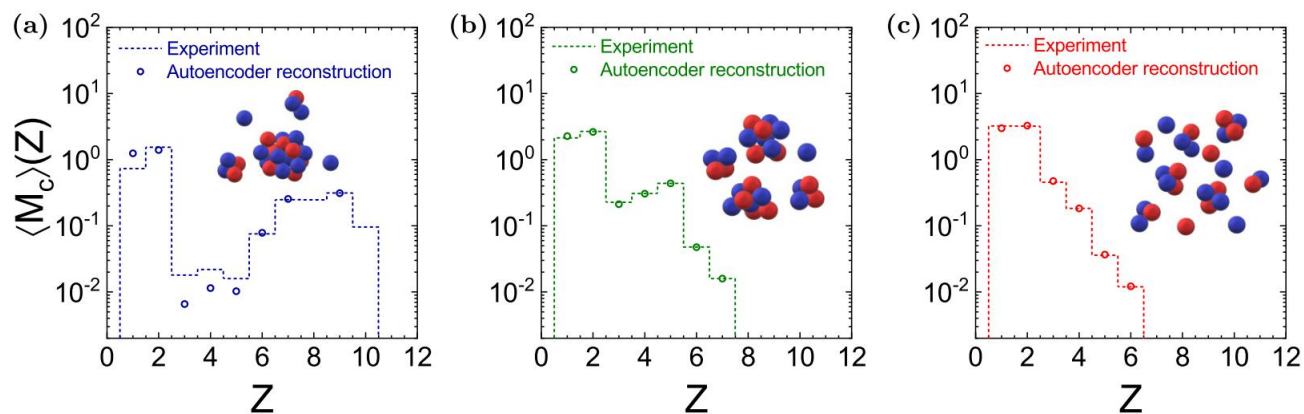


PLB 822 (2021) 136669, Y.J Wang, F.P. Li, Q.F. Li, H.L. L`u, and K. Zhou

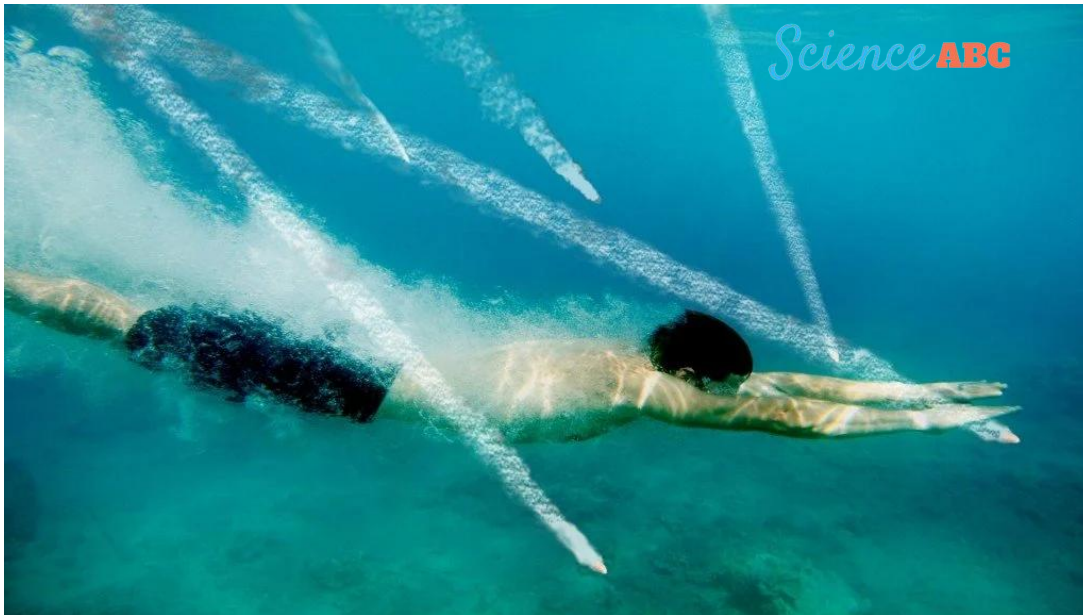


## Nuclear liquid-gas phase transition with machine learning

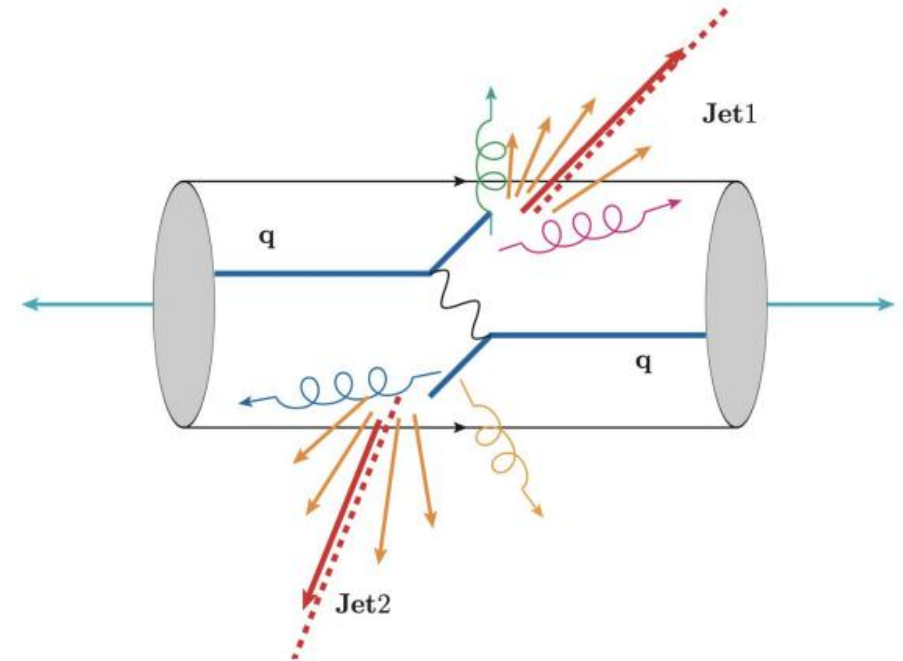
Rui Wang <sup>1,2,\*</sup> Yu-Gang Ma,<sup>1,2,†</sup> R. Wada,<sup>3</sup> Lie-Wen Chen <sup>4</sup> Wan-Bing He,<sup>1</sup> Huan-Ling Liu,<sup>2</sup> and Kai-Jia Sun<sup>3,5</sup>



## Can Being Underwater Protect You From Bullets?

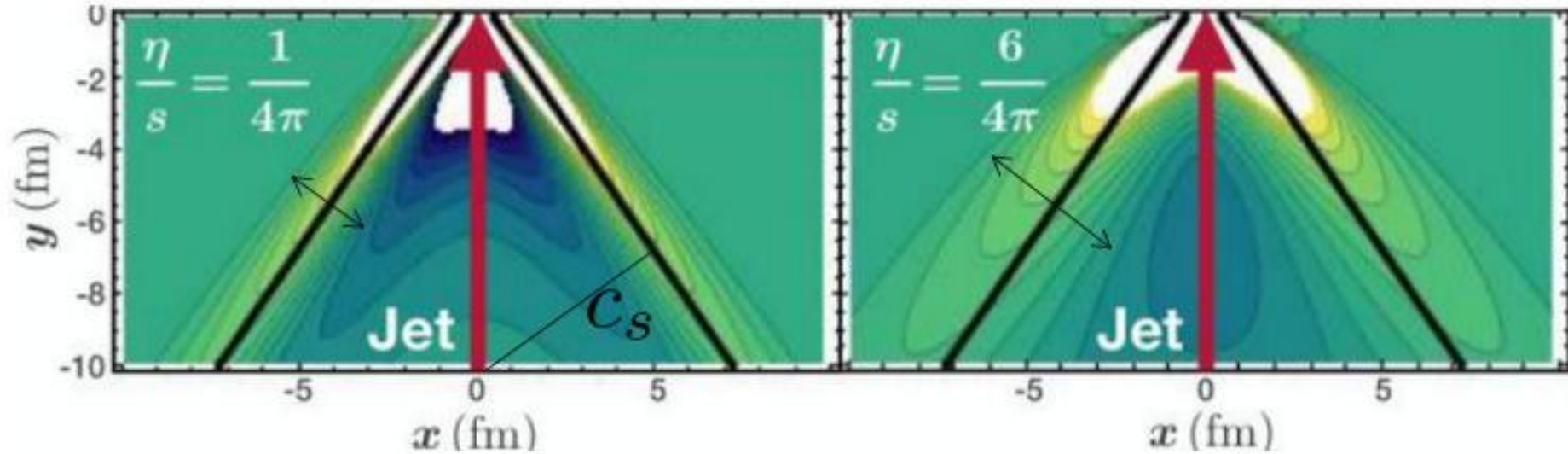


“ If the bullet is shot from an angle of 30 Degrees, then being underwater in the range of 3-5 feet (0.9-1.5 meters) can ensure safety from most guns.



Jet quenching in hot QGP

# The nuclear EoS and Mach Cone



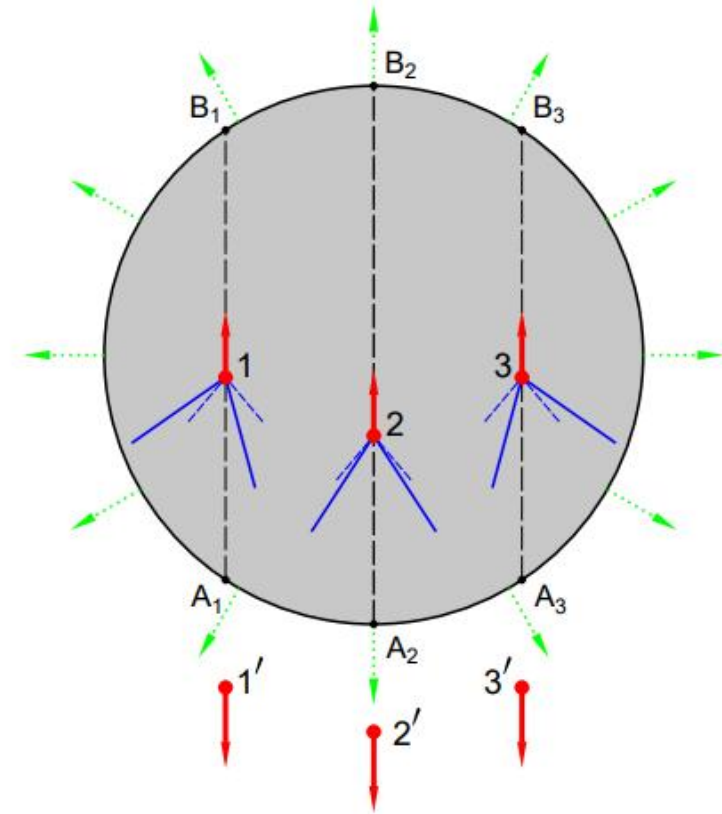
R.B. Neufeld. PRC79,054909(09')

Nuclear EoS:  $c_s^2 = \frac{dP}{d\epsilon} = \sin^2 \theta$

Shear Viscosity: width of the shock wave

# Difficulties in looking for Mach Cones in HIC

- Random production locations and **propagating directions** relative to collective flow
- Tilted by different **path length** and **collective flow**



L.M. Satarov, H. Stoecker, I.N. Mishustin,  
PLB 627 (2005) 64-70



# Training data: CoLBT(LBT + CLVisc)

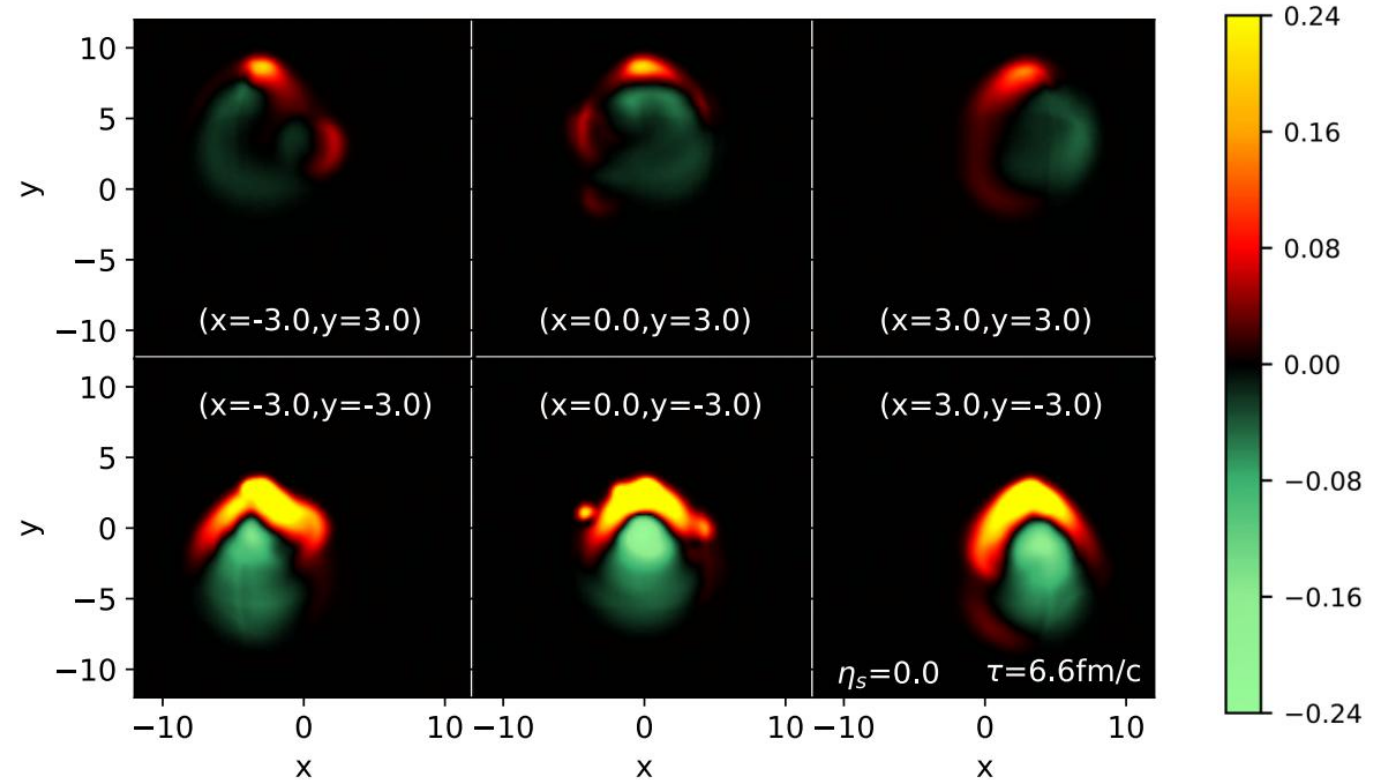
$$p\partial f(p) = -C(p) \quad (p \cdot u > p_{cut}^0)$$

$$\partial_\mu T^{\mu\nu}(x) = j^\nu(x)$$

$$j^\nu = \sum_i p_i^\nu \delta^{(4)}(x - x_i) \theta(p_{cut}^0 - p \cdot u)$$

**LBT:** YY He, T Luo, XN Wang, Y Zhu,  
PRC 91 (2015) 054908, PRC 97 (2018) 1, 019902

**CLVisc:**  
LG Pang, Q Wang, XN Wang, PRC 86 (2012) 024911  
LG Pang, H Petersen, XN Wang, PRC 97 (2018) 6,  
064918  
XY Wu, GY Qin, LG Pang, XN Wang, PRC 105 (2022)  
3, 034909



**CoLBT:**

W Chen, T Luo, SS Cao, LG Pang, XN Wang,  
PLB 777 (2018) 86-90



# LBT: Linear Boltzmann Transport

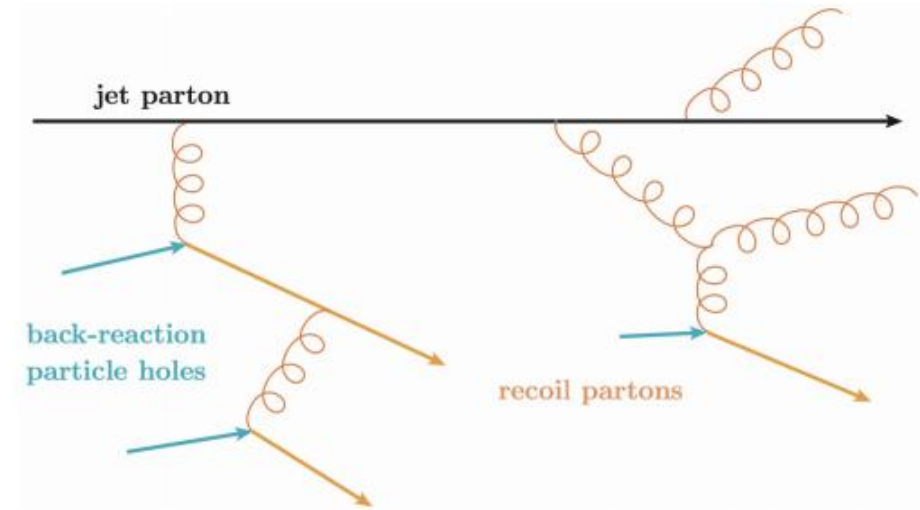
$$p_1 \partial f_1 = - \int dp_2 dp_3 dp_4 (f_1 f_2 - f_3 f_4) |M_{12 \rightarrow 34}|^2 (2\pi)^4 \delta^4(\sum_i p^i) + inelastic$$

Medium-induced gluon(HT):

$$\frac{dN_g}{dz d^2 k_{\perp} dt} \approx \frac{2C_A \alpha_s}{\pi k_{\perp}^4} P(z) \hat{q} (\hat{p} \cdot u) \sin^2 \frac{k_{\perp}^2 (t - t_0)}{4z(1-z)E}$$

Tracked partons:

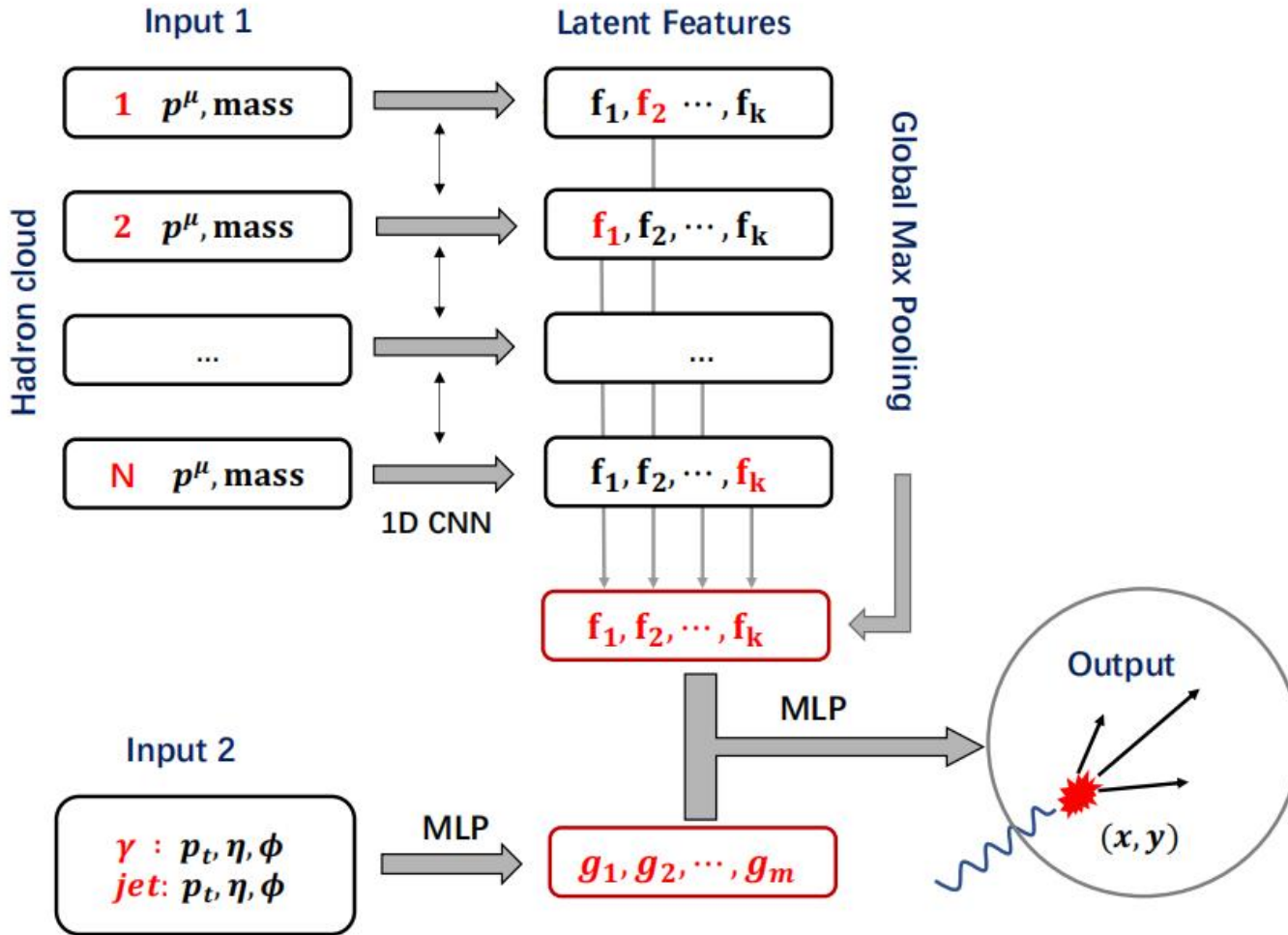
- Jet shower partons
- Thermal recoil partons
- Radiated gluons
- Negative partons(Back reaction induced by energy-momentum conservation)



YY He, T Luo, XN Wang, Y Zhu, PRC 91 (2015) 054908, PRC 97 (2018) 1, 019902



# DL assisted jet tomography (gamma-jet)



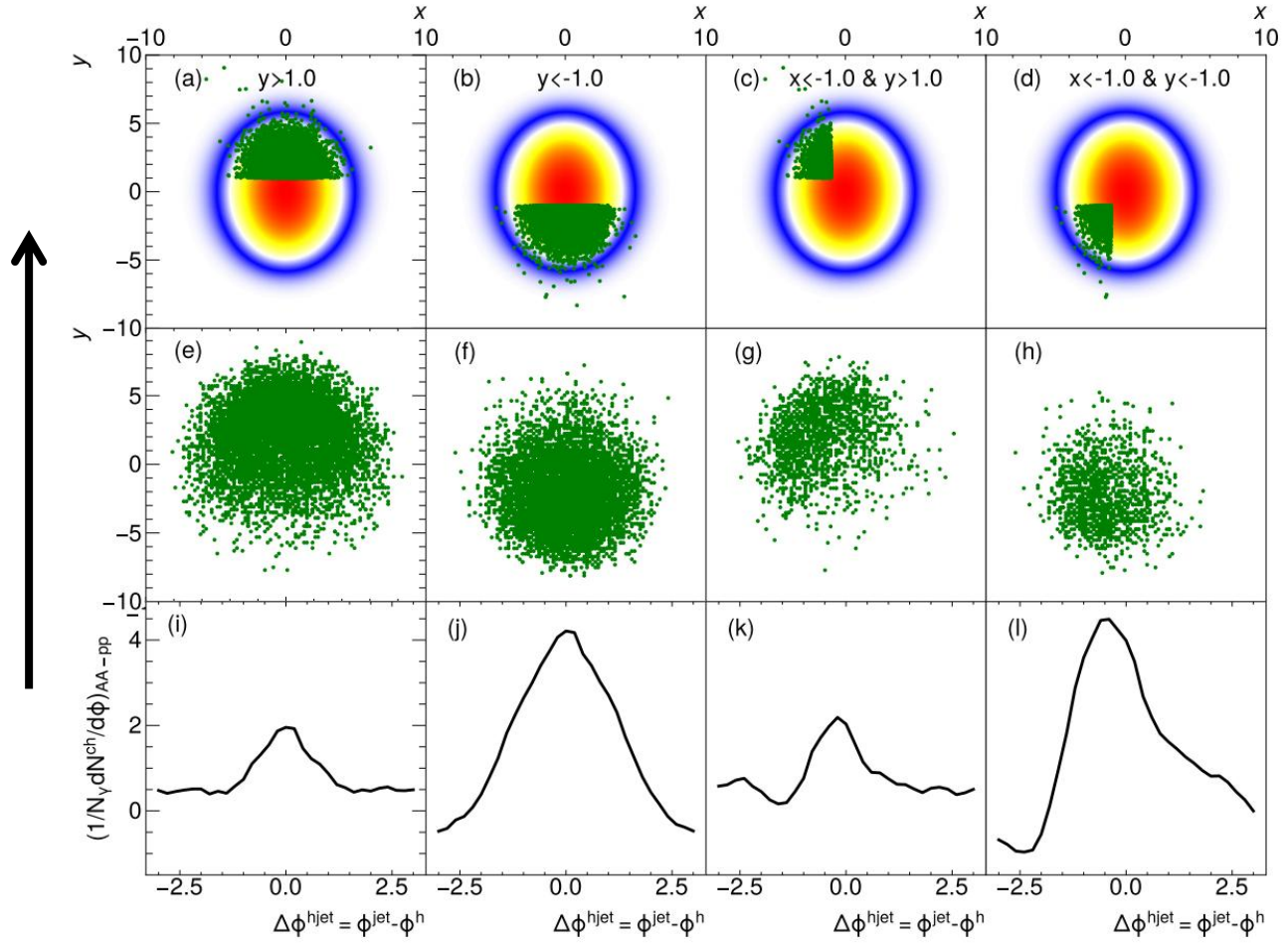
$ij \rightarrow kl$	$ M _{ij \rightarrow kl}^2$	
$gg \rightarrow gg$	$\frac{9}{2}g_s^4 \left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right)$	(A-1)
$gg \rightarrow q\bar{q}$	$\frac{3}{8}g_s^4 \left(\frac{4t^2+u^2}{9tu} - \frac{t^2+u^2}{s^2}\right)$	(A-2)
$gq \rightarrow gq$	$g_s^4 \left(\frac{s^2+u^2}{t^2} - \frac{4}{9} \frac{s^2+u^2}{su}\right)$	(A-3)
$g\bar{q} \rightarrow g\bar{q}$		
$q_i q_j \rightarrow q_i q_j$		
$q_i \bar{q}_j \rightarrow q_i \bar{q}_j$	$\frac{4}{9}g_s^4 \frac{s^2+u^2}{t^2}, \quad i \neq j$	(A-4)
$\bar{q}_i q_j \rightarrow \bar{q}_i q_j$		
$\bar{q}_i \bar{q}_j \rightarrow \bar{q}_i \bar{q}_j$		
$q_i q_i \rightarrow q_i q_i$	$\frac{4}{9}g_s^4 \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} - \frac{2}{3} \frac{s^2}{tu}\right)$	(A-5)
$\bar{q}_i \bar{q}_i \rightarrow \bar{q}_i \bar{q}_i$		
$q_i \bar{q}_i \rightarrow q_j \bar{q}_j$	$\frac{4}{9}g_s^4 \frac{t^2+u^2}{s^2}$	(A-6)
$q_i \bar{q}_i \rightarrow q_i \bar{q}_i$	$\frac{4}{9}g_s^4 \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} - \frac{2}{3} \frac{u^2}{st}\right)$	(A-7)
$q\bar{q} \rightarrow gg$	$\frac{8}{3}g_s^4 \left(\frac{4t^2+u^2}{9tu} - \frac{t^2+u^2}{s^2}\right)$	(A-8)

$$(x_i^{\text{net}}, y_i^{\text{net}}) = f(\{\vec{p}\}_i, \theta),$$

Z Yang, YY He, W Chen, WY Ke, LG Pang, XN Wang, EPJC 83 (2023) 7, 652

# DL assisted jet tomography

The Jet direction



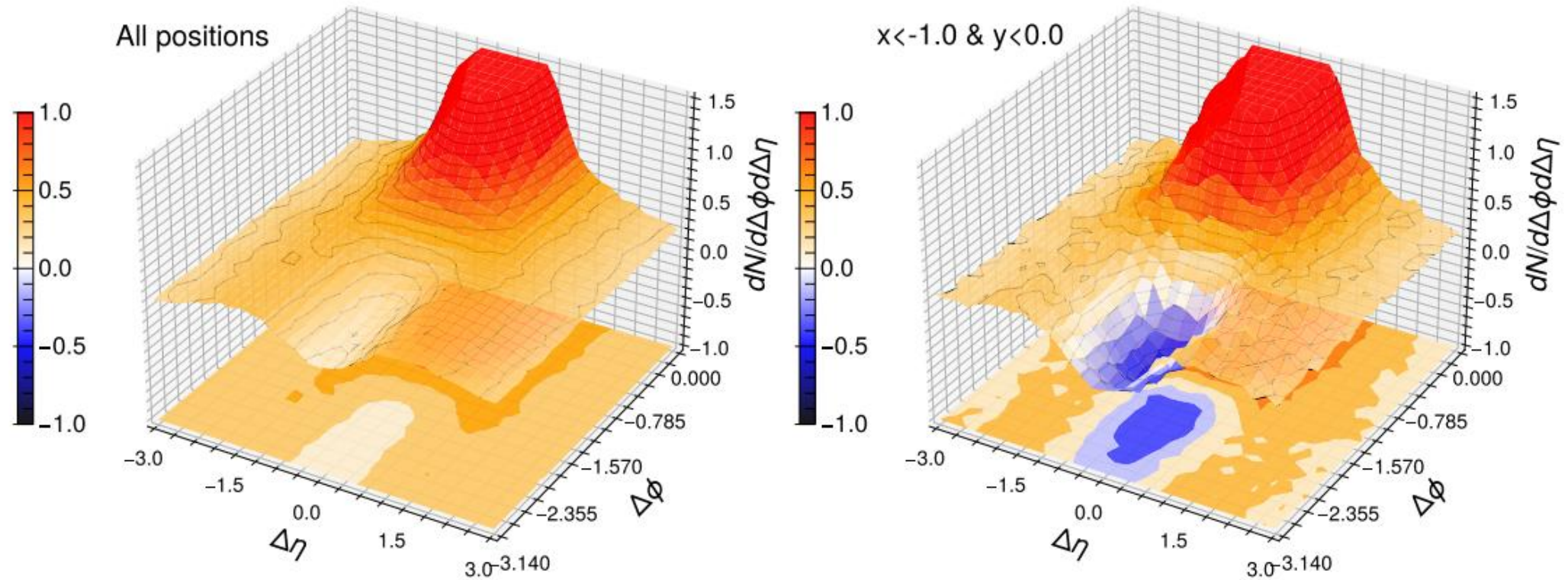
Network predictions

True locations

Jet hadron correlation for selected events whose locations are constrained to specific regions using DL assisted jet tomography

Z Yang, YY He, W Chen, WY Ke, **LG Pang**, XN Wang, EPJC 83 (2023) 7, 652

# Enhance the Diffusion Wake signal

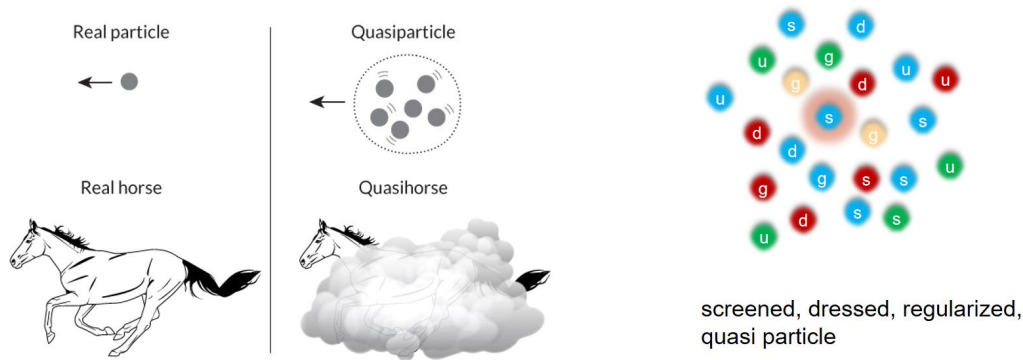


Z Yang, YY He, W Chen, WY Ke, LG Pang, XN Wang, EPJC 83 (2023) 7, 652

Z Yang, T Luo, W Chen, LG Pang, XN Wang, PRL 130 (2023) 5, 052301



# Effective theory: DL For Quasi Particle Mass



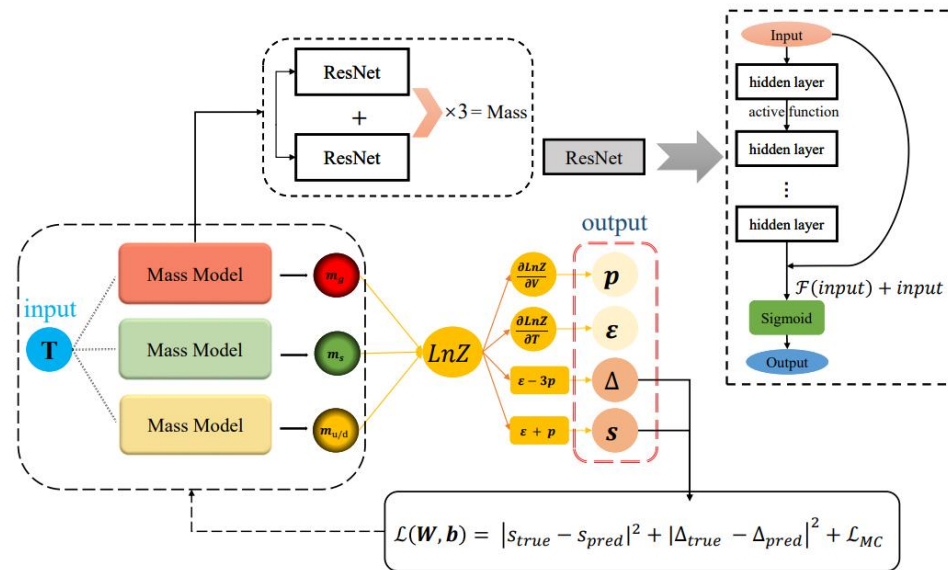
FuPeng Li, HL Lu, LG Pang, GY Qin, PLB 2023

$$\ln Z(T) = \ln Z_g(T) + \ln Z_{u,d}(T) + \ln Z_s(T),$$

Fermi-Dirac distributions,

$$\ln Z_g(T) = -\frac{16V}{2\pi^2} \int_0^\infty p^2 dp \ln \left[ 1 - \exp \left( -\frac{1}{T} \sqrt{p^2 + m_g^2(T)} \right) \right], \quad (2)$$

$$\ln Z_{q_i}(T) = +\frac{12V}{2\pi^2} \int_0^\infty p^2 dp \ln \left[ 1 + \exp \left( -\frac{1}{T} \sqrt{p^2 + m_{q_i}^2(T)} \right) \right], \quad (3)$$



quarks,  $m_s(T, \theta_2)$  for strange quark and  $m_g(T, \theta_3)$  for gluons, where  $\theta_1, \theta_2$  and  $\theta_3$  are the parameters in DNN shown in Fig. 1.

The resulting pressure and energy density are computed using the following statistical formulae,

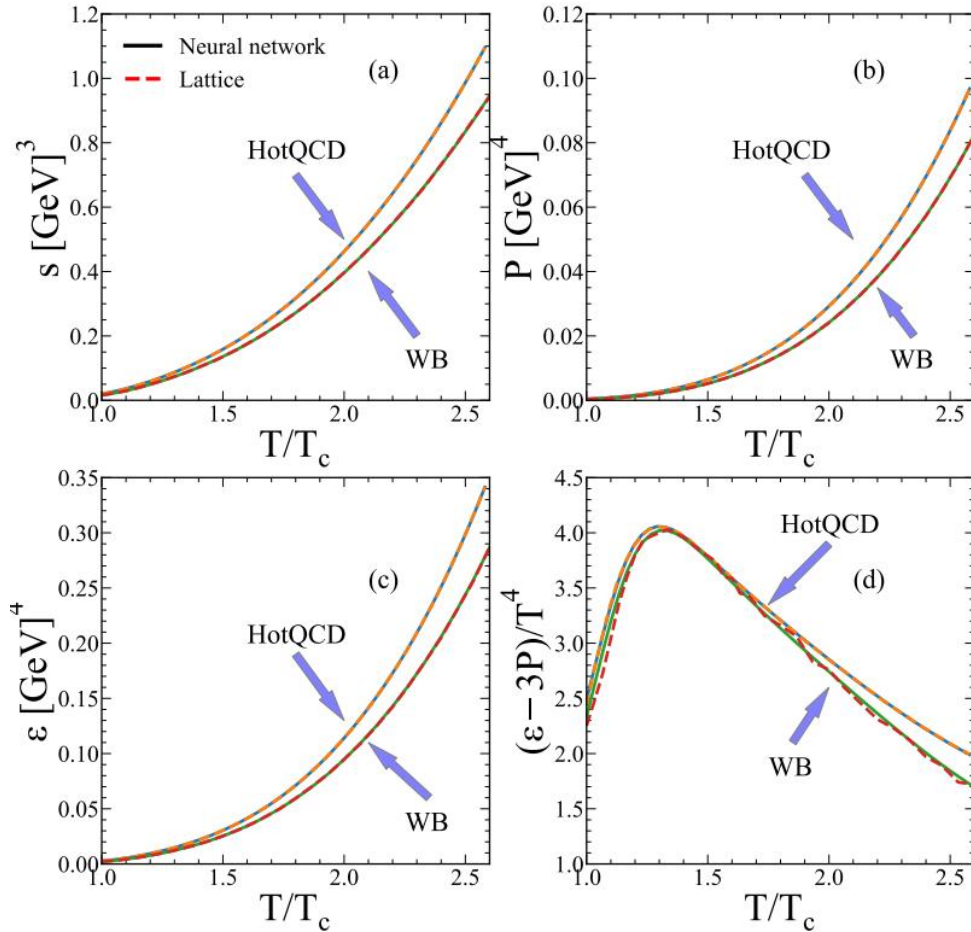
$$P(T) = T \left( \frac{\partial \ln Z(T)}{\partial V} \right)_T, \quad (5)$$

$$\epsilon(T) = \frac{T^2}{V} \left( \frac{\partial \ln Z(T)}{\partial T} \right)_V, \quad (6)$$

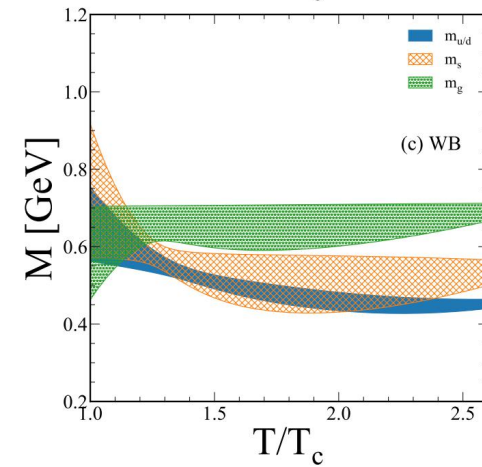
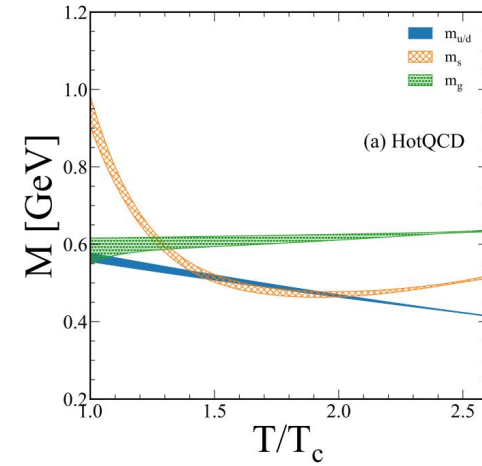


# The learned quasi parton mass

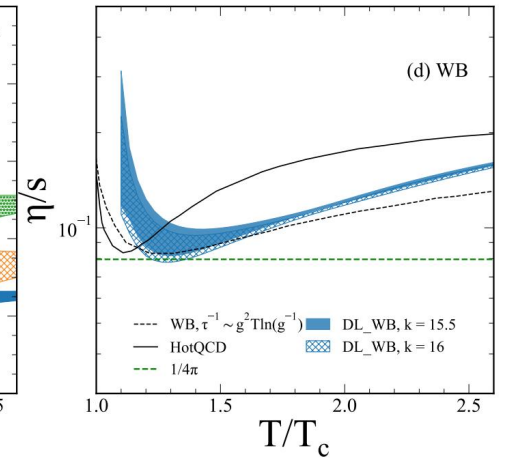
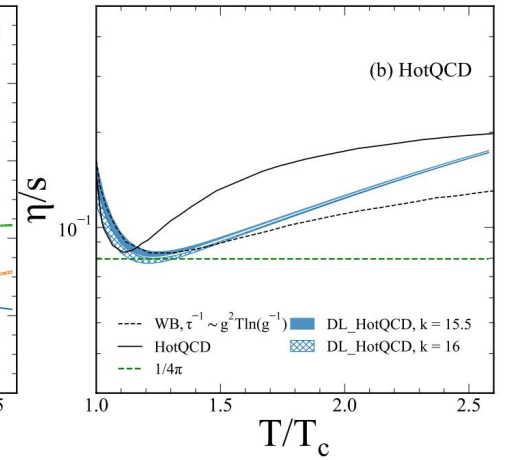
### EoS vs Lattice QCD



### Learned Mass



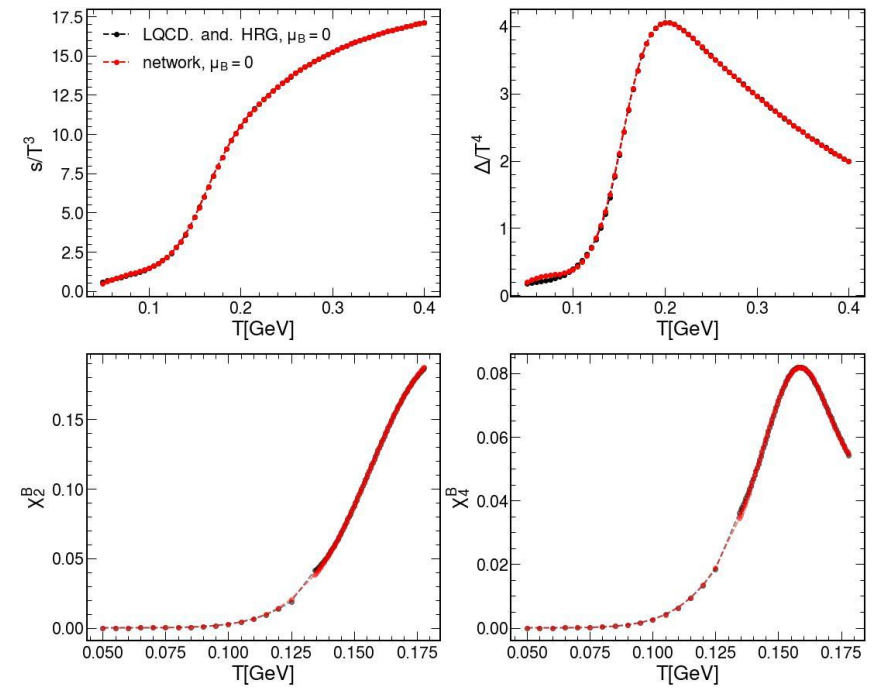
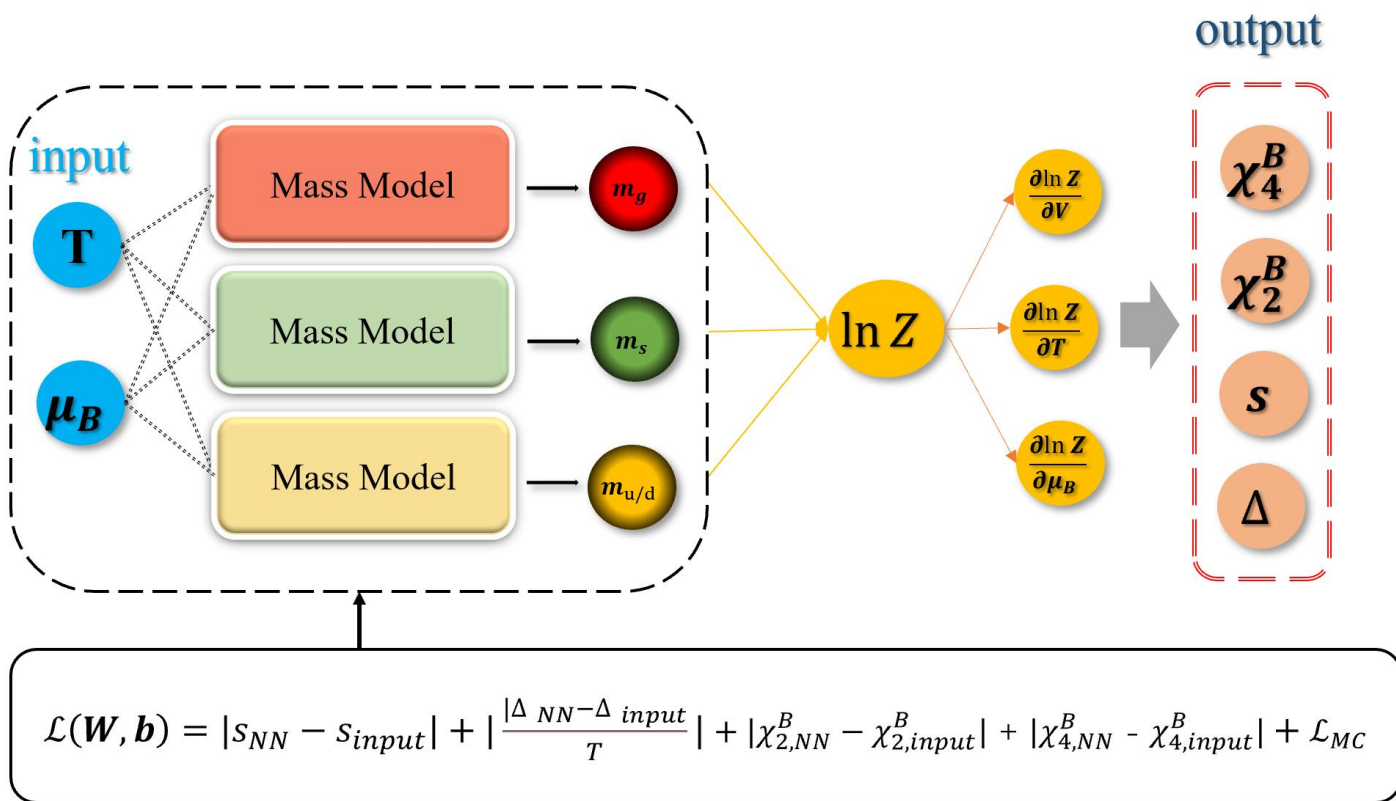
### $\eta/s$



FuPeng Li, HL Lu, LG Pang, GY Qin, PLB 2023



# Looking for CEP using Quasi Parton Model



**Model:**  
 Deep learning Quasi Parton Model  
 Effective theory of strongly coupled QGP and nuclear matter at finite baryon density

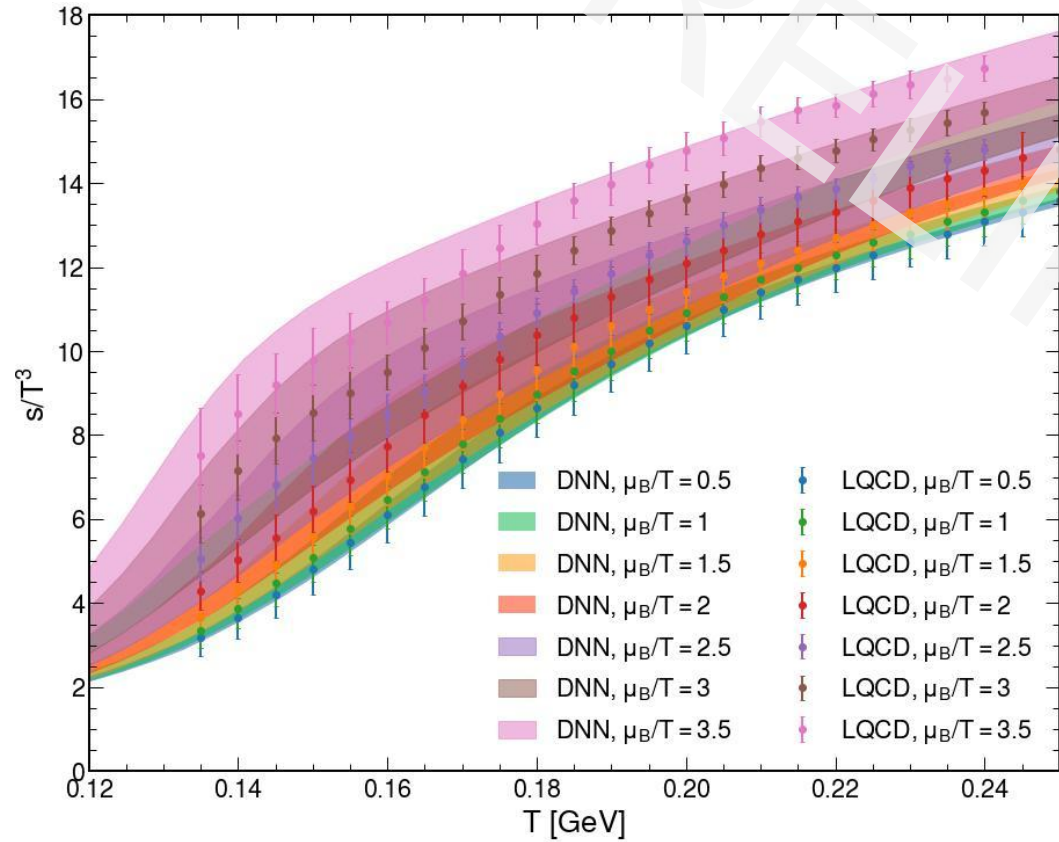
**Training data: Lattice QCD + HRG**  
 PRD 95, 054504 (2017)  
 PRL 118, 182301 (2017)  
 PRD 90, 094503 (2014)



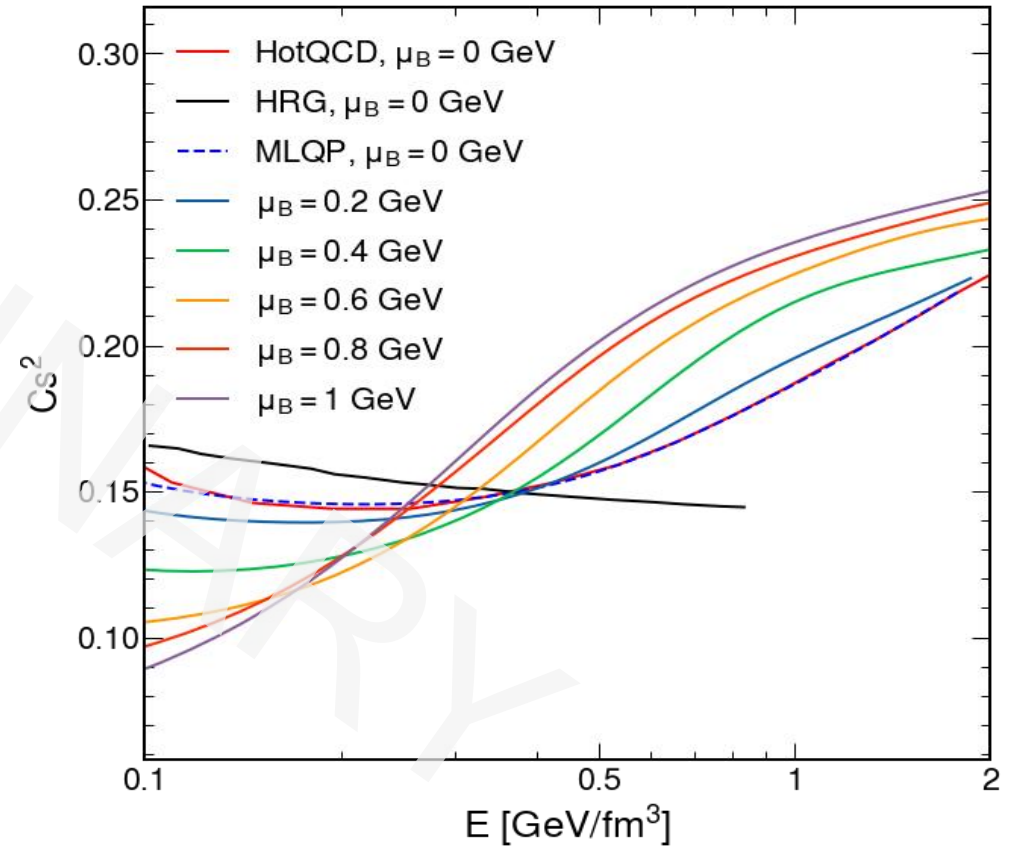


# Predictions of DL quasi parton model

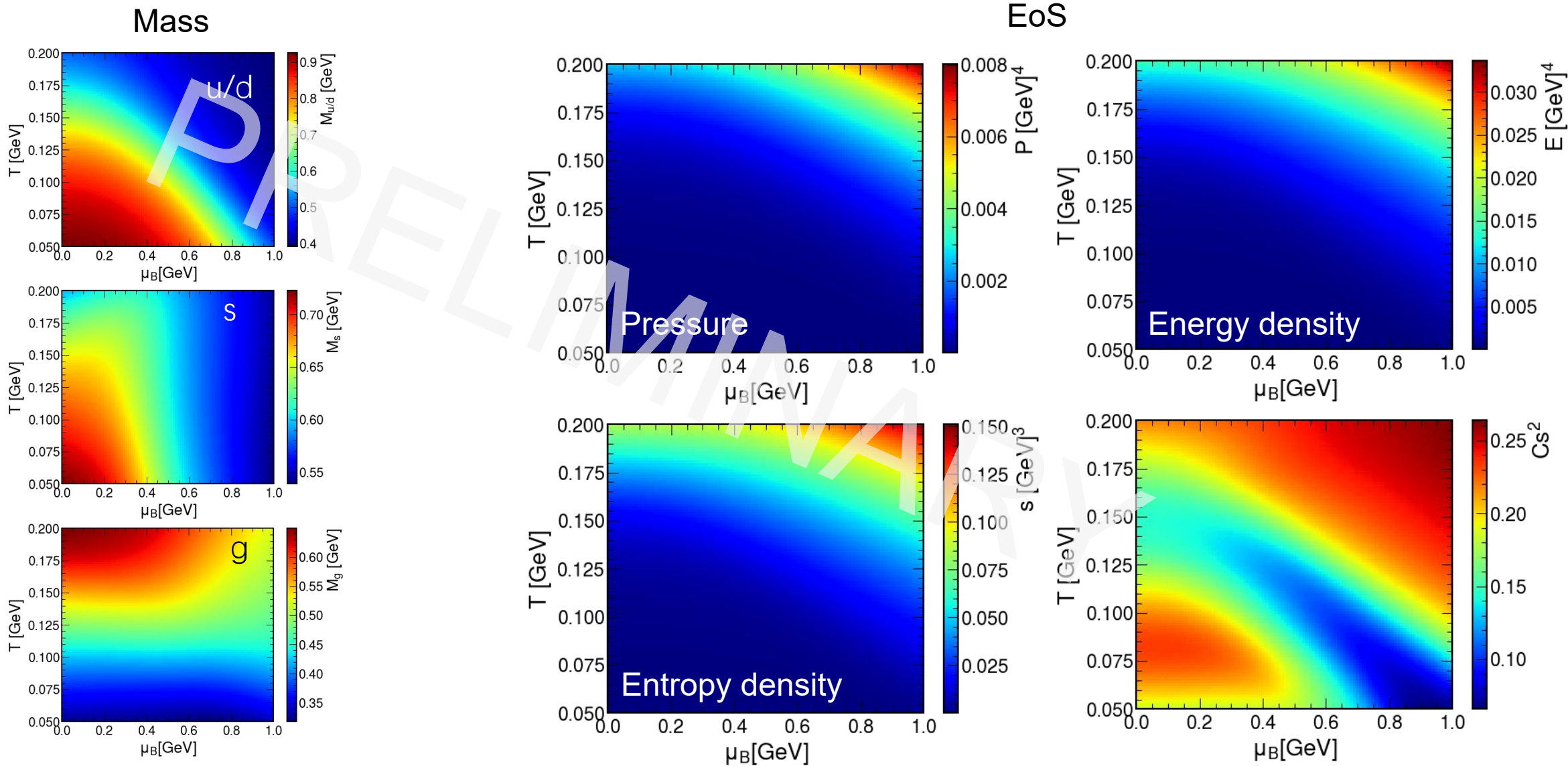
### Entropy density



### Speed of sound



# The learned Mass and EoS





# Reviews

## Colloquium: Machine learning in nuclear physics

Amber Boehnlein, Markus Diefenthaler, Nobuo Sato, Malachi Schram, Veronique Ziegler, Cristiano Faneli, Morten Hjorth-Jensen, Tanja Horn, Michelle P. Kuchera, Dean Lee, Witold Nazarewicz, Peter Ostroumov, Orginos, Alan Poon, Xin-Nian Wang, Alexander Scheinker, Michael S. Smith, and Long-Gang Pang  
Rev. Mod. Phys. **94**, 031003 – Published 8 September 2022

Article    References    No Citing Articles    PDF    HTML    Export Citation

### ABSTRACT

Advances in machine learning methods provide tools that have broad applicability in scientific research. These techniques are being applied across the diversity of nuclear physics research topics, leading

## Exploring QCD matter in extreme conditions with Machine Learning

Kai Zhou (Frankfurt U., FIAS), Lingxiao Wang (Frankfurt U., FIAS), Long-Gang Pang (CCNU, Wuhan, Inst. Part. Phys.), Shuzhe Shi (Stony Brook U.)

Mar 27, 2023

146 pages

e-Print: [2303.15136](#) [hep-ph]

## High energy nuclear physics meets Machine Learning

Wan-Bing He (Fudan U., Shanghai and Fudan U.), Yu-Gang Ma (Fudan U., Shanghai and Fudan U.), Long-Gang Pang, Huichao Song (CCNU, Wuhan, Inst. Part. Phys. and Hua-Zhong Normal U., LQLP and Peking U.), Kai Zhou (Frankfurt U., FIAS) (Mar 12, 2023)

e-Print: [2303.06752](#) [hep-ph]

### HEPML-LivingReview

#### A Living Review of Machine Learning for Particle Physics

Modern machine learning techniques, including deep learning, is rapidly being applied, adapted, and developed for high energy physics. The goal of this document is to provide a nearly comprehensive list of citations for those developing and applying these approaches to experimental, phenomenological, or theoretical analyses. As a living document, it will be updated as often as possible to incorporate the latest developments. A list of proper (unchanging) reviews can be found within. Papers are grouped into a small set of topics to be as useful as possible. Suggestions are most welcome.

[download](#) [review](#)

The purpose of this note is to collect references for modern machine learning as applied to particle physics. A minimal number of categories is chosen in order to be as useful as possible. Note that papers may be referenced in more than one category. The fact that a paper is listed in this document does not endorse or validate its content - that is for the community (and for peer-review) to decide. Furthermore, the classification here is a best attempt and may have flaws - please let us know if (a) we have missed a paper you think should be included, (b) a paper has been misclassified, or (c) a citation for a paper is not correct or if the journal information is now available. In order to be as useful as possible, this document will continue to evolve so please check back before you write your next paper. If you find this review helpful, please consider citing it using `\cite{hepmlivingreview}` in HEPML.bib.

- Reviews
  - Modern reviews
    - [Jet Substructure at the Large Hadron Collider: A Review of Recent Advances in Theory and Machine Learning \[DOI\]](#)
    - [Deep Learning and its Application to LHC Physics \[DOI\]](#)





# Summary

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- For soft probes, DL serves as an EoS-meter
- For hard probes, DL assisted jet tomography aids in the investigation of QCD EoS through Mach cones
- DL and auto-diff are widely used to represent unknown functions to construct effective theories of QCD EoS.
- DL quasi parton model are extended to finite  $\mu_B$  region



# Active learning to map out unphysical EoS

$(\mu_{BC}, \alpha_{\text{diff}}, w, \rho) \mapsto P(T, \mu_B) \mapsto \{\text{acceptable, unstable, acausal}\}.$

4 parameters from 3D Ising model

QCD EoS

Labels for classification

Acceptable = Stable + Causal

$$P, s, \varepsilon, n_B, \chi_2^B, \left(\frac{\partial S}{\partial T}\right)_{n_B} > 0,$$

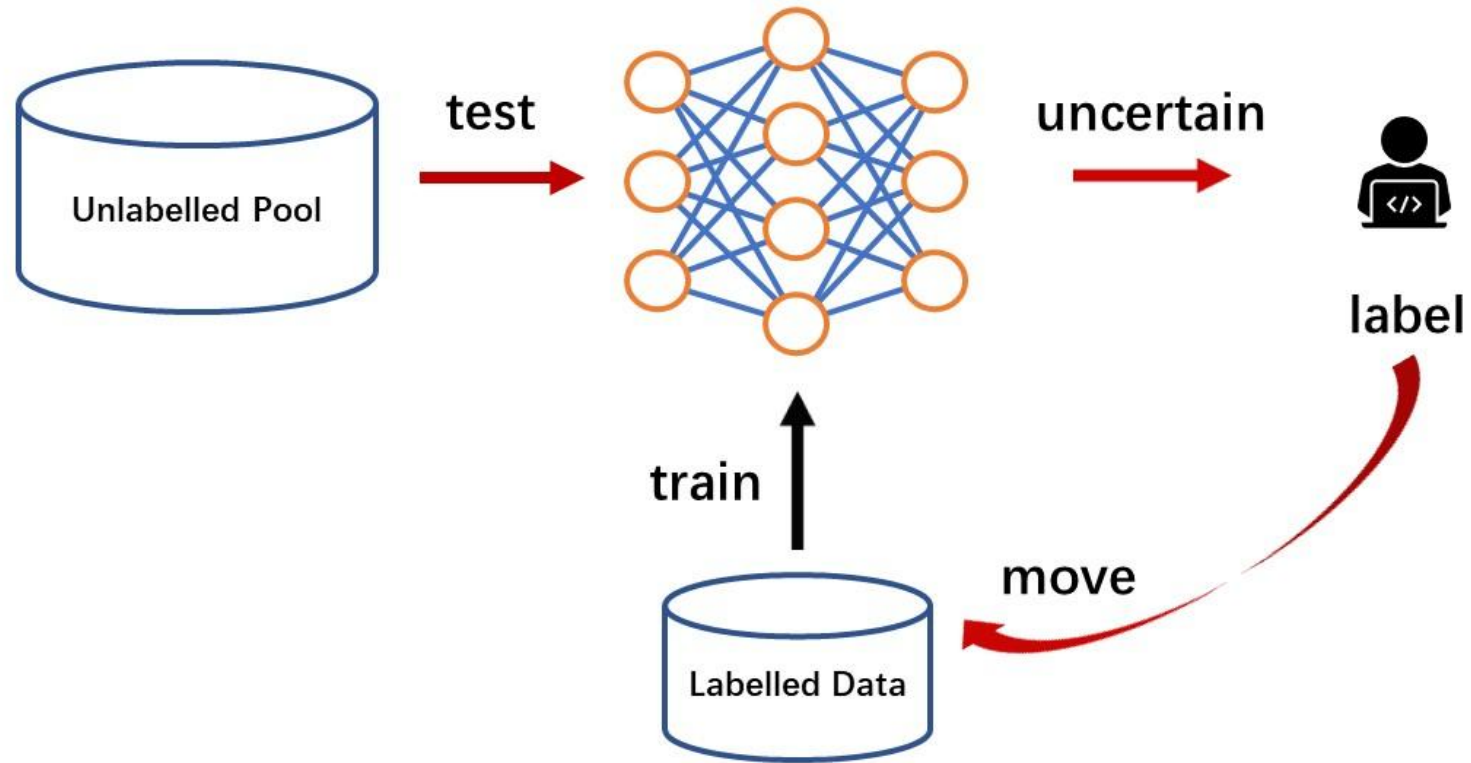
$$0 \leq c_s^2 \leq 1.$$

D. Mroczek, M. Hjorth-Jensen, J. Noronha-Hostler, P. Parotto, C. Ratti, and R. Vilalta, PRC 107, 054911





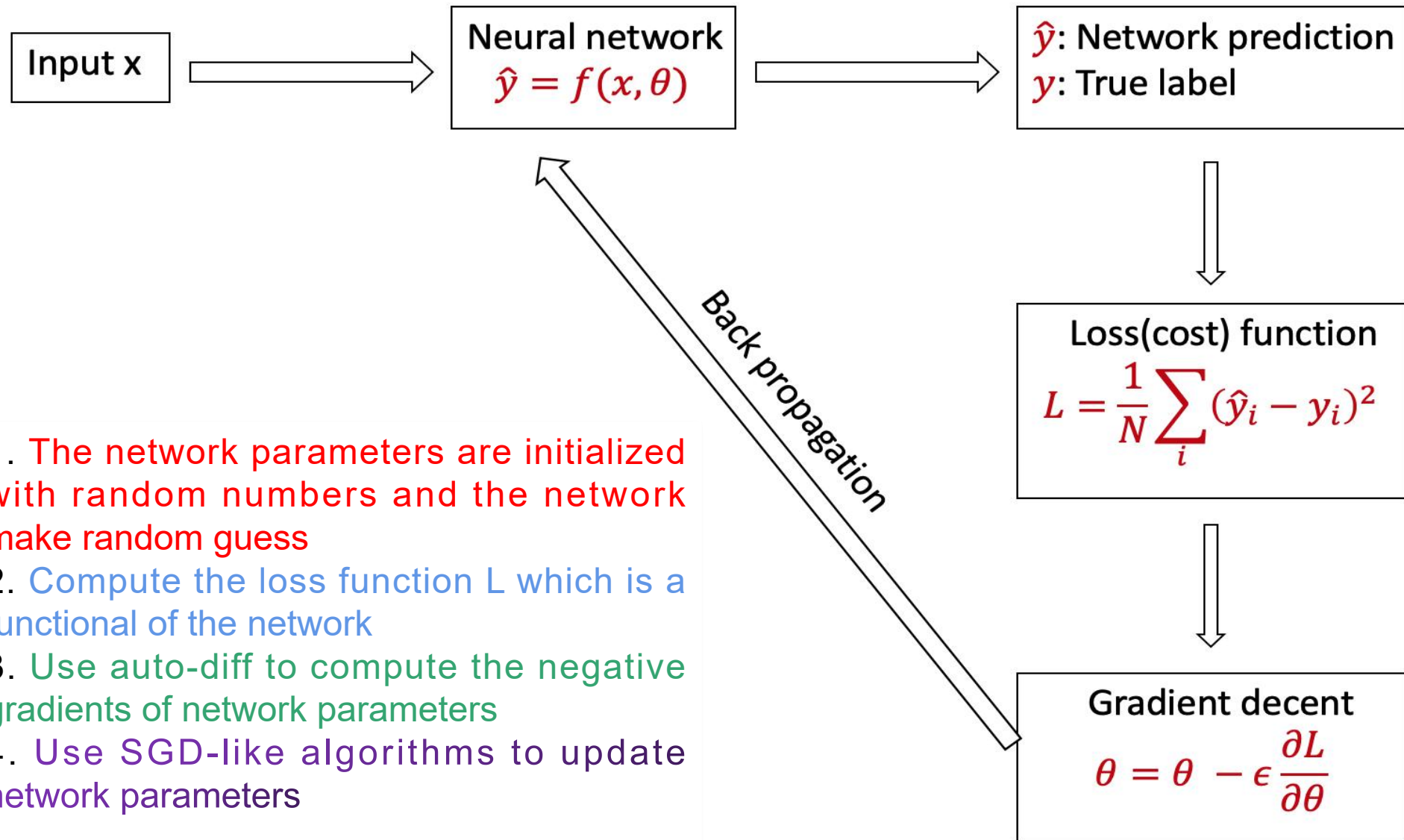
# Active learning procedure



D. Mroczek, M. Hjorth-Jensen, J. Noronha-Hostler, P. Parotto, C. Ratti, and R. Vilalta, PRC 107, 054911



# How does the network learn

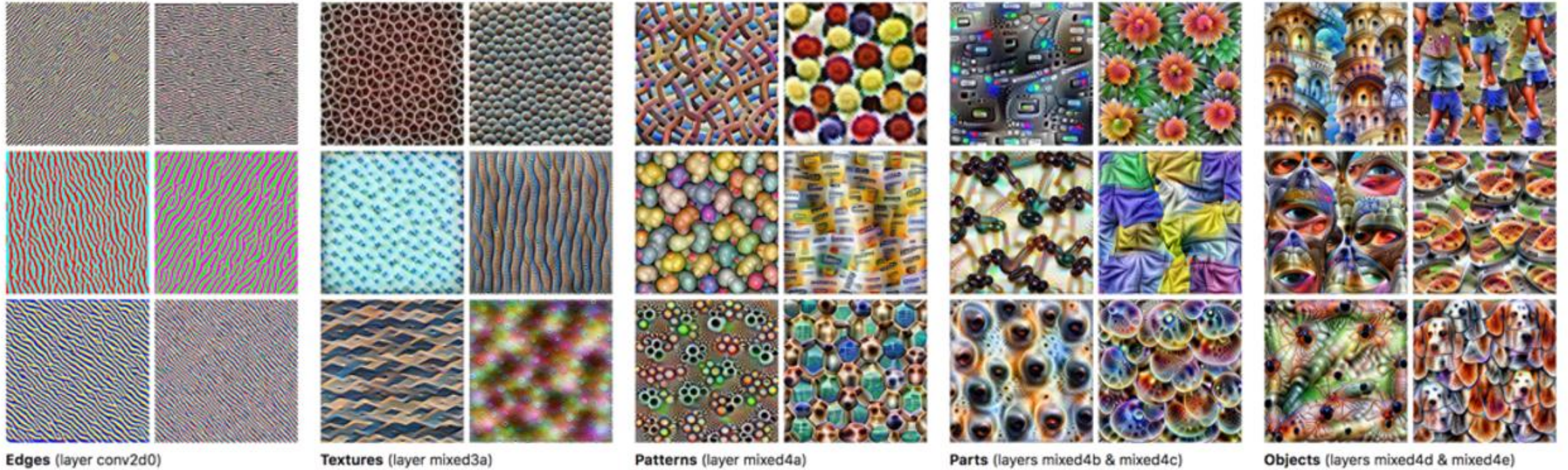


1. The network parameters are initialized with random numbers and the network make random guess
2. Compute the loss function  $L$  which is a functional of the network
3. Use auto-diff to compute the negative gradients of network parameters
4. Use SGD-like algorithms to update network parameters



# What has been learned (Global interpretation)

Olah, et al., "Feature Visualization", Distill, 2017.



shallow layers

deep layers



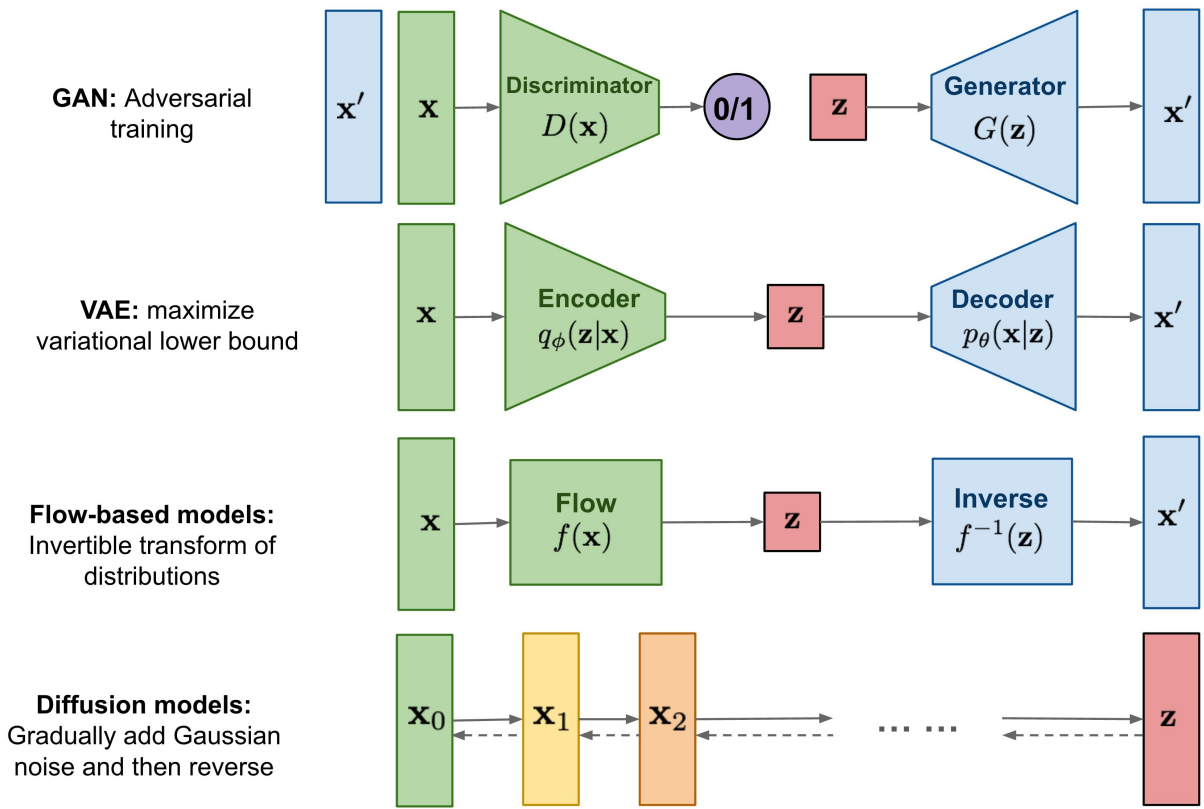
# Local interpretation



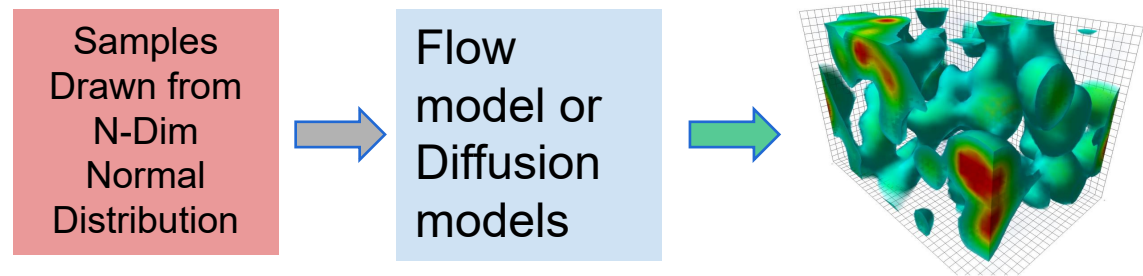
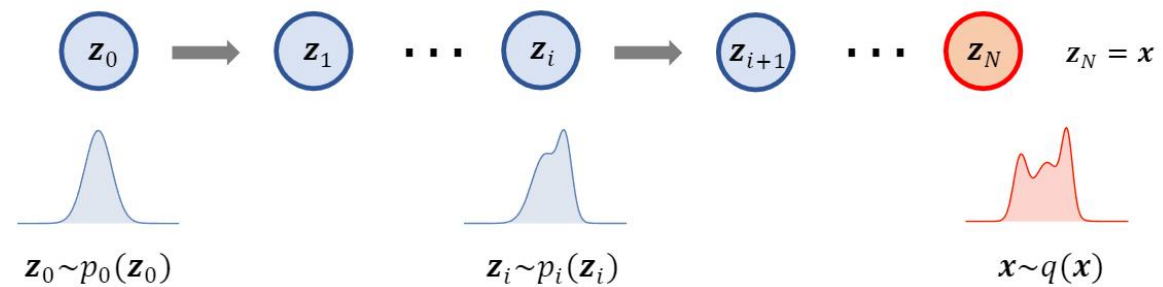
- **Ablation studies:** LIME or Prediction Difference Analysis. **M. Tulio Ribeiro, et. al.** "Why should I trust you?"
- **Class activation map:** map the deep layers to the input image, look for the most important region for decision making. **BoLei Zhou, et. al.** "Learning Deep Features for discriminative localization"
- **Layer-wise relevance propagation:** set the relevance of the output layer to 1, propagate the relevance to the input data, to look for the most important region for decision making.



# Generative models: MC sampling



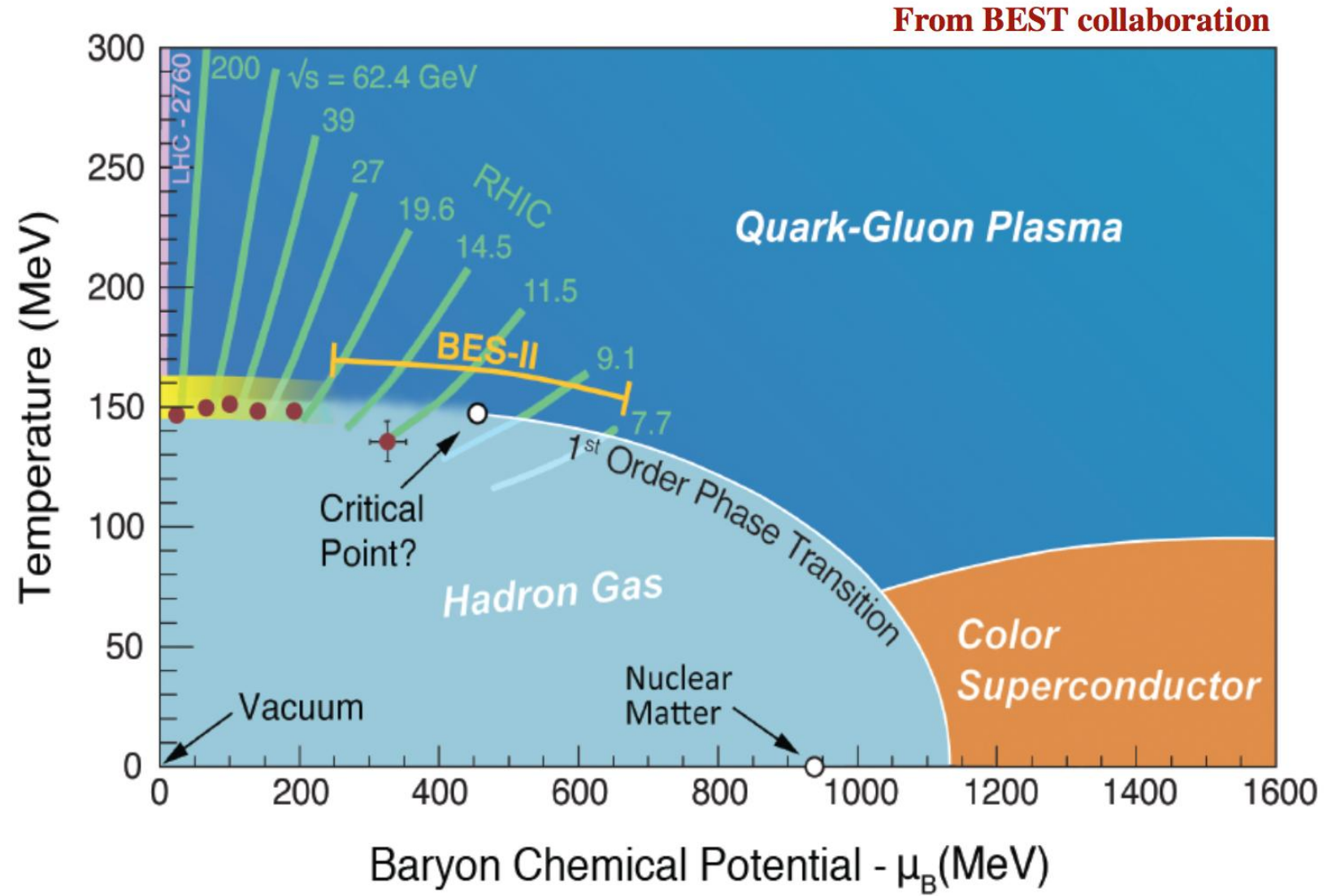
Similar to Box Muller algorithm



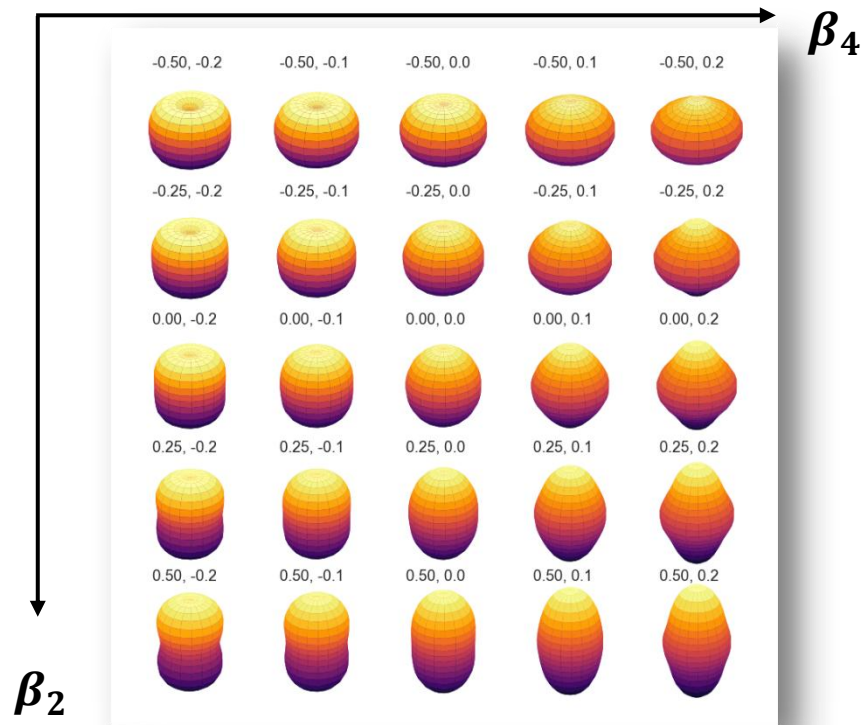
Flow-based generative models for Markov chain Monte Carlo in lattice field theory  
 Albergo, Kanwar, Shanahan 1904.1207



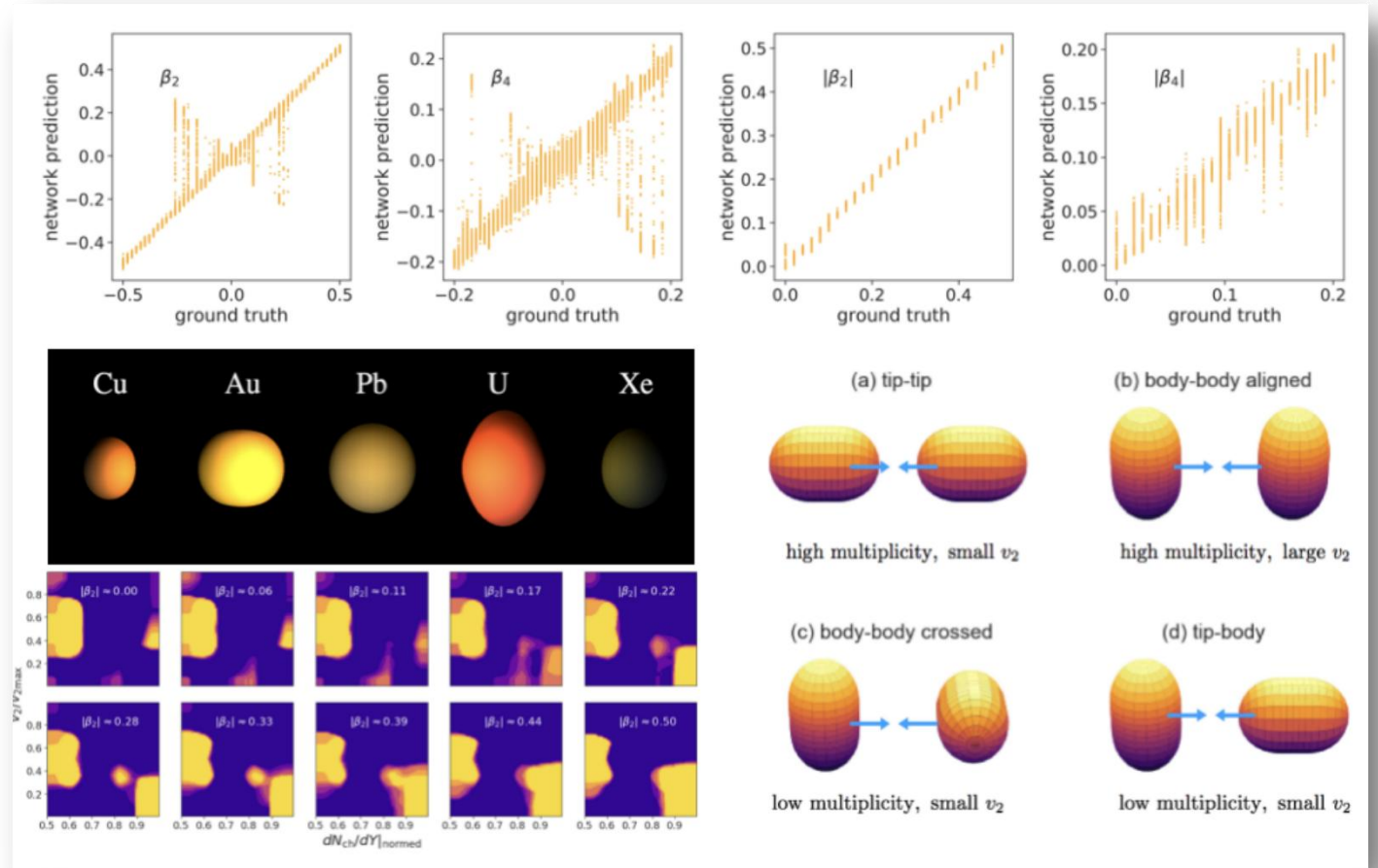
# Explore QCD phase structure using HIC



# Determining nuclear deformation



Data: Trento + Matching

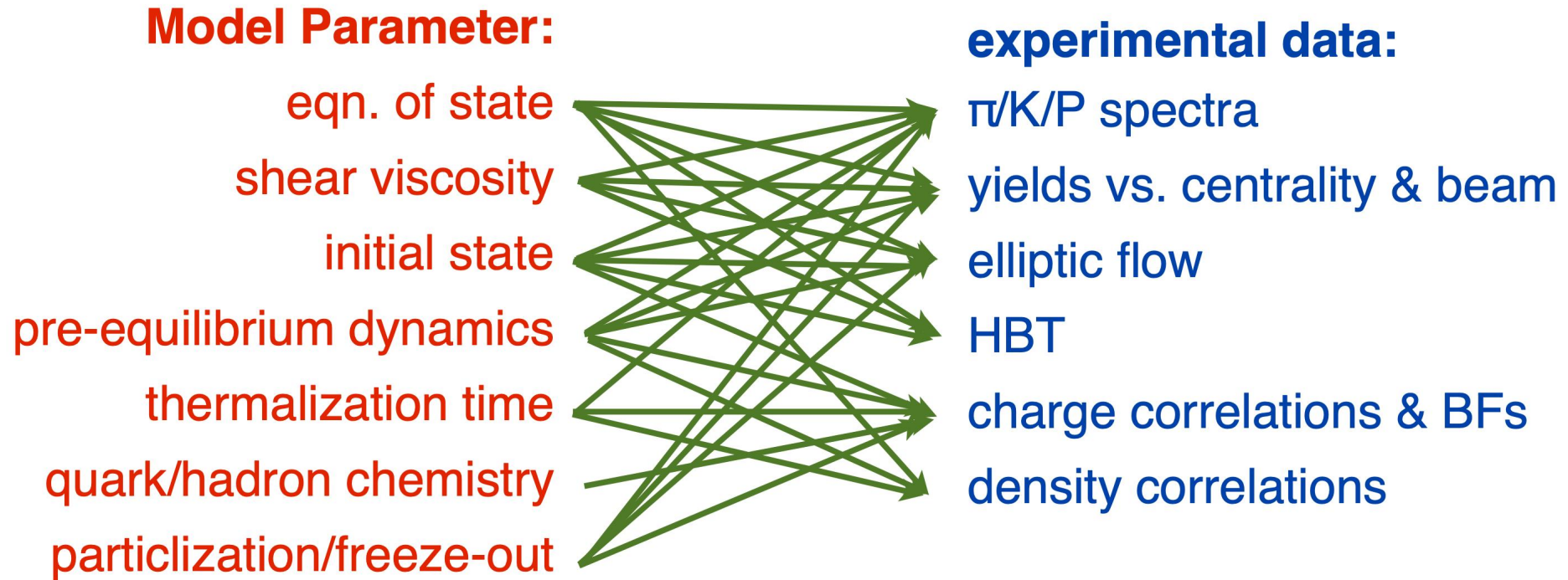


L.-G. Pang, K. Zhou and X.-N. Wang, arXiv:1906.06429



# Challenges

Fig from S. Bass QM2017 (Bayesian method)



(1) Multiple parameters entangle with multiple observables

(2) Different parameter combinations describe the same data





# Bayesian analysis QCD EoS

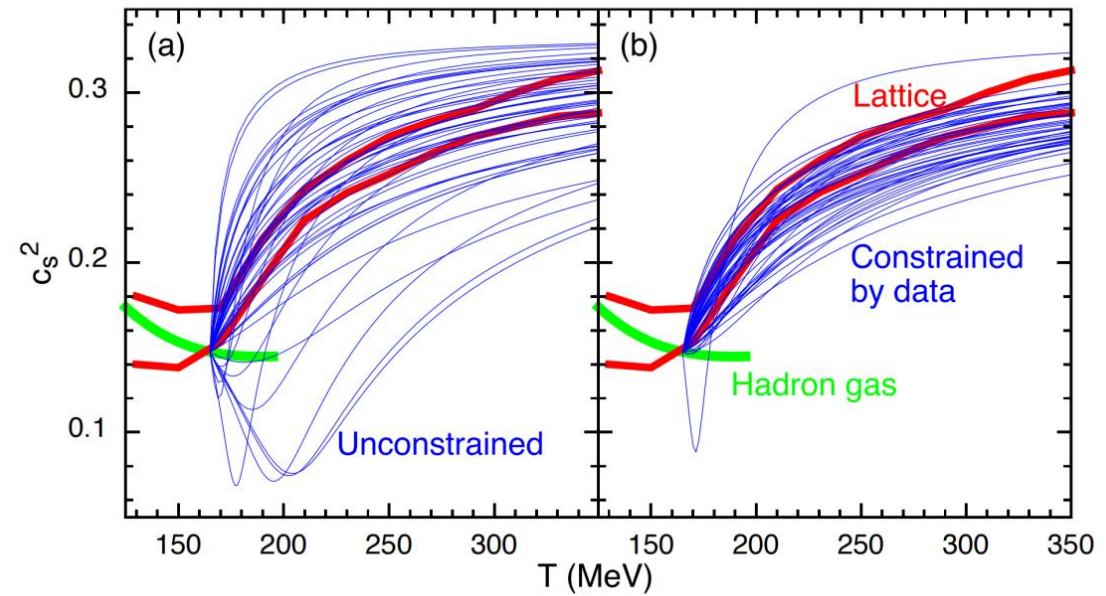
The  $c_s^2$  is parameterized as a function of energy density in the following,

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2} \quad (2.12)$$

where  $X_0 = \sqrt{12} R X' c_s(\epsilon_h)$ ,  $x \equiv \ln \frac{\epsilon}{\epsilon_h}$ ,  $\epsilon_h$  is the energy density at  $T = 165$  MeV,  $R$  and  $X'$  are the two parameters in the EoS to be determined. Randomly choosing  $R$  and  $X'$  from the range  $-0.9 < R < 2$  and  $0.5 < X' < 5$  generate the unconstrained EoS that varies in a large region between  $c_s^2 = 0.05$  and  $c_s^2 = 0.33$ , as shown in Fig. 2.4-a. This corresponds to the a priori distribution of  $c_s^2$  parameters together with other 12 parameters in the model  $P(\theta)$ .

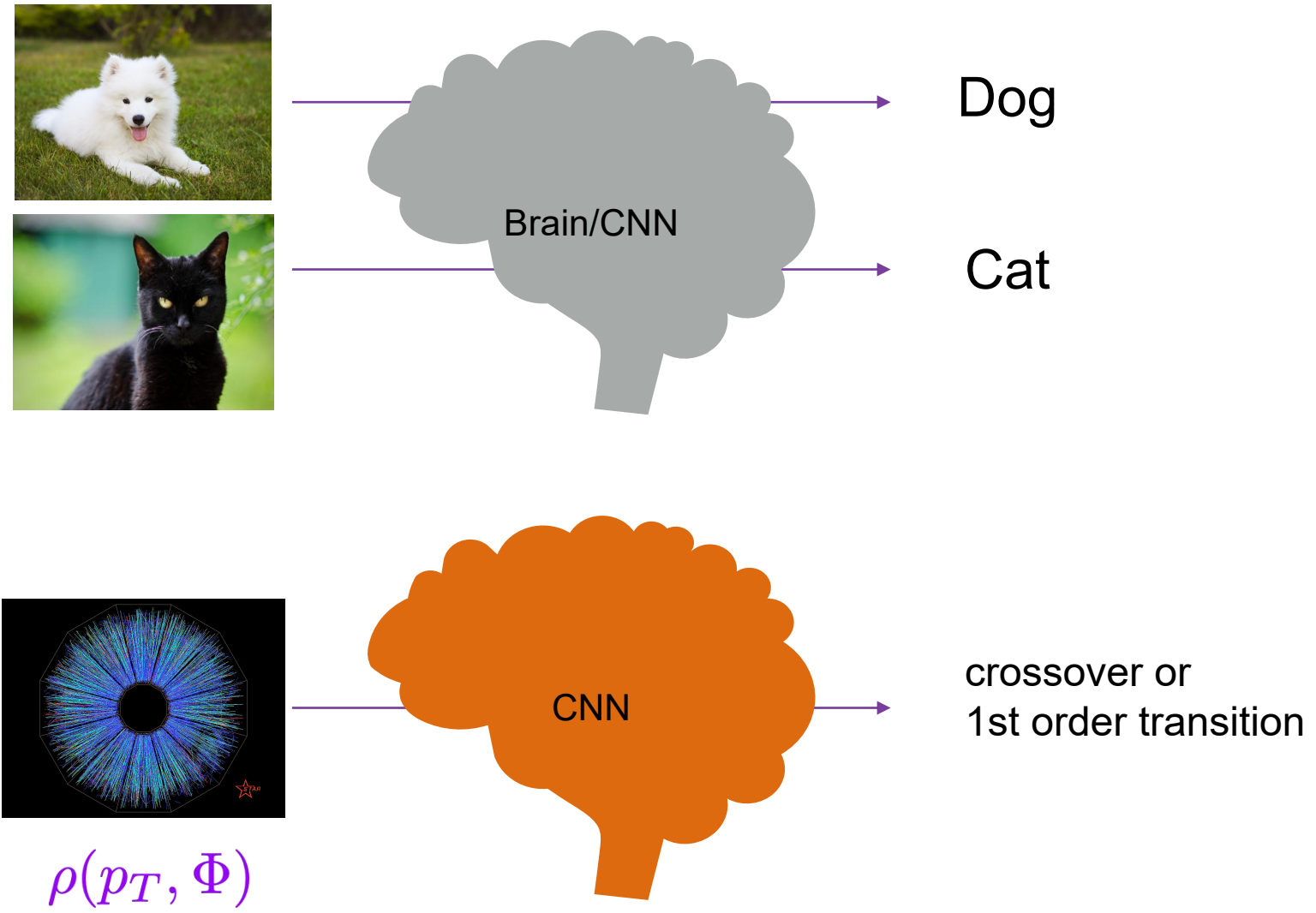
Likelihood: 
$$P(D|\theta) = \prod_i \exp\left(-\frac{(z_i(\theta) - z_{i,\text{exp}})^2}{2}\right)$$

Posterior: 
$$P(\theta | D) \propto P(D | \theta)P(\theta)$$



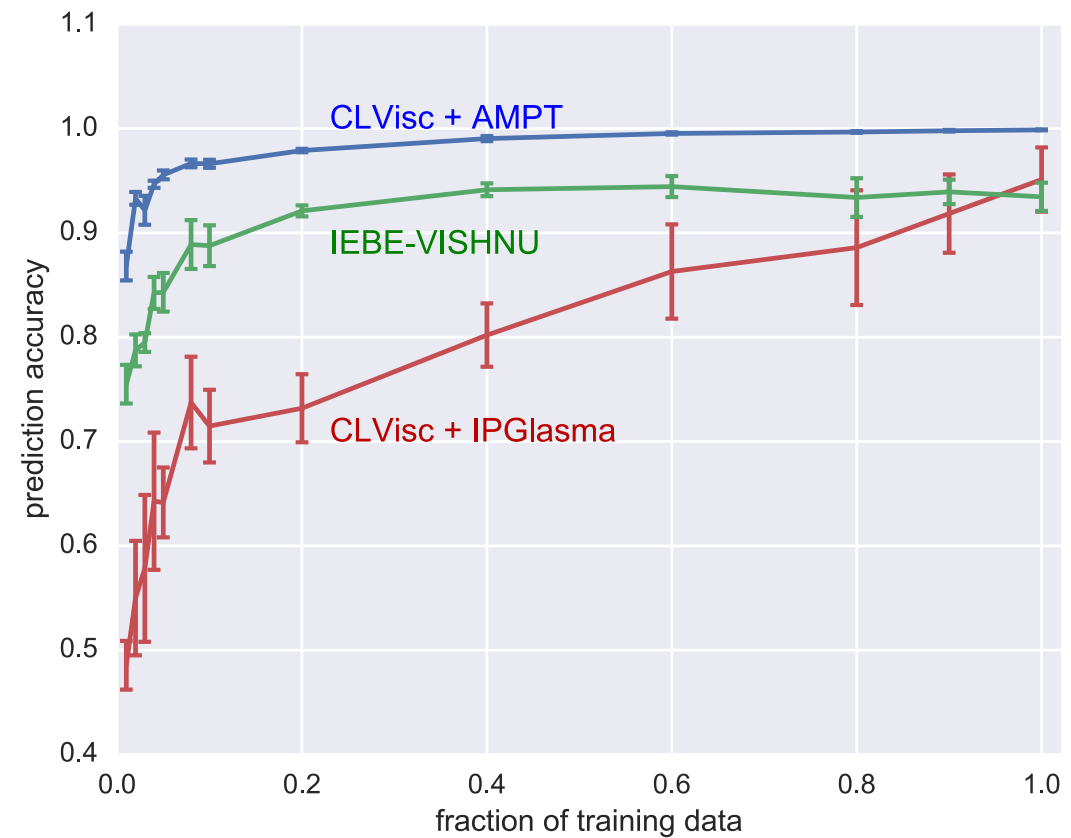
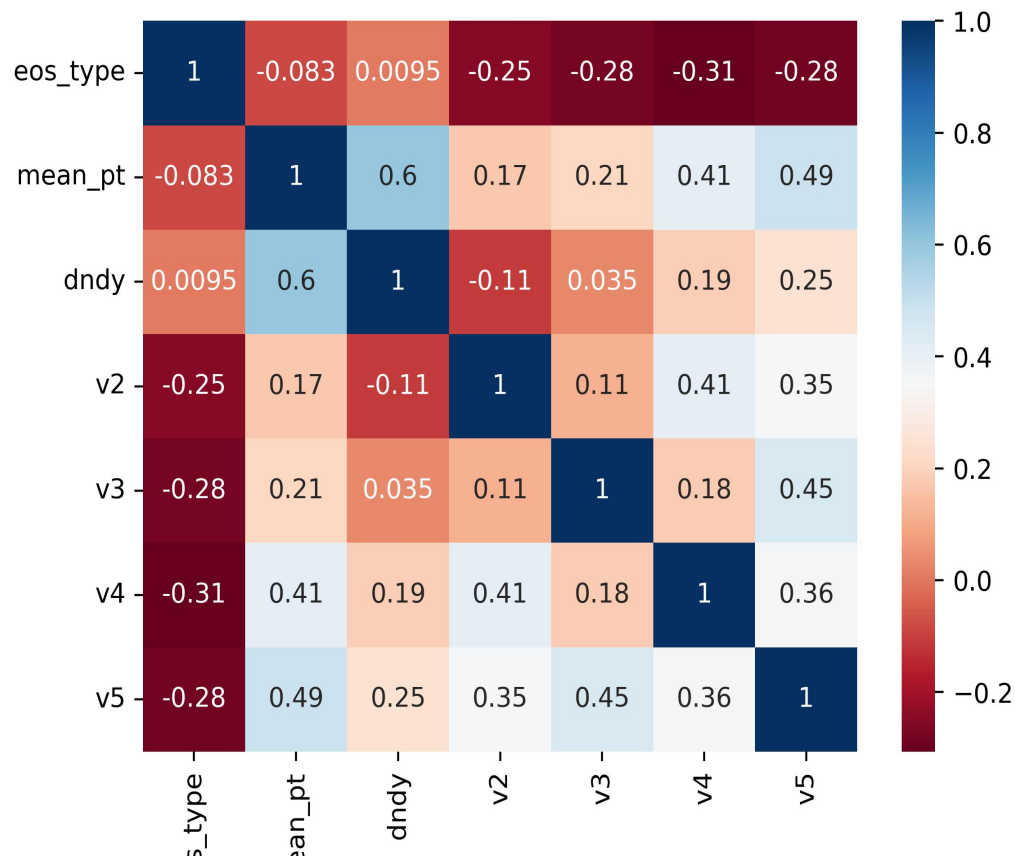
S. Pratt, E. Sangaline, P. Sorensen, H. Wang, PRL. 114 (2015) 202301.

# Excellent pattern recognition of deep learning

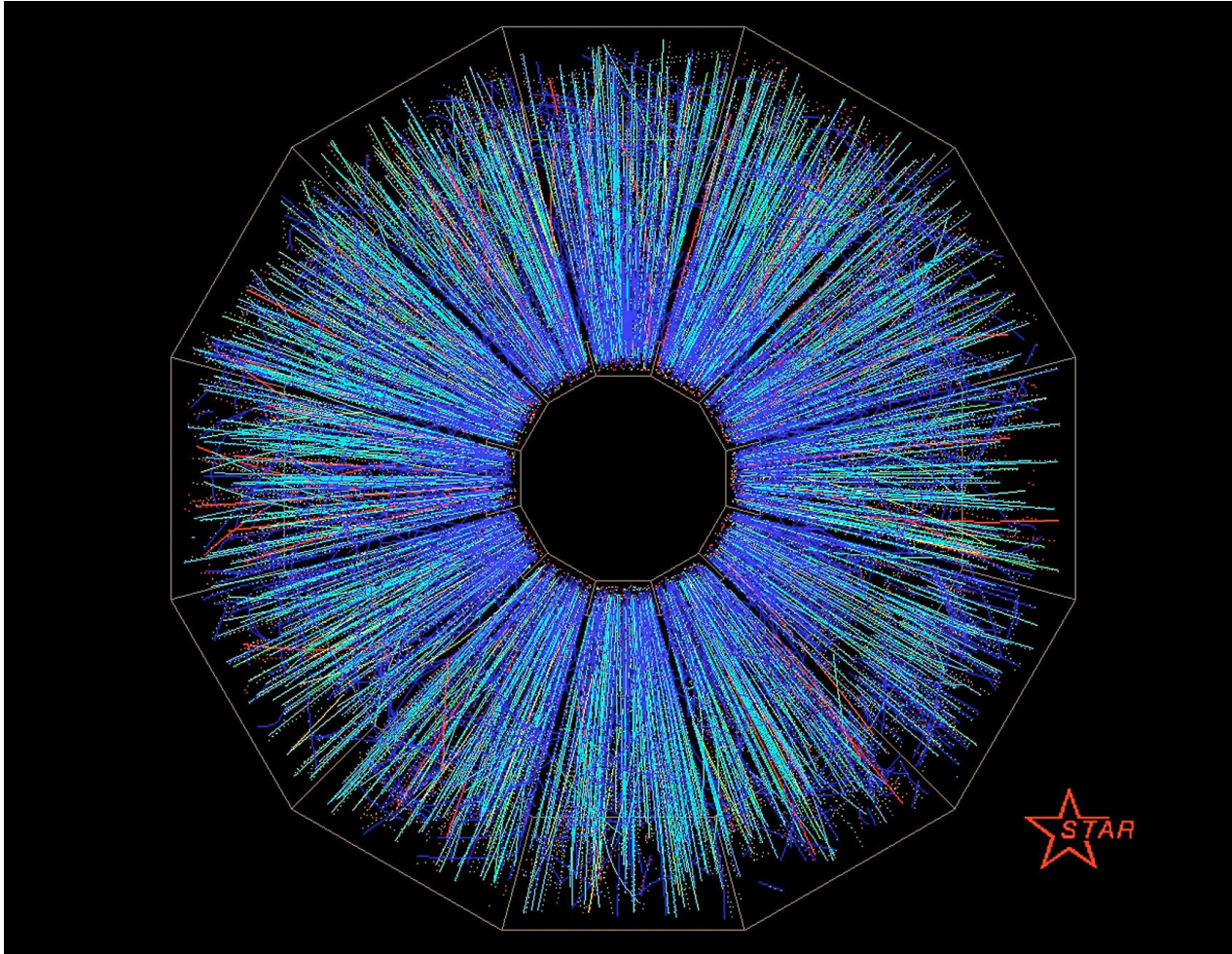




# DL with CNN for EoS classification



# Data representation



- Images: histograms
  - $(px, py)$  or  $(pt, \phi)$
  - $(px, py, pz)$
  - $(pt, \phi, \eta)$
- Point cloud: particle list

E	Px	Py	Pz	pid
6.84	1.07	4.5	6.83	211
68.92	0.75	0.64	68.91	2212
40.4	0.06	0.54	40	321
...				



# Capture more local correlations

## Dynamical Edge Convolution Network

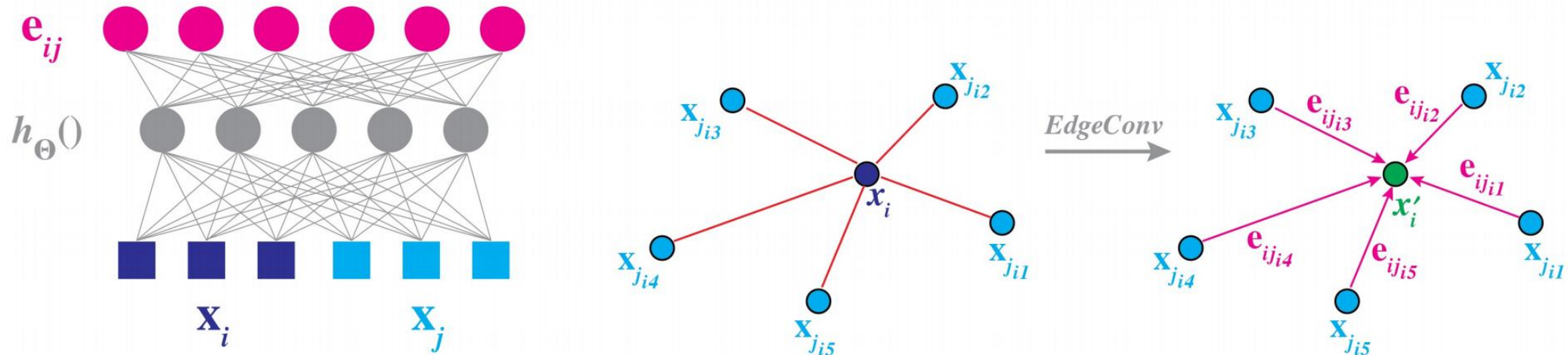
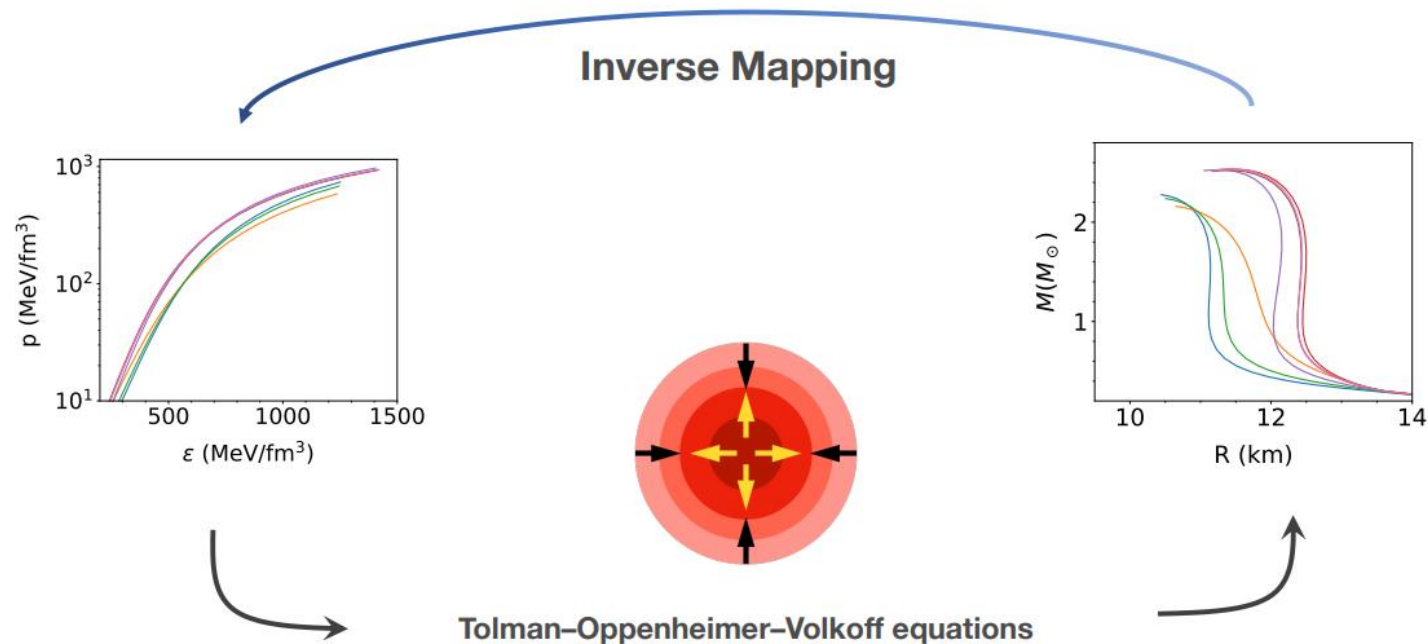


Fig. 2. **Left:** Computing an edge feature,  $e_{ij}$  (top), from a point pair,  $x_i$  and  $x_j$  (bottom). In this example,  $h_{\theta}()$  is instantiated using a fully connected layer, and the learnable parameters are its associated weights. **Right:** The EdgeConv operation. The output of EdgeConv is calculated by aggregating the edge features associated with all the edges emanating from each connected vertex.

# TOV Equation and Nuclear EoS from DL

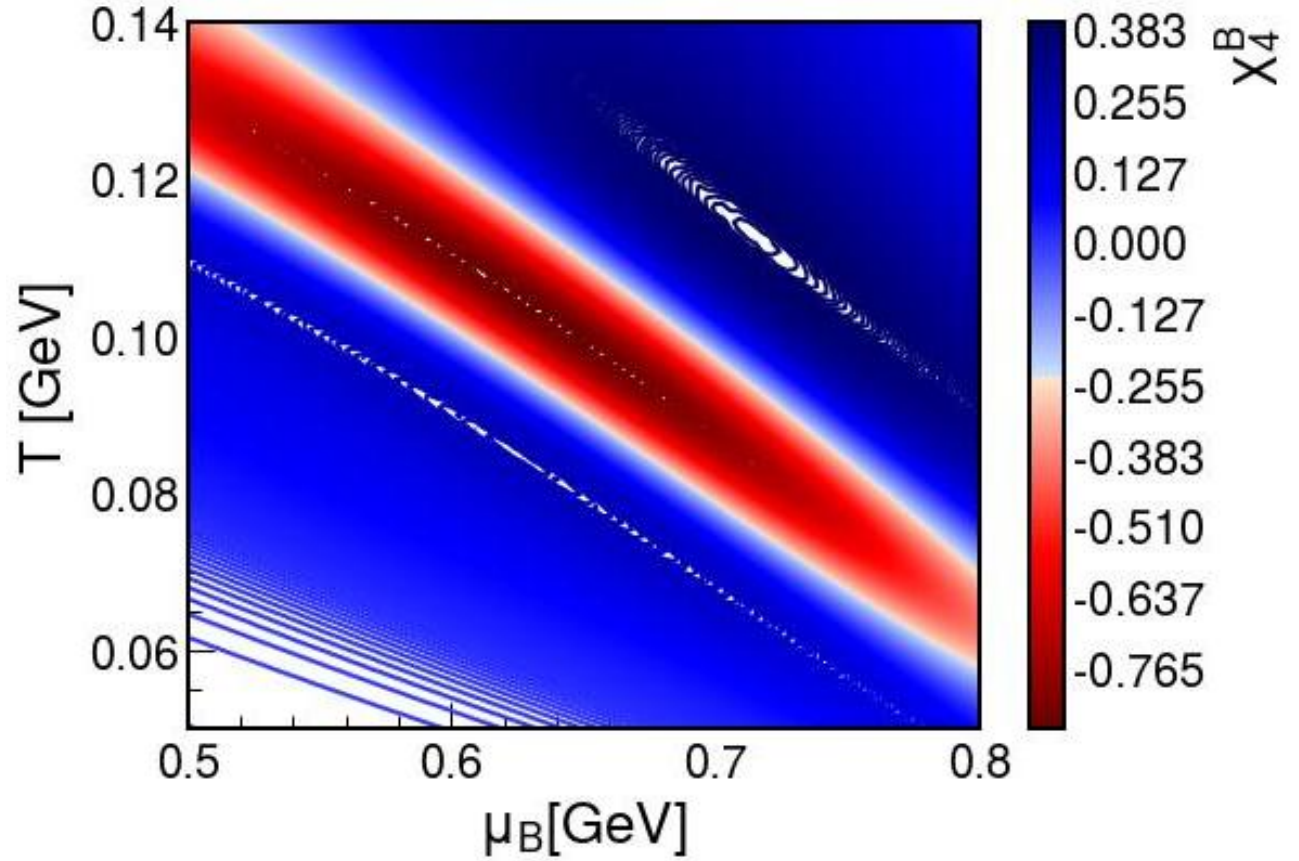
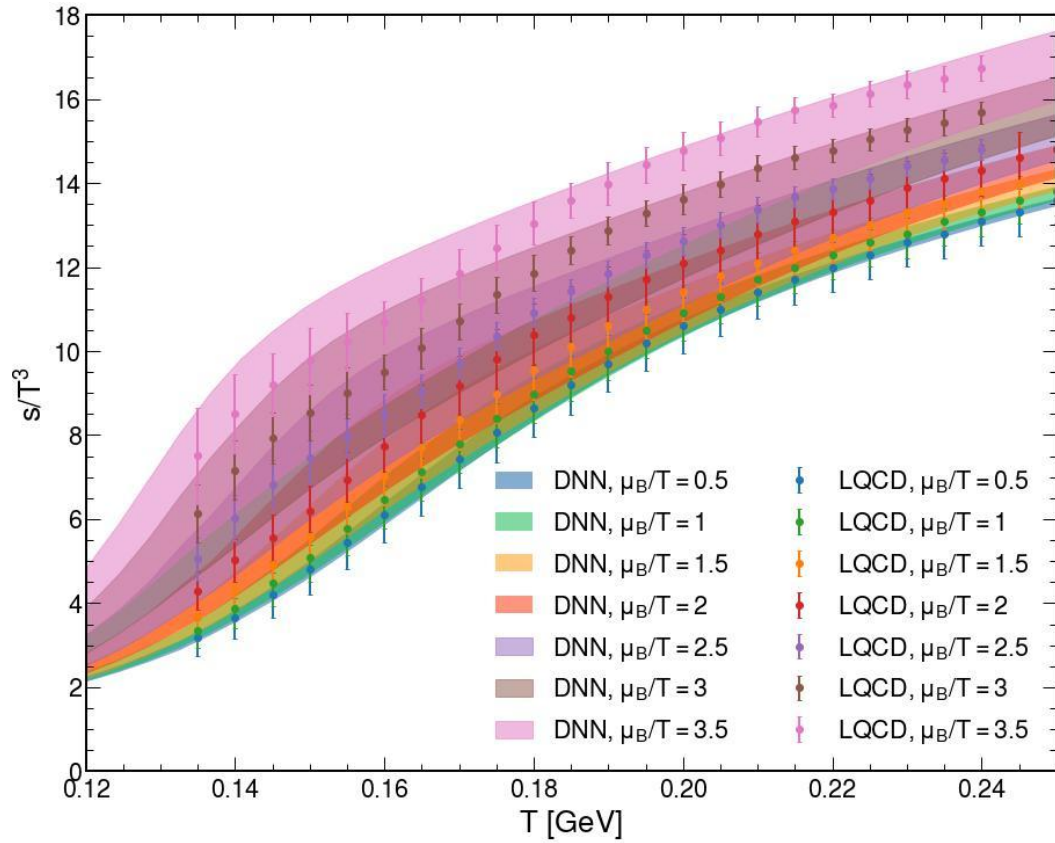
$$\frac{dp}{dr} = -G \frac{m(r)\epsilon(r)}{r^2} \left(1 + \frac{p(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 p(r)}{m(r)}\right) \left(1 - \frac{2Gm(r)}{r}\right)^{-1},$$
$$\frac{dm}{dr} = 4\pi r^2 \epsilon,$$



S. Soma, L. Wang, S. Shi, H. Stöcker, K. Zhou, PRD 107, (2023) 083028



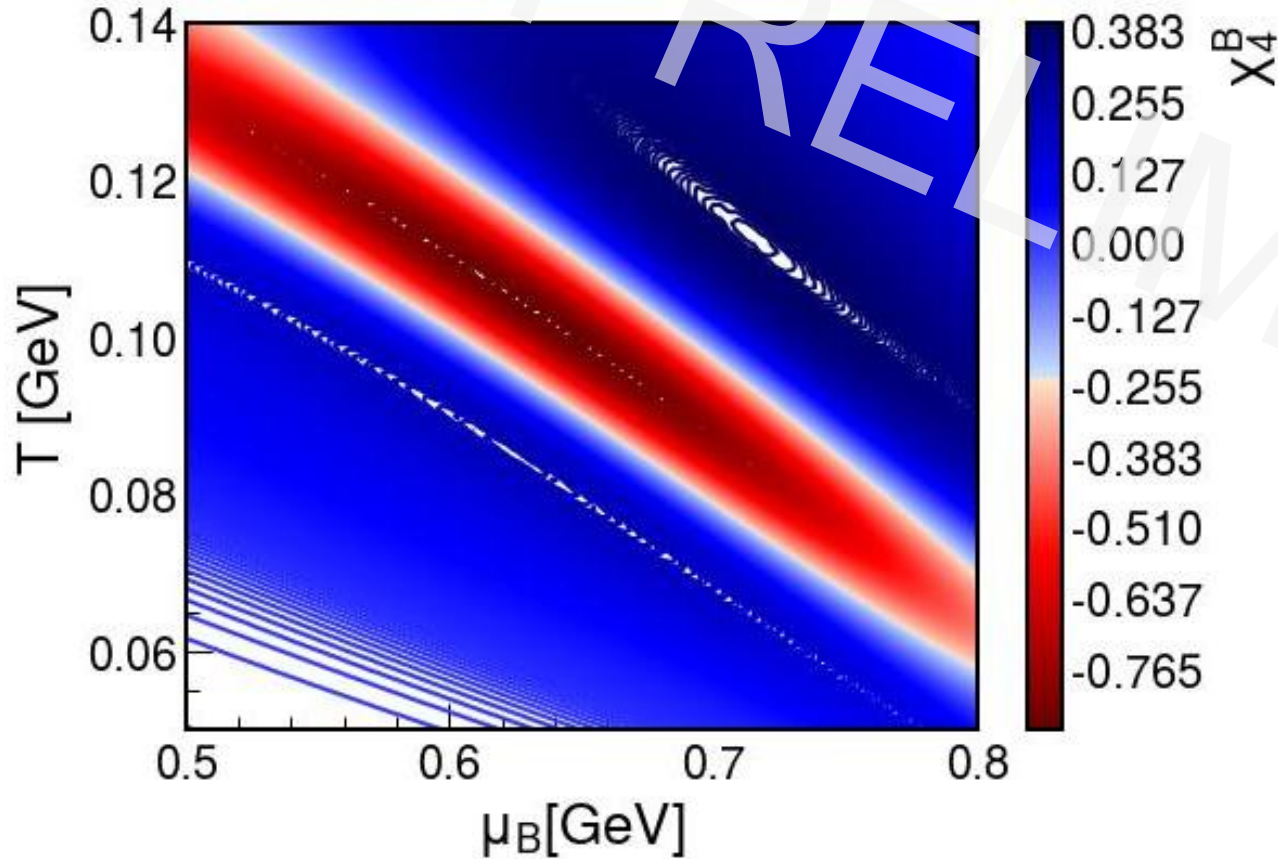
# method





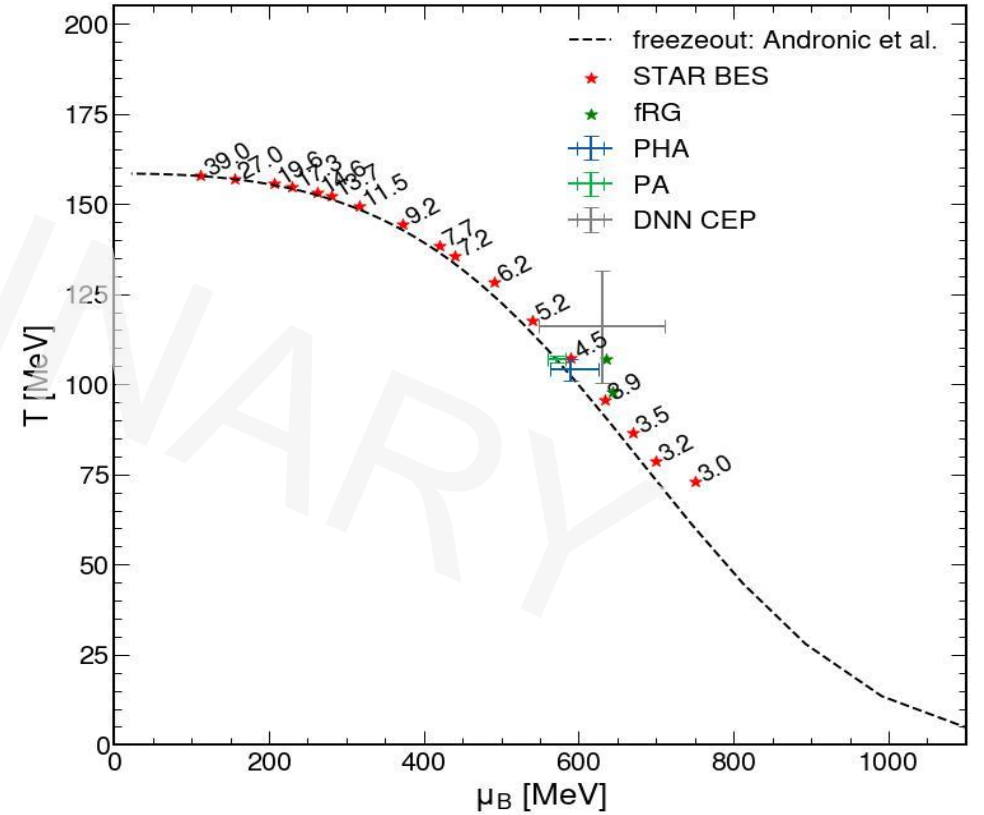
# Predictions of DL quasi parton model

### Susceptibility



$$\chi_k^{\text{lattice}} = \partial^k(p/T^4)/\partial(\mu/T)^k.$$

### Location of CEP

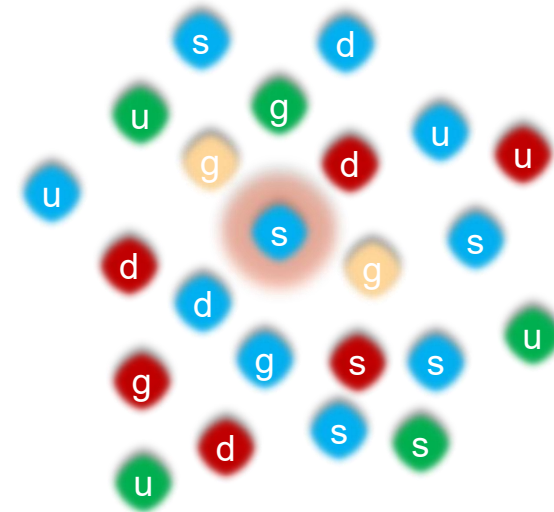
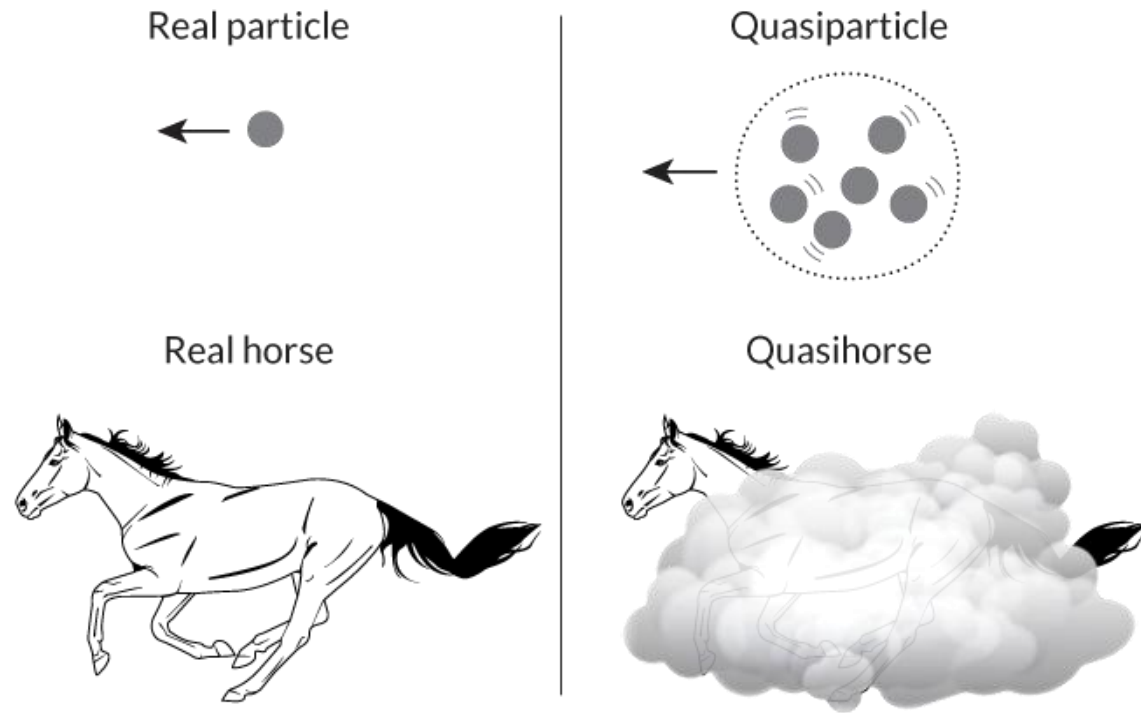


$$(\mu_B, T) : (630.1 \pm 81.42, 115.9 \pm 15.46)\text{MeV},$$



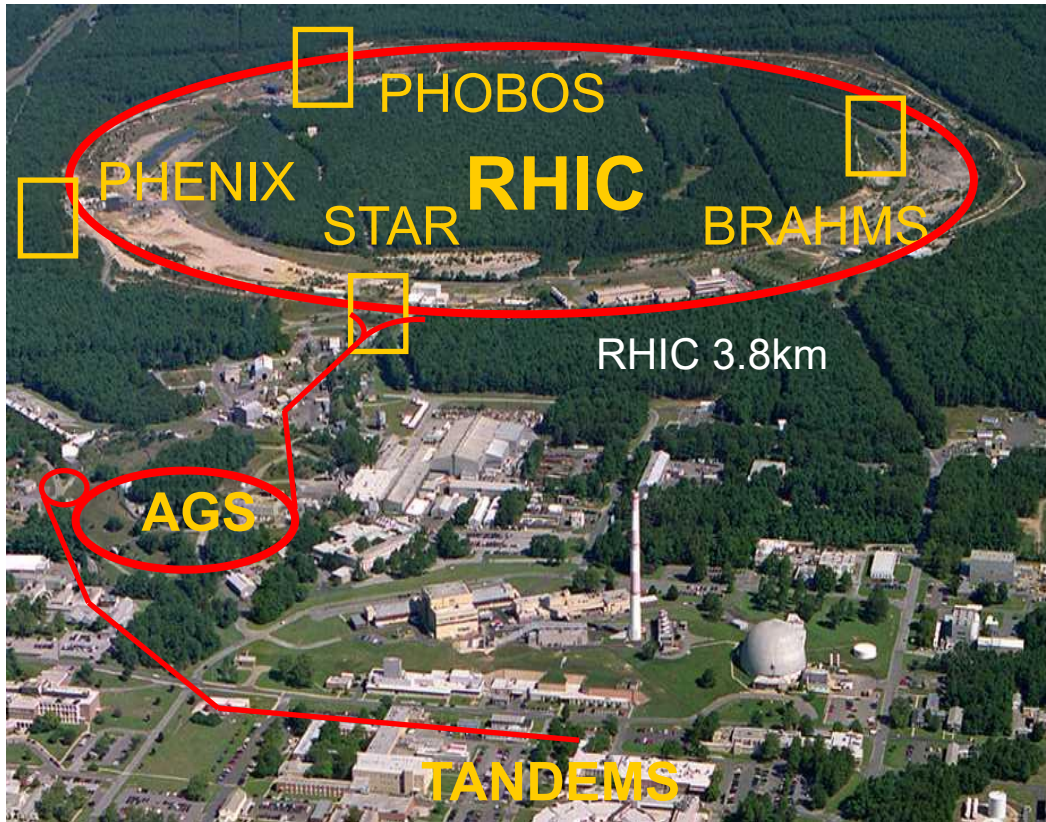


# Effective theory: DL Quasi parton model

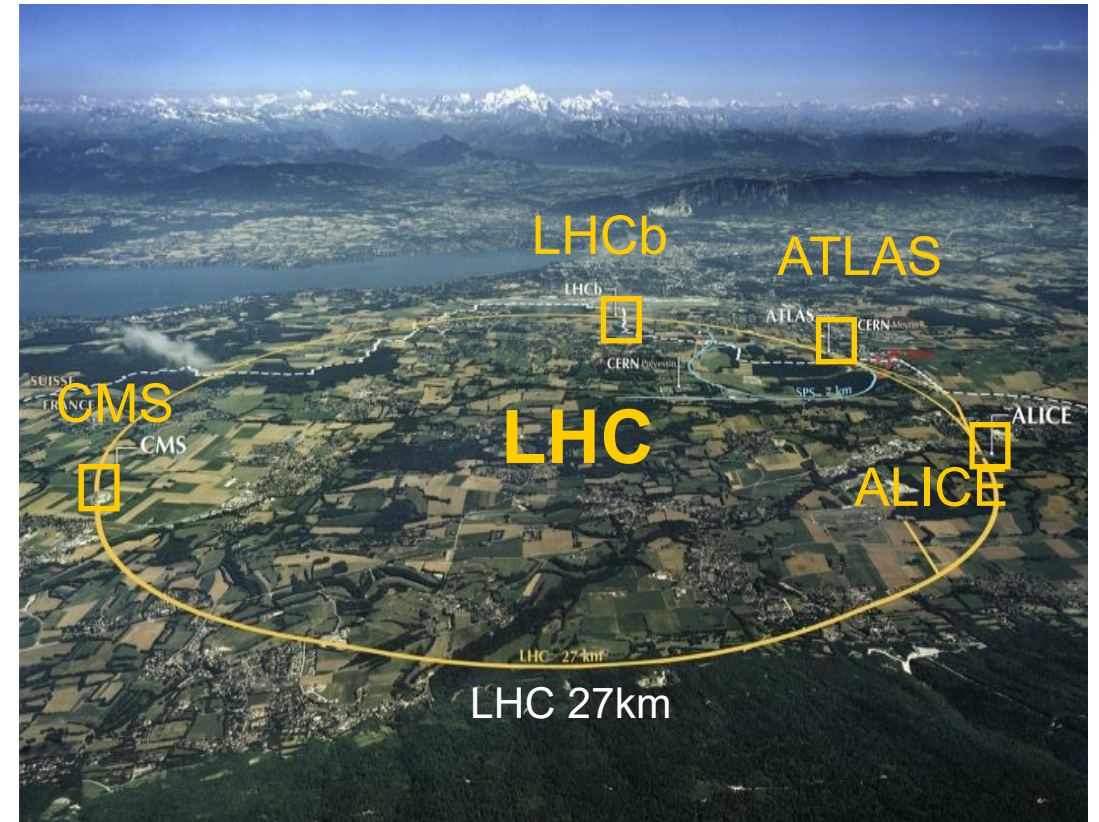


screened, dressed, regularized,  
quasi particle

# Nuclear Matter at extreme conditions

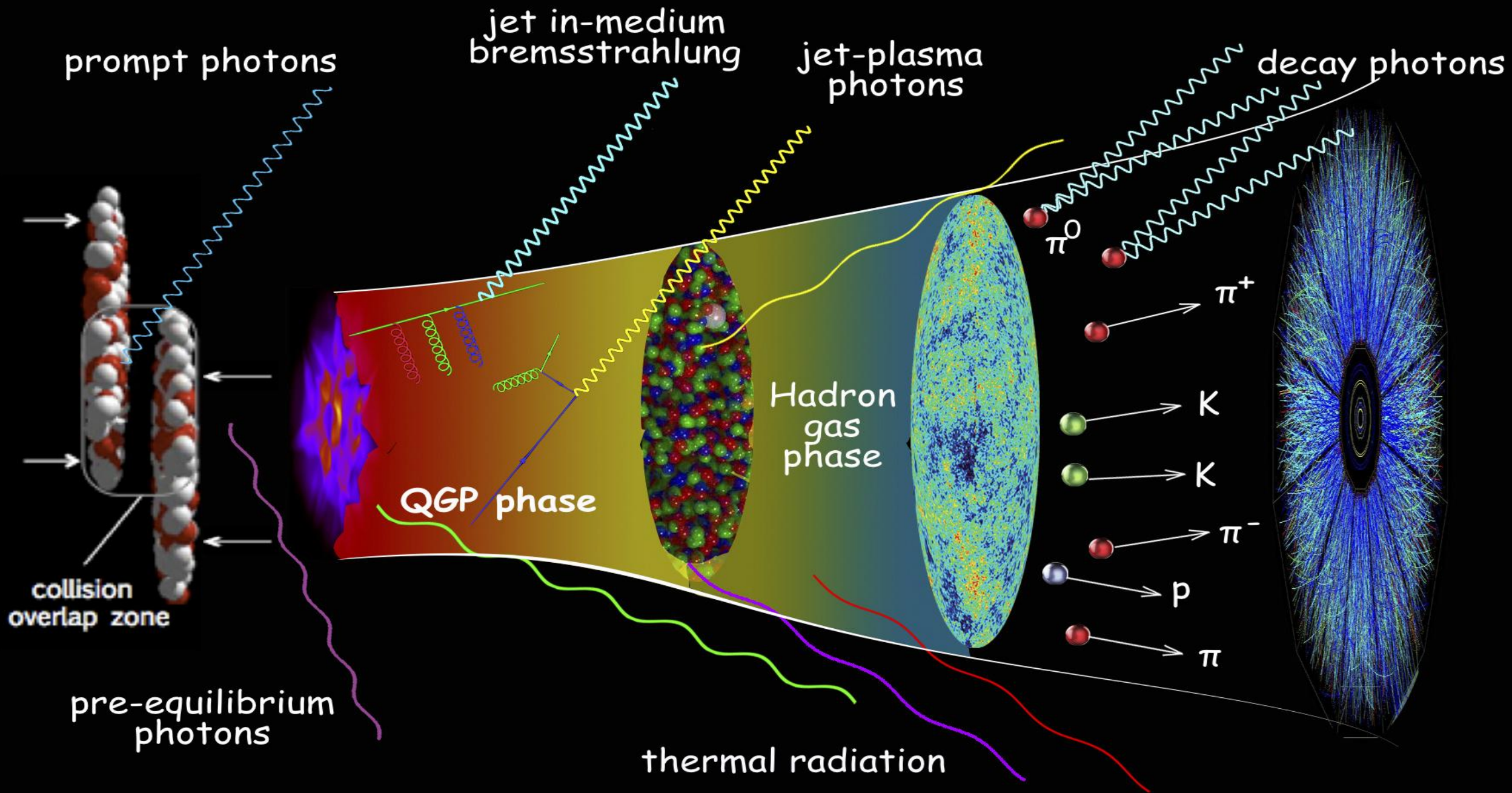


Au speed  $\sim 99.99\% c$



Pb speed  $\sim 99.9999\% c$



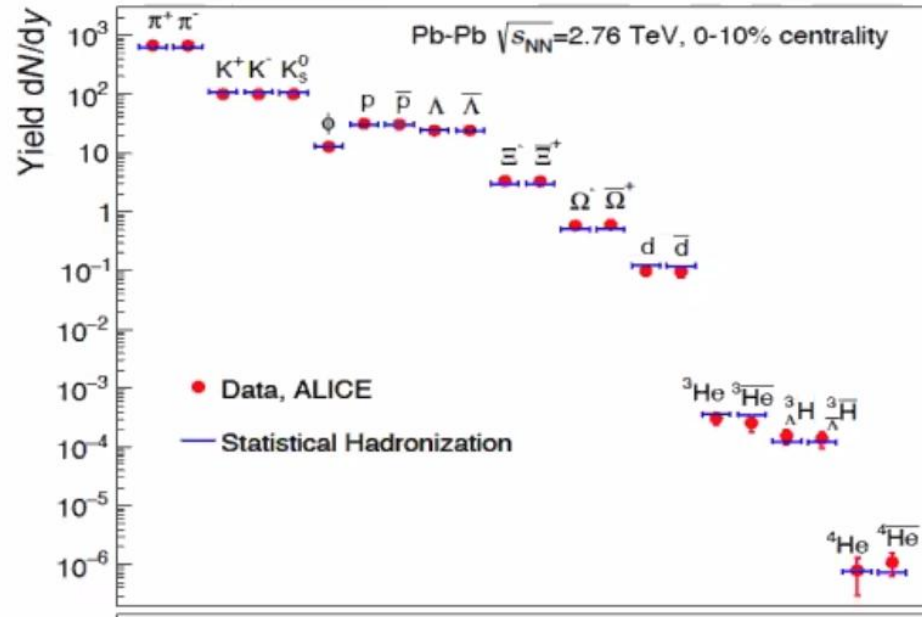




# How many hadrons are produced

## statistical hadronization of (u,d,s) hadrons

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321



Best fit:

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

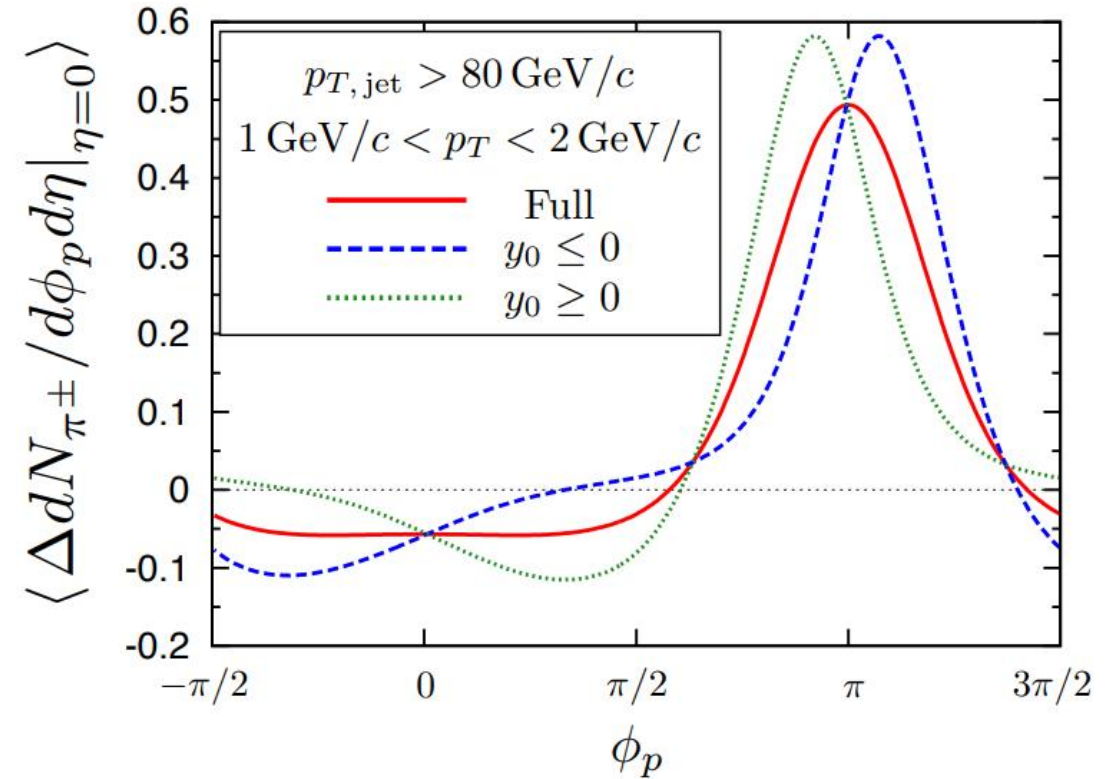
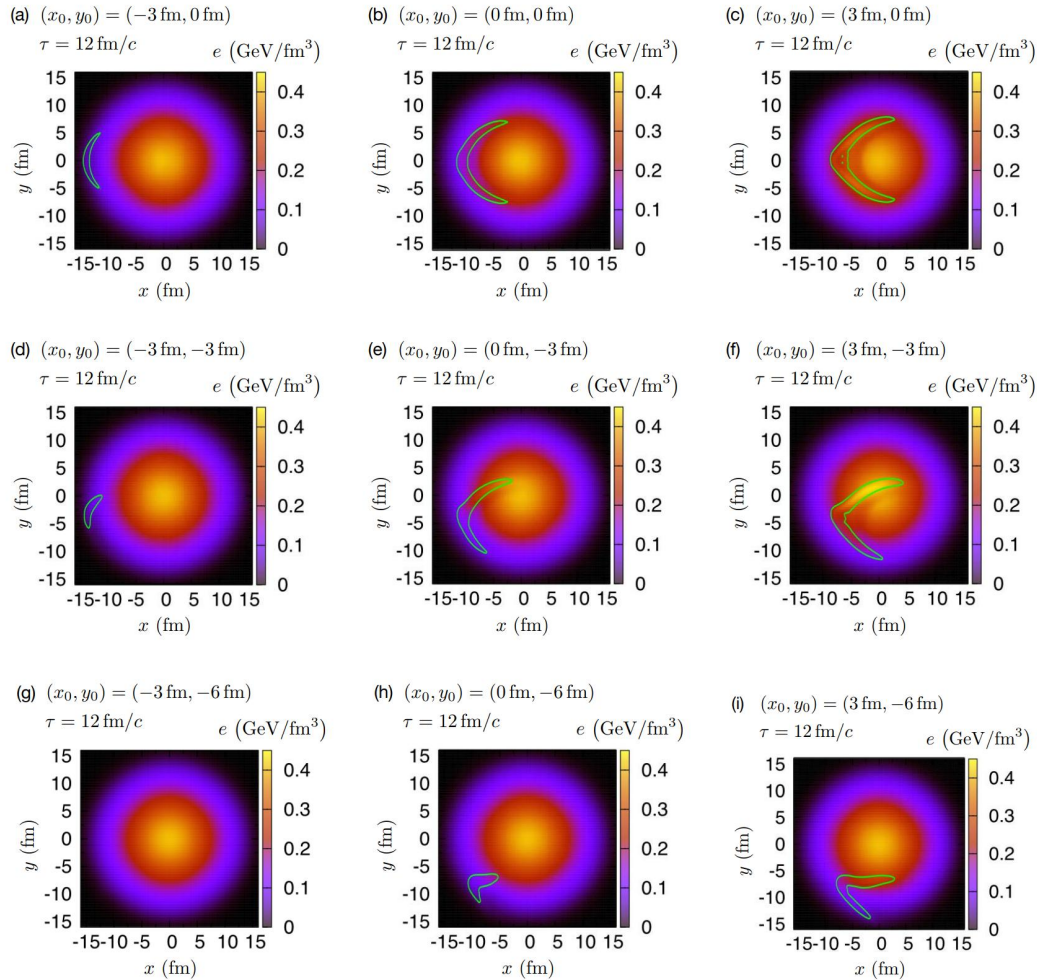
*S*-matrix treatment of inter-

$$n_i = (2s_i + 1)4\pi \int p^2 \left[ e^{(\sqrt{p^2 + m_i^2} - \mu_i)/T} \pm 1 \right]^{-1} dp$$

1. At LHC, equal amounts of matter and anti-matter are produced
2. At BES region, more protons than anti protons



# If it is possible to locate the initial jets



Y Tachibana, T Hirano, PRC 93 (2016) 5, 054907