Functional renormalization group approach to some problems of condensed matter physics

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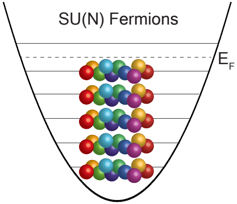
Physics: Quantum gases

Why cold atoms are interesting?

- ✓ Ultracold gases are “simple” systems
- ✓ Both fermions (^{6}Li, ^{87}Sr, $\ldots$) and bosons (^{87}Rb, ^{23}Na, $\ldots$) can be studied
- ✓ Interaction can be tuned to arbitrary values (via resonance scattering)
- ✓ Various internal symmetries $(SU(N), SO(N), \ldots)$ can be realized

- o ^{171}Yb $I = 1/2$ $SU(2)$
- o ^{173}Yb $I = 5/2$ $SU(6)$
- o ^{87}Sr $I = 9/2$ $SU(10)$
- o ^{171}Yb $+^{173}$ Yb $SU(2) \times SU(6)$



pic. by Sonderhouse

Scales in cold atoms

Lenght scales (in units of Bohr radius)

- Van der Waals lenght $\ell_{VdW} \sim 10 \div 100$
- s-wave scattering length $a \sim 10 \div 200$
- interparticle distance $\ell \sim 800 \div 3000$
- de Broglie wavelength $\ell_T \sim (1 \div 4) \times 10^4$
- size of the system $L \sim 10^5$

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Energy scales (e.g. ^{6}Li)

- $T_c \sim 0.16\ T_F \sim 0.1 \mu K$
- $T_c \sim 10^{-11}$eV

  where $T_F \sim n^{2/3}$

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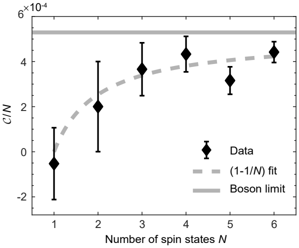
HTS vs Gases

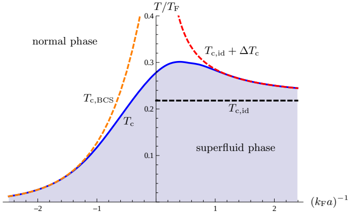
- $T_c^{\text{HTS}} \sim 0.01\ T_F^{\text{HTS}} \sim 100K$

cold atoms are "hot" matter

Key observables



Tan's contact $n(k) \sim \mathcal{C}k^{-4}$,
^{173}Yb [Song, PRX'20].



The phase diagram of interacting
$SU(2)$ Fermi gas [pic. by Boettcher]

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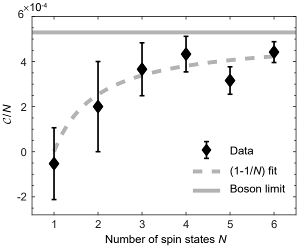

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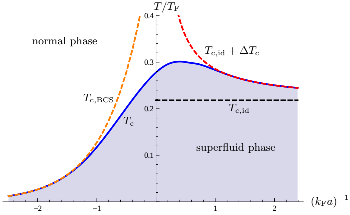

The phase diagram of interacting $SU(2)$ Fermi gas [pic. by Boettcher]

Method	T_c/T_F
Nascimbene (Exp)	0.157(15)
Horikoshi (Exp)	0.17(1)
Ku (Exp)	0.167(13)
Goulko (QMC)	0.171(5)
Floerchinger (FRG)	0.248
Haussmann (LW)	0.160

The critical temperature of superfluid (*2nd order*) phase transition in $SU(2)$ systems in the unitary regime: $k_F a \to -\infty$.

Method	Δ/T_F
Schirotzek (Exp)	0.44(3)
Carlson (QMC)	0.50(5)
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The superfluid gap Δ at $T = 0$ in $SU(2)$ systems in the unitary regime

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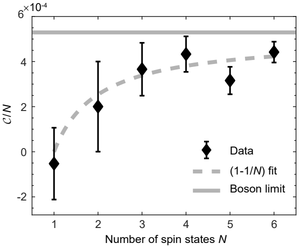

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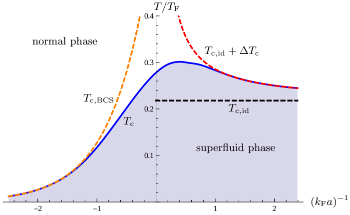

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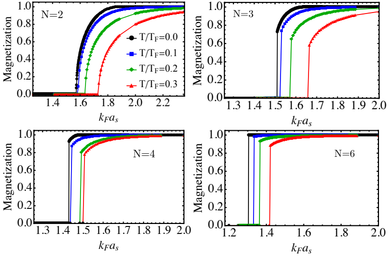


Magnetization in $SU(N)$ system within the HF-method [Cazalilla,New J. Phys'23]

- MF and HF predict 2nd order phase transition for $SU(2)$, and the 1st o.p.t. for $SU(N > 3)$, due to the cubic term in the Landau free energy

$$\mathcal{F} = aM^2 + bM^3 + cM^4$$

- Qualitative agreement with QMC

Computation of free energy

✓ The partition functional

$$Z[J] = \int \mathcal{D}\psi \; e^{-S[\psi]+J\psi}$$

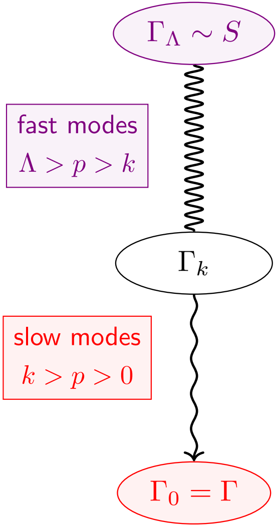
✓ The free energy (depends on φ: magnetization, superfluid density, superconducting gap, and etc.)

$$\Gamma[\varphi] = J_\varphi \varphi - \ln Z[J_\varphi], \quad \varphi = \frac{\delta}{\delta J} \ln Z[J] \Big|_{J=J_\varphi} .$$

solving the theory = computing $\Gamma[\varphi]$

Functional RG in a nutshell

From functional integral to functional DE



Flow equation (Wetterich, Moris)

✓ $\Gamma_{k=\Lambda} = S$ – microscopic action – input

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\,\mathrm{Tr}\left\{(\Gamma_k^{(2)}[\varphi] + R_k)^{-1}\partial_k R_k\right\}$$

✓ $\Gamma_{k=0} = \Gamma$ – macroscopic free energy – output

Some features:

- The flow equation is exact
- Non-perturbative approximation is possible
- Equilibrium and far-from-equilibrium systems

Treatment of large spin fermions

Flow equations

The flow system must be solved

$$\partial_k \mathrm{V}_k(\rho) = k^{d+1} \left\{ \frac{1/n^2}{k^2 + \mathrm{V}_k' + 2\rho \mathrm{V}_k''} + \frac{(n+1)(n-2)/(2\,n^2)}{k^2 + \mathrm{V}_k' + 4\rho \mathrm{W}_k} + \frac{(n-1)/(2\,n)}{k^2 + \mathrm{V}_k'} \right\},$$

$$\partial_k \mathrm{W}_k(\rho) = k^{d+1} \left\{ \frac{(1-2/n)\,\mathrm{W}_k^2}{(k^2 + \mathrm{V}_k')^3} + \frac{9(n+2)(n-4)\,\mathrm{W}_k^2}{n^2(k^2 + \mathrm{V}_k' + 4\rho \mathrm{W}_k)^3} - \frac{(\mathrm{W}_k + 2\,(1-1/n)\rho \mathrm{W}_k')}{4\rho(k^2 + \mathrm{V}_k')^2} \right.$$
$$- \frac{1/n^2}{(k^2 + \mathrm{V}_k' + 2\rho \mathrm{V}_k'')^2} \left(2\mathrm{W}_k''\,\rho + 5\,\mathrm{W}_k' + \frac{\mathrm{W}_k}{\rho} - \frac{\mathrm{V}_k''}{2\rho_1} + \frac{\left(\mathrm{V}_k'' + 4\,\mathrm{W}_k'\,\rho + 4\,\mathrm{W}_k\right)^2}{2\rho_1\left(\mathrm{V}_k'' - 2\,\mathrm{W}_k\right)} \right)$$
$$\left. + \frac{1/n^2}{(k^2 + \mathrm{V}_k' + 4\rho \mathrm{W}_k)^2} \left(\frac{n^2+4}{4\rho}\,\mathrm{W}_k - \frac{n^2+6}{2}\,\mathrm{W}_k' - \frac{\mathrm{V}_k''}{2\rho_1} + \frac{\left(\mathrm{V}_k'' + 4\,\mathrm{W}_k'\,\rho + 4\,\mathrm{W}_k\right)^2}{2\rho_1\left(\mathrm{V}_k'' - 2\,\mathrm{W}_k\right)} \right) \right\}.$$

Discontinuous phase transition

The Ginzburg-Landau (GL) action as an input

$$S = \mathrm{tr}[\partial\varphi^\dagger\,\partial\varphi] - m_\Lambda^2\,\mathrm{tr}[\varphi^\dagger\varphi] + g_{1\Lambda}\big(\mathrm{tr}[\varphi^\dagger\varphi]\big)^2 + g_{2\Lambda}\,\mathrm{tr}[\varphi^\dagger\varphi\varphi^\dagger\varphi]$$

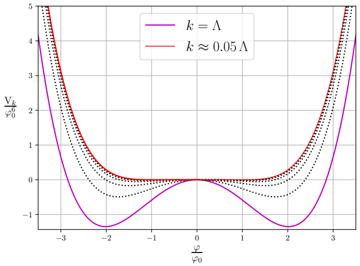
φ – $N \times N$ antisymmetric matrix

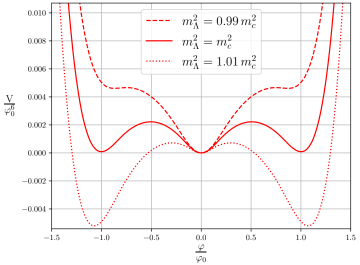
Mean field prediction:

- Continuous phase transition (2nd order) at $m_\Lambda^2 = 0$ for all N (N is even).
- Scaling behaviour with the MF critical exponents.

Functional RG prediction: [Hnatic,Kalagov,Nucl.Phys.B'23]

- Discontinuous phase transition (1st order) for all $N > 2$. But continuous one for $N = 2$ (as expected).
- No scaling behaviour = No critical exponents.

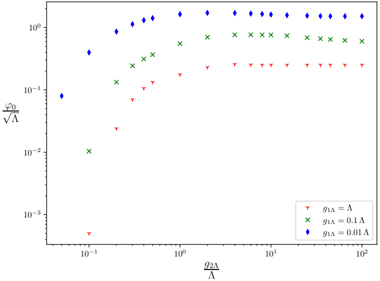




The gap

The fermion excitation spectrum bellow the transition point

$$E(p) = \sqrt{\left(\frac{p^2}{2m} - \mu\right) + \Delta^2}, \quad \Delta \sim \varphi_0$$



Two regimes are possible:

- ✓ Weak-coupling regime – $g_{1\Lambda}, g_{2\Lambda}$ are small – the jump φ_0 of the superfluid density is barely discernible.
- ✓ Strong-coupling regime – $g_{1\Lambda}, g_{2\Lambda}$ are large – the first-order transition becomes more pronounced.

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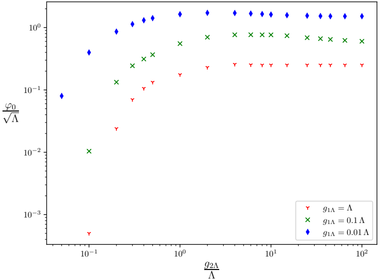

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The generic structure of the established pattern persists for all $N > 2$ up to the limit $N \to \infty$.

Outlook

✓ So what did we do?

- $SU(N>2)$ symmetry + GL approach close to the criticality
- interaction are treated within the functional RG

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The end!