Self-consistent Description of Graphene Quantum Amplifier

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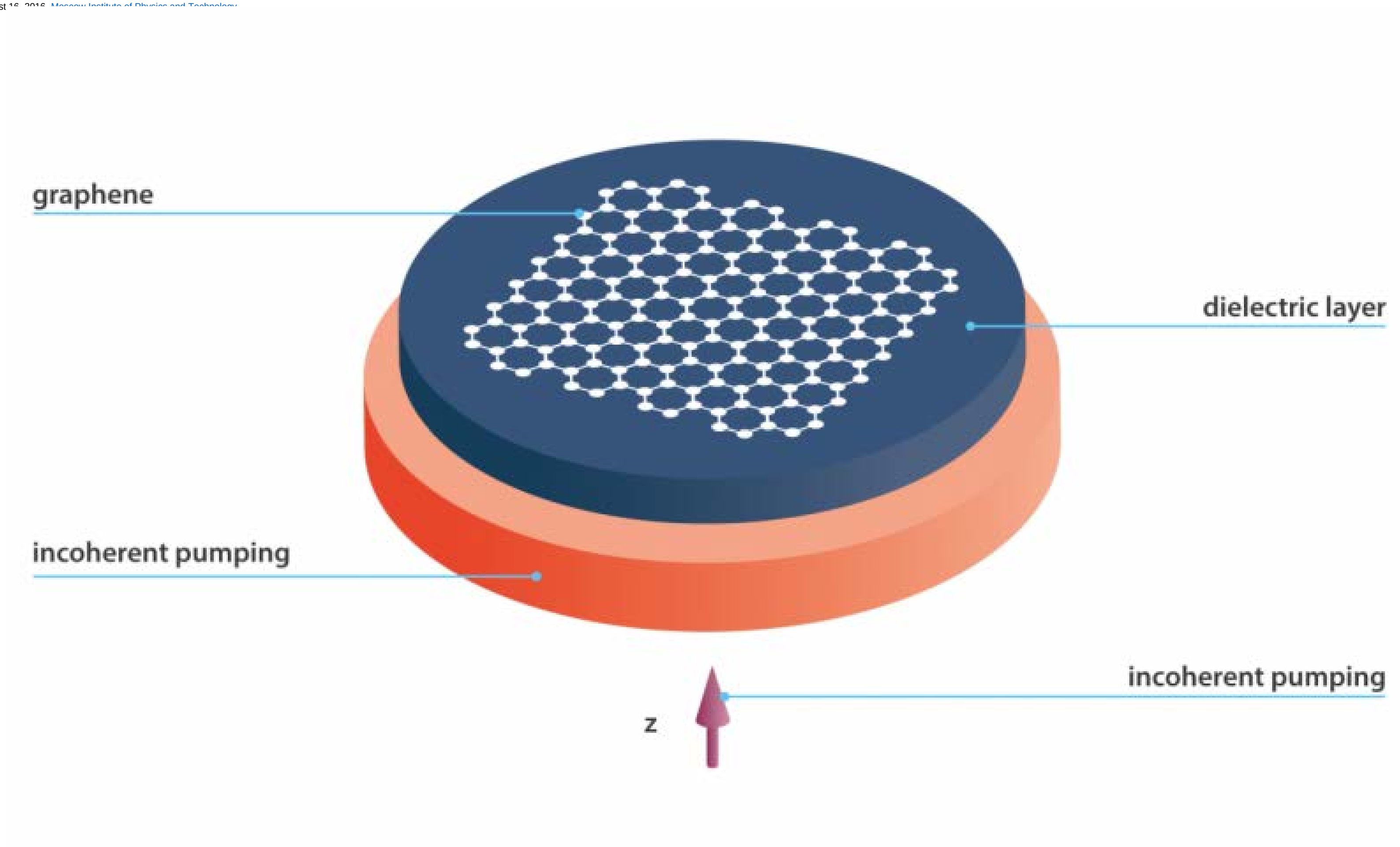
Scientists have proposed a graphene-based sensor that can 'sniff out' a single molecule of explosives. Credit: Moscow Institute of Physics and Technology

'Sniffer plasmons' could detect explosives

August 16, 2016, Moscow Institute of Physics and Technology



Read more at: https://phys.org/news/-sniffer-plasmons-explosives.html#jCp

Plasmons, reminder

Bulk plasmon

Collective excitation
of conducting electrons.

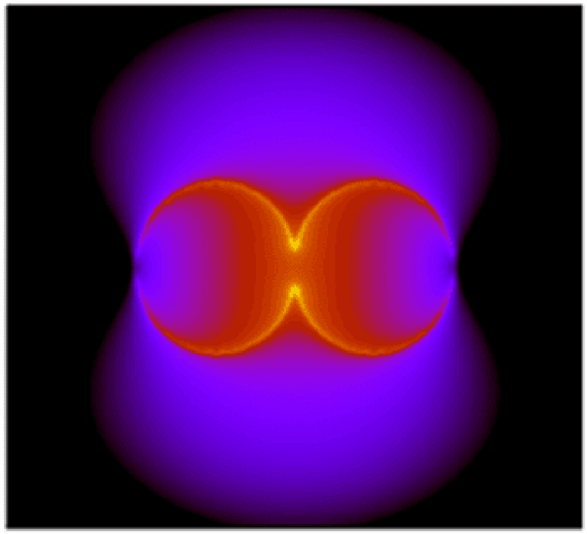
Plasmon energy depends
only on electron density.

$$\omega_p = \sqrt{\frac{4\pi n e^2}{m_e}}$$

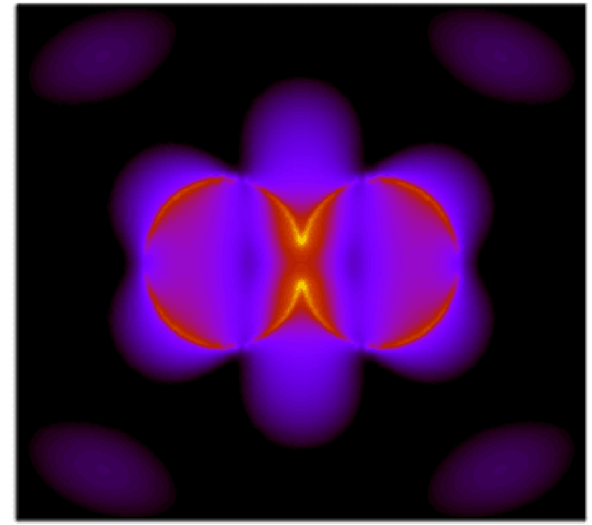
Surface plasmon

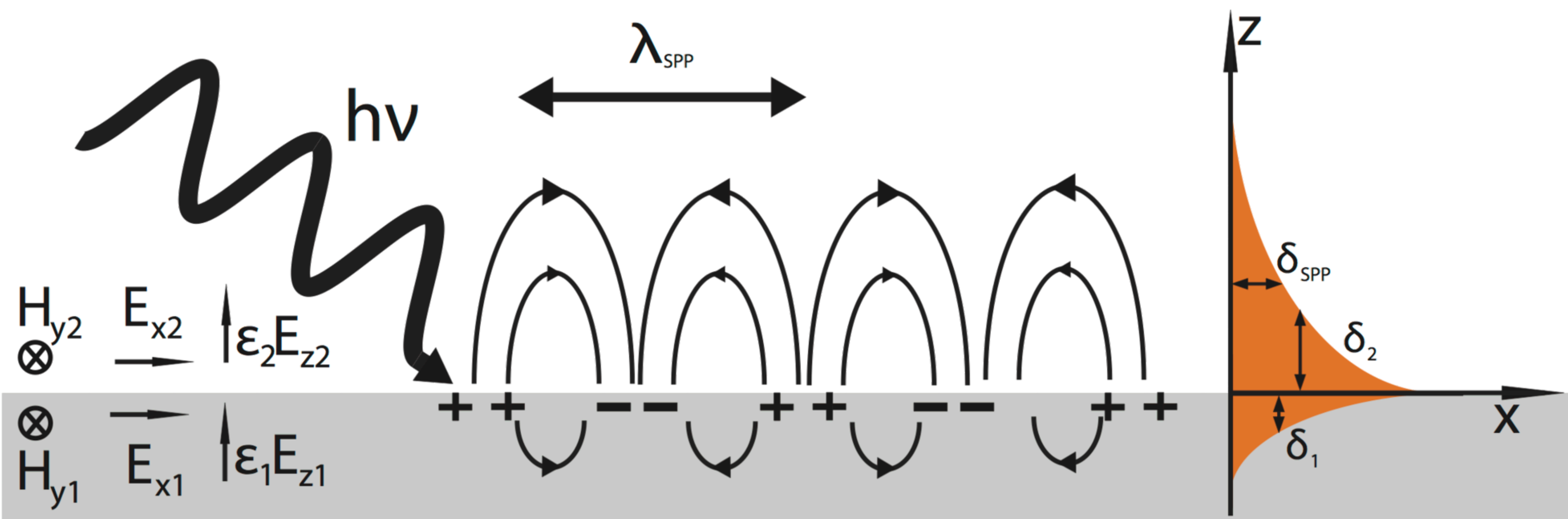
Wave nature.

Charge density waves on surface.



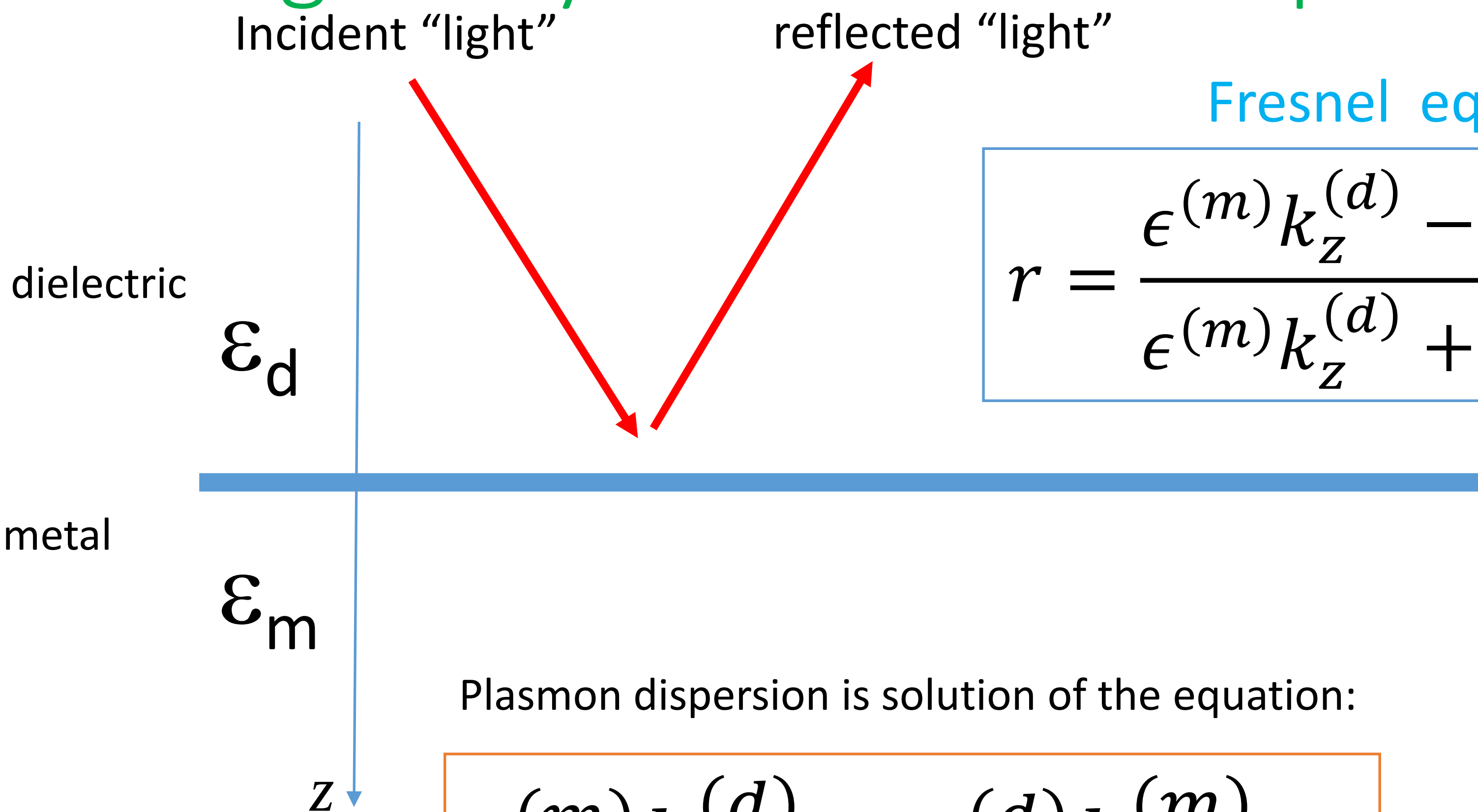
bulk plasmons





Surface plasmon

Surface plasmon – complex singularity of reflection amplitude



Fresnel equation:

$$r = \frac{\epsilon^{(m)} k_z^{(d)} - \epsilon^{(d)} k_z^{(m)}}{\epsilon^{(m)} k_z^{(d)} + \epsilon^{(d)} k_z^{(m)}}$$

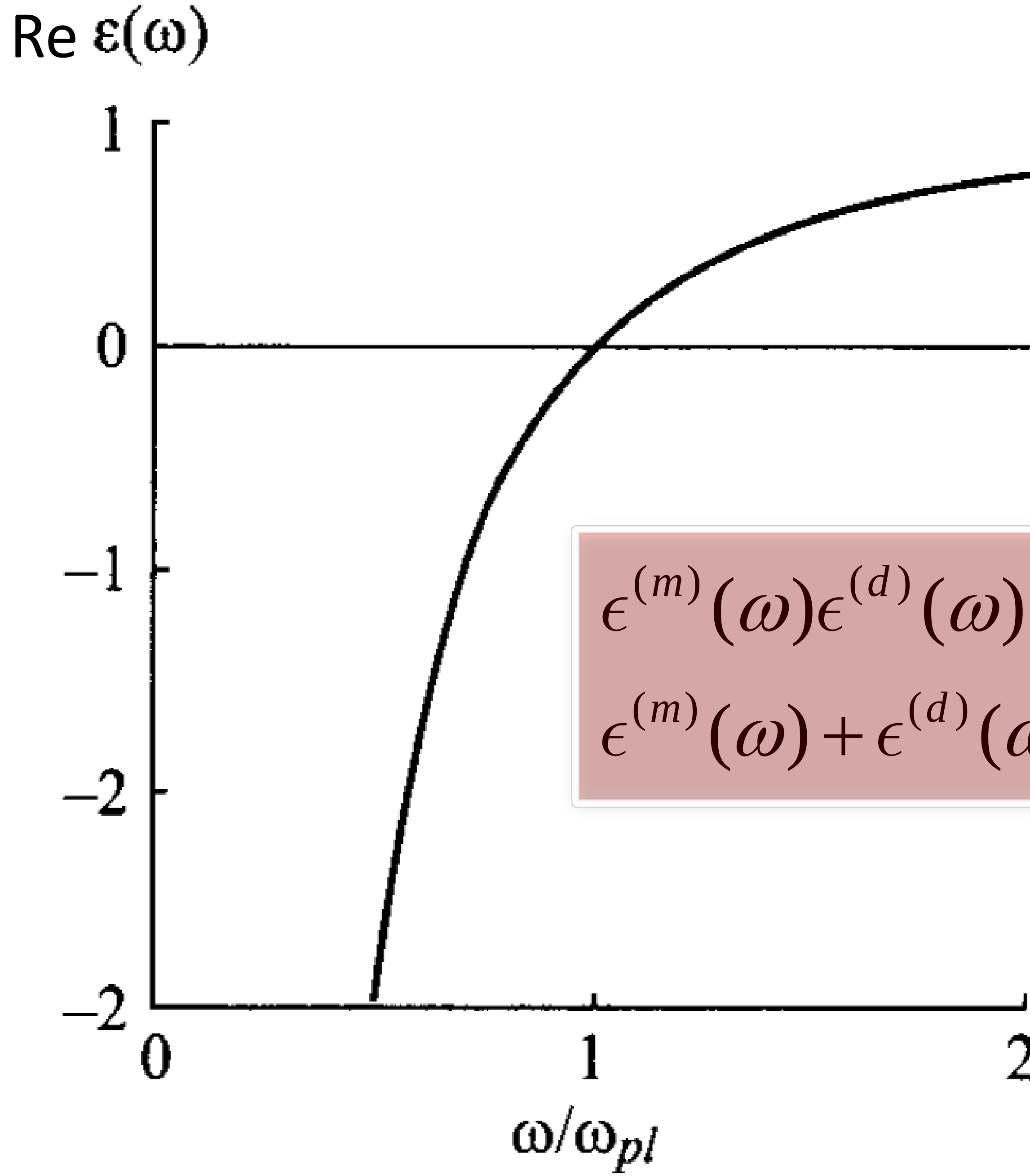
Plasmon dispersion is solution of the equation:

$$\epsilon^{(m)} k_z^{(d)} + \epsilon^{(d)} k_z^{(m)} = 0$$

$$k_z^{(d)} = \sqrt{\epsilon^{(d)}(\omega)\left(\frac{\omega}{c}\right)^2 - k_\perp^2}$$

$$\epsilon^{(m)}(\omega)\epsilon^{(d)}(\omega) < 0$$

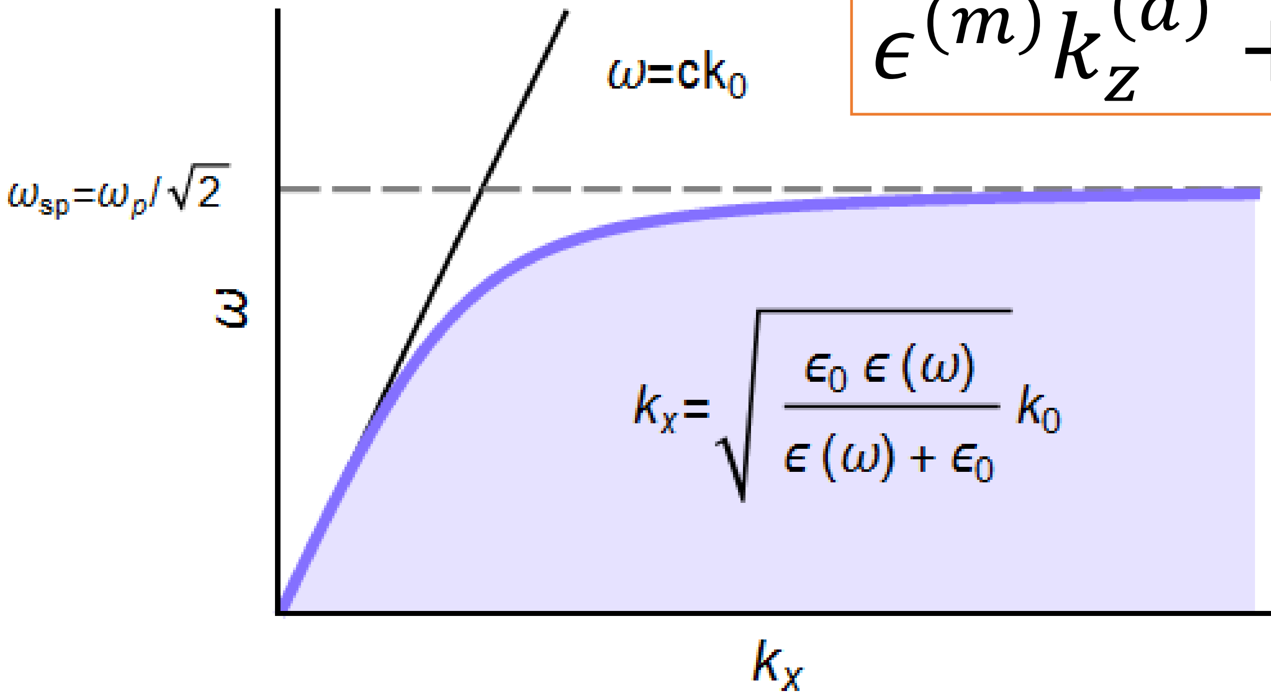
$$\epsilon^{(m)}(\omega) + \epsilon^{(d)}(\omega) < 0$$



$$\epsilon^{(m)}(\omega)\epsilon^{(d)}(\omega) < 0$$

$$\epsilon^{(m)}(\omega) + \epsilon^{(d)}(\omega) < 0$$

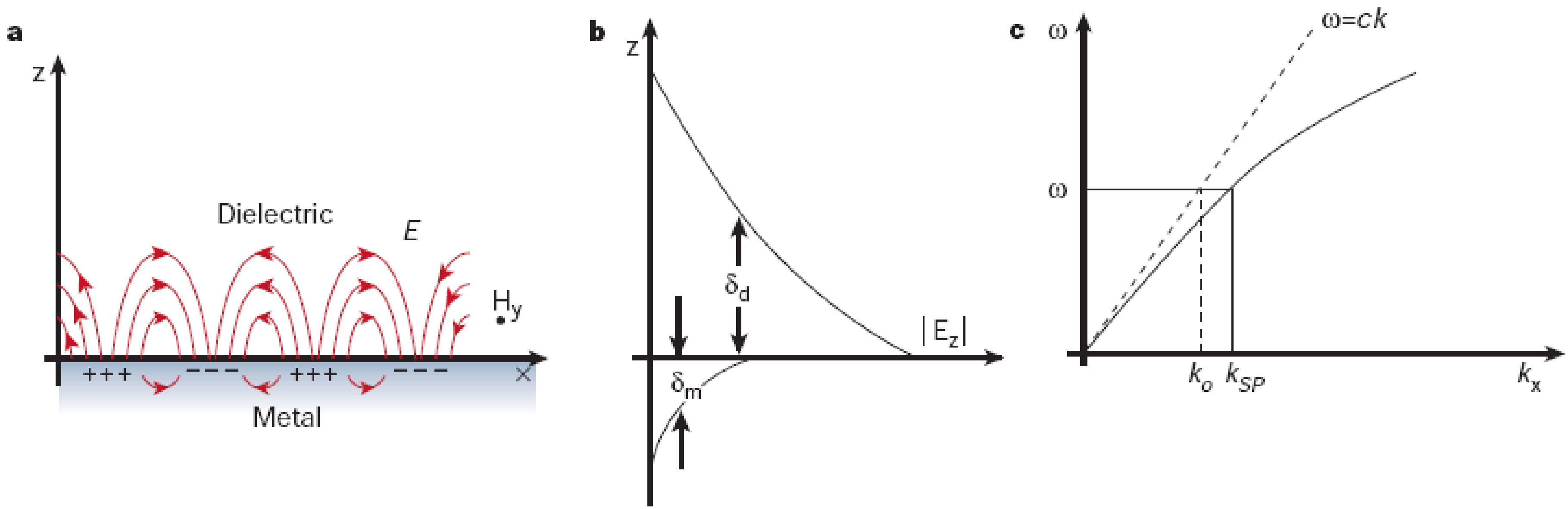
$$\epsilon^{(m)} k_z^{(d)} + \epsilon^{(d)} k_z^{(m)} = 0$$



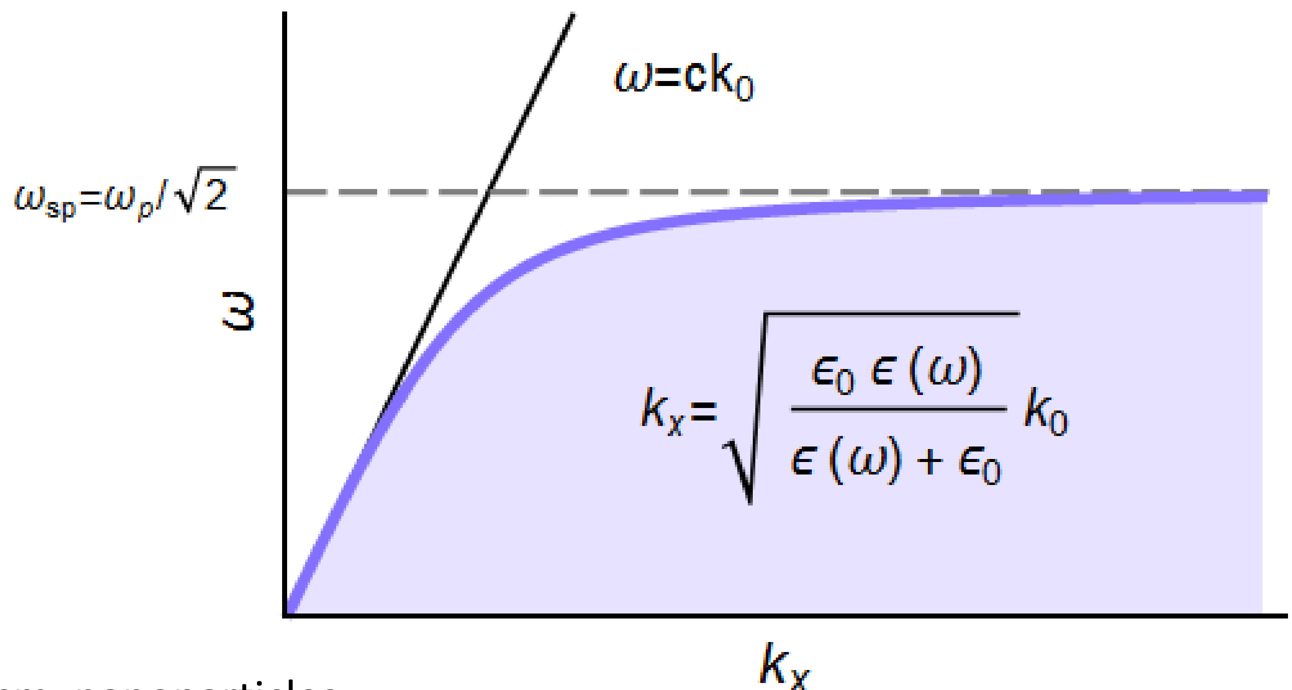
$$\epsilon^{(d)} \equiv \epsilon_0$$

$$\text{Re}\, \epsilon^{(m)} \equiv \epsilon(\omega)$$

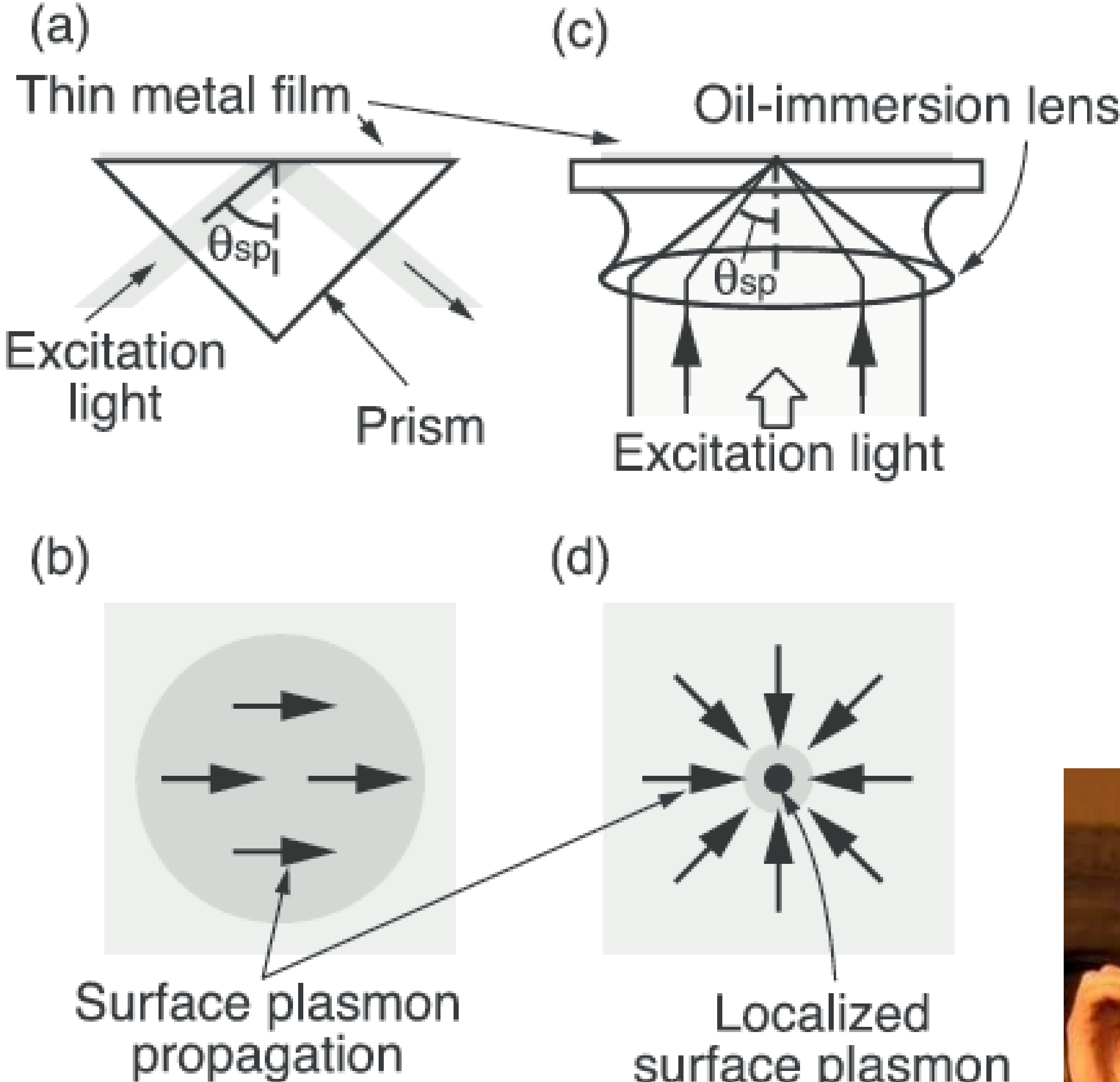
$$k_0 = \omega / c$$

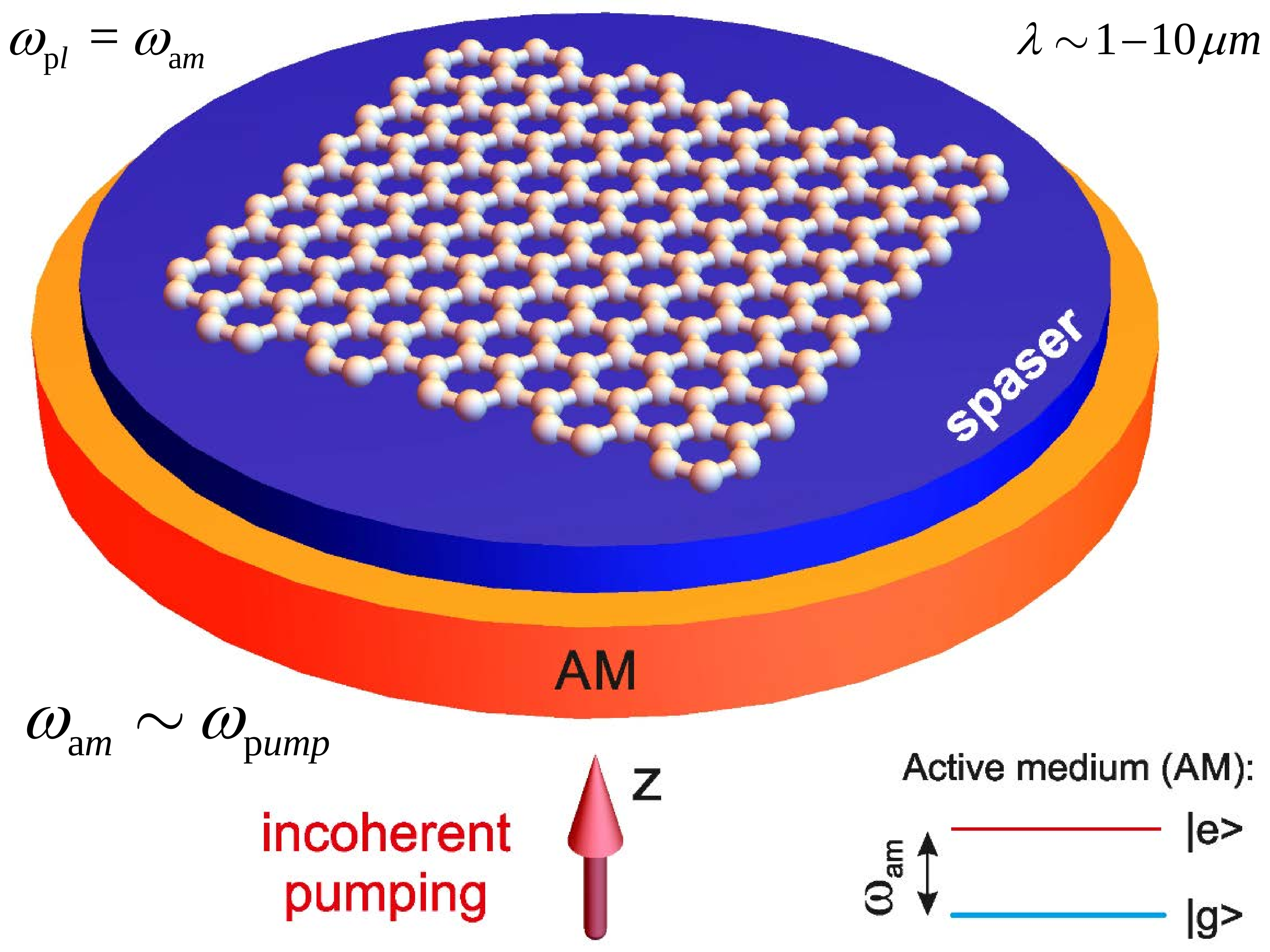


One does not simply excite plasmon… by an electromagnetic wave…



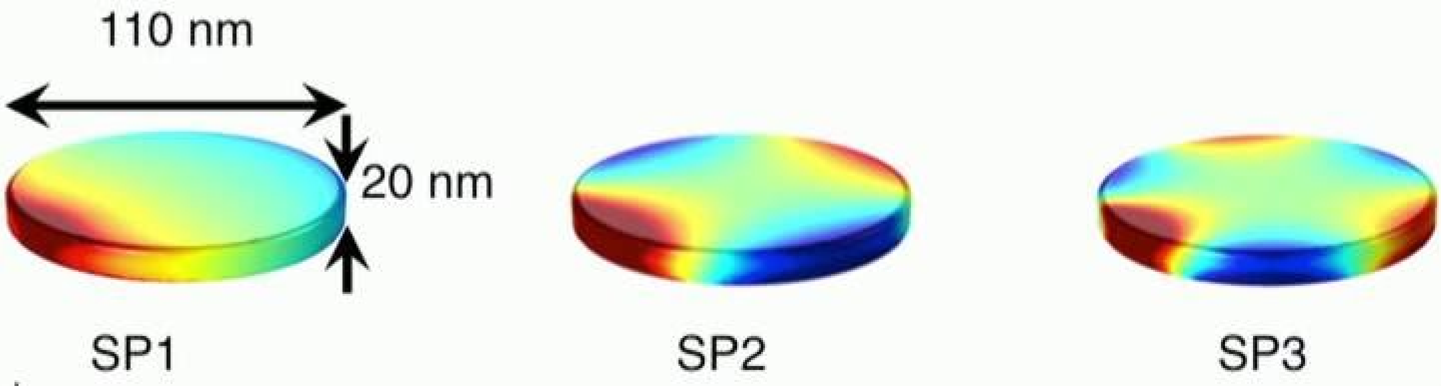
Momentum mismatch… Prism, nanoparticles…


(a)
Thin metal film
θsp
Excitation light
Prism
(c)
Oil-immersion lens
θsp
Excitation light
(b)
Surface plasmon propagation
(d)
Localized surface plasmon

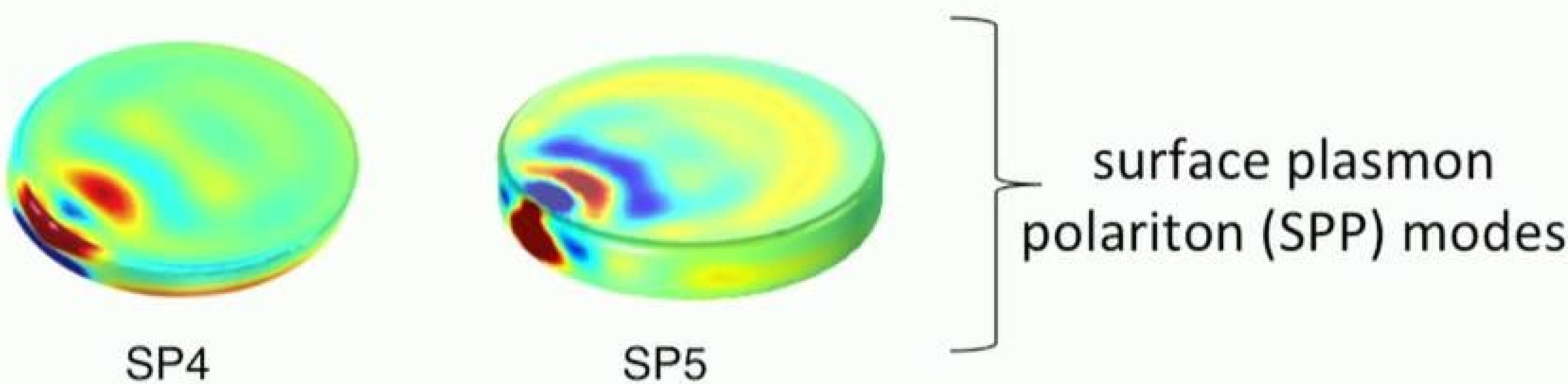

$\omega_{pl} = \omega_{am}$
$\lambda \sim 1-10\mu m$
spaser
AM
$\omega_{am} \sim \omega_{pump}$
z
incoherent
pumping
Active medium (AM):
|e>
ω_{am}
|g>

Surface plasmons in one Al nanodisk

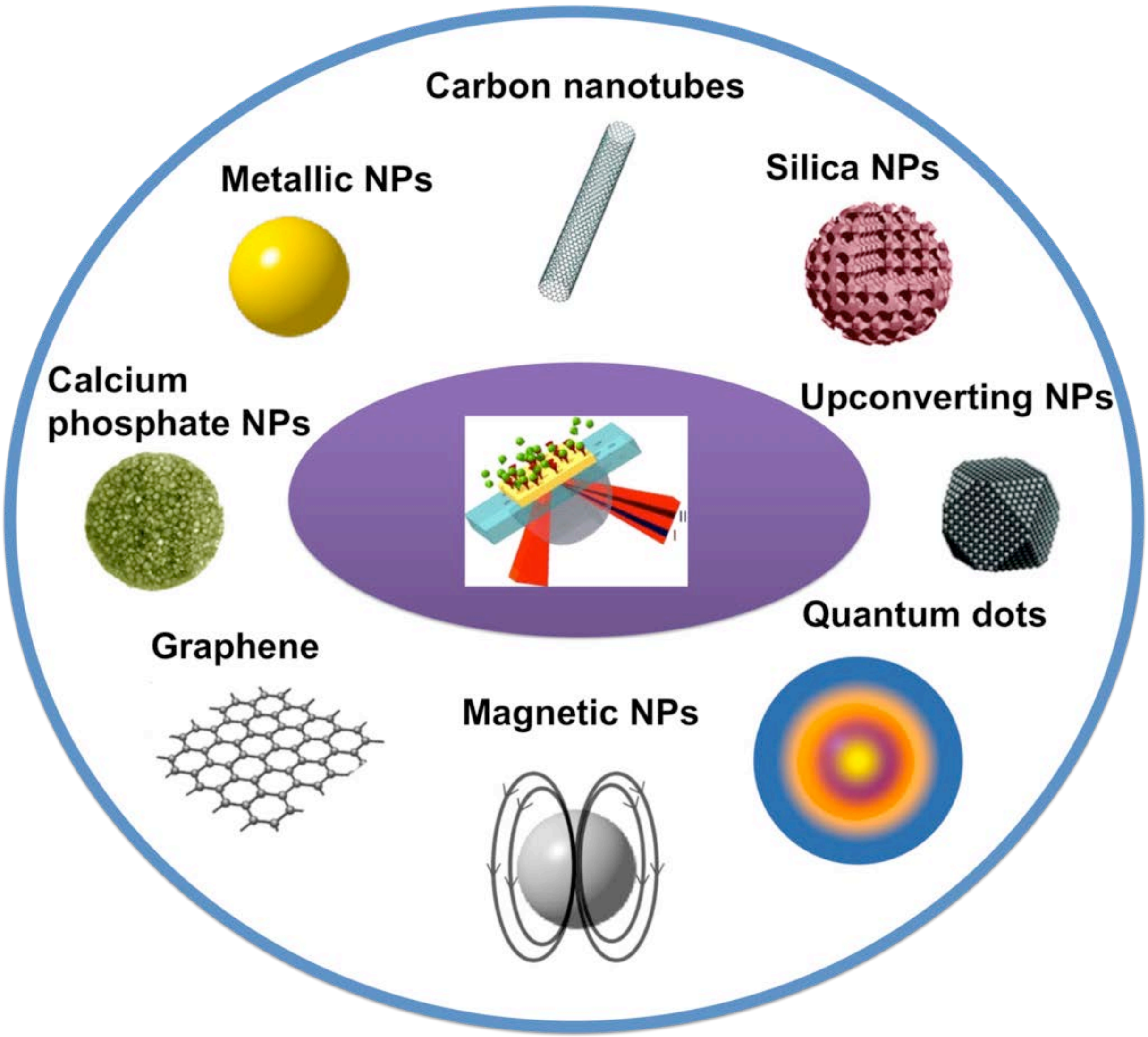
- electric field simulation normal to the surface



OVERHEATING PROBLEM



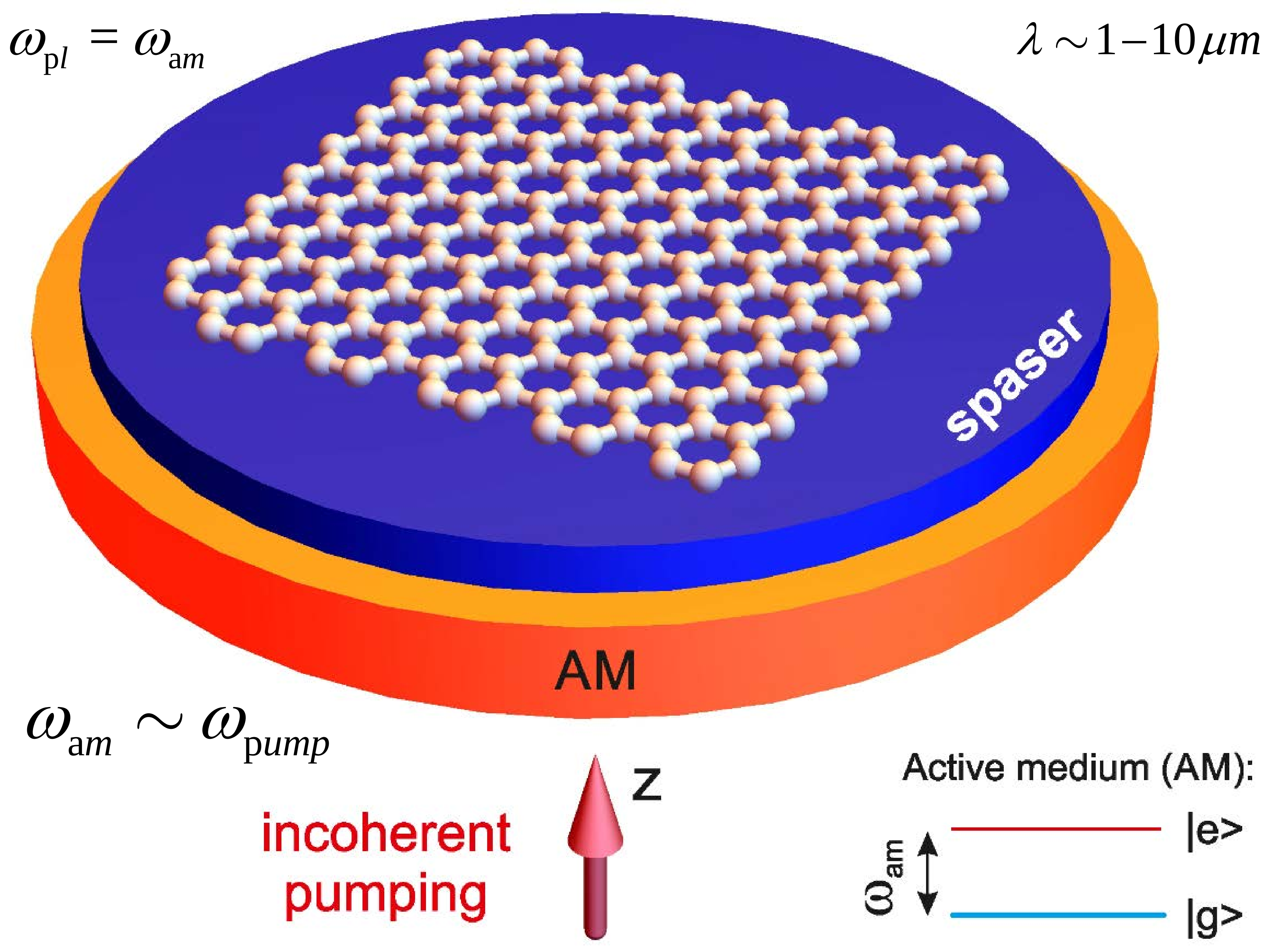
Alternative materials for plasmons



We account for the first time quantum correlations and dissipation effects that allows describing such regimes of quantum plasmonic amplifier as

- Surface plasmon emitting diode and
- surface plasmon amplifier by stimulated emission of radiation.

Switching between these generation types is possible in situ with variance of graphene Fermi-level.


$\omega_{pl} = \omega_{am}$
$\lambda \sim 1-10\mu m$
spaser
AM
$\omega_{am} \sim \omega_{pump}$
z
incoherent pumping
Active medium (AM):
|e>
ω_{am}
|g>

$\lambda \sim 1-10\mu m$

$$n_{th} = \left(\exp\left(\hbar\omega / k_{\mathrm{B}}T\right)-1\right)^{-1} \ll 1$$

Main equations:

Plasmon relaxation

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\mathrm{p}l}}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right) + \sum_{k}\frac{\gamma_{\mathrm{a}m}^{\mathrm{d}ecay}}{2}\left(2\hat{\sigma}_{k}\hat{\rho}\hat{\sigma}_{k}^{+} - \hat{\sigma}_{k}^{+}\hat{\sigma}_{k}\hat{\rho} - \hat{\rho}\hat{\sigma}_{k}^{+}\hat{\sigma}_{k}\right) +$$

$$+\sum_{k}\frac{\gamma_{\mathrm{a}m}^{\mathrm{d}ephasing}}{2}\left(\hat{\sigma}_{z,k}\hat{\rho}\hat{\sigma}_{z,k} - \hat{\rho}\right) + \sum_{k}\frac{\gamma_{\mathrm{a}m}^{\mathrm{p}ump}}{2}\left(2\hat{\sigma}_{k}^{+}\hat{\rho}\hat{\sigma}_{k} - \hat{\sigma}_{k}\hat{\sigma}_{k}^{+}\hat{\rho} - \hat{\rho}\hat{\sigma}_{k}\hat{\sigma}_{k}^{+}\right).$$

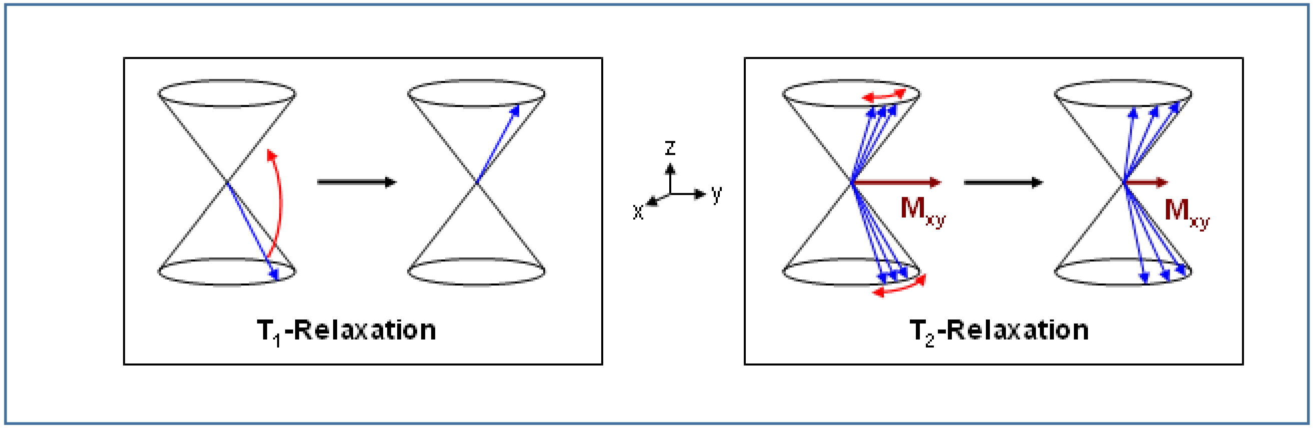
?

$$\lambda \sim 1-10\mu m$$

$$n_{th} = \left(\exp\left(\hbar\omega / k_{\mathrm{B}}T\right)-1\right)^{-1} \ll 1$$

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\mathrm{pl}}}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right) + \sum_{k}\frac{\gamma_{\mathrm{am}}^{decay}}{2}\left(2\hat{\sigma}_{k}\hat{\rho}\hat{\sigma}_{k}^{+} - \hat{\sigma}_{k}^{+}\hat{\sigma}_{k}\hat{\rho} - \hat{\rho}\hat{\sigma}_{k}^{+}\hat{\sigma}_{k}\right) +$$
$$+\sum_{k}\frac{\gamma_{\mathrm{am}}^{dephasing}}{2}\left(\hat{\sigma}_{z,k}\hat{\rho}\hat{\sigma}_{z,k} - \hat{\rho}\right) + \sum_{k}\frac{\gamma_{\mathrm{am}}^{pump}}{2}\left(2\hat{\sigma}_{k}^{+}\hat{\rho}\hat{\sigma}_{k} - \hat{\sigma}_{k}\hat{\sigma}_{k}^{+}\hat{\rho} - \hat{\rho}\hat{\sigma}_{k}\hat{\sigma}_{k}^{+}\right).$$

“working” levels of the active medium – two levels. TLS is equivalent to spin-1/2…



Up-down flip… Coherence of rotation…

About the Hamiltonian…

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\text{pl}}}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right) + \sum_k \frac{\gamma_{\text{am}}^{decay}}{2}\left(2\hat{\sigma}_k\hat{\rho}\hat{\sigma}_k^{+} - \hat{\sigma}_k^{+}\hat{\sigma}_k\hat{\rho} - \hat{\rho}\hat{\sigma}_k^{+}\hat{\sigma}_k\right) +$$

$$+\sum_k \frac{\gamma_{\text{am}}^{dephasing}}{2}\left(\hat{\sigma}_{z,k}\hat{\rho}\hat{\sigma}_{z,k} - \hat{\rho}\right) + \sum_k \frac{\gamma_{\text{am}}^{pump}}{2}\left(2\hat{\sigma}_k^{+}\hat{\rho}\hat{\sigma}_k - \hat{\sigma}_k\hat{\sigma}_k^{+}\hat{\rho} - \hat{\rho}\hat{\sigma}_k\hat{\sigma}_k^{+}\right).$$

Jaynes–Cummings-like model. Rotating wave approximation…

$$\hat{H} = \hbar\omega_{\text{pl}}\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right) + \sum_k \hbar\omega_{\text{am}}\hat{\sigma}_k^{+}\hat{\sigma}_k + \sum_k \hbar\Omega_{\text{R}}(k)\left(\hat{a}^{+}\hat{\sigma}_k + \hat{a}\hat{\sigma}_k^{+}\right)$$

plasmons — active medium — interaction

$$\hat{H} = \hbar\omega_{\mathrm{pl}}\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right) + \sum_k \hbar\omega_{\mathrm{am}}\hat{\sigma}_k^{+}\hat{\sigma}_k + \sum_k \hbar\Omega_{\mathrm{R}}(k)\left(\hat{a}^{+}\hat{\sigma}_k + \hat{a}\hat{\sigma}_k^{+}\right)$$

How to simplify the model?
Collective operators… Idea taken from Dicke-model.

$$\hat{J} = \sum_k \hat{\sigma}_k$$

$$\hat{J}^{+} = \sum_k \hat{\sigma}_k^{+}$$

$$\hat{J}_z = \sum_k \hat{\sigma}_{z,k}$$

Simplify???

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\mathrm{pl}}}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{+} - \hat{a}^{+}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{+}\hat{a}\right) + \sum_{k}\frac{\gamma_{\mathrm{am}}^{decay}}{2}\left(2\hat{\sigma}_{k}\hat{\rho}\hat{\sigma}_{k}^{+} - \hat{\sigma}_{k}^{+}\hat{\sigma}_{k}\hat{\rho} - \hat{\rho}\hat{\sigma}_{k}^{+}\hat{\sigma}_{k}\right) +$$

$$+\sum_{k}\frac{\gamma_{\mathrm{am}}^{dephasing}}{2}\left(\hat{\sigma}_{z,k}\hat{\rho}\hat{\sigma}_{z,k} - \hat{\rho}\right) + \sum_{k}\frac{\gamma_{\mathrm{am}}^{pump}}{2}\left(2\hat{\sigma}_{k}^{+}\hat{\rho}\hat{\sigma}_{k} - \hat{\sigma}_{k}\hat{\sigma}_{k}^{+}\hat{\rho} - \hat{\rho}\hat{\sigma}_{k}\hat{\sigma}_{k}^{+}\right).$$

Jaynes–Cummings-like model

$$\hat{H} = \hbar\omega_{\mathrm{pl}}\left(\hat{a}^{+}\hat{a} + \frac{1}{2}\right) + \sum_{k}\hbar\omega_{\mathrm{am}}\hat{\sigma}_{k}^{+}\hat{\sigma}_{k} + \sum_{k}\hbar\Omega_{\mathrm{R}}(k)\left(\hat{a}^{+}\hat{\sigma}_{k} + \hat{a}\hat{\sigma}_{k}^{+}\right)$$

plasmons | active medium | interaction

Collective operators for active medium…

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\mathrm{p}l}}{2}\mathcal{L}[\hat{a},\hat{a}^{+}] + \frac{\gamma_{\mathrm{a}m}^{\mathrm{d}ecay}}{2}\mathcal{L}[\hat{J},\hat{J}^{+}] +$$

$$\frac{\gamma_{\mathrm{a}m}^{\mathrm{d}ephasing}}{2}\mathcal{L}[\hat{J}_{z},\hat{J}_{z}^{+}] + \frac{\gamma_{\mathrm{a}m}^{\mathrm{p}ump}}{2}\mathcal{L}[\hat{J}^{+},\hat{J}],$$

$$\mathcal{L}[\hat{A},\hat{A}^{+}] = 2A\hat{\rho}\hat{A}^{+} - \hat{A}^{+}\hat{A}\hat{\rho} - \hat{\rho}\hat{A}^{+}\hat{A}$$

We preserve only “the most important” active medium molecules…

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\mathrm{p}l}}{2}\mathcal{L}[\hat{a},\hat{a}^{+}] + \frac{\gamma_{\mathrm{am}}^{\mathrm{d}ecay}}{2}\mathcal{L}[\hat{J},\hat{J}^{+}] +$$

$$\frac{\gamma_{\mathrm{am}}^{\mathrm{d}ephasing}}{2}\mathcal{L}[\hat{J}_z,\hat{J}_z^{+}] + \frac{\gamma_{\mathrm{am}}^{\mathrm{p}ump}}{2}\mathcal{L}[\hat{J}^{+},\hat{J}],$$

Active medium:

$$\gamma_{\mathrm{am}}^{\mathrm{d}ecay} \sim 10^{11}s^{-1}$$

$$\gamma_{\mathrm{am}}^{\mathrm{d}ephasing} \sim 10^{12}s^{-1}$$

$$\gamma_{\mathrm{am}}^{\mathrm{p}ump} \sim 10^{13}s^{-1}$$

current density

$$\sim 10kA/cm^2$$

Jacob B Khurgin and Greg Sun. Injection pumped single mode surface plasmon generators: threshold, linewidth, and coherence. *Optics Express*, 20(14):15309–15325, 2012

Jacob B Khurgin and Greg Sun. Practicality of compensating the loss in the plasmonic waveguides using semiconductor gain medium. *Appl. Phys. Lett.*, 100(1):011105, 2012.

For colloidal quantum dots and dye molecles which are pumped by external electromagnetic field corresponding field intensity is

$$E \simeq 10^5 V/m$$

maximum possible pumping rate: $\gamma_{\mathrm{am}}^{\mathrm{pump}} \sim 10^{13}s^{-1}$

??

- 1) How to relate interaction parameters with experimental setup?
- 2) How to extract useful information from the density matrix equation?

Interaction (Rabi) constant Ω_R

Electric field amplitude

$$\Omega_R = -d_{12}E / \hbar,$$

Dipole matrix element of a dye molecule

Interaction (Rabi) constant Ω_R

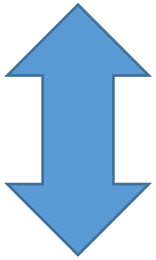
$$\Omega_R = -d_{12}E/\hbar,$$

$$\nabla\times\nabla\times E - \frac{\omega_{pl}^2}{c^2}\varepsilon(r)E = 0,$$

$$\frac{1}{8\pi}\int\left[\frac{\partial(\varepsilon'\omega)}{\partial\omega}(EE^*) + (HH^*)\right]dV = \hbar\omega_{pl}.$$

Quantization condition

$$\frac{1}{8\pi}\int\left[\frac{\partial(\varepsilon'\omega)}{\partial\omega}\left(EE^*\right)+\left(HH^*\right)\right]dV=\hbar\omega_{\mathrm{pl}}.$$



$$\oint_{\mathcal{H}(p,q)=E} p_i dq_i = n_i h,$$

Rabi constant Ω_{R}

$$\Omega_{R} = -d_{12}E/\hbar,$$

$$\frac{1}{8\pi}\int\left[\frac{\partial\left(\varepsilon'\omega\right)}{\partial\omega}\left(EE^{*}\right)+\left(HH^{*}\right)\right]dV = \hbar\omega_{\mathrm{p}l}.$$

$$\Omega_{R} = -d_{12}\omega_{\mathrm{p}l}\sqrt{\frac{2k^{3}}{\pi\hbar\left[-\pi k\frac{\partial}{\partial\omega}(\sigma''\omega)\Big|_{\omega_{\mathrm{p}l}}+\omega_{\mathrm{p}l}\right]}},$$

Rabi constant Ω_R

$$\Omega_R = -d_{12}E/\hbar,$$

$$\Omega_R = -d_{12}\omega_{pl}\sqrt{\frac{2k^3}{\pi\hbar\left[-\pi k\frac{\partial}{\partial\omega}(\sigma''\omega)\Big|_{\omega_{pl}} + \omega_{pl}\right]}},$$

$$\sigma(\omega, E_F)/e^2 = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\frac{\hbar\omega - 2E_F}{2k_BT}\right] +$$

$$\frac{i}{\pi}\left\{\frac{8k_BT\ln\left[2\cosh\left(\frac{E_F}{2k_BT}\right)\right]}{\hbar\left(\omega+\frac{i}{\tau}\right)} + \log\left[\frac{\left(\hbar(\omega+\frac{i}{\tau})+2E_F\right)^2}{\left(\hbar(\omega+\frac{i}{\tau})+2E_F\right)^2+(2k_BT)^2}\right]\right\}.$$

Yu.E. Lozovik *et al*

Rabi constant Ω_R

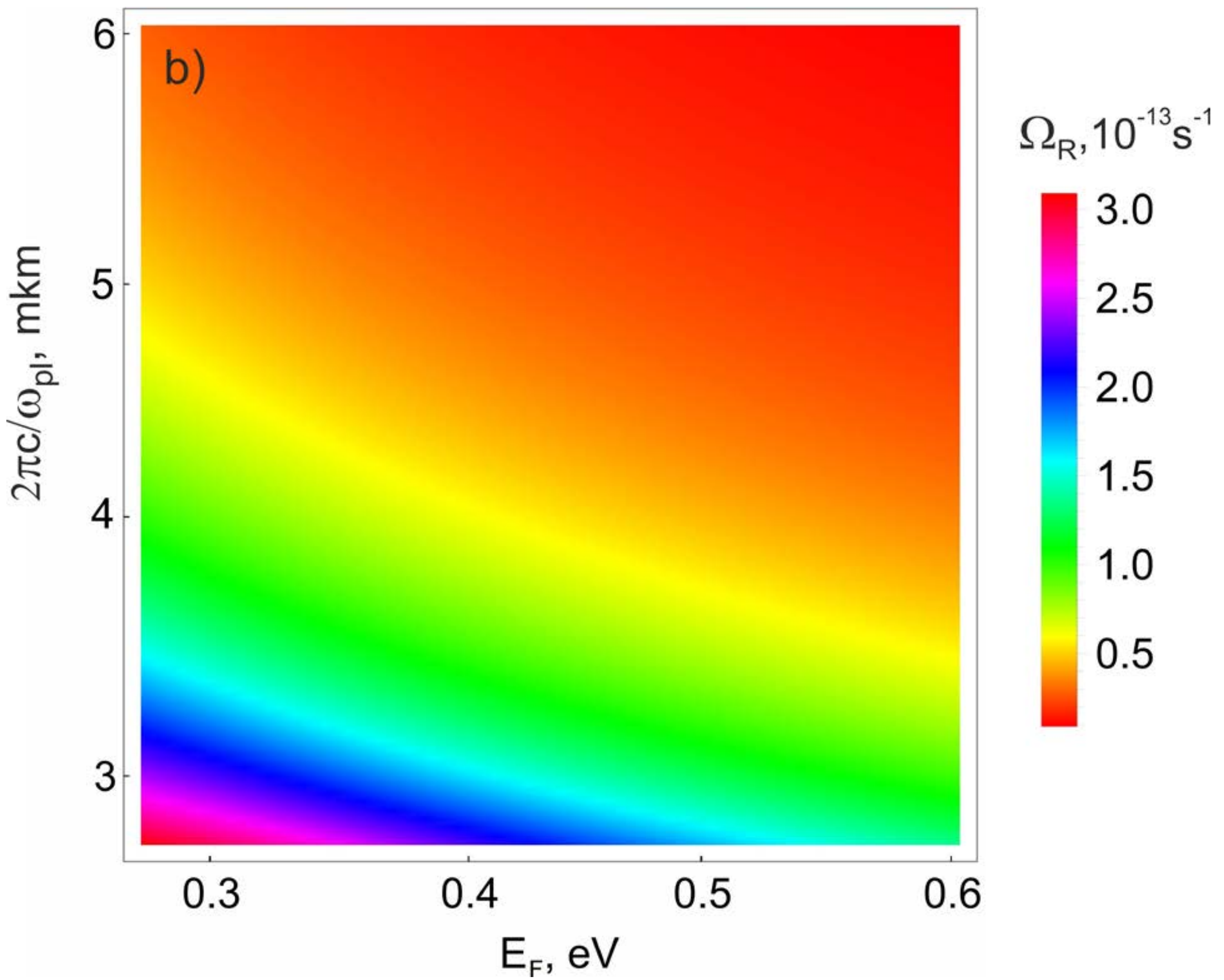
$$\Omega_R = -d_{12} E / \hbar,$$

$$\Omega_R = -d_{12}\omega_{pl} \sqrt{\frac{2k^3}{\pi\hbar\left[-\pi k \frac{\partial}{\partial\omega}(\sigma''\omega)\Big|_{\omega_{pl}} + \omega_{pl}\right]}},$$

$E_F \simeq 0.5eV$ $\lambda_{pl} = 5\mu m$

$$\boxed{\Omega_R = 0.21 \cdot 10^{13} s^{-1}}$$

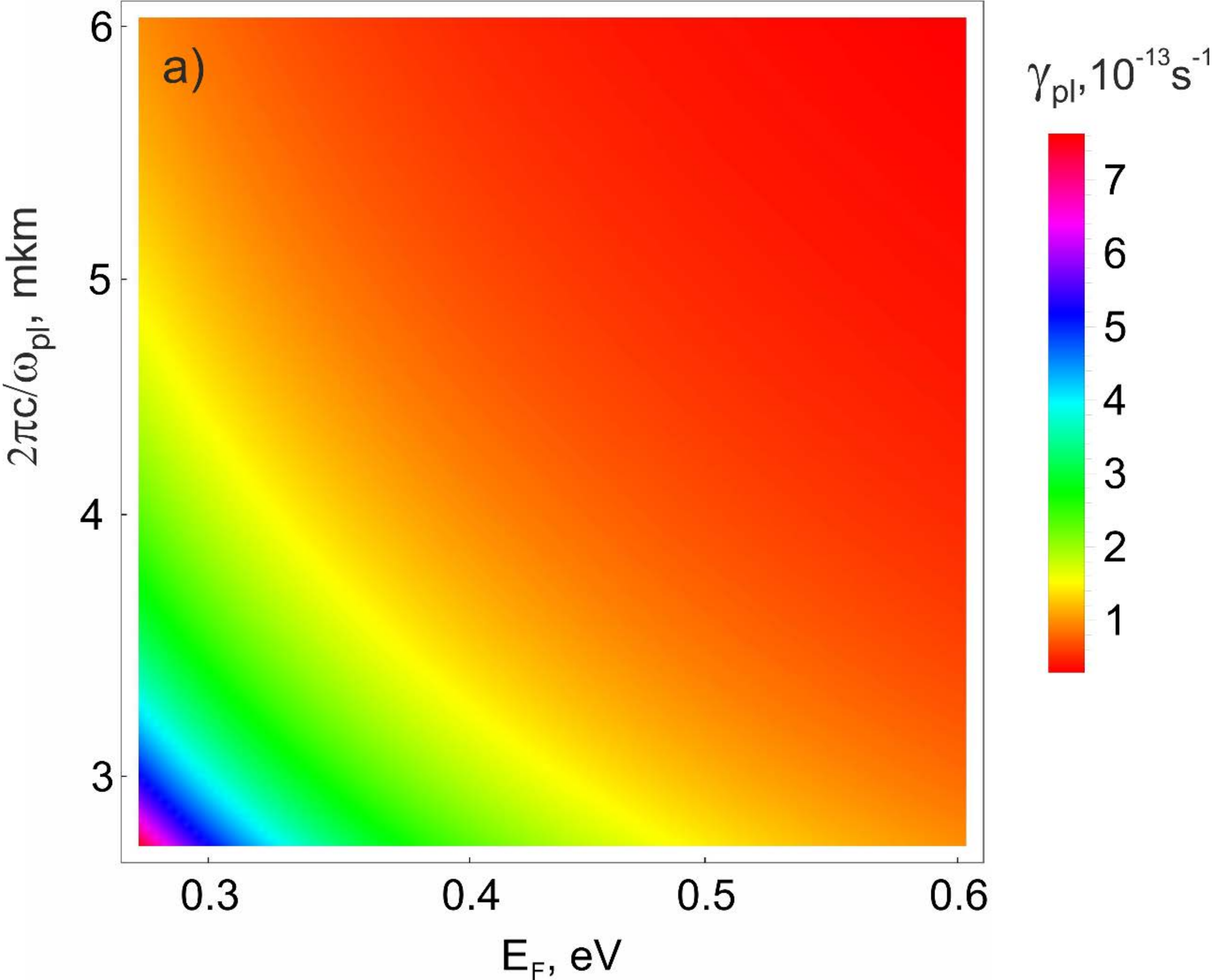
Tunable coherent plasmon generator



damping rate of surface plasmon: $\gamma_{\mathrm{p}l}$

$$\gamma_{\mathrm{p}l} = \frac{\omega_{\mathrm{p}l}}{4\pi} \int \varepsilon'' \left(E E^* \right) dV / W$$

$$\gamma_{\mathrm{p}l} = \frac{2\pi k \omega_{\mathrm{p}l} \sigma'}{-\pi k \frac{\partial}{\partial \omega} (\sigma'' \omega) \Big|_{\omega_{\mathrm{p}l}} + 0.5 \omega l}.$$



a)
6
5
4
3
$2\pi c/\omega_{pl}$, mkm
0.3
0.4
0.5
0.6
E_F, eV
$\gamma_{pl}, 10^{-13}s^{-1}$
7
6
5
4
3
2
1

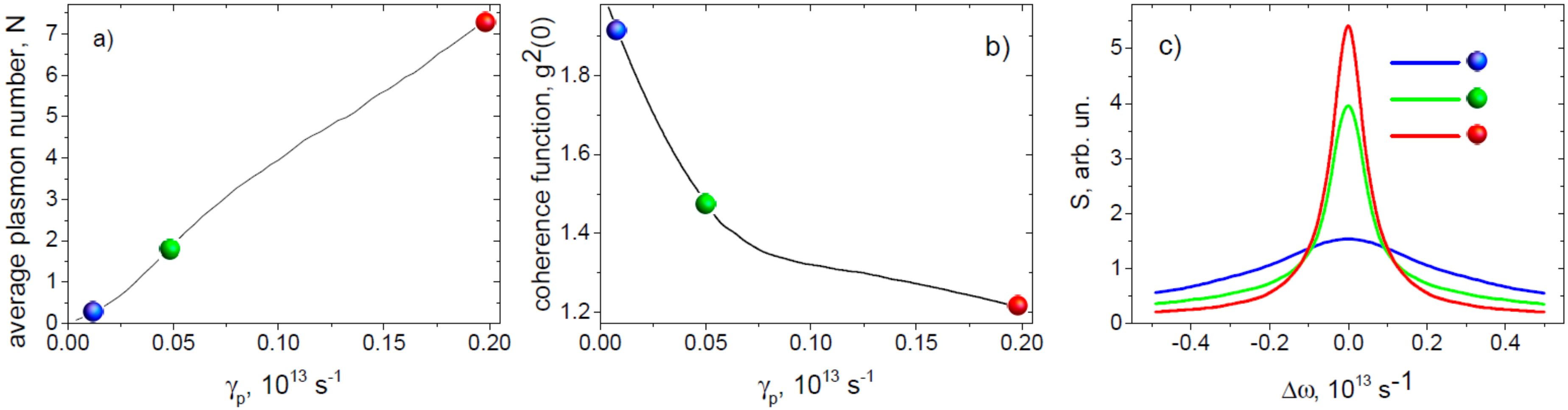


FIG. 3. (Color online) From left to right (a-c): the dependence of average number of excited plasmons N on the pumping rate γ_p, the dependence of second order correlation function $g^{(2)}$ on the pumping rate γ_p and spectrum of the plasmon field $S(\omega)$ at different values of pumping rates. Color balls, •, •, and, •, correspond to pumping rates at which spectra have been calculated. Parameters of graphene are the following: $E_{\rm F} \simeq 0.5eV$, $\lambda_{\rm pl} = 5\,\mu$m, (corresponding Rabi constant and plasmon decay rate are $\Omega_{\rm R} = 0.21 \cdot 10^{13} s^{-1}$ and $\gamma_{\rm pl} = 0.46 \cdot 10^{13} s^{-1}$, respectively).

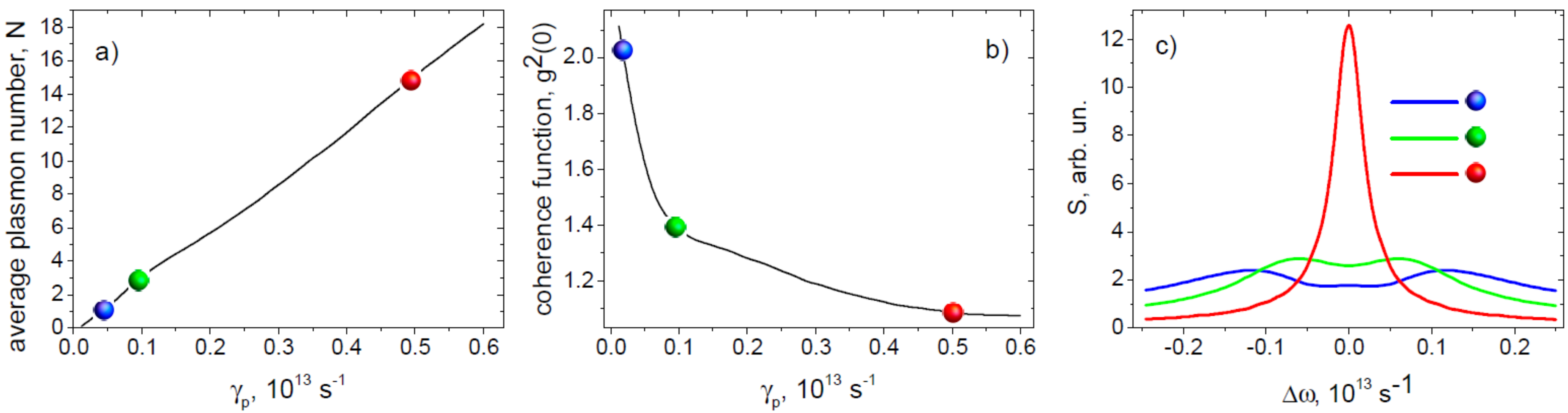


FIG. 4. The same as in Fig. 3. Parameters of graphene are the following: $E_{\rm F} \simeq 0.5eV$, $\lambda_{\rm pl} = 4\,\mu$m, (corresponding Rabi constant and plasmon decay rate are $\Omega_{\rm R} = 0.43 \cdot 10^{13} s^{-1}$ and $\gamma_{\rm pl} = 0.64 \cdot 10^{13} s^{-1}$, respectively).

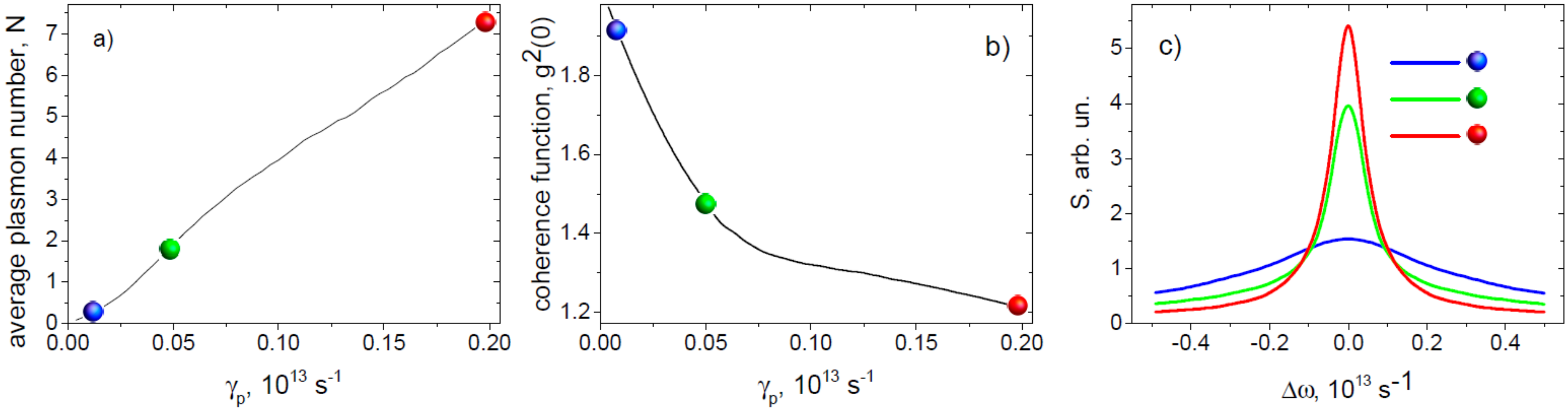
Coherence function

$$g^{(2)}(\tau) = \langle \hat{a}^{+}(t+\tau)\hat{a}^{+}(t+\tau)\hat{a}(t)\hat{a}(t)\rangle$$

Spectral function

$$S(t) = \langle \hat{a}^{+}(t+\tau)\hat{a}(t) \rangle$$

Plasmon creation and annihilation operators



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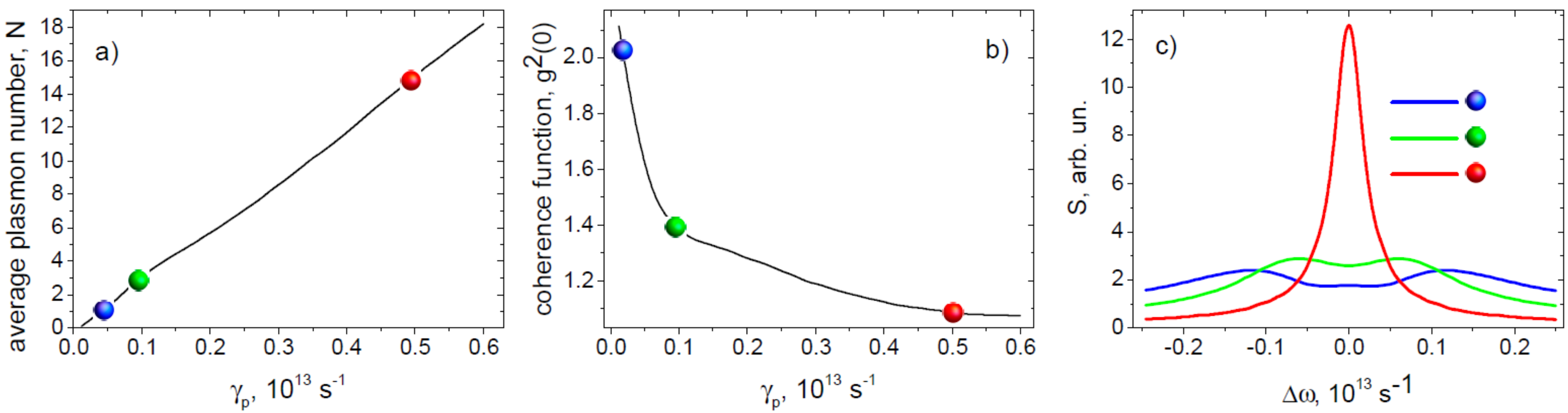

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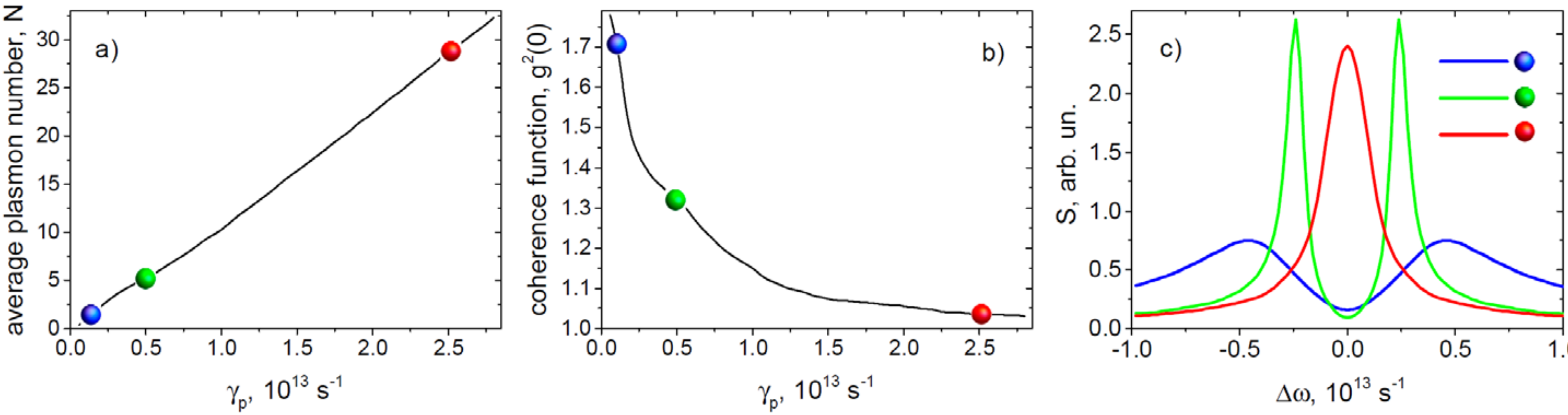
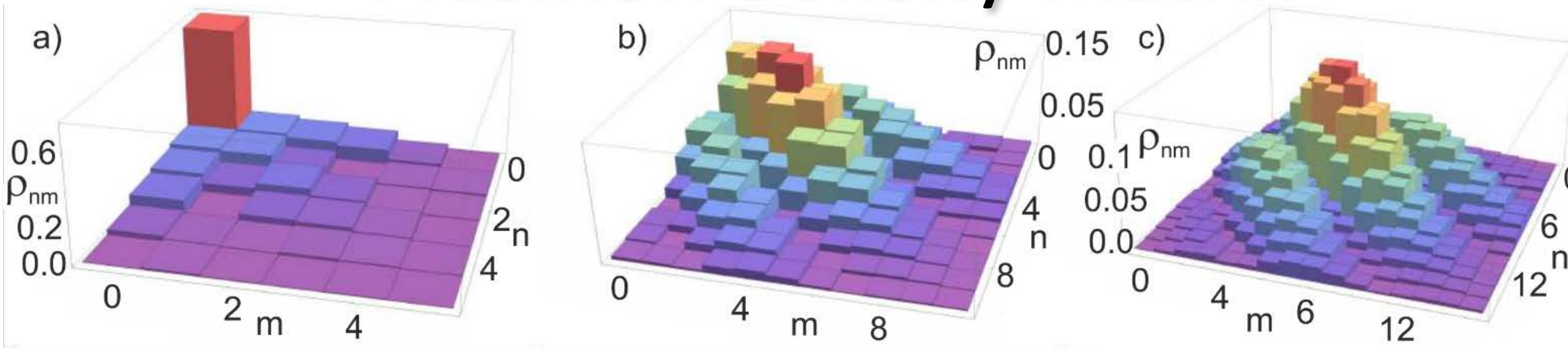


FIG. 5. (Color online) The same as in Fig. 3. Parameters of graphene are the following: $E_{\rm F} \simeq 0.4eV$, $\lambda_{\rm pl} = 3\,\mu$m, (corresponding Rabi constant and plasmon decay rate are $\Omega_{\rm R} = 1.56 \cdot 10^{13} s^{-1}$ and $\gamma_{\rm pl} = 1.76 \cdot 10^{13} s^{-1}$, respectively).

Plasmon density matrix



Wigner function where x and y are Re and Im part of the coherent state

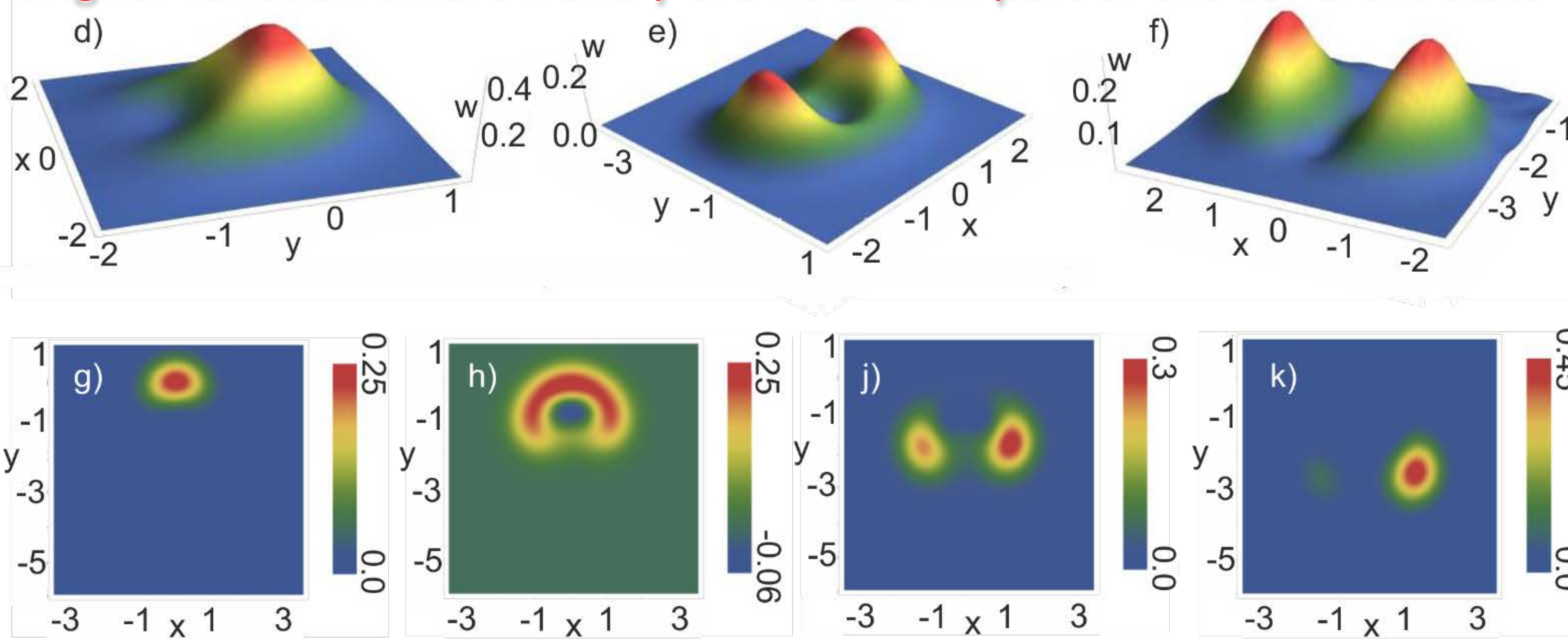
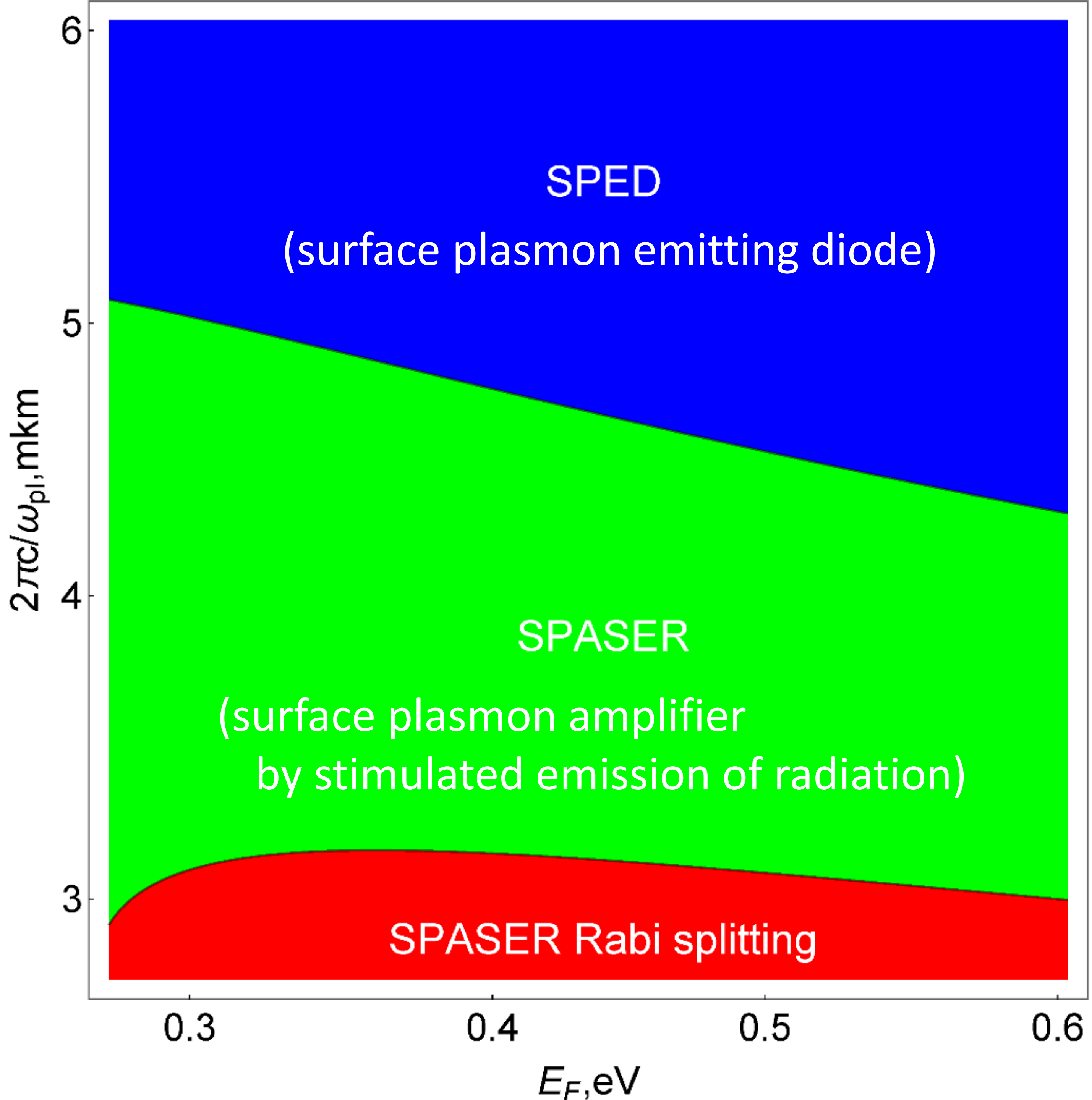


TABLE I. Active mediums and working frequency

$2\pi c/\omega_{\mathrm{am}}$, μm	Active medium
1.5 - 5	HgTe colloidal QDs [83, 84]
2 - 4	PbSe colloidal QDs [85, 86]
2.5	Cr^{2+} dopped ZnS, ZnSe, CdSe [87]
3 - 5	Transition-metal-doped nanocrystalline QDs [88]
3.5	SiGe QDs [89]
4.5	Fe^{2+} dopped ZnSe, CdMnTe [87]
4.7	InGaAs/GaAs quantum box structure [90]


SPED
(surface plasmon emitting diode)
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6
5
4
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0.3
0.4
0.5
0.6

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right] + \frac{\gamma_{\mathrm{p}l}}{2}\mathcal{L}[\hat{a},\hat{a}^{+}] + \frac{\gamma_{\mathrm{a}m}^{\mathrm{d}ecay}}{2}\mathcal{L}[\hat{J},\hat{J}^{+}] +$$

$$\frac{\gamma_{\mathrm{a}m}^{\mathrm{d}ephasing}}{2}\mathcal{L}[\hat{J}_{z},\hat{J}_{z}^{+}] + \frac{\gamma_{\mathrm{a}m}^{\mathrm{p}ump}}{2}\mathcal{L}[\hat{J}^{+},\hat{J}],$$

$$\mathcal{L}[\hat{A},\hat{A}^{+}] = 2A\hat{\rho}\hat{A}^{+} - \hat{A}^{+}\hat{A}\hat{\rho} - \hat{\rho}\hat{A}^{+}\hat{A}$$

$$\frac{d\rho_S(t)}{dt} = -i\left[H, \rho_S(t)\right] + \\ + \sum_k \gamma_k \left(A\rho(t)A^\dagger - \frac{1}{2}\left\{A^\dagger A, \rho(t)\right\} \right)$$

Quantum analog of Chepmen-Colmogorov equation for Markov processes:

$$\partial P[\psi, t] / \partial t = i\int dx \left\{ \frac{\delta}{\delta\psi(x)}(G(\psi)(x)) - \frac{\delta}{\delta\psi^*(x)}(G(\psi)^*(x)) \right\} P[\psi, t] + \\ + \int D\tilde{\psi} D\tilde{\psi}^* \left\{ W[\psi \mid \tilde{\psi}]P[\tilde{\psi}, t] - W[\tilde{\psi} \mid \psi]P[\psi, t] \right\}$$

$$G(\psi) = \left[\left(H - \frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k \right) + \frac{i}{2}\sum_k \gamma_k \left\langle \psi \middle| A_k^\dagger A_k \middle| \psi \right\rangle \right] \left| \psi \right\rangle$$

$$W[\psi \mid \tilde{\psi}] = \sum_k \gamma_k \left\langle \tilde{\psi} \middle| A_k^\dagger A_k \middle| \tilde{\psi} \right\rangle \delta\left(\frac{A_k \left| \tilde{\psi} \right\rangle}{\left\langle \tilde{\psi} \middle| A_k^\dagger A_k \middle| \tilde{\psi} \right\rangle} - \left| \psi \right\rangle \right)$$

$$\frac{d}{dt}x(t) = g(x(t)), \quad x(t) \in \mathbb{R}^d$$

$$\frac{\partial}{\partial t}P(n,t) = \sum_{n'=-\infty}^{+\infty} [W(n|n',t)P(n',t) - W(n'|n,t)P(n,t)]$$

$$\frac{\partial}{\partial t}T(x,t|x',t') = -\frac{\partial}{\partial x_i}[g_i(x)T(x,t|x',t')]$$

$$+ \int dx'' [W(x|x'')T(x'',t|x',t') - W(x''|x)T(x,t|x',t')] .$$

Jump probability for a Markov process

$$\frac{d\rho_S(t)}{dt} = -i\left[H,\rho_S(t)\right] +$$

$$+\sum_k \gamma_k \left(A\rho(t)A^\dagger - \frac{1}{2}\left\{A^\dagger A, \rho(t)\right\} \right)$$

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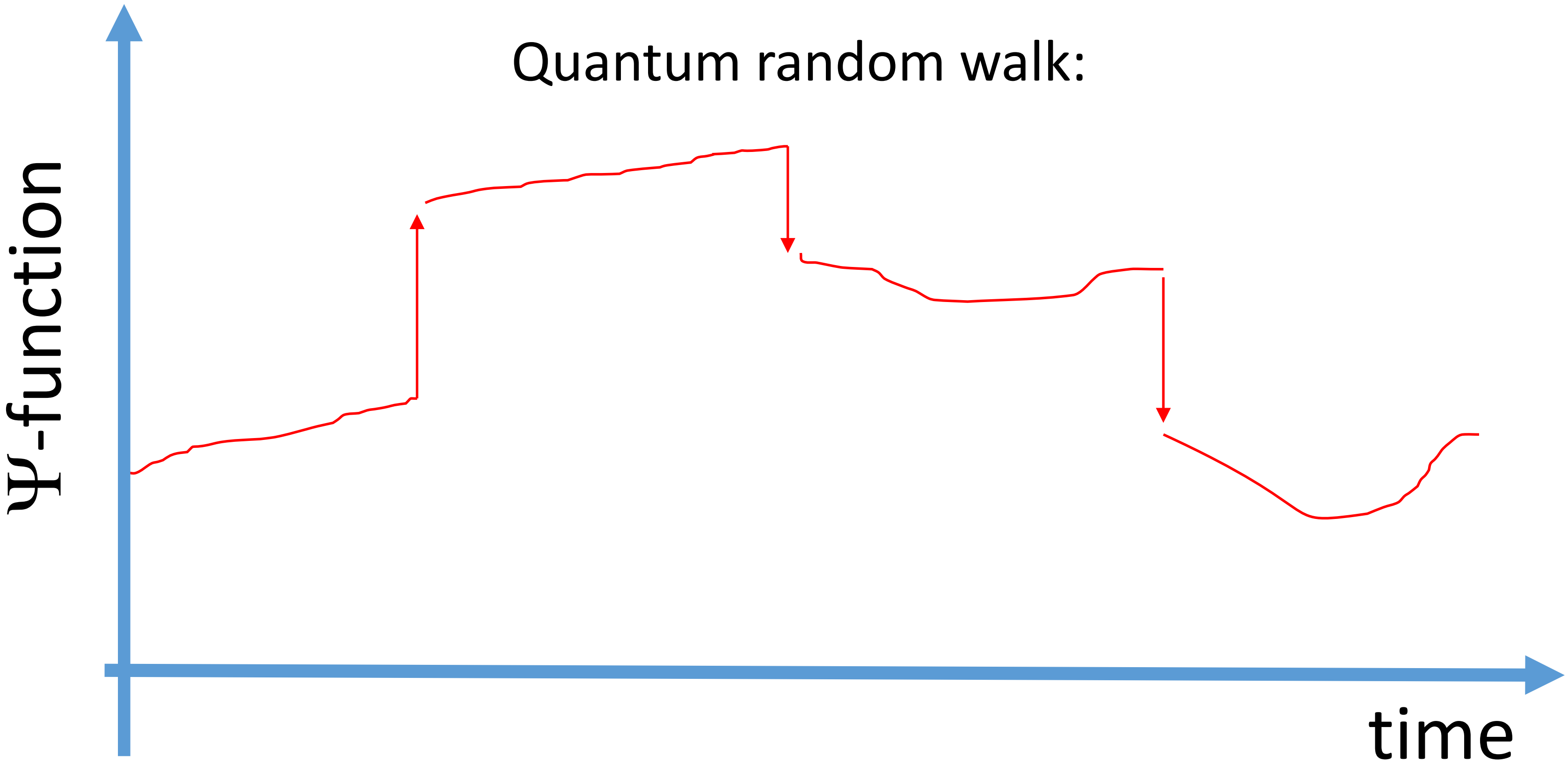
$$+\int D\tilde{\psi} D\tilde{\psi}^* \left\{ W[\psi\,|\,\tilde{\psi}]P[\tilde{\psi},t] - W[\tilde{\psi}\,|\,\psi]P[\psi,t] \right\}$$

$$G(\psi) = \left[\left(H - \frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k \right) + \frac{i}{2}\sum_k \gamma_k \left\langle \psi \middle| A_k^\dagger A_k \middle| \psi \right\rangle \right] \left|\psi\right\rangle$$

$$W[\psi\,|\,\tilde{\psi}] = \sum_k \gamma_k \left\langle \tilde{\psi} \middle| A_k^\dagger A_k \middle| \tilde{\psi} \right\rangle \delta\left(\frac{A_k\left|\tilde{\psi}\right\rangle}{\left\langle \tilde{\psi} \middle| A_k^\dagger A_k \middle| \tilde{\psi} \right\rangle} - \left|\psi\right\rangle \right)$$

Non Hermit Hamiltonian

$$H=\left(H_0-\frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k\right)+\frac{i}{2}\sum_k \gamma_k \langle\psi|A_k^\dagger A_k|\psi\rangle$$



$$\frac{d}{dt}\rho_S(t) = -i[H, \rho_S(t)] + \sum_i \gamma_i \left(A_i \rho_S(t) A_i^\dagger - \frac{1}{2} A_i^\dagger A_i \rho_S(t) - \frac{1}{2} \rho_S(t) A_i^\dagger A_i \right)$$

$$F[\tilde{\psi}, \tau] = 1 - \exp\left(- \int_0^\tau ds\, \Gamma[g_s(\tilde{\psi})] \right) = 1 - ||\exp(-i\hat{H}\tau)\tilde{\psi}||^2$$

Random Change of the wave function:

Probability:

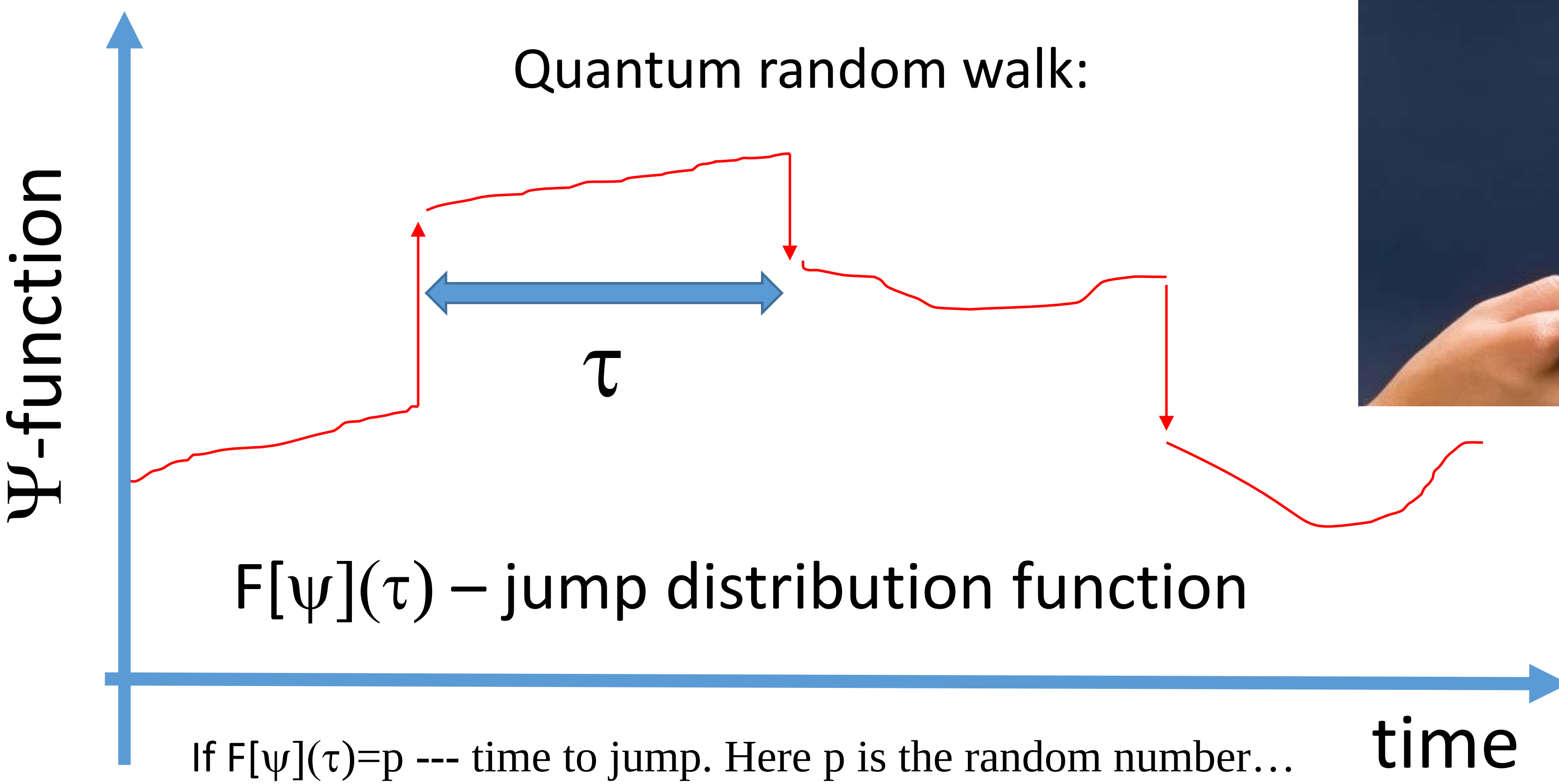
$$\tilde{\psi} \longrightarrow \psi = \frac{A_i \tilde{\psi}}{||A_i \tilde{\psi}||}$$

$$p_i = \frac{\gamma_i ||A_i \tilde{\psi}||^2}{\Gamma[\tilde{\psi}]}$$

Non Hermit Hamiltonian

$$H=\left(H_0-\frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k\right)+\frac{i}{2}\sum_k \gamma_k \langle\psi|A_k^\dagger A_k|\psi\rangle$$

Quantum random walk:



F[ψ](τ) – jump distribution function

If F[ψ](τ)=p --- time to jump. Here p is the random number…

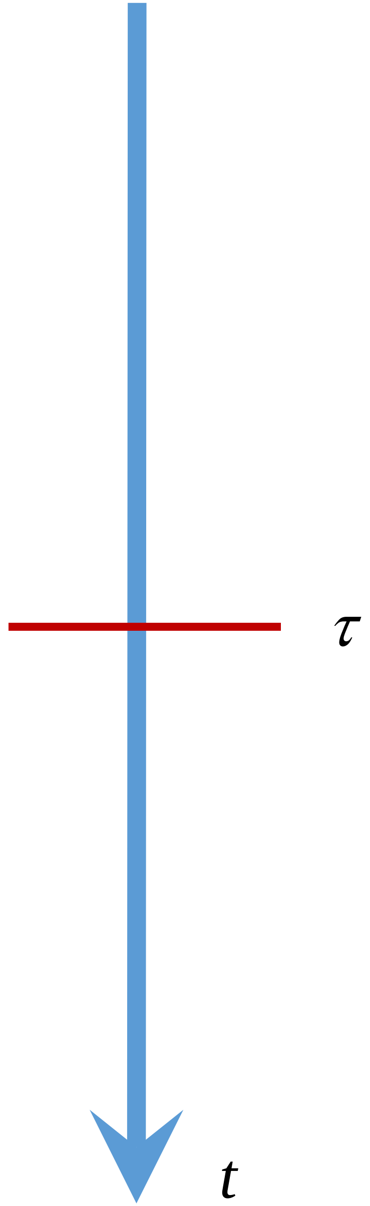
Quantum “Monte-Carlo” simulation

$$\psi(t)=\frac{\exp\left(-i\left(H-\frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k\right)t\right)\tilde{\psi}}{\left\|\exp\left(-i\left(H-\frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k\right)t\right)\tilde{\psi}\right\|}$$

$$F[\psi,\tau]=1-\left\|\exp\left(-i\left(H-\frac{i}{2}\sum_k \gamma_k A_k^\dagger A_k\right)t\right)\tilde{\psi}\right\|^2$$

$$\tilde{\psi}\to\frac{A_k\left|\psi\right\rangle}{\left\langle\psi\right|A_k^\dagger A_k\left|\psi\right\rangle}$$

$$p_k=\frac{\gamma_k\left\langle\psi\right|A_k^\dagger A_k\left|\psi\right\rangle}{\sum_k \gamma_k\left\langle\psi\right|A_k^\dagger A_k\left|\psi\right\rangle}$$



THE THEORY OF OPEN QUANTUM SYSTEMS

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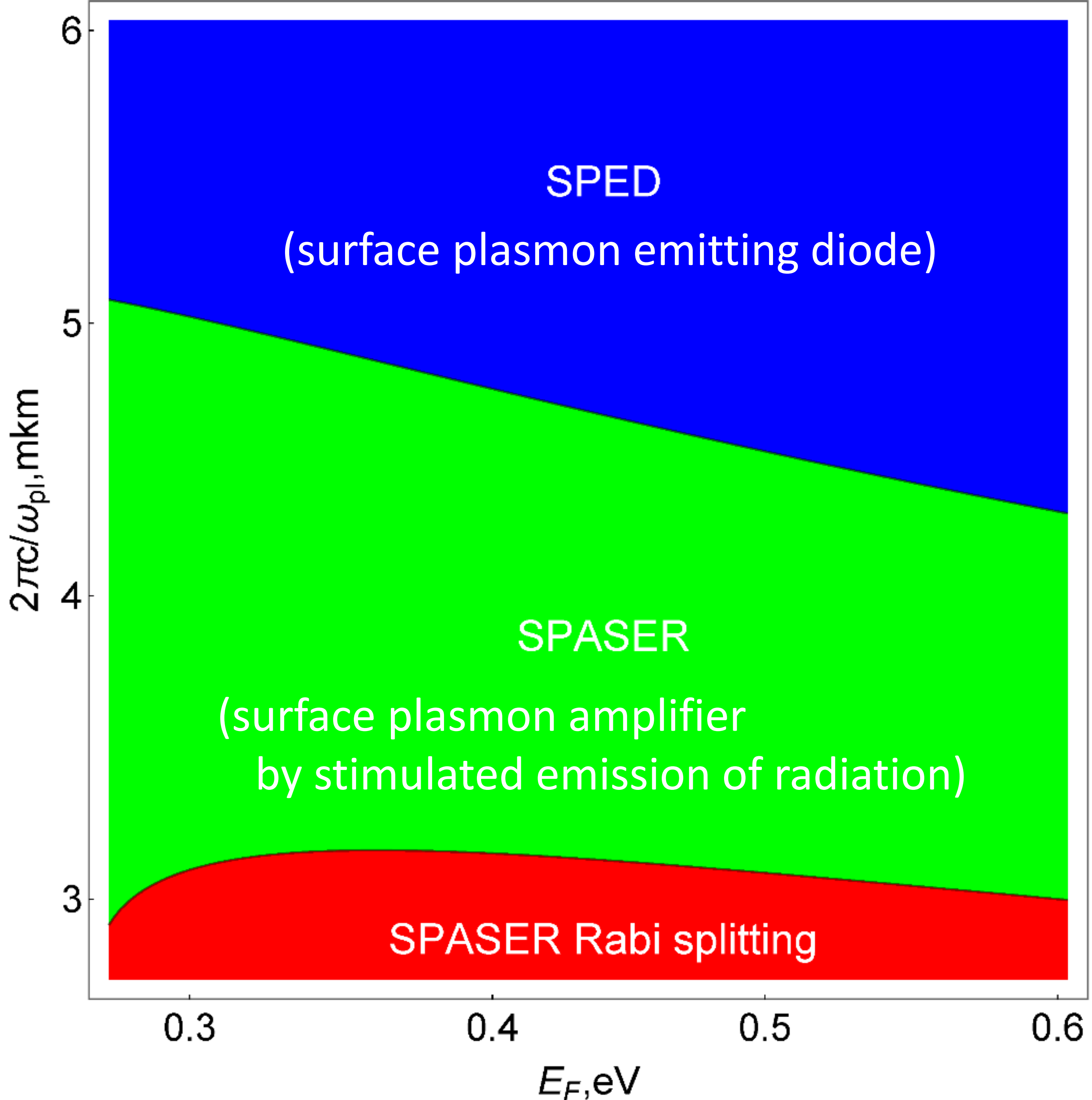
and
Istituto Italiano per gli Studi Filosofici

In conclusion,

We have shown doing self-consistent quantum calculations that

- graphene is the promising material for applications in state-of-the-art active and passive plasmonic devices that allow *in situ* tuning of parameters.
- High graphene conductivity dependence on Fermi-level and frequency allows switching between the plasmon generation types such as SPED and SPASER.
- We have found the generation spectrum and the second order correlation function, which predicts laser statistics.
- We provide explicit expressions for interaction and dissipation parameters through material constants and geometry.



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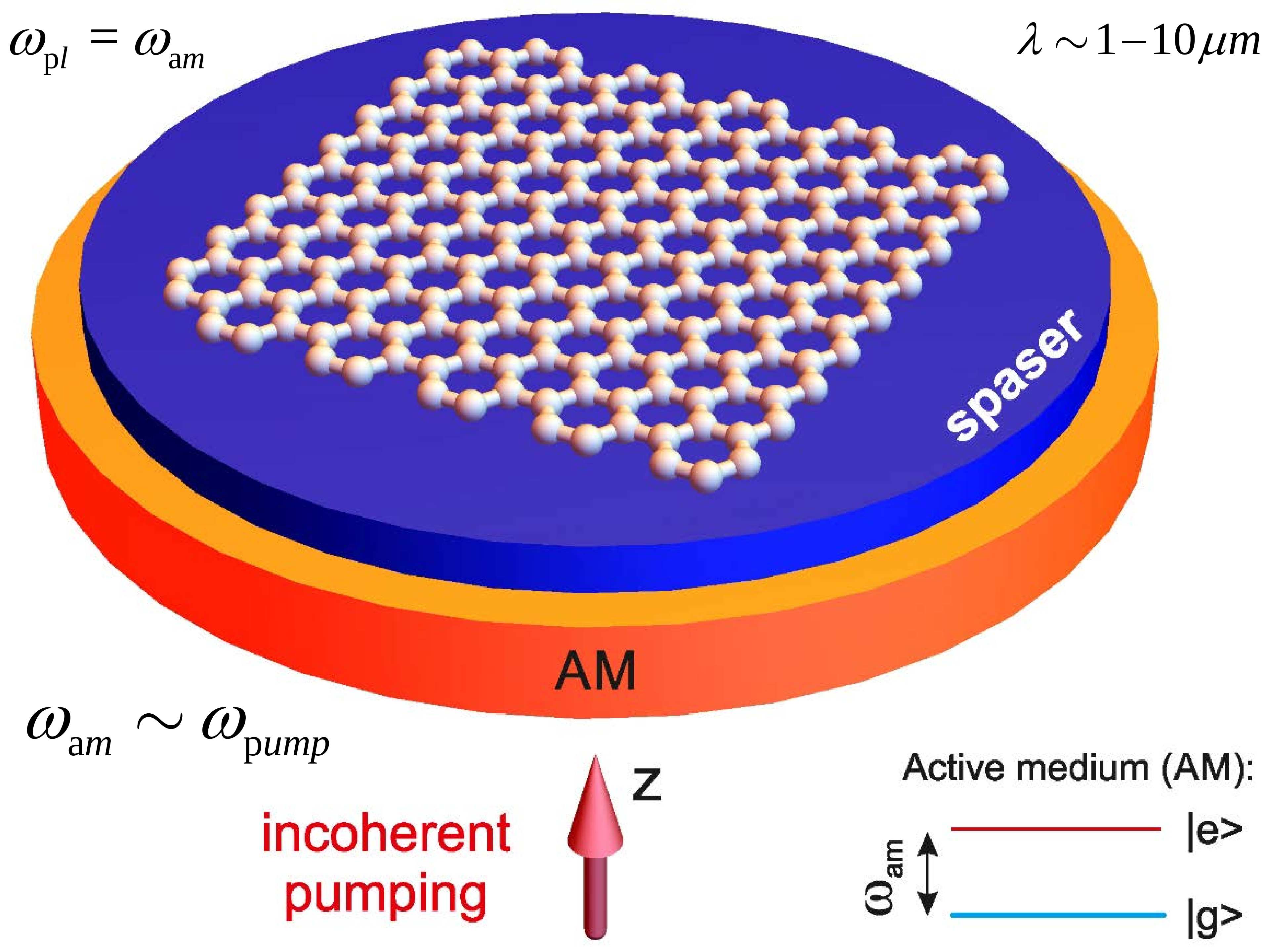

$\omega_{pl} = \omega_{am}$
$\lambda \sim 1-10\mu m$
spaser
AM
$\omega_{am} \sim \omega_{pump}$
z
incoherent pumping
Active medium (AM):
|e>
ω_{am}
|g>