SIMULATIONS OF HEAVY ION COLLISIONS WITH A QCD PHASE TRANSITION

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Today

- Motivation
- 3-fluid hydrodynamics
- Hybrid models
- Coarse graining

Tomorrow

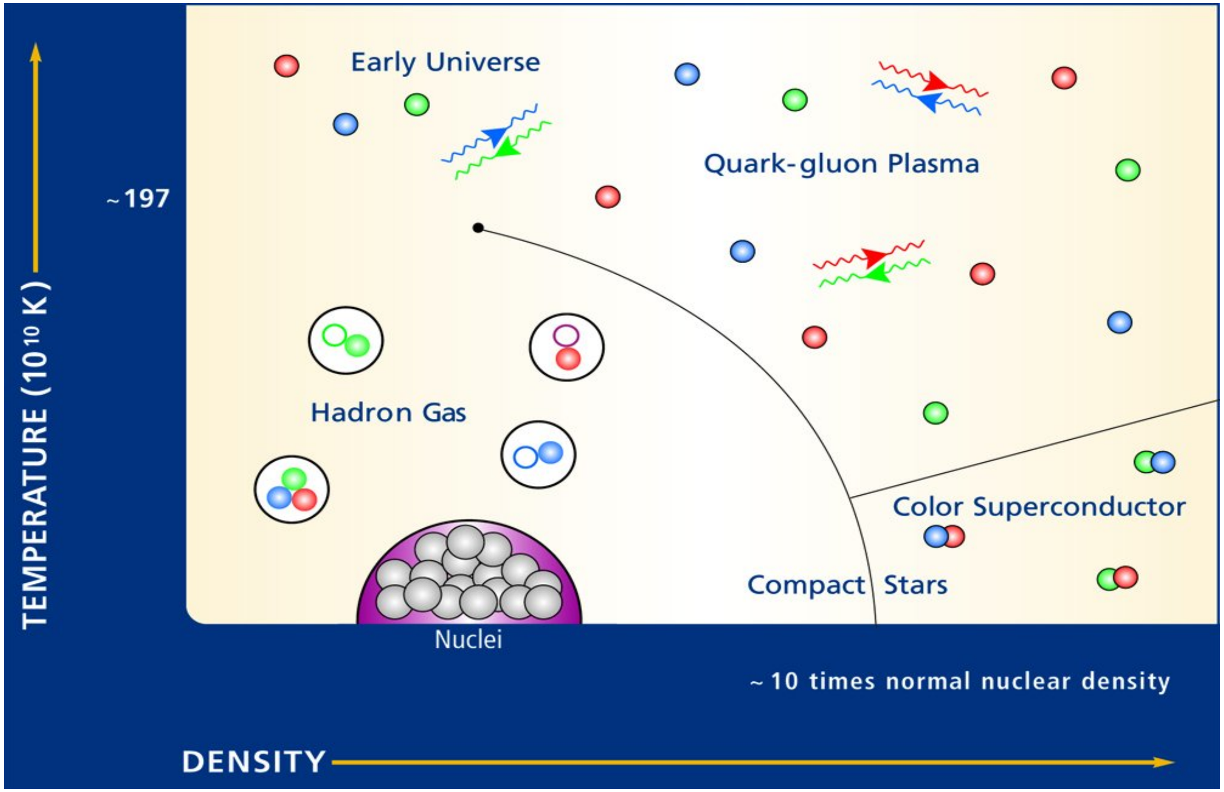
- Quark Molecular Dynamics
- Chiral Hydrodynamics
- Chiral Particle Dynamics

Thanks

- Hannah Petersen
- Jan Steinheimer
- Yuri Karpenko
- Pasi Huovinen
- Hendrik van Hees
- Marlene Nahrgang
- Stefan Endres

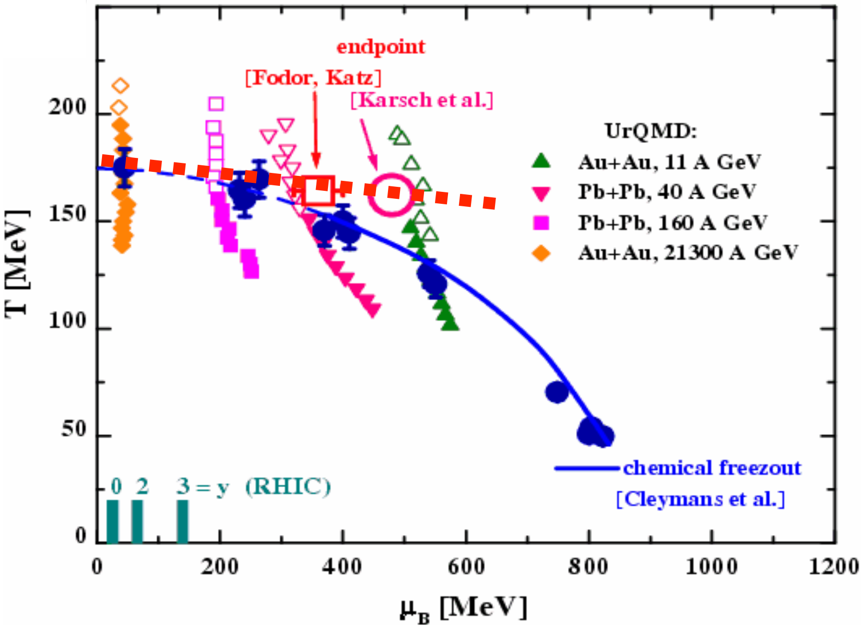
QCD Phase Diagram: Sketch

http://www.ice.csic.es/en/graphics/phase.jpg

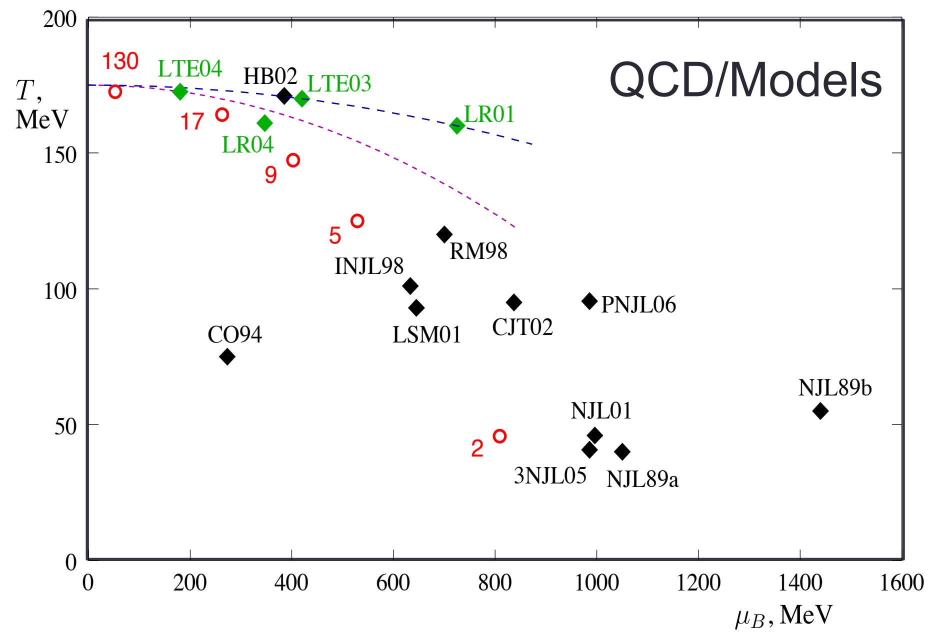


In heavy ion collisions heated and compressed nuclear matter is produced under controlled conditions

QCD Phase Diagram: „Reality“



L. Bravina, M.B., et al., JPG 1999
I. Arsene et al., PRC 2007



M. Stephanov PoS 2006

- Except for $\mu_B \rightarrow 0$, many features are unknown
- Order of PT, critical points, dof (Quarkyonic matter?)

Signals of the Phase Transition

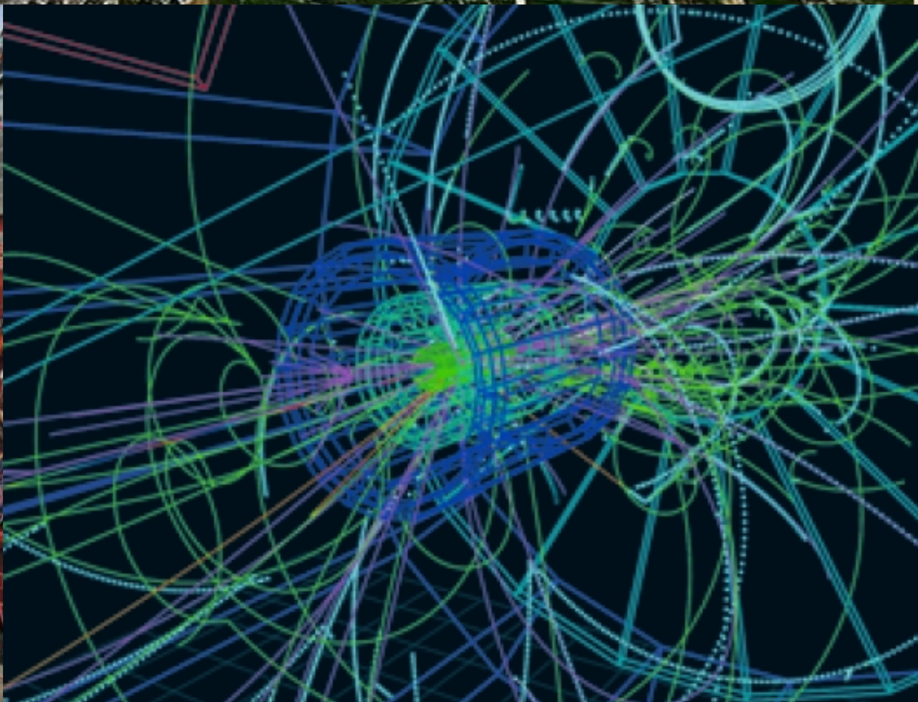
Potential Signals

- Correlation length
- Fluctuations
- Softening of the EoS
- Delayed expansion
- Parton coalescence
- Change of the dof
- Entropy production
- Fragmentation

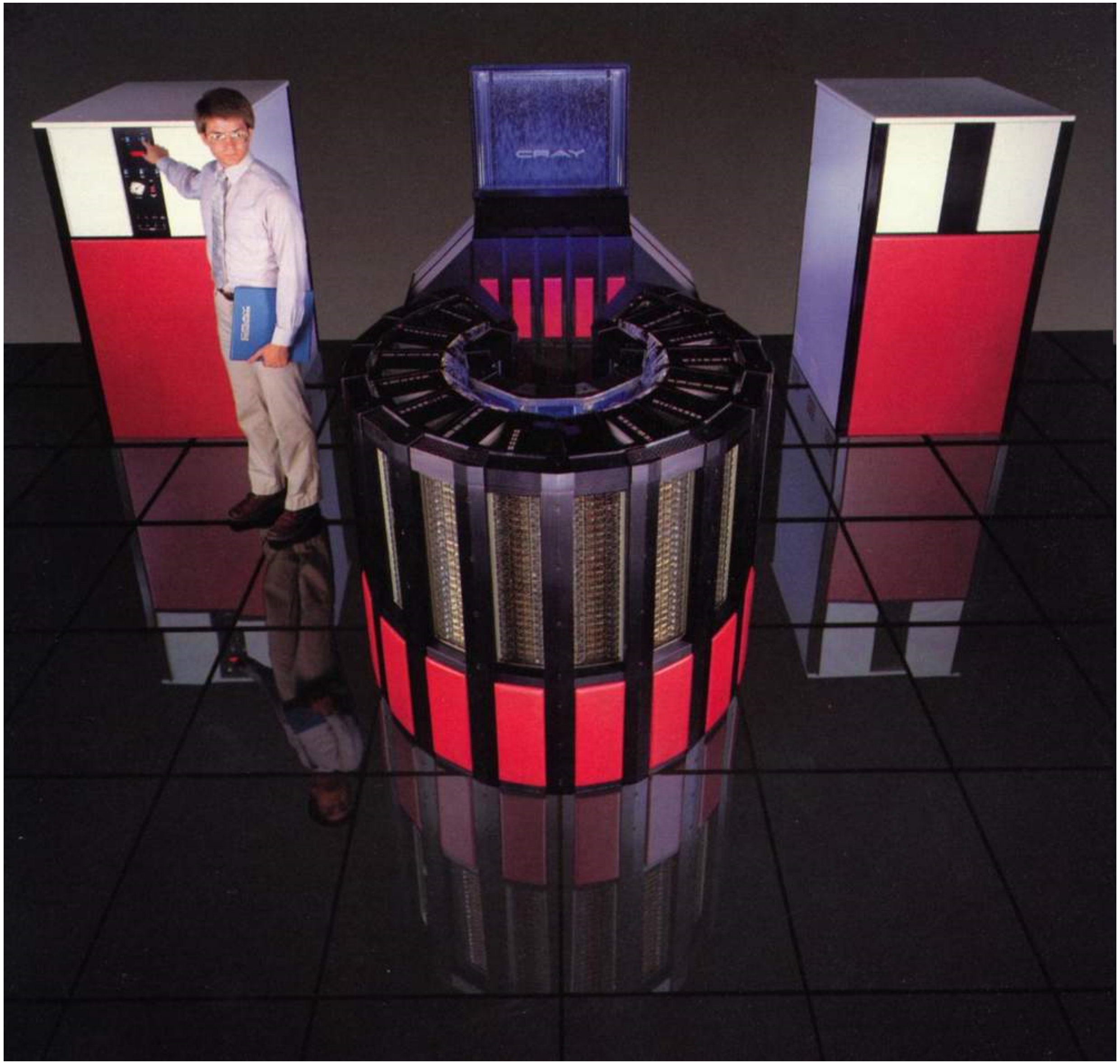
Potential Observables

- Non-Gaussian fluctuations
- Charge ratio fluctuations
- Irregular v_1 vs E_{lab}
- HBT and/or Photons
- Elliptic flow, exotic mesons
- Thermalization
- Pion yield
- Cluster formation / v_n


ALICE
CMS
ATLAS
LHCb



Need for Simulations



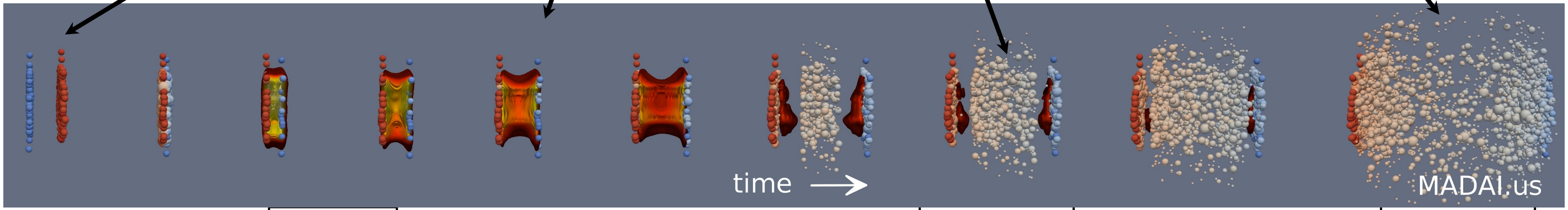
Time Evolution of Heavy Ion Collisions

Nuclei at 99 % speed of light

Quark Gluon Plasma

Hadronic Rescattering

Measurable Fragments in the detector



$1x\ 10^{-23}$ s

10×10^{-23} s

30×10^{-23} s

Nonequilibrium initial state dynamics → Relativistic Hydrodynamics/ Parton dynamics → Hadron Transport

Hybrid approaches are very successful for the description of the dynamics

Hannah Petersen, special issue JPG, arXiv:1404.1763

Models

Ab-initio simulations, e.g. lattice QCD are not possible for dynamical systems

Effective approaches are needed

Phase transitions out of equilibrium are difficult to describe

- Hybrid models (UrQMD, NexSpherio, hydro+JAM, MUSIC, Nonaka)
- 3 fluid hydrodynamics (Ivanov, Brachmann)
- Multi-phase models (AMPT)
- Quark molecular dynamics (qMD, Martens)
- Parton Cascades + Coales. (Molnar)
- Chiral hydrodynamics (Dumitru, Nahrgang)

THREE FLUID HYDRO

Motivation

1-fluid hydrodynamics

- Instantaneous local equilibration between projectile and target nuclei
- → unrealistically large energy densities (pressure) and baryon densities
- → too large flow

3-fluid hydrodynamics

- Set-up: Target and projectile nuclei (fluid 1+2). Fluid 3: Fireball
- Introduce gradual deceleration, pump energy into fireball fluid
- Merge fluids when locally in equilibrium
- → realistic energy and baryon densities

Equations of motion

Split Energy-Momentum Tensor and currents into individual fluids

Introduce source terms for the coupling

Solve larger set of equations, including conservation equations for total energy and current

From 1-fluid equations to 3-fluids:

$$\partial_\mu T^{\mu\nu} = 0 \quad T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

$$\partial_\mu j_i^\mu = 0 \qquad j^\mu = n u^\mu$$

Split T and j into different fluids:

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} + \cdots + T_N^{\mu\nu} \quad ,$$

$$j^\mu = j_1^\mu + j_2^\mu + \cdots + j_N^\mu \quad .$$

Each individual T and j is not conserved (source terms)

$$\partial_\mu T_l^{\mu\nu} = F_l^\nu \quad , \qquad F_1^\nu + F_2^\nu + \cdots + F_N^\nu = 0$$

$$\partial_\mu j_l^\mu = S_l \qquad S_1 + S_2 + \cdots + S_N = 0$$

Mishustin et al, NPA 494 (1989) 595
Toneev et al, Phys.Part.Nucl.Lett. 2 (2005) 288
Brachmann et al, NPAA619 (1997) 391

Coupling between the fluids

Split Energy-Momentum Tensor and currents into individual fluids

Introduce source terms for the coupling

Solve larger set of equations, including conservation equations for total energy and current

(for details see ref. [8]))

8) L.M. Satarov, preprint IAE-4562/2, Moscow, 1988

Energy/momentum loss of fluid I per time per volume via the collision rate

$$v_{M\o ller} = \sqrt{(u_1^\mu\, u_{2\mu})^2 - 1}$$

$$F_l^\nu = n_1\, n_2 \left\langle v_{M\o ller} \int\limits_{\tilde p'_{||}>0} \mathrm{d}\sigma_{\mathrm{NN}\to\mathrm{NX}}\, (p'-p)^\nu \right\rangle \quad , \quad \mathrm{d}\sigma_{\mathrm{NN}\to\mathrm{NX}} = \sigma^{inv}_{\mathrm{NN}\to\mathrm{NX}} \frac{\mathrm{d}^3 p'}{E'}$$

Splitting the integral into moments yields (E‘,p‘ are after the collision) and neglecting <.>)

$$F_1^\nu = \frac{1}{2} n_1\, n_2\, v_{M\o ller} \left[(p_2 - p_1)^\nu \sigma_P(s) \;-\; (p_2 + p_1)^\nu \sigma_E(s)\right]$$

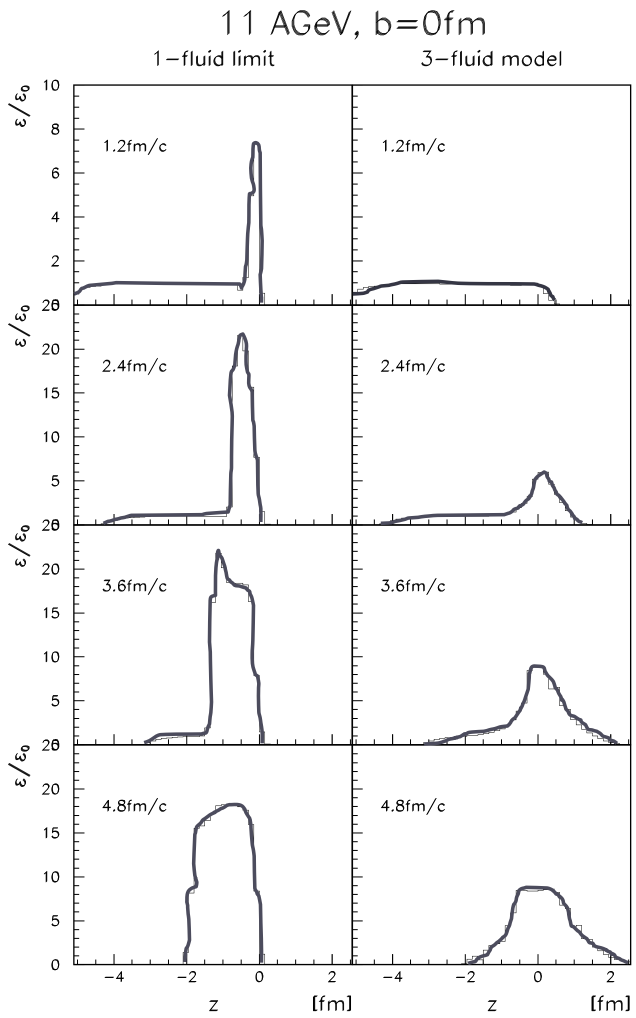
$$F_2^\nu = \frac{1}{2} n_2\, n_1\, v_{M\o ller} \left[(p_1 - p_2)^\nu \sigma_P(s) \;-\; (p_1 + p_2)^\nu \sigma_E(s)\right]$$

$$\sigma_E(s) = \int\limits_{\tilde p'_{||}>0} \mathrm{d}\sigma_{\mathrm{NN}\to\mathrm{NX}} \left(1 - \frac{\tilde E'}{\tilde E}\right) \quad , \quad \sigma_P(s) = \int\limits_{\tilde p'_{||}>0} \mathrm{d}\sigma_{\mathrm{NN}\to\mathrm{NX}} \left(1 - \frac{\tilde p'_{||}}{\tilde p_{||}}\right)$$

Sigma_E and sigma_P are parametrized from experimental data

Mishustin et al, NPA 494 (1989) 595
Brachmann et al, NPAA619 (1997) 391

Energy deposition of the Projectiles



1-fluid/3-fluid comparison

- Less energy density
- Transition of the projectile
- Different time of highest compression
- → Less flow
- → Lower temperatures

Brachmann et al, NPAA619 (1997) 391

3 minute break

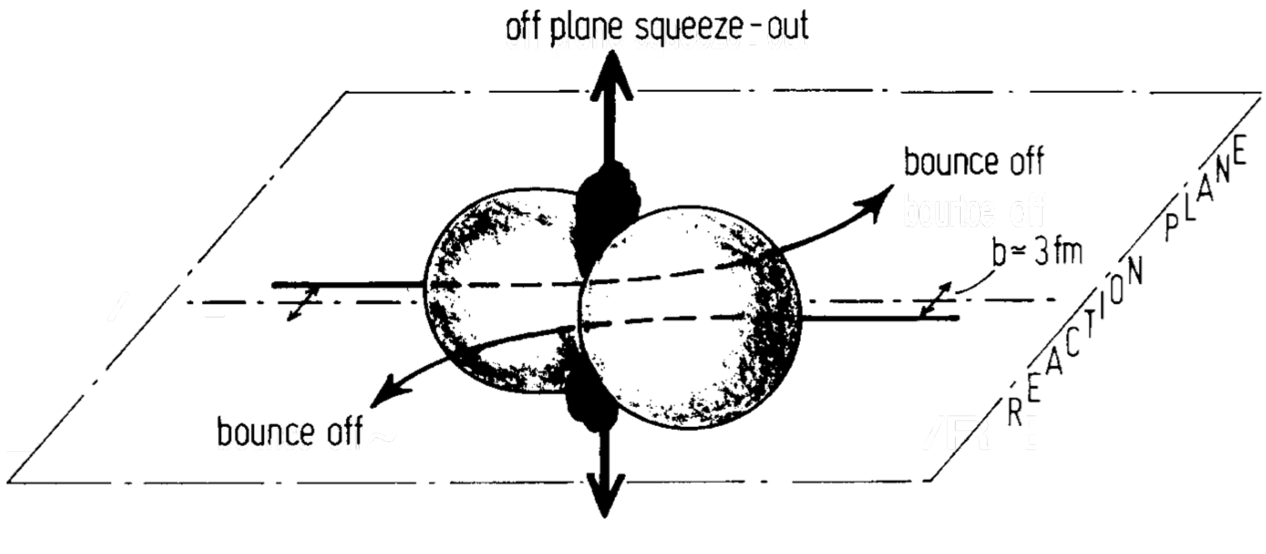
- Discuss with your neighbor

how to measure the pressure in the central collision region

THREE FLUID HYDRO

Results

Directed Flows

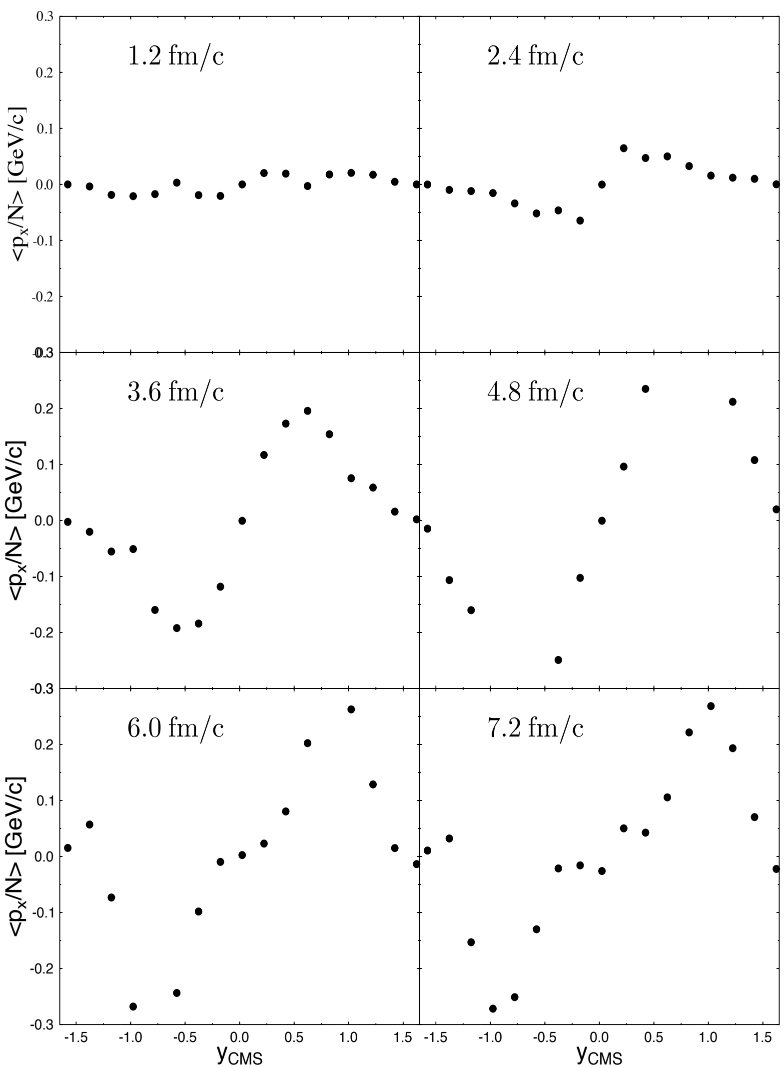


- The squeeze-out (v_2) and the bounce-off (v_1) are related to the pressure of the system

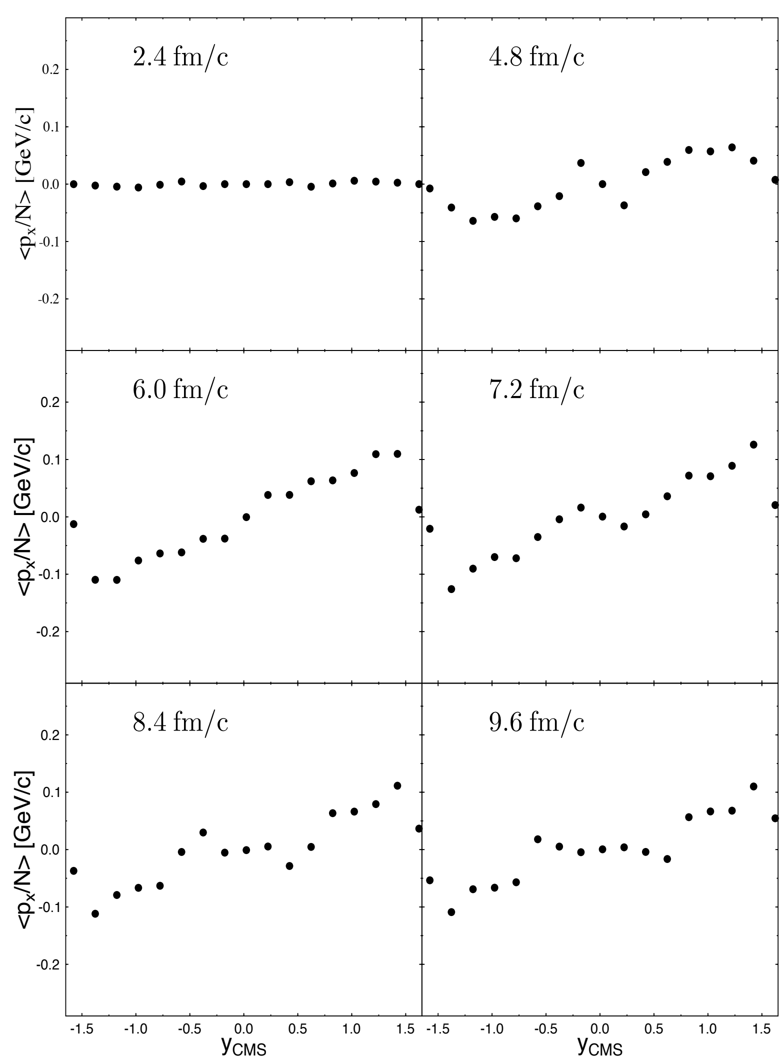
$$\mathbf{p} \propto \int_t \int_A P(\rho, S)\, \mathrm{d}\mathbf{A}\, \mathrm{d}t\,.$$

Directed Flow (aka v1)

Au+Au (11 AGeV), b=3fm, 1-Fluid-Limit



Au+Au (11 AGeV), b=3fm, 3-Fluid-Model



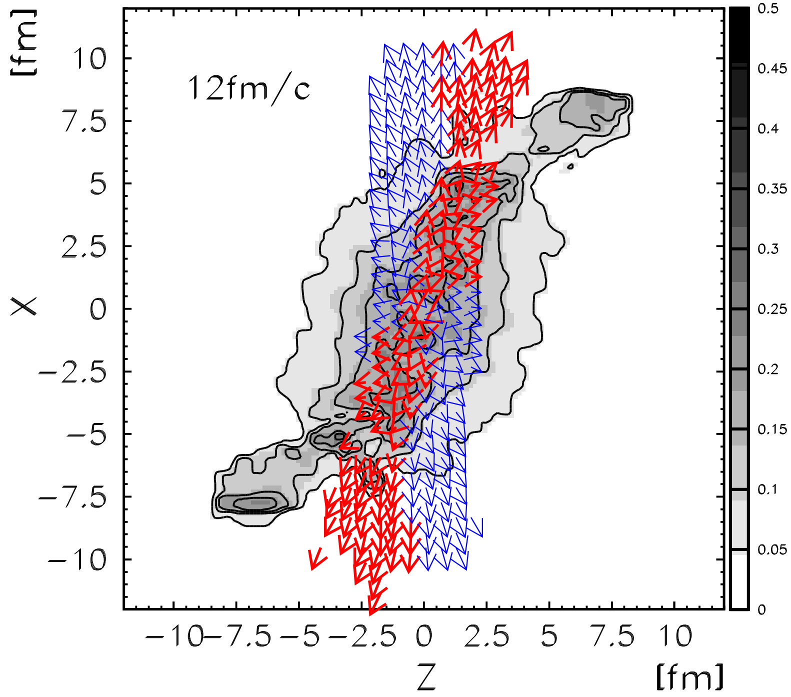
Brachmann et al, NPAA619 (1997) 391

Brachmann, Phys.Rev. C61 (2000) 024909

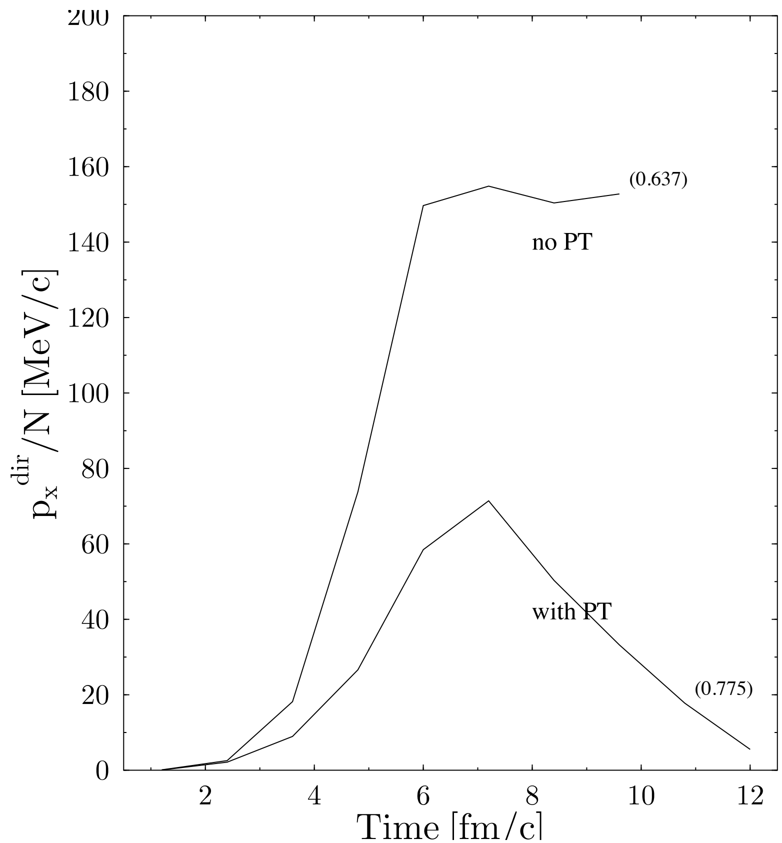
Directed Flow

Definition and interpretation

$$\frac{p_x^{\mathrm{dir}}}{N} \equiv \left(\int \mathrm{d}y \, \frac{\mathrm{d}N}{\mathrm{d}y}\right)^{-1} \int \mathrm{d}y \, \frac{\mathrm{d}N}{\mathrm{d}y} \, \left\langle \frac{p_x}{N}(y) \right\rangle \, \mathrm{sgn}(y)$$

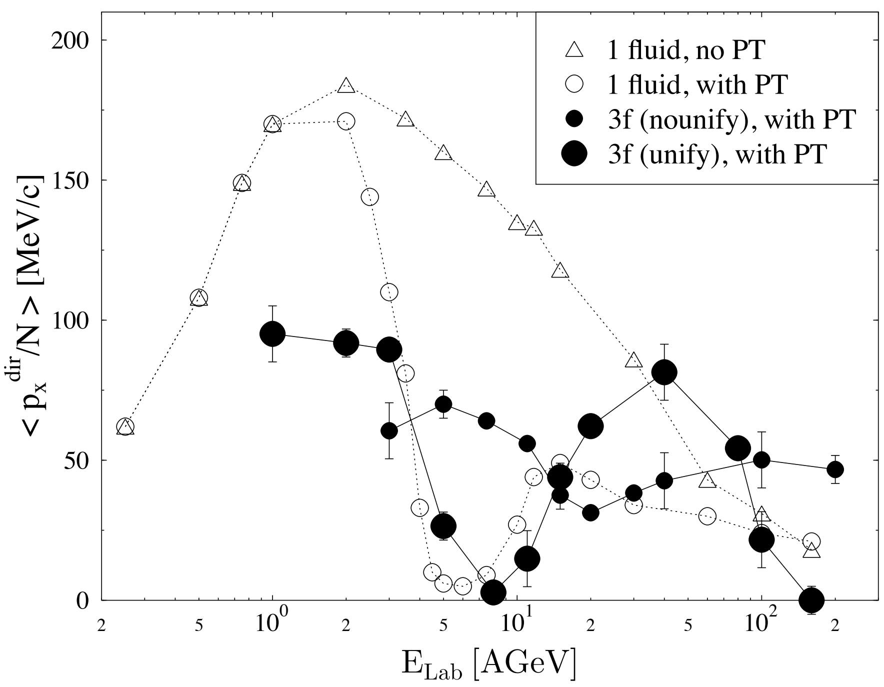


Time dependence



Directed Flow Excitation Function

1-fluid/3-fluid Comparison



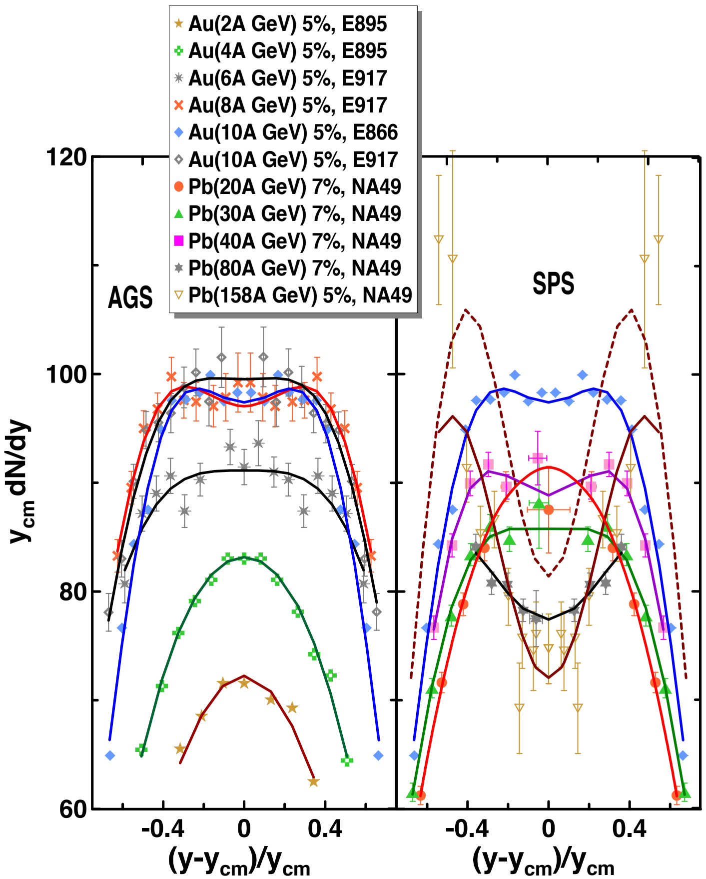
Importance of fluid unification

- Shift of the flow minimum
- Dissapearance of the minumum, if unification of the fluids is not included!
- Minimum (softest point) in reach of NICA

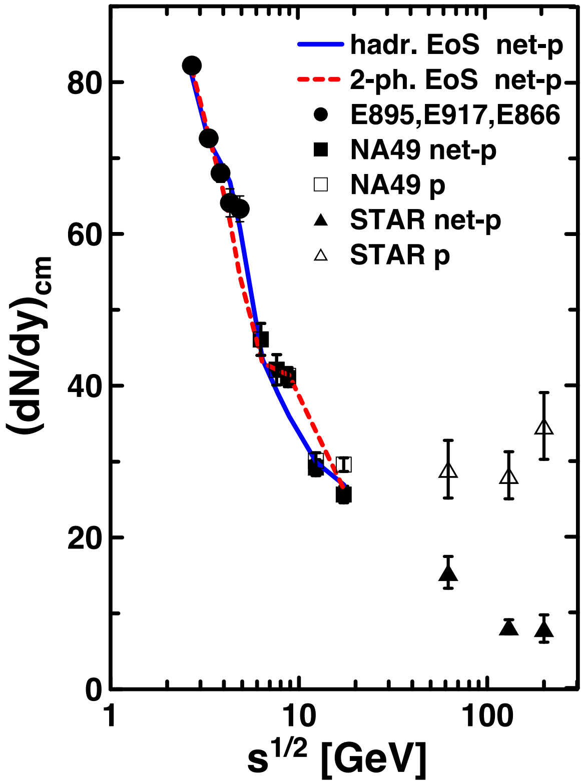
Brachmann, Phys.Rev. C61 (2000) 024909

Stopping and Rapidity Densities

Proton rapidity spectra

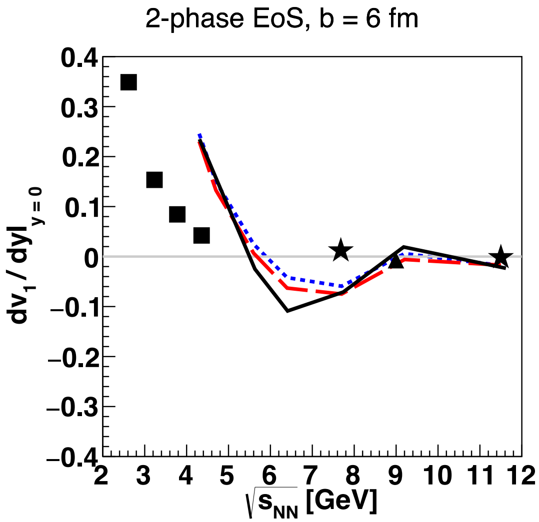


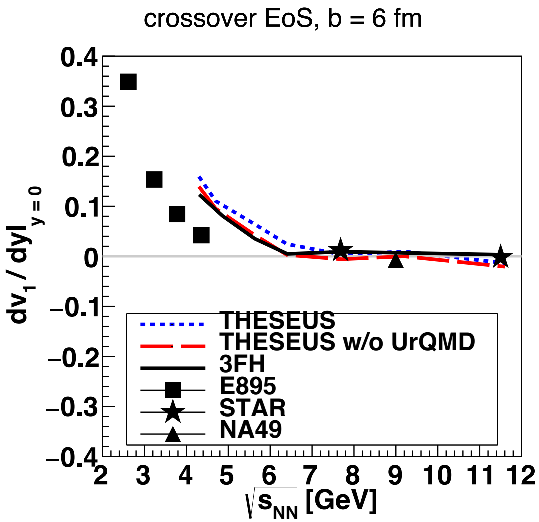
Midrapidity proton yields



Ivanov, Phys.Lett. B690 (2010) 358-362

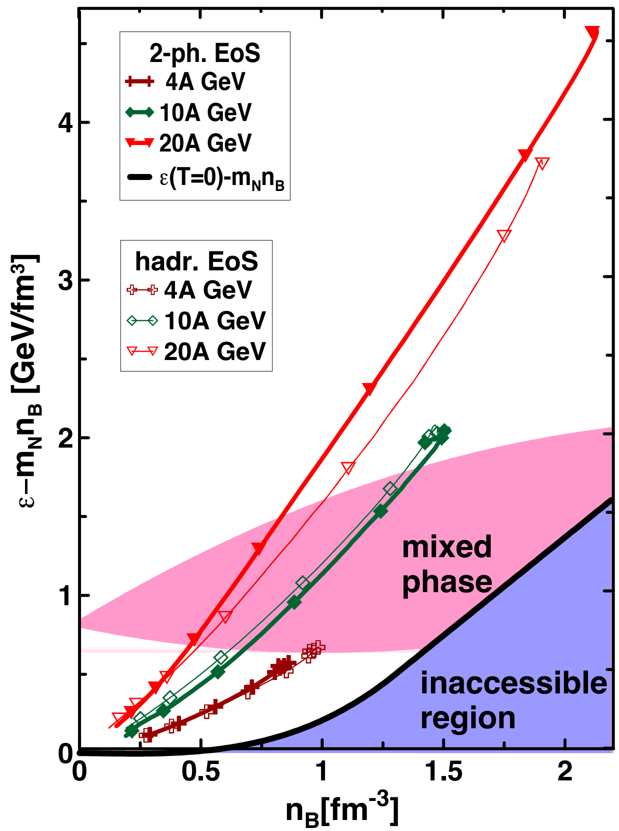
Flow excitation function: Irregularities





P. Batyuk et al, Phys.Rev. C94 (2016) 044917

Coincides with mixed phase



Ivanov, Phys.Lett. B690 (2010) 358-362

HYBRID APPROACHES

UrQMD hybrid model as an example

Hybrid Approaches

Combines relativistic hydrodynamics with relativistic Boltzmann equation.

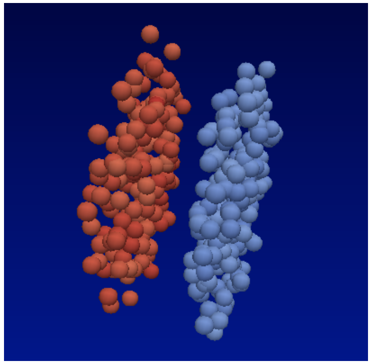
Phase transition proceeds in hydro stage

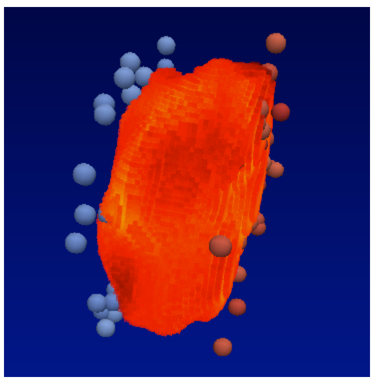
Assumption:
local equilibrium

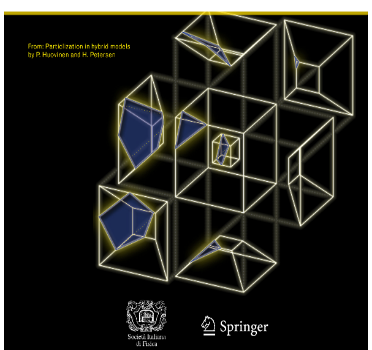
- 2D hydro +UrQMD
  S. Bass, A. Dumitru, M. Bleicher,
  Phys.Rev.C60:021902,1999
- NexSpherio
  F. Grassi, T. Kodama, Y. Hama,
  J.Phys.G31:S1041-S1044,2005
- 3D Hydro+JAM
  T. Hirano, U. Heinz, D. Kharzeev, Y. Nara,
  Phys.Lett.B636:299-304,2006
- 3D Hydro + UrQMD
  C. Nonaka, S.A. Bass,
  Nucl.Phys.A774:873-876,2006
- UrQMD 3.3
  H. Petersen, J. Steinheimer, M. Bleicher,
  Phys. Rev. C 78:044901, 2008
- EPOS+Hydro+UrQMD
  K. Werner, M. Bleicher, T. Pierog,
  Phys.Rev. C82 (2010) 044904
- MUSIC
  B. Schenke, S. Jeon, C. Gale, ...
  Nucl.Phys. A855 (2011) 303-306

H.Petersen, et al, PRC78 (2008) 044901
P. Huovinen, H. P. EPJ A48 (2012) 171

UrQMD hybrid approach







- Initial State:
  - Initialization of two nuclei
  - Non-equilibrium hadron-string dynamics
  - Initial state fluctuations are included naturally

- 3+1d Hydro +EoS:
  - **SHASTA** ideal relativistic fluid dynamics
  - Net baryon density is explicitly propagated
  - Equation of state at finit μ_B

- Final State:
  - Hypersurface at constant energy density
  - Hadronic rescattering and resonance decays within UrQMD

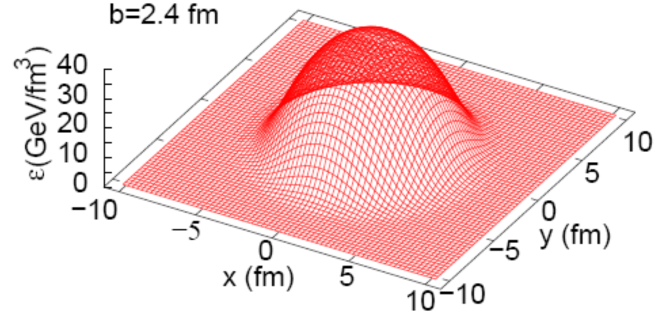
(J.Steinheimer et al., PRC 77,034901,2008)

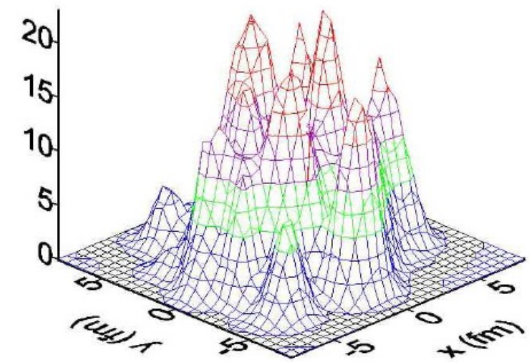
Initial State

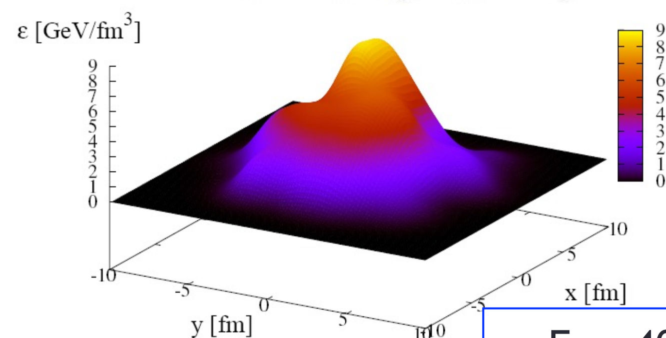
- Contracted nuclei have passed through each other

$$t_{start} = \frac{2R}{\gamma v}$$

  - Energy is deposited
  - Baryon currents have separated
- Energy-, momentum- and baryon number densities are mapped onto the hydro grid
- **Event-by-event fluctuations** are taken into account
- Spectators are propagated separately in the cascade

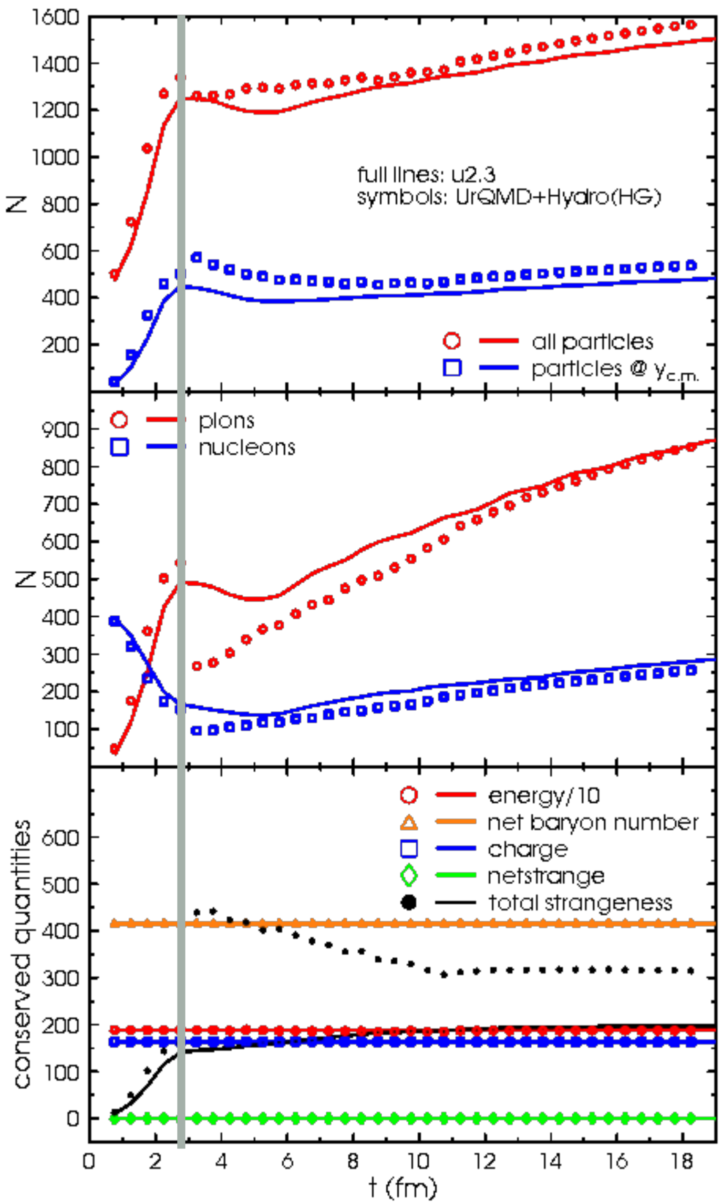






(nucl-th/0607018, nucl-th/0511021)

E_{lab}=40 AGeV
b=0 fm



Time Evolution

Central Pb+Pb collisions at 40A GeV:

- Number of particles decreases in the beginning due to resonance creation
- Qualitative behaviour very similar in both calculations

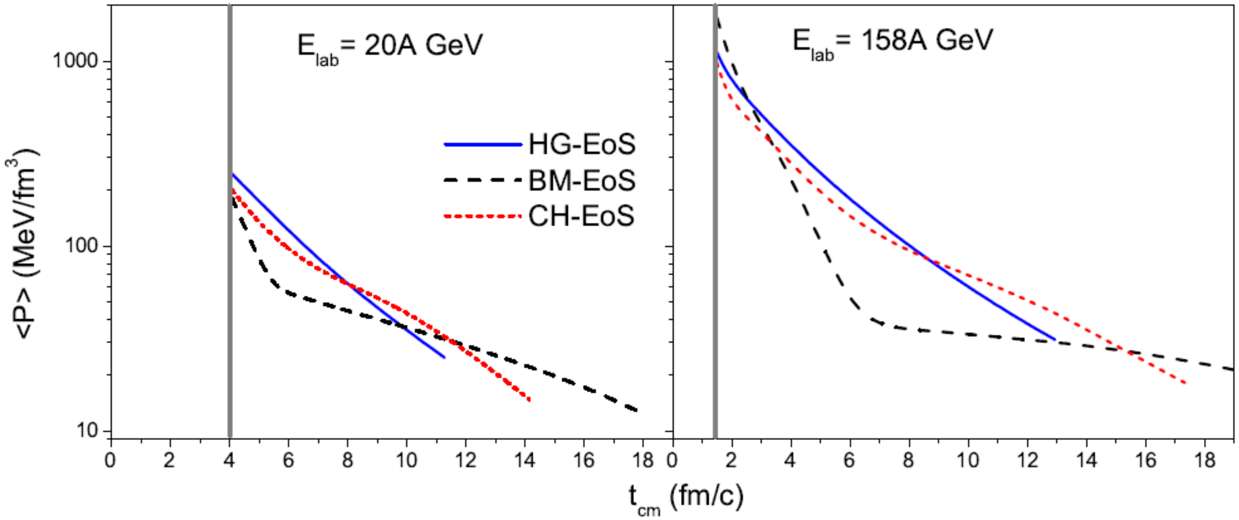
→ UrQMD **equilibrates** to a rather large degree

Equations of State

Ideal relativistic one fluid dynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu (n u^\mu) = 0$$

- HG: **Hadron gas** including the same degrees of freedom as in UrQMD (all hadrons with masses up to 2.2 GeV)
- CH: **Chiral EoS** from quark-meson model with first order transition and critical endpoint
- BM: **Bag Model EoS** with a strong first order phase transition between QGP and hadronic phase



D. Rischke et al., NPA 595, 346, 1995,

D. Zschiesche et al., PLB 547, 7, 2002

Papazoglou et al., PRC 59, 411, 1999

J. Steinheimer, et al., JPG 38 (2011) 035001

Hadronization, Particlization, Decoupling

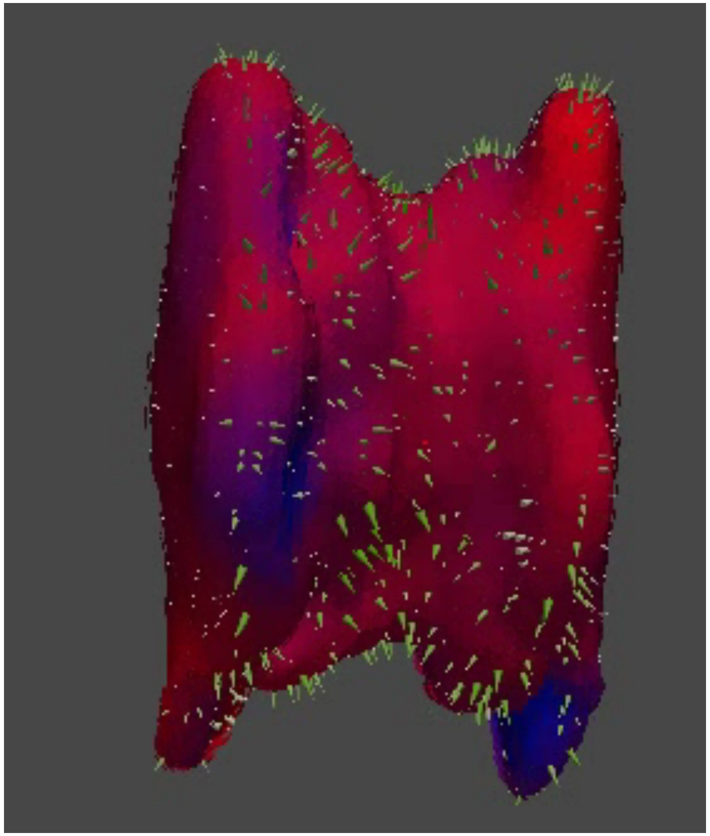
Particlization

- Experiment: **finite number** of hadrons in detectors (conservation laws!)
- **Hadronization** controlled by the equation of state
- Sampling of particles →**Cooper-Frye** equation:

$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)p^\mu d\sigma_\mu$$

- →E,**p**,t,**x** on hypersurface

Sophisticated event-by-event 3D hypersurface

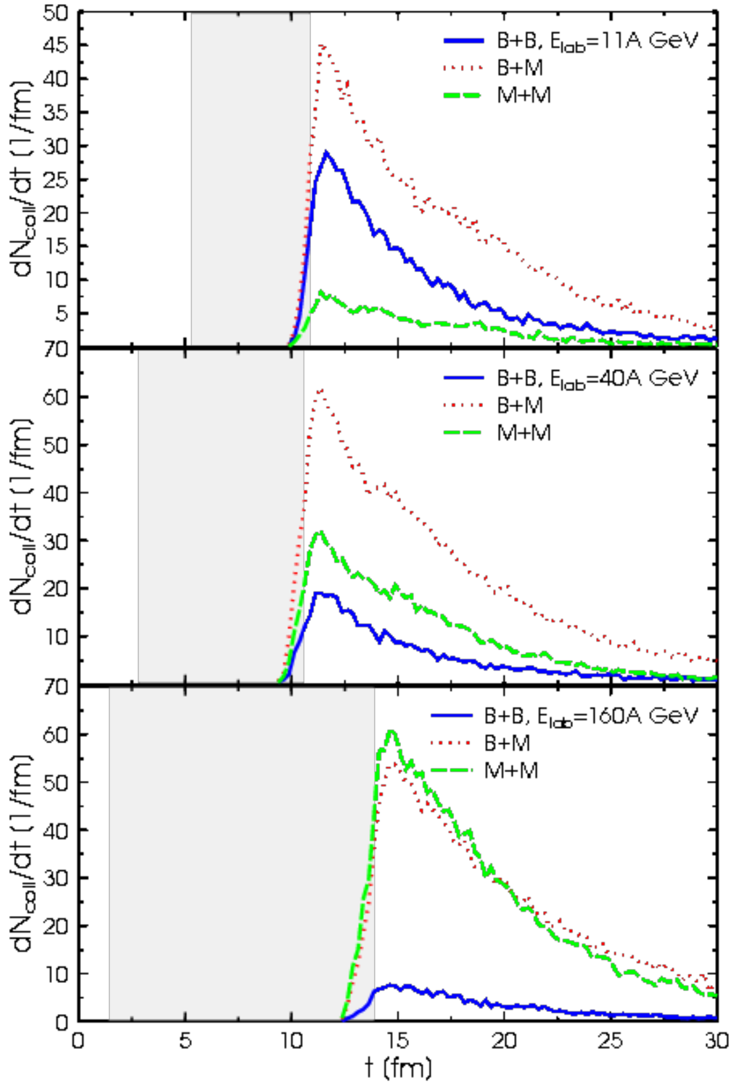


Petersen, Huovinen, arXiv:1206.3371

Decoupling stage

Collisions and decays

- Final propagation via Relativistic Boltzmann equation: $\left(p^{\mu}\partial_{\mu}\right)f = I_{coll}$
- Substantial amount of final state interactions
- Decoupling duration is on the order of 5 fm/c (central Au+Au/Pb+Pb)



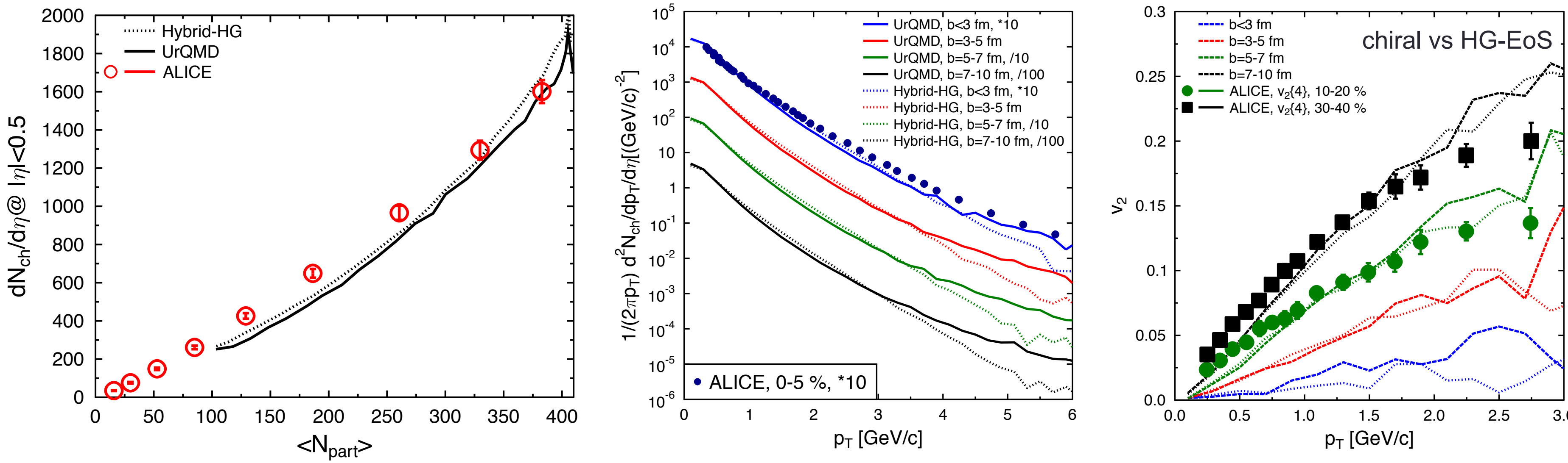
3 minute break

- Discuss with your neighbor

problems at the interfaces between different stages

HYBRID APPROACHES

Results



Hybrid model at LHC
(Pb+Pb, 2.76 TeV)

Excellent description of
- centrality dependence
- transverse momenta
- elliptic flow.

H. Petersen, Phys.Rev. C84 (2011) 034912

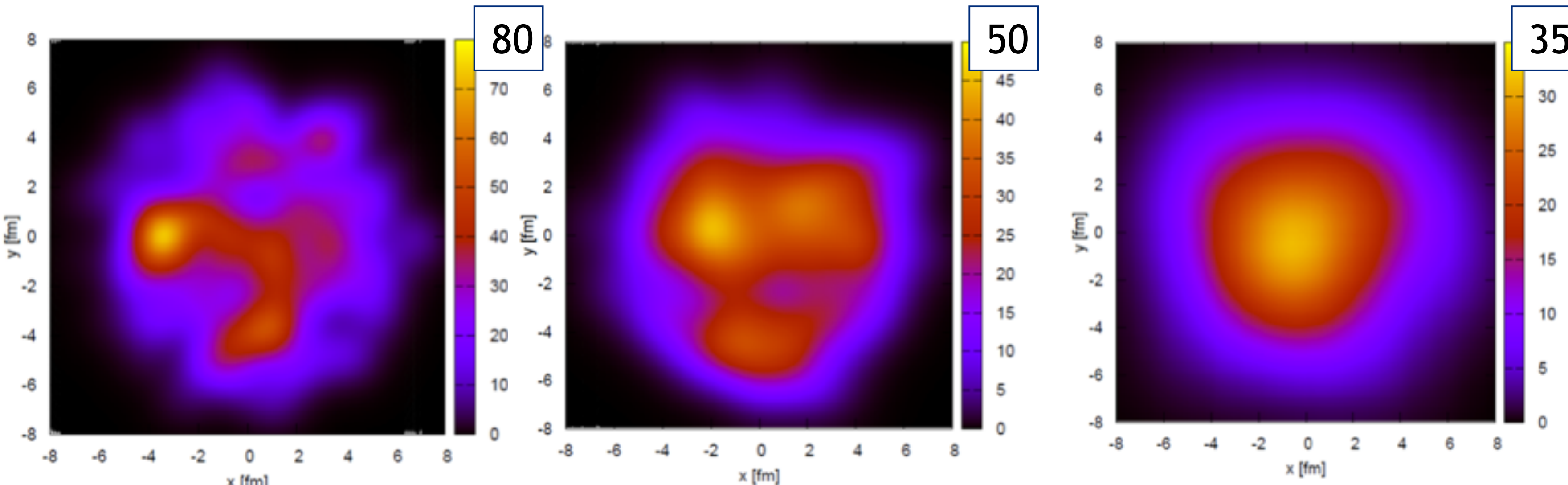
Initial State

From H. Petersen

- Energy-, momentum- and baryon number densities are mapped onto the hydro grid using for each particle

$$\epsilon(x,y,z) = \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \frac{\gamma_z}{\sigma^3} E_p \exp - \frac{(x-x_p)^2 + (y-y_p)^2 + (\gamma_z(z-z_p))^2}{2\sigma^2}$$

- Changing σ leads to different granularities, but also changes in the overall profile

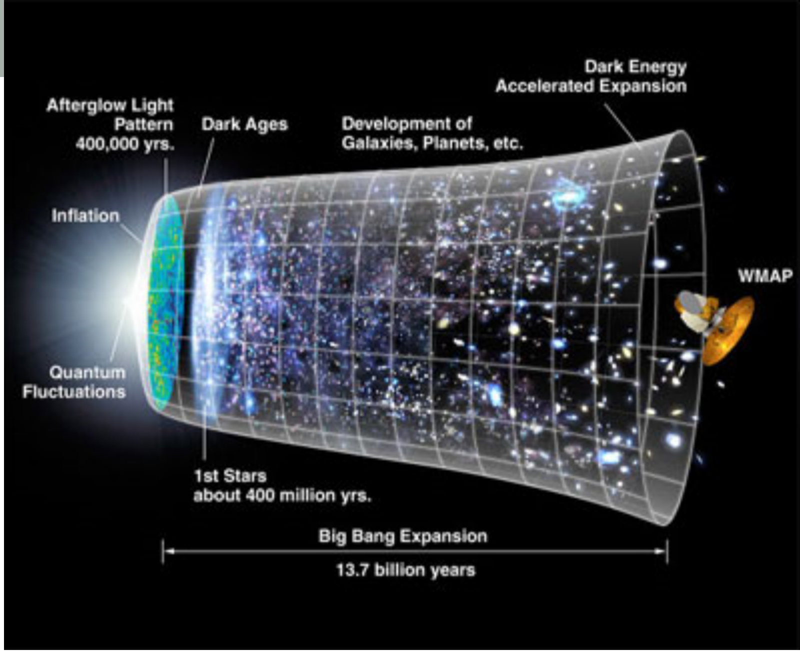


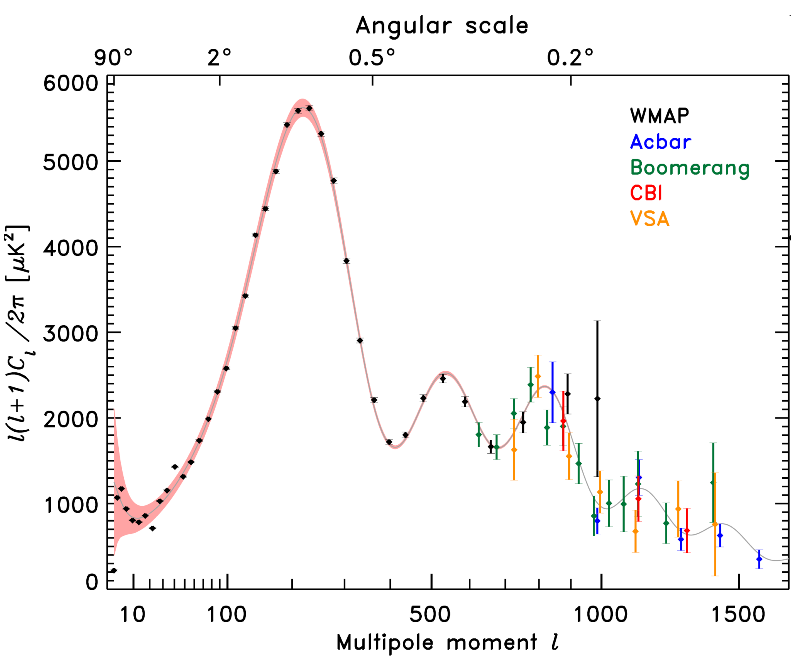
σ = 0.8 fm | σ = 1.0 fm | σ = 2.0 fm

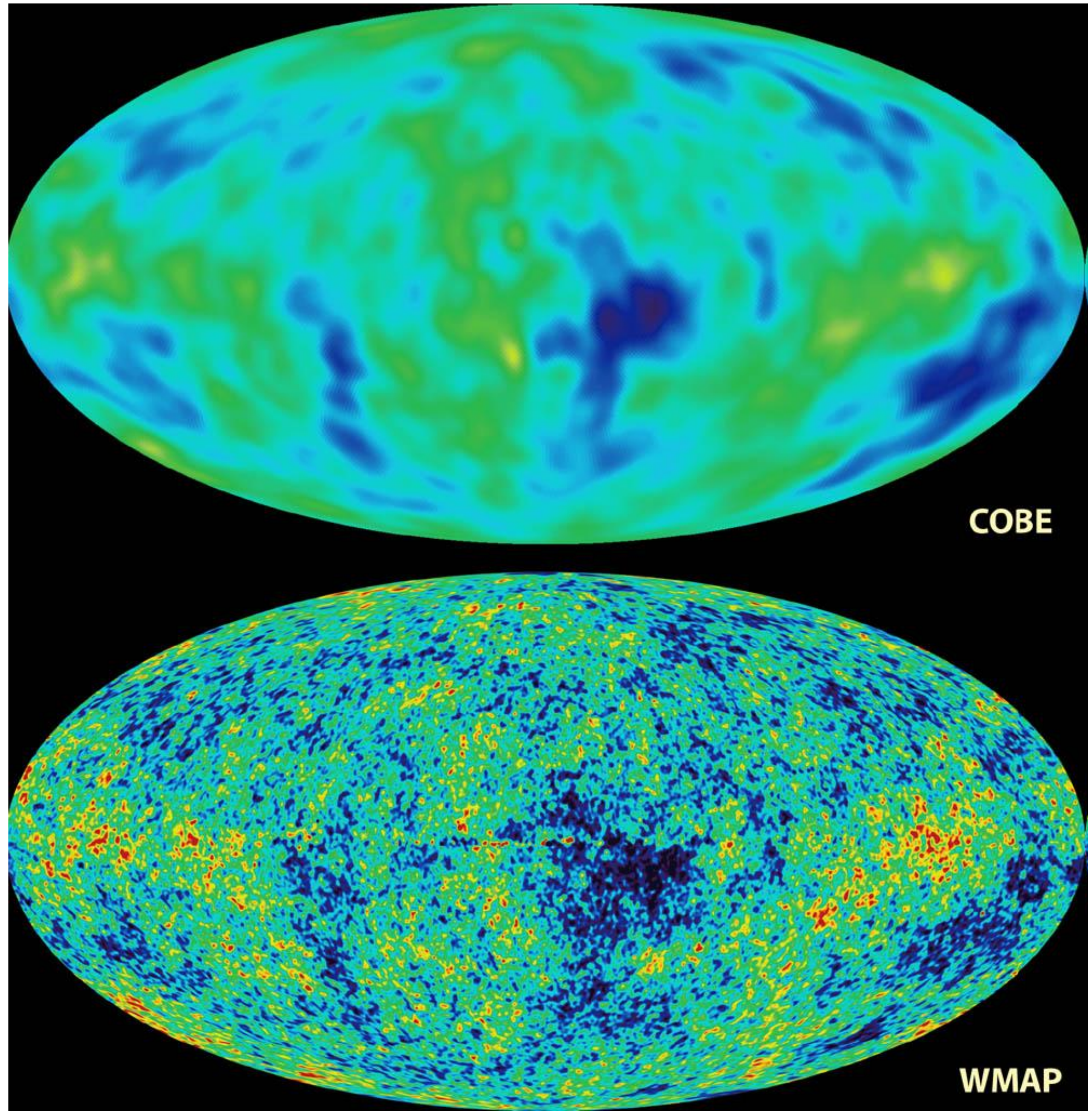
H.P. et al, PRC 83, 014916, 2011 and JPG 38, 045102, 2011

- How does changing the starting time affect the picture?

Idea: Angular correlation



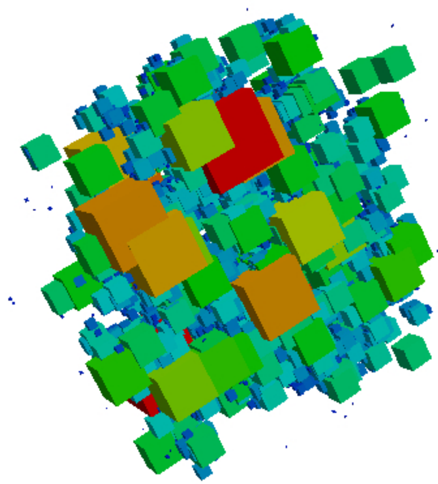




From H. Petersen

Sources of Fluctuations

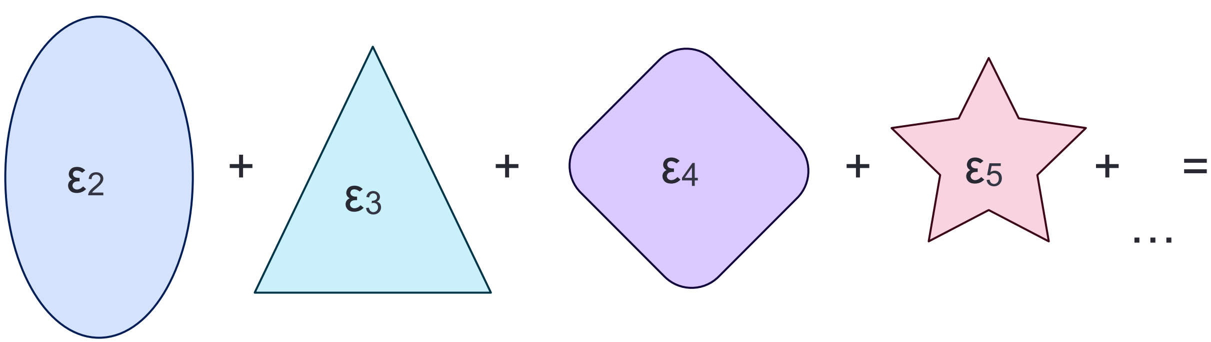
- Granularity is driven by
  - position of nucleons
  - distribution of collisions
  - type of interaction
  - degree of thermalization

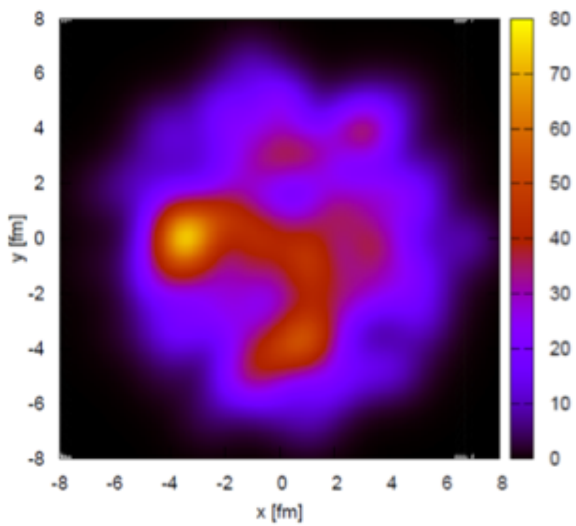


UrQMD @ 200 AGeV

- How to quantify the fluctuating shape of the initial state?

→ Fourier-expansion in position space



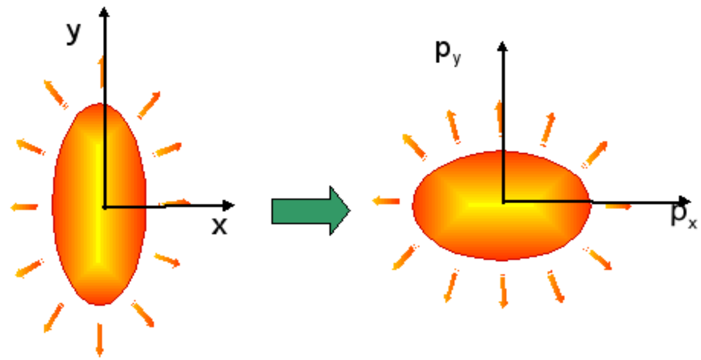


From H. Petersen

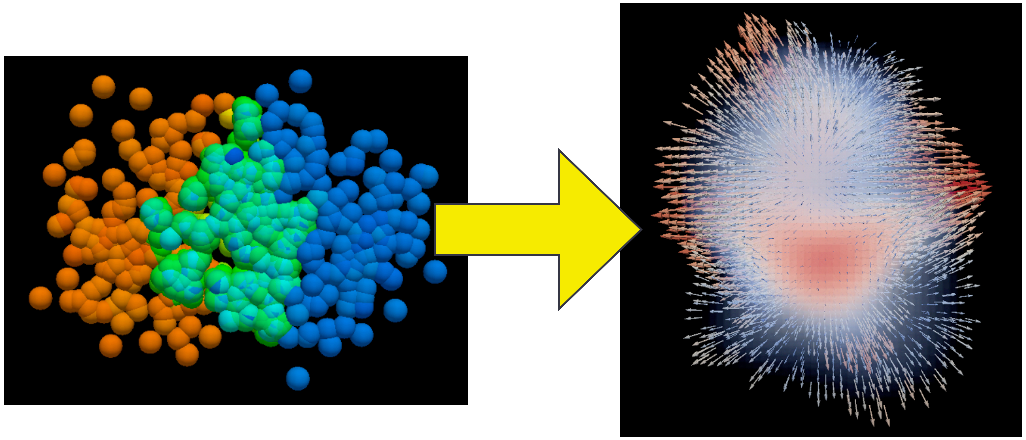
Anisotropic Flow – Higher Order Fourier Coefficients

Simplified picture:

Position-space anisotropy
→ Momentum-space anisotropy



Real picture:
Complicated state, mean free paths,…



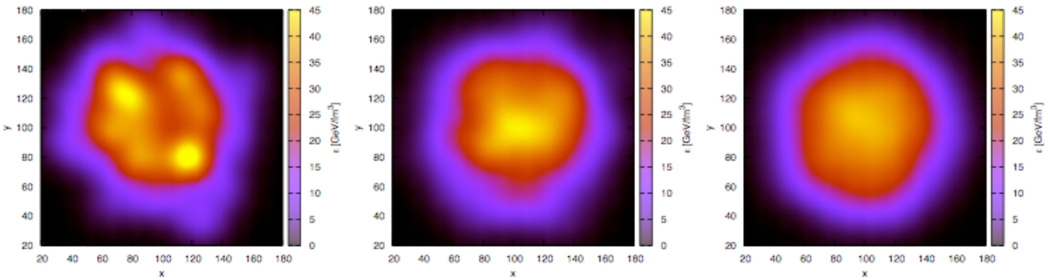
by MADAI.us

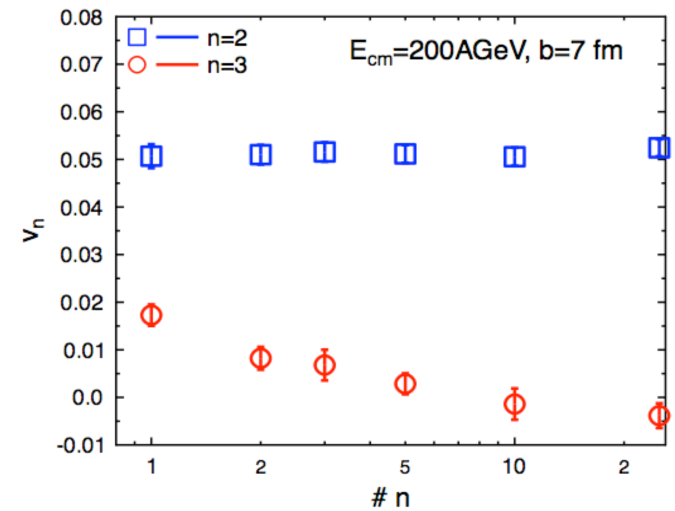
Use these coefficients to learn about the initial state

From H. Petersen

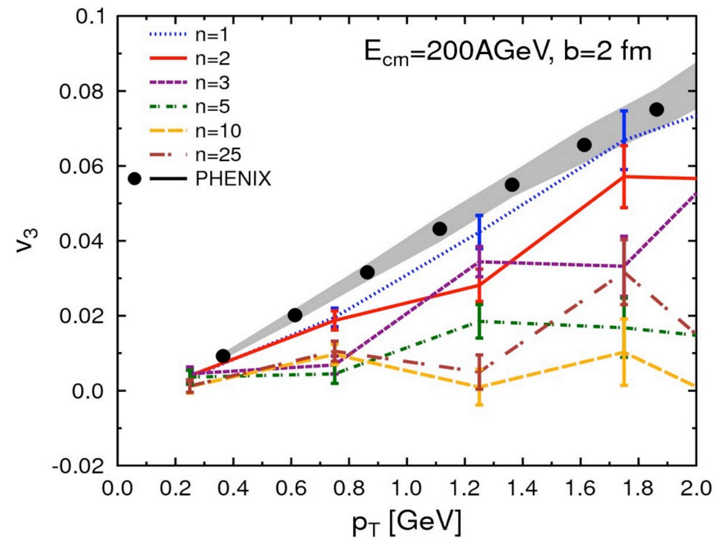
Constraining Granularity

H.P. et al, J.Phys.G G39 (2012) 055102

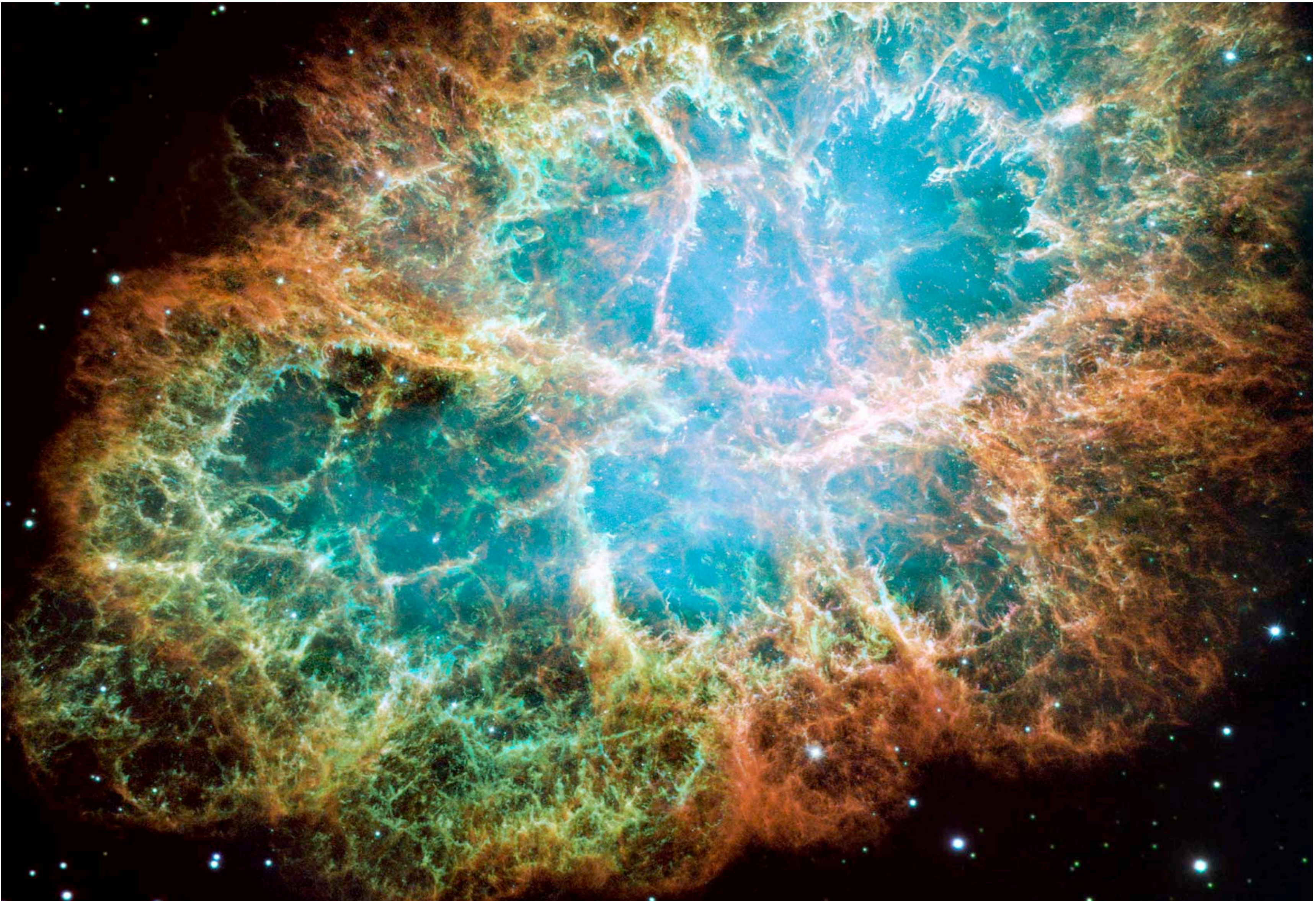




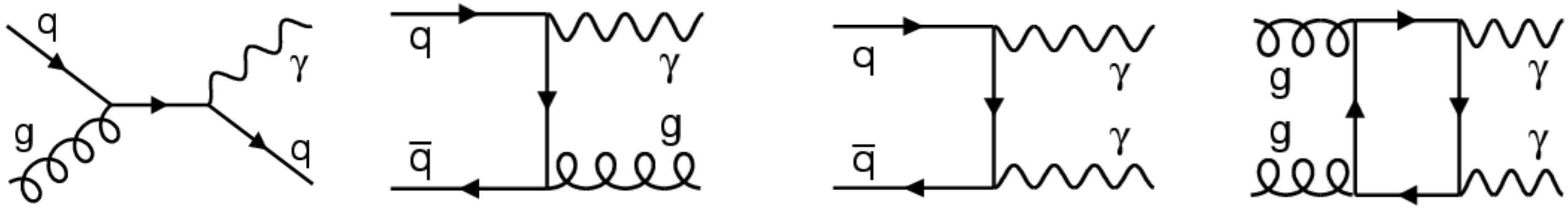
- Triangular flow is **very sensitive** to amount of initial state fluctuations
- It is important to have final state particle distributions to apply **same analysis** as in experiment
- Single-event initial condition provides best agreement with PHENIX data
- Does that imply that the initial state is well-described by binary nucleon interactions +PYTHIA?
- Lower bound for fluctuations!



Use Photons to Learn More



Photons: Direct Messengers from the QGP



→ from QGP: sensitivity to parton density and temperature

→ from initial state: sensitivity to PDFs (gluon!)

→ Compare to hadronic channels, i.e. $\pi+\rho \rightarrow \gamma+\pi$, …

Cross section Refs

[1] E.g. Aurenche, Fontannaz *et. al*, PRD **73**, 094007 (2006)

[2] Turbide, Rapp and Gale, PRC **69**, 014903 (2004); Turbide, Gale *et al.*, PRC **72**, 014906 (2005); Liu and Werner, arXiv:0712.3612 [hep-ph]; Vitev and Zhang, arXiv:0804.3805 [hep-ph]; Haglin, PRC **50**, 1688 (1994); Haglin, JPG **30**, L27 (2004), Chatterjee *et al.*, Nucl. Phys. A **830** (2009) 503C

[3] Dumitru, Bleicher, Bass, Spieles, Neise, Stöcker and Greiner, PRC **57**, 3271 (1998); Huovinen, Belkacem, Ellis and Kapusta, PRC **66**, 014903 (2002); Li, Brown, Gale and Ko, arXiv:nucl-th/9712048; Bratkovskaya and Cassing, NPA **619**, 413 (1997); Bratkovskaya, Kiselev and Sharkov, arXiv:0806.3465 [nucl.th]

Photon Rates: Hadronic and Partonic

- Hadronic rate parametrization:

$$E\frac{dR}{d^3p} = A\exp\left(\frac{B}{(2ET)^C} - D\frac{E}{T}\right)$$

S. Turbide, R. Rapp, C. Gale, Phys. Rev. C69 (2004) 014903

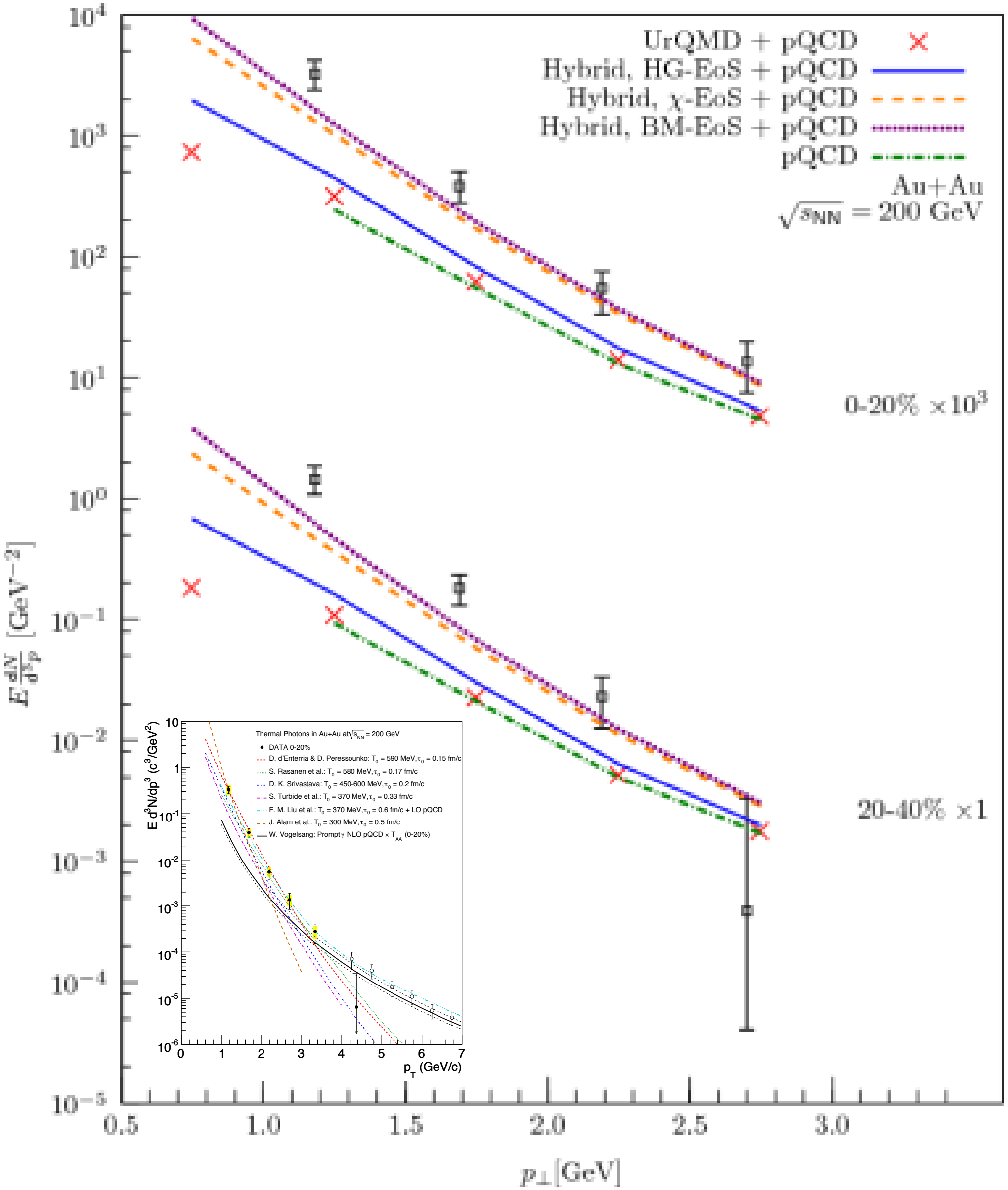
- QGP rate:

$$E\frac{dR}{d^3p} = \sum_{i=1}^{N_f} q_i^2 \frac{\alpha_{\rm em}\alpha_{\rm S}}{2\pi^2} T^2 \frac{1}{e^x+1}\left(\ln\left(\frac{\sqrt{3}}{g}\right) + \frac{1}{2}\ln(2x) + C_{22}(x) + C_{\rm brems}(x) + C_{\rm ann}(x)\right)$$

P. Arnold, G. Moore, L. Yaffe, JHEP 0112 (2001)009

Insert all rates into the hybrid model and compare to data.

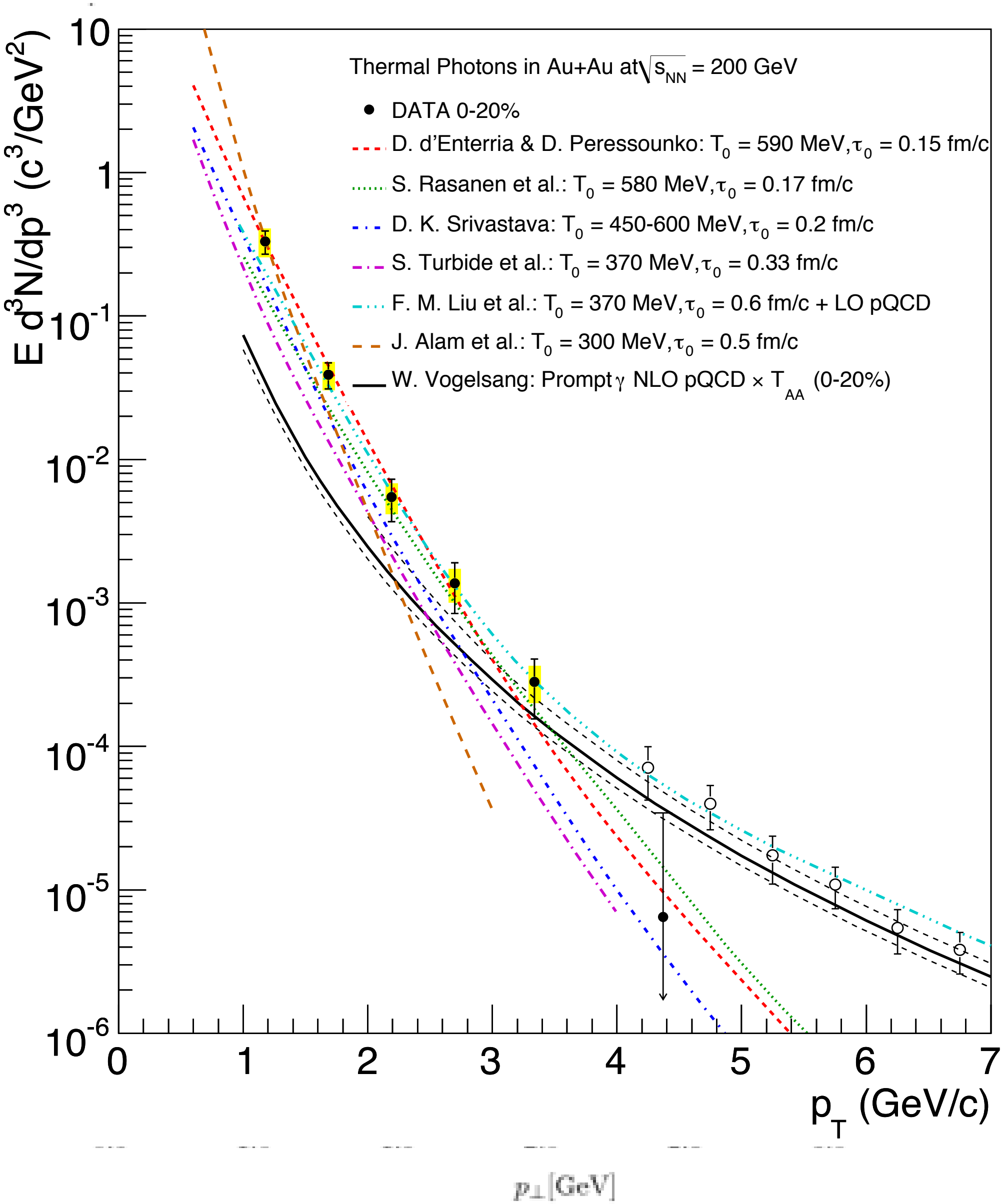
Temperature and dof: Photons



- Clear separation hadronic vs. partonic
- partonic calc. fit data
- Reasons for missing contributions in UrQMD/Hadron gas:
  - late equilibration,
  - hadronic rates,
  - shorter life time

Data points from:
PHENIX, PRC 81 (2010) 034911
fig: Bäuchle, MB, PRC 82 (2010) 064901

Temperature and dof: Photons



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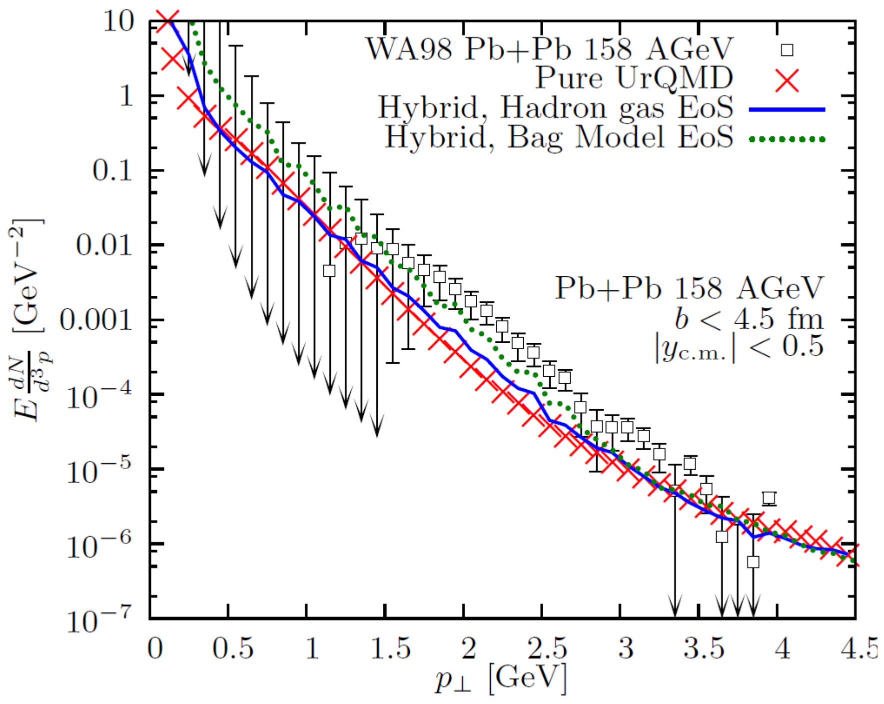
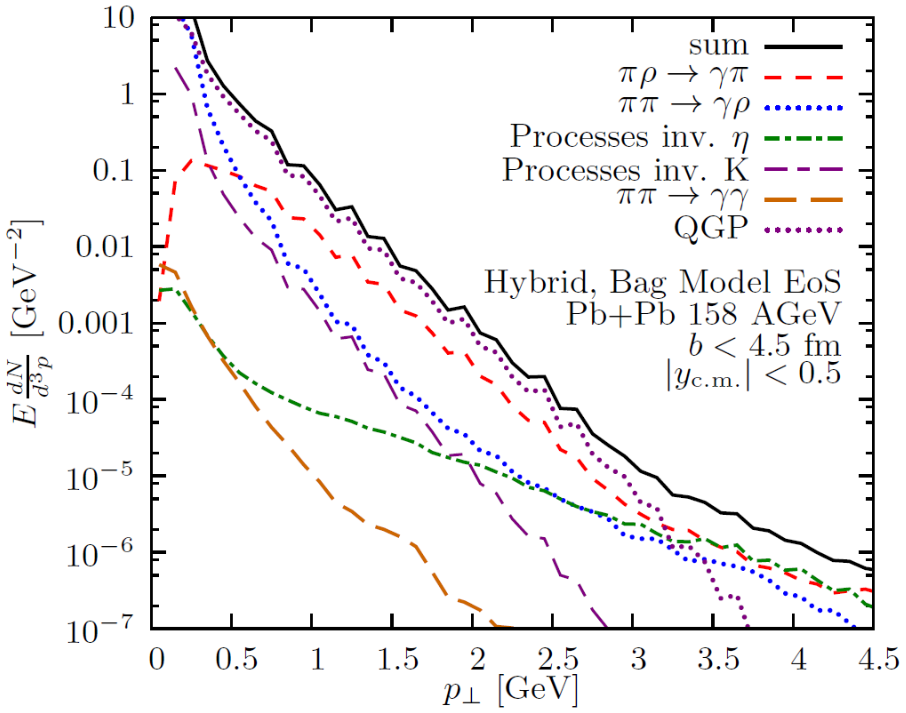
  - hadronic rates,
  - shorter life time

Data points from:
PHENIX, PRC 81 (2010) 034911
fig: Bäuchle, MB, PRC 82 (2010) 064901

Is there QGP?



Comparisons



Hybrid, QGP: Channels

Bauechle, Bleicher, Phys.Rev. C81 (2010) 044904

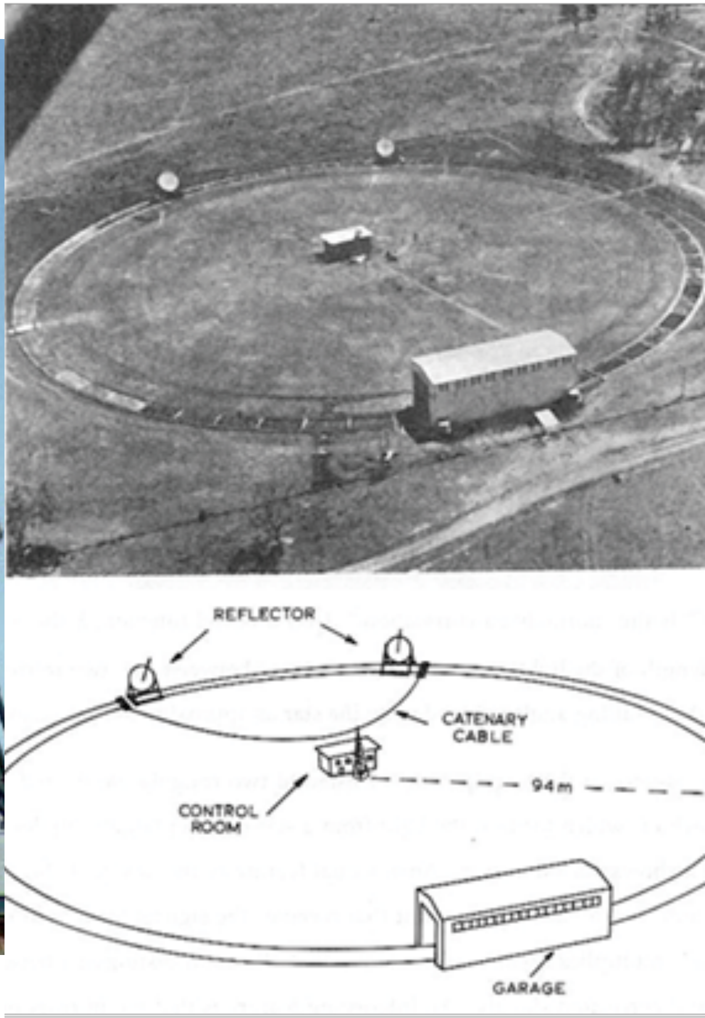
3 minute break

- Discuss with your neighbor

how to measure the life time of the fireball

HBT Correlations

Hanbury-Brown-Twiss Correlations

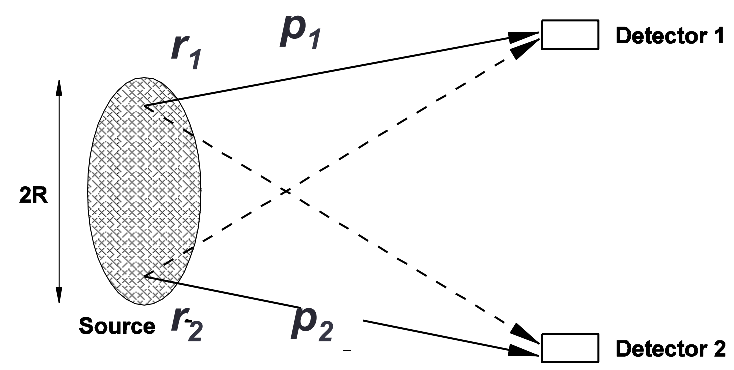


1. Aerial photo and illustration of the original HBT
ıey have been extracted from Ref.[1].

From M. Lisa

HBT correlations: Idea

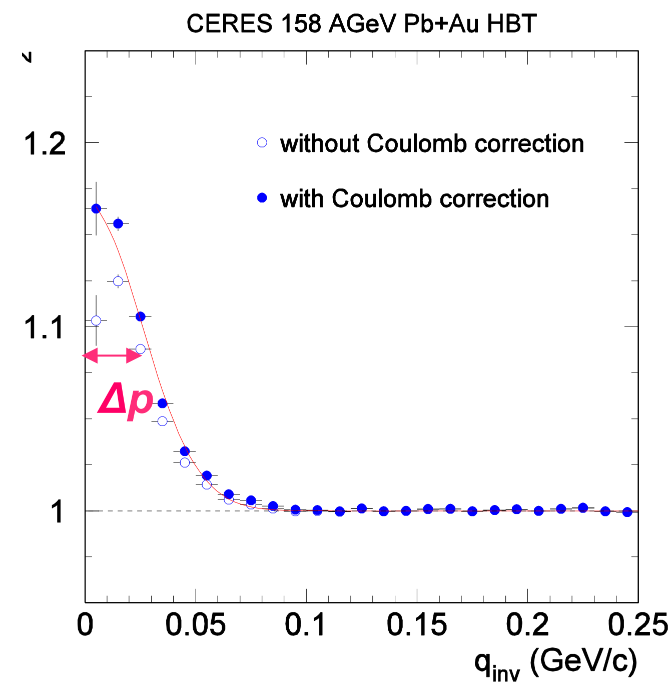
(R. Hanbury-Brown, R.Q. Twiss, 1956)



Bose-Einstein-statistics leads to short range correlations of bosons in momentum

$$C_2(\vec{p}_1,\vec{p}_2) = \frac{P_2(\vec{p}_1,\vec{p}_2)}{P_1(\vec{p}_1)\cdot P_1(\vec{p}_2)}$$
$$= 1 + \chi(\vec{p}_2 - \vec{p}_1)$$

χ allows to obtain information on the emission source (Imaging, Gauss-Source)



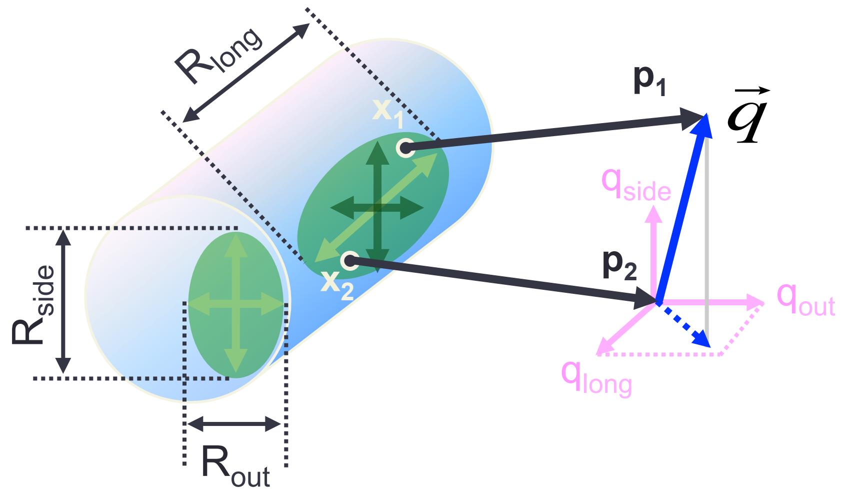
In heavy ion collisions: Pions, Kaons, ...

$$\Delta r = \frac{\hbar c}{\Delta p} = \frac{197\,\mathrm{MeV/c}}{\Delta p}\,\mathrm{fm}$$

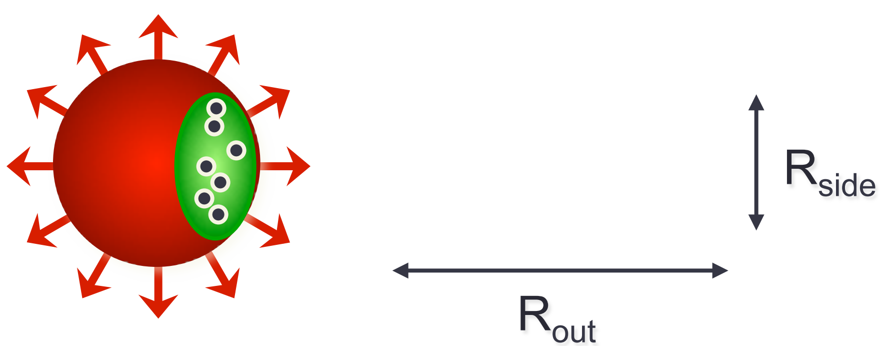
Meaning of Components

From M. Lisa

- Two particle interferometry: Image and emission duration



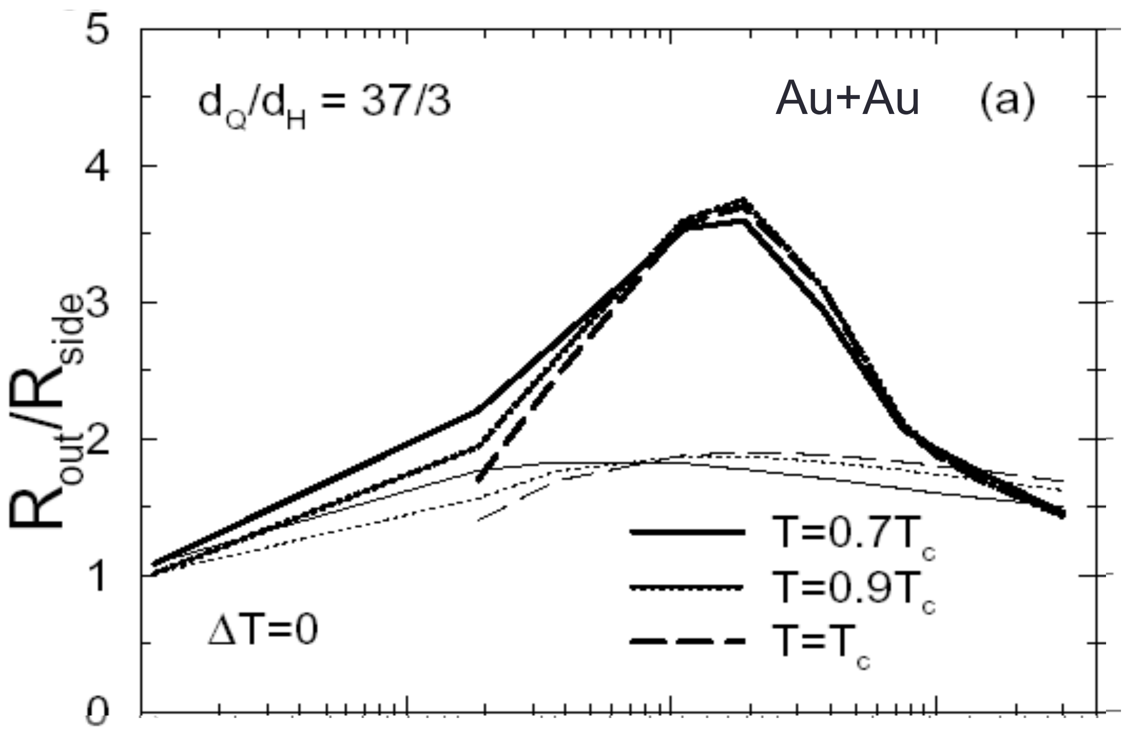
R_{out}/R_{side}-ratio measures emission time of the system



Pratt-Bertsch (“out-side-long”) coordinanates allow to obtain space and time information

1-fluid Hydro Prediction

- Mixed phase should lead to drastic increase in life time, visible in R_o/R_s ratio

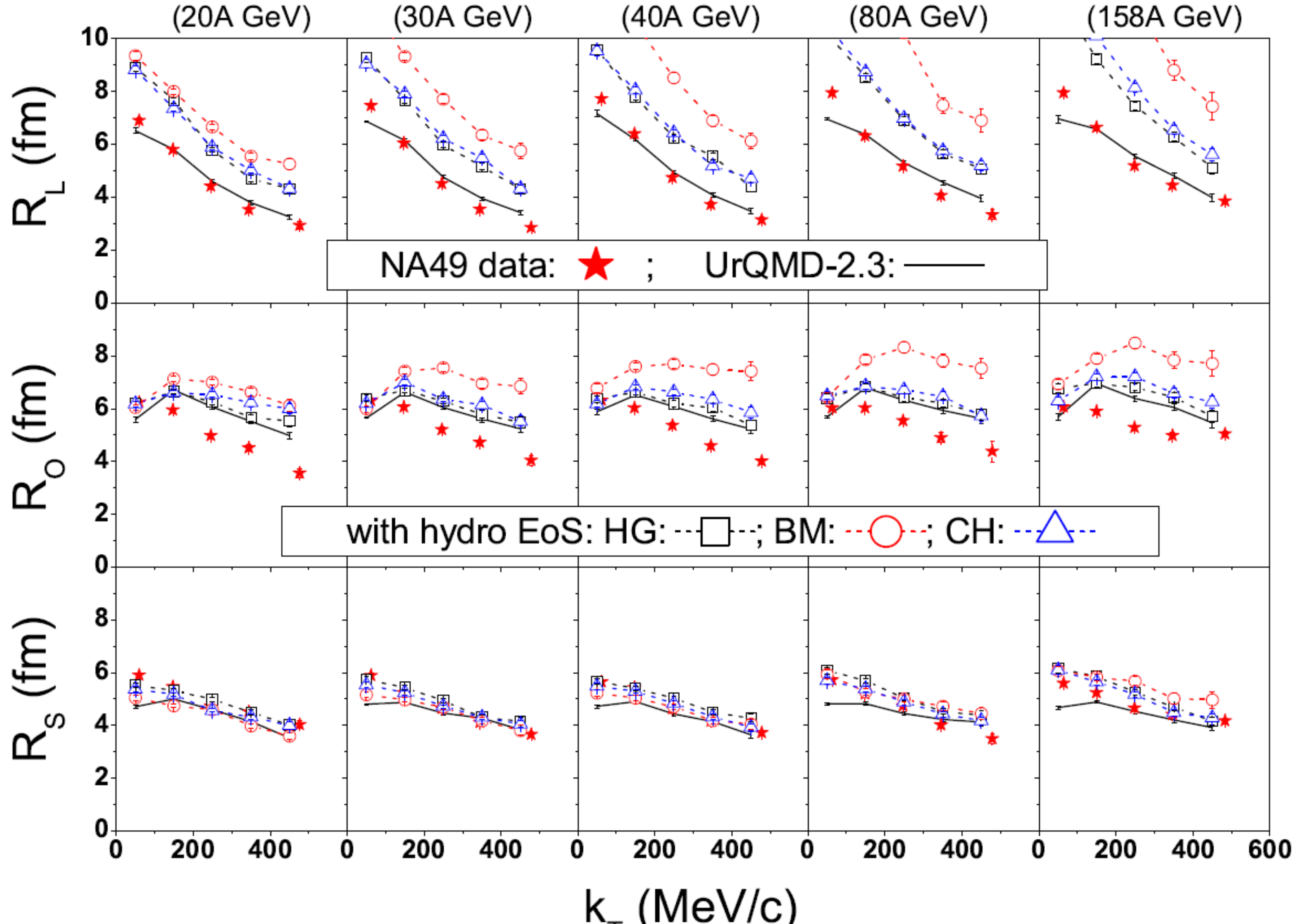


~ beam energy

- 10 times increased life time
- Factor 2-4 increased R_{out}/R_{side} ratio

From: Rischke, Gyulassy, Nucl.Phys.A608:479-512,1996

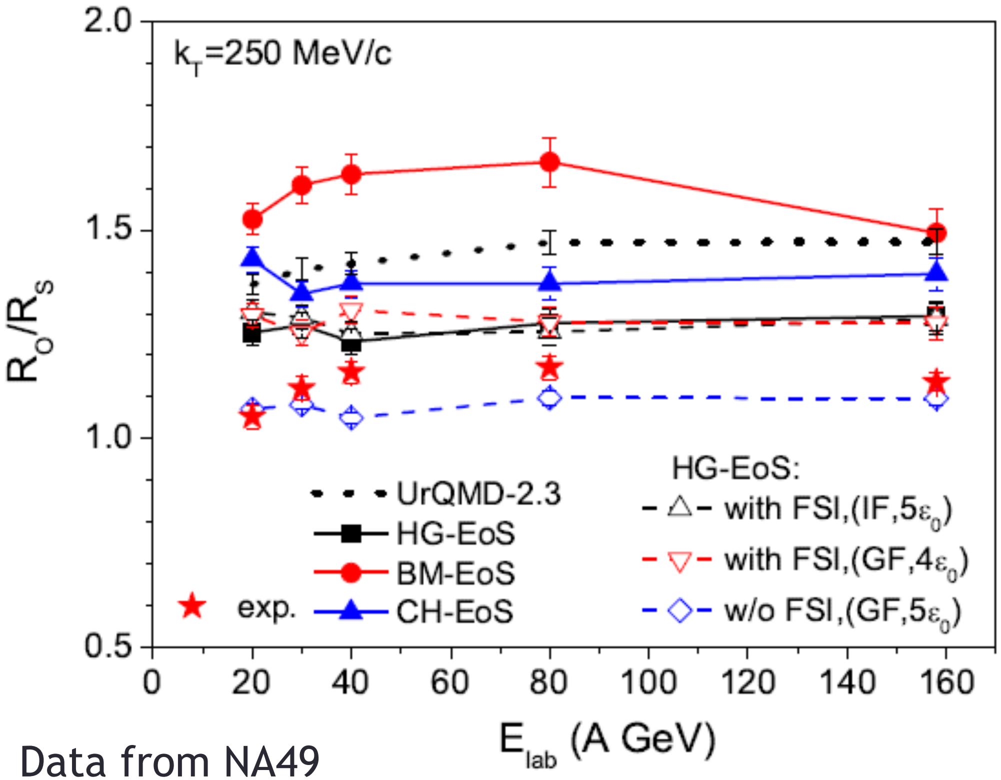
HBT radii → Lifetime



Q. Li et al., Phys.Lett. B674 (2009) 111-116

Hydro evolution leads to larger radii, esp. with phase transition

R_O/R_S Ratio



Data from NA49

- Hydro phase leads to smaller ratios
- Hydro to transport transition does not matter, if final **rescattering** is taken into account
- **EoS dependence** is visible, but not as strong as previuosly predicted (factor of 5)

Q. Li et al., Phys.Lett. B674 (2009) 111-116

Summary: Hydro and Hybrid Models

- Hybrid approaches have become the „Standard Model“ for Heavy Ion collisions
- Can not be used at low energies, because initial/intermediate state separation is not well defined

- Angular correlations constrain initial state
- Photon yields support the existence of QGP
- HBT correlations may indicate increased life times

3 minute break

- Discuss with your neighbor

Is there a (simple) alternative to hydro/hybrid transport to explore in-medium effects/phase transitions?

COARSE GRAINING

Results

Hadronic models

1. **Kinetic theory**
   - Realized in **transport models** (here UrQMD)
   - Effective solution of the **Boltzmann equation**
   - Physics input and parameters: cross-sections (total and partial), resonance parameters, string fragmentation scheme
   - **"On-shell" quasi-particles** on classical trajectories
   - Collision term includes **elastic & inelastic scatterings** (e.g. $\pi\pi \to \rho$) and **resonance decays** (e.g. $\mathrm{N}^* \to N + \pi$)

   $\hookrightarrow$ But: Incoherent summation over processes, missing off-shell dynamics, restricted to lower densities (no multi-particle interactions) $\to$ **Medium effects only partially implemented**
2. **Hadronic many-body theory**
   - Calculate particle **self-energies** using **quantum field theory**
   - **Coherent summation**: Accounts for quantum interference

   $\hookrightarrow$ But: Restricted to equilibrated matter, assumes heat bath

$\to$ **Two sides of the same medal!**

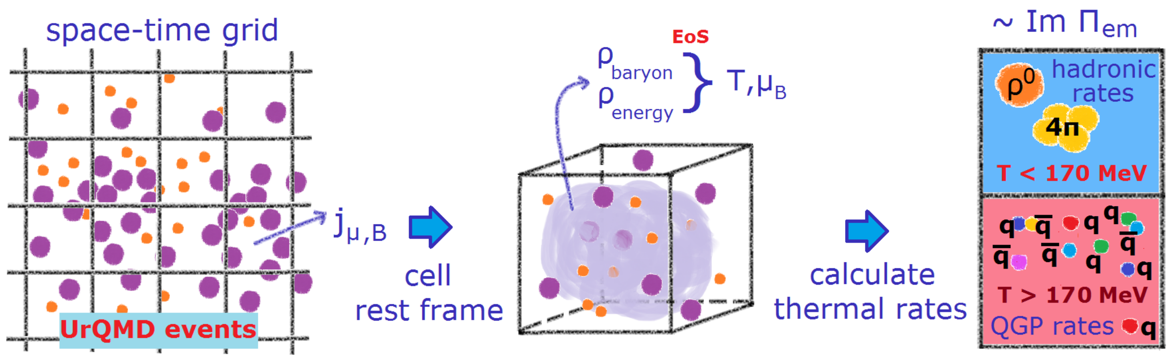
Coarse graining

- Goal: One approach for all energies, realistic evolution of the reaction, but limited number of variables
  - $\hookrightarrow$ Combining a realistic 3+1 dimensional microscopic expansion of the system with macroscopic description of the dilepton emission
- **Coarse-graining = Reduction of information** $\rightarrow$ System uniquely determined by (local) energy and particle densities
- Microscopic description $\rightarrow$ Necessary to average over many simulation events
- Sufficiently large number of events $\rightarrow$ **Distribution function** $f(\vec{x}, \vec{p}, t)$ **takes a smooth form**

$$f(\vec{x}, \vec{p}, t) = \left\langle \sum_h \delta^3(\vec{x} - \vec{x}_h(t))\delta^3(\vec{p} - \vec{p}_h(t)) \right\rangle$$

- UrQMD model constitutes a non-equilibrium approach
  - $\hookrightarrow$ Equilibrium quantities have to be extracted locally at each space-time point

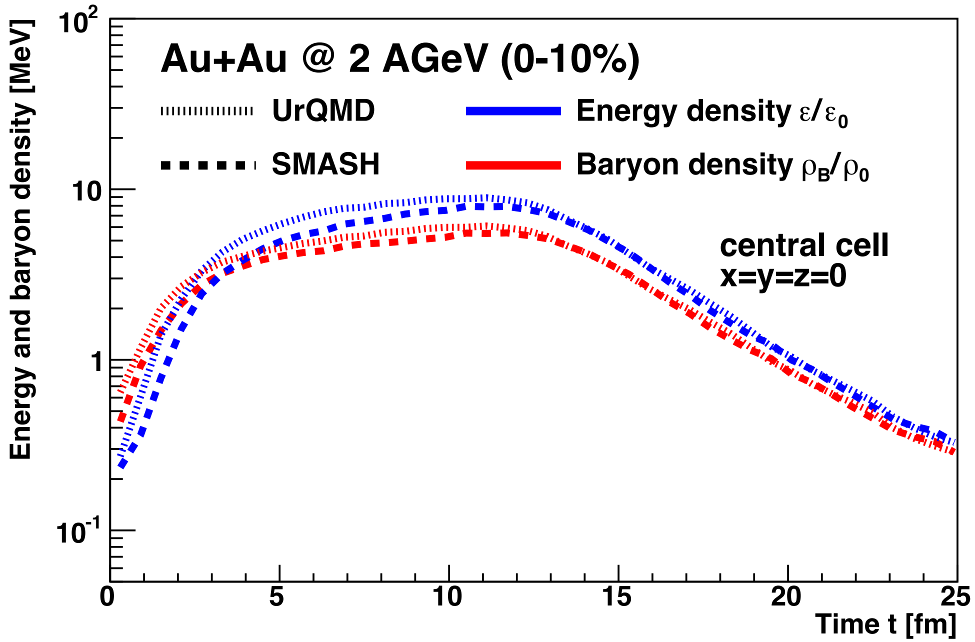
Coarse graining



- First proposed by Huovinen et al. [Phys. Rev. C66, 014903 (2002)]
- Put ensemble of UrQMD events on **grid of space-time cells**
- Determine baryon and energy density and use Eckart's definition to determine the **rest frame** properties
  $\rightarrow$ Use **equation of state** to calculate T and μ_B
- Two EoS: **Free hadron gas** with UrQMD-like degrees of freedom + **Lattice EoS** for $T > 170$ MeV

[D. Zschiesche et al., Phys. Lett. B547, 7 (2002); M. He et al., Phys. Rev. C 85 (2012)]

Advantages



- **Robustness of the evolution** $\rightarrow$ Microscopic details differ, but evolution of energy and particle densities similar
- **Medium effects straightforward** in terms of T and μ_{B} $\leftrightarrow$ *But*: Assumption of local equilibrium necessary

Accounting for non-equilibrium

→ To which extent is equilibrium obtained in the dynamics?

→ How can one deal with deviations from equilibrium?

- **Macroscopic descriptions** → *Equilibrium* usually introduced as ad-hoc assumption
- **Transport models** → *Non-equilibrium* normal case at any stage
- Two aspects have to be taken into account:
  1. Kinetic non-equilibrium → momentum-space anisotropies
  2. Chemical non-equilibrium → overdense pionic system → finite pion chemical potential μ_π

⇒ Calculate "effective" energy density and determine μ_π in Boltzmann approximation

Di-lepton emission

- Emission is calculated for each cell of 4-dim. grid
- Electromagnetic emission is **related to the imaginary part of the retarded current-current correlator** $\Pi_{\mathrm{em}}^{(\mathrm{ret})}$ as

[R. Rapp, J. Wambach, Adv. Nucl. Phys. 25, 1 (2000)]

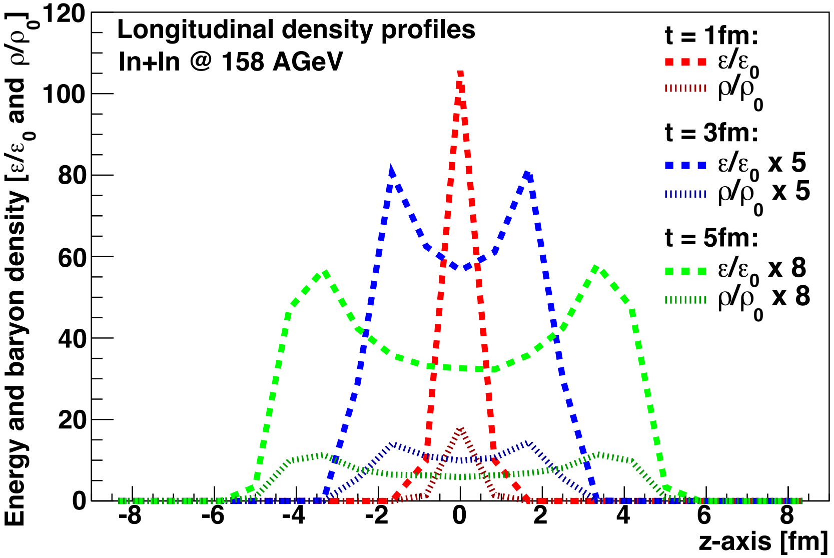
$$\frac{\mathrm{d}N_{ll}}{\mathrm{d}^4x\mathrm{d}^4q} = -\frac{\alpha_{\mathrm{em}}^2 L(M)}{\pi^3 M^2} f_{\mathrm{B}}(q;T) \times \mathrm{Im}\,\Pi_{\mathrm{em}}^{(\mathrm{ret})}(M,\vec{q};\mu_{\mathrm{B}},T),$$

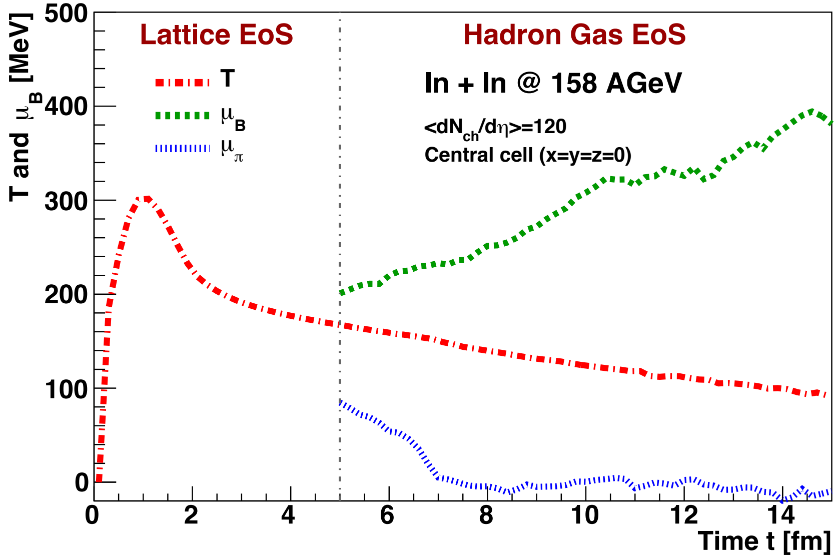
$$q_0\frac{\mathrm{d}N_{\gamma}}{\mathrm{d}^4x\mathrm{d}^3q} = -\frac{\alpha_{\mathrm{em}}}{\pi^2} f_{\mathrm{B}}(q;T) \times \mathrm{Im}\,\Pi_{\mathrm{em}}^{T,(\mathrm{ret})}(q_0=|\vec{q}|;\mu_{\mathrm{B}},T).$$

- Include ρ and ω spectral functions from HMBT (Rapp et al.), meson gas contributions and lattice rates for the QGP
- Non-thermal dilepton contributions (π, η, ϕ) directly from UrQMD + freeze-out ρ and ω (if $T < 50\,\mathrm{MeV}$)

$\hookrightarrow$ For more details about the CG-approach see PRC 91, 054911 (2015); PRC 92, 014911 (2015) and PRC 93, 054901 (2016)

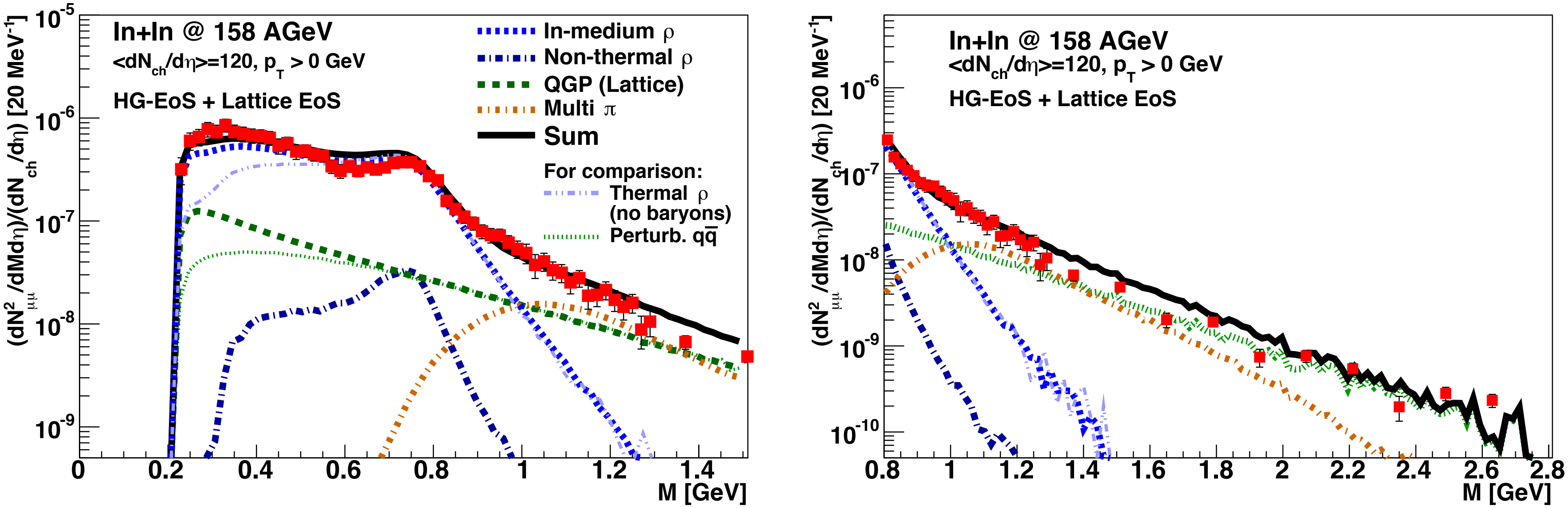
Baseline comparison at SPS





- The coarse-graining of UrQMD input gives **realistic and nuanced picture** of the collision evolution $\rightarrow$ Detailed space-time description of temperature and chemical potential
- At SPS one reaches temperatures significantly above T_c in combination with moderate values of μ_B
- <u>Note</u>: Right plot shows maxima of T and μ (central cell), not average $\rightarrow$ Different values for each space-time cell!

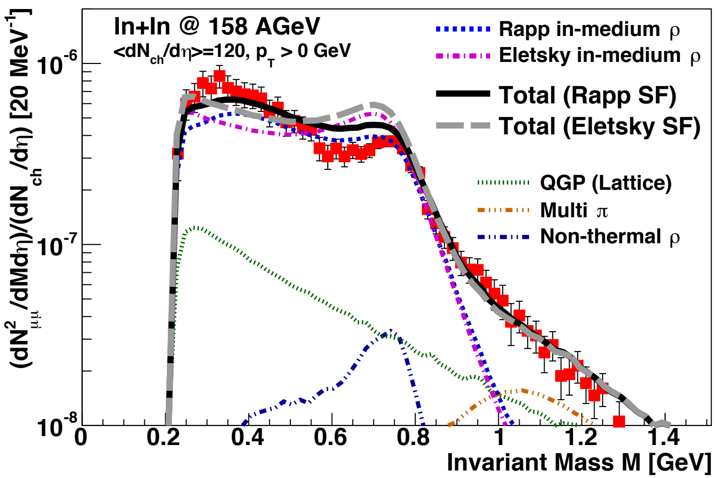
NA60 – di-lepton spectra

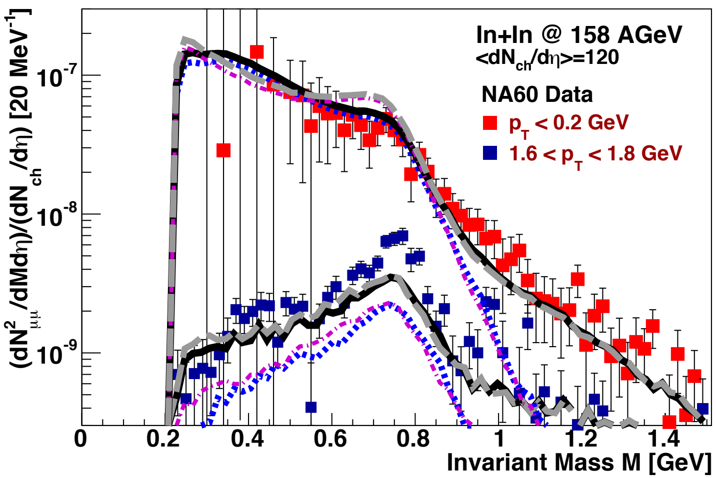


- ρ shows **broadening compared to case without baryons**
- QGP and multi-pion annihilation are the relevant sources in the intermediate mass region
- For M > 1.5 GeV$/c^2$ QGP contribution clearly dominates
- **Duality** between hadronic and partonic emission rates?

↪ Results agree with fireball + hydro calculations; differences in dynamics

Comparison of different spectral functions

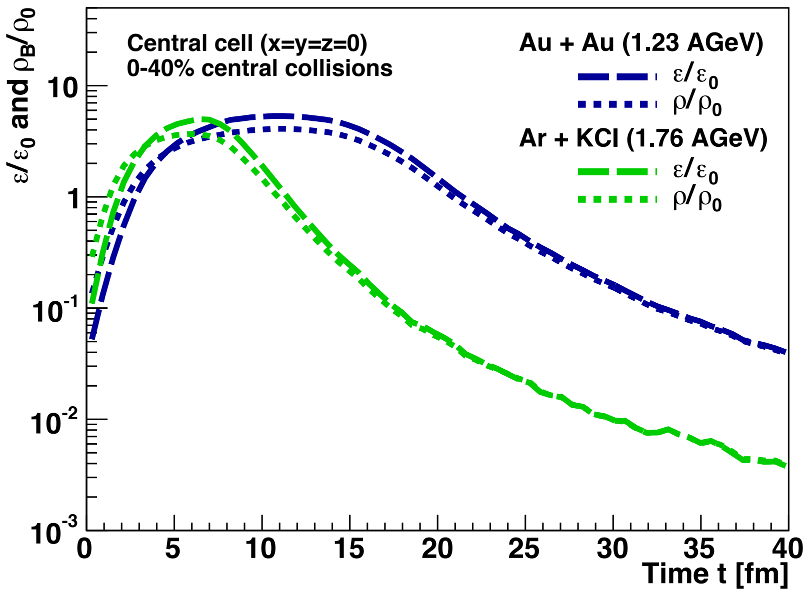


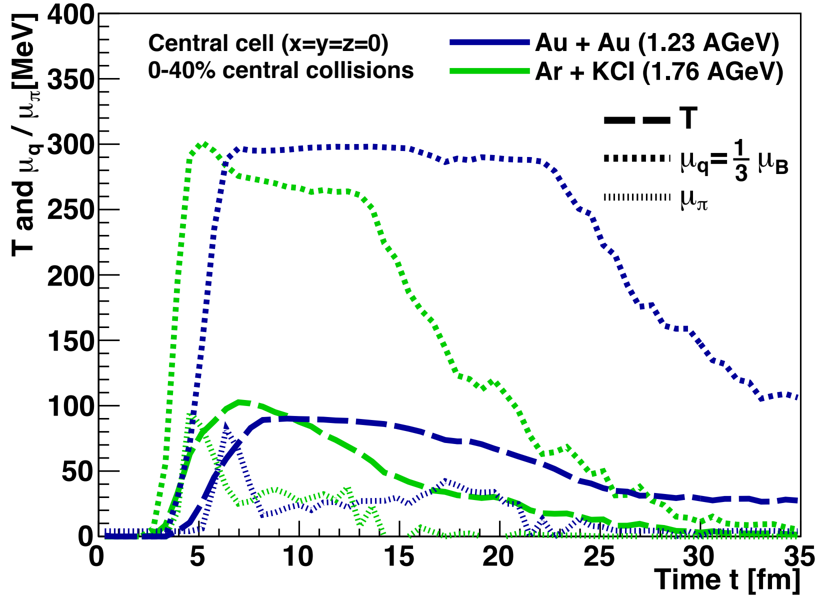


- HMBT results $\leftrightarrow$ ρ spectral function obtained using **empirical scattering amplitudes from resonance dominance**

  [V. L. Eletsky et al., Phys. Rev. C64, 035303 (2001)]
- Not enough broadening due to low-density expansion of the self energies $\rightarrow$ Overshoots data at peak, underestimates for lower masses

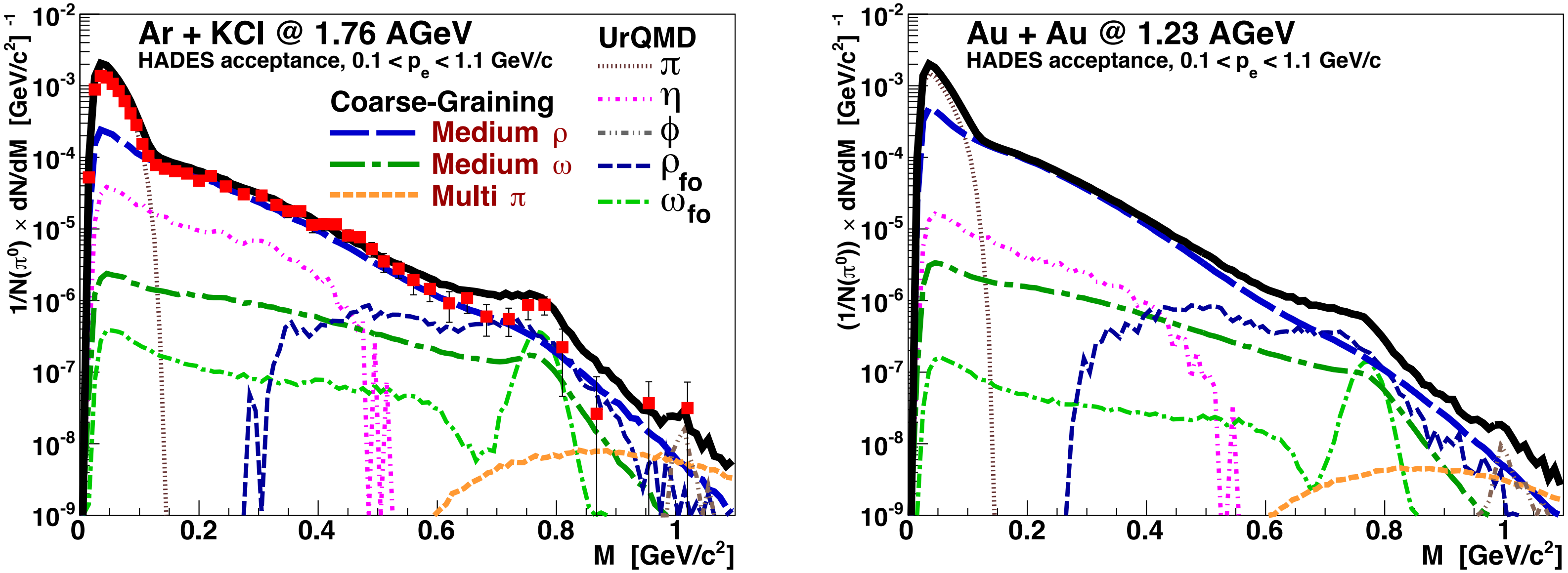
SIS 18, low T, high μ_B





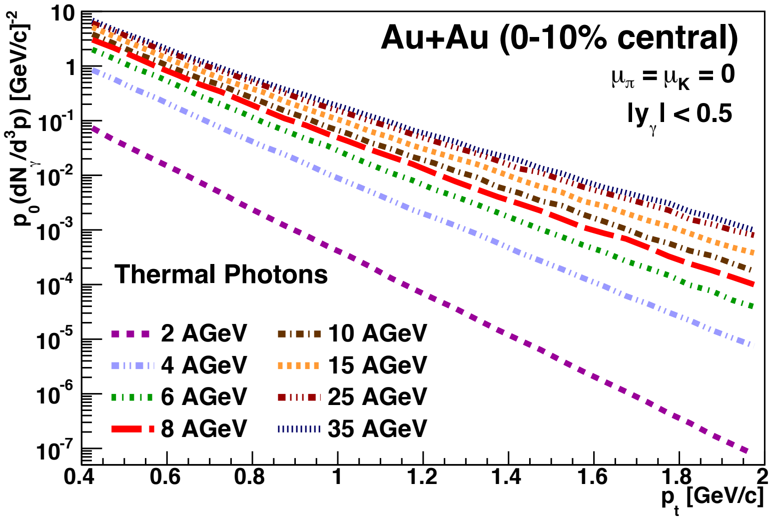
- **Very slow evolution** of the fireball
  - ↪ T and μ_B remain roughly constant for up to 20 fm/c!
- Moderate temperatures and very **high baryon density** respectively baryochemical potential → Ideal situation to study in-medium modifications

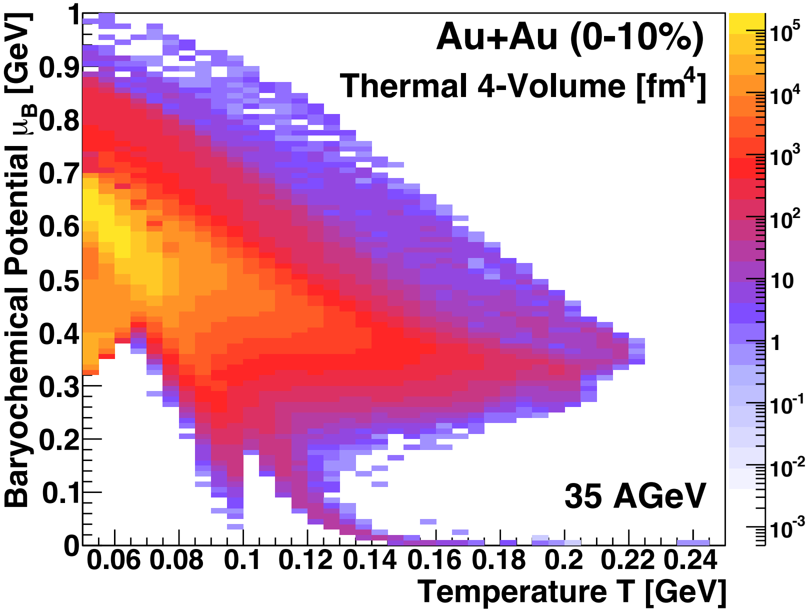
HADES vs. c.g. UrQMD

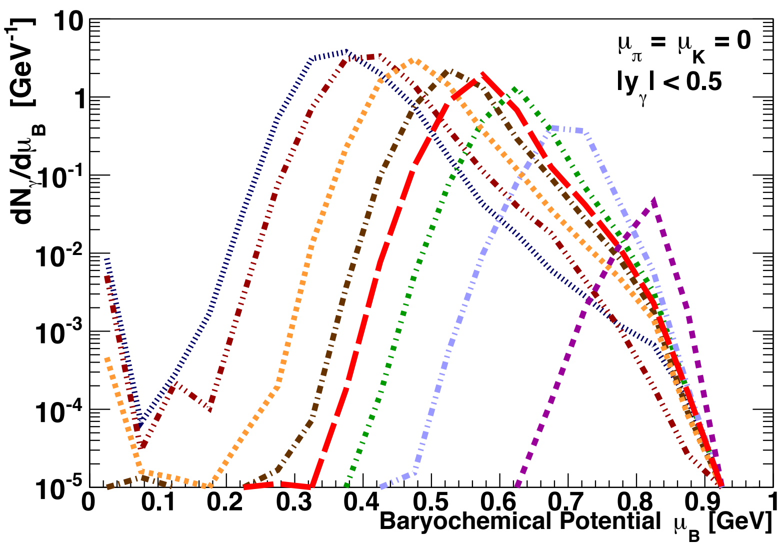


- **Significant in-medium broadening** of the ρ spectral function, causing a strong increase of the dilepton yield below the pole mass
- Low-mass enhancement **increases with system size**
- Low temperatures $\rightarrow$ Higher masses and pole mass peak suppressed

FAIR and photons







- Unexplored **transition region**
  - ↪ Hadron gas ↔ QGP
  - ↪ high μ_B ↔ moderate μ_B
- High luminosities enable **systematic studies**
  - ↪ Energy, momentum, centrality, system size, ...

Summary: Coarse graining

- Allows to employ QFT in the analysis
- Alternative to hybrid and 3-fluid at low energies.
- No EoS in the dynamics, but in the emission rates.
- Works excellent for leptons
- Photon yields support the existence of QGP
- Charm studies under way

CHIRAL HYDRODYNAMICS

Nonequilibrium Chiral Fluid Dynamics (NχFD, PNχFD)

Aim: Explore signals for QCD first order phase transition and critical end point

Model: Ideal quark fluid coupled to Polyakov-quark-meson model

$$\mathcal{L} = \bar{q}\left[i\left(\gamma^\mu \partial_\mu - i g_s \gamma^0 A_0\right) - g\left(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi}\right)\right] q + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2 - U\left(\sigma, \vec{\pi}\right) - \mathcal{U}(\ell, \bar{\ell})$$

Propagate chiral fields and Polyakov loop explicitly via Langevin equations of motion

$$\partial_\mu \partial^\mu \sigma + \eta_\sigma(T)\partial_t \sigma + \frac{\partial V_{\rm eff}}{\partial \sigma} = \xi_\sigma \qquad \eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}}{\partial \ell} = \xi_\ell$$

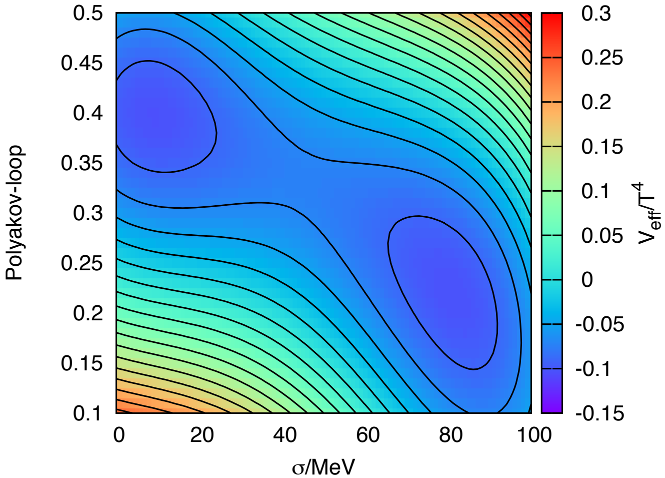
Energy-momentum exchange between fields and fluid described by source terms

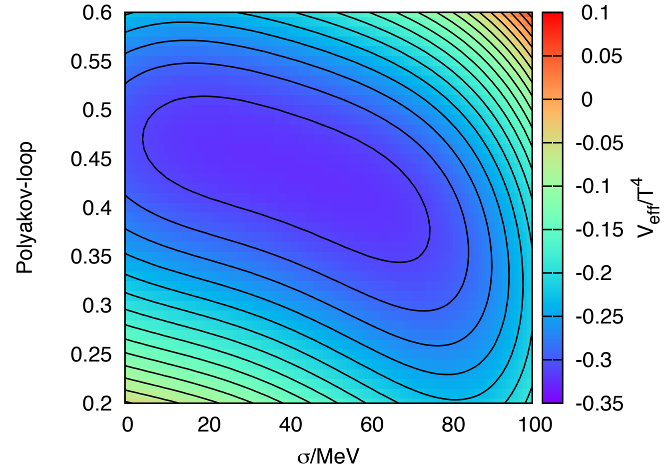
$$\partial_\mu T^{\mu\nu}_{\rm q} = \left(\frac{\partial \Omega_{q\bar{q}}}{\partial \sigma} + \eta_\sigma \partial_t \sigma\right)\partial^\nu \sigma + \left(\frac{\partial \Omega_{q\bar{q}}}{\partial \ell} + \eta_\ell \partial_t \ell T^2\right)\partial^\nu \ell$$

Solve with 3+1 dim. hydro, using SHASTA and staggered leap frog for the fields

M.Nahrgang, S.Leupold, C.Herold, M.Bleicher, PRC 84 (2011); M.Nahrgang, S.Leupold, M.Bleicher, PLB 711 (2012); M.Nahrgang, C.Herold, S.Leupold, M.Bleicher, I.Mishustin arXiv:1105.1962

CP vs. 1st order by adjustment of g





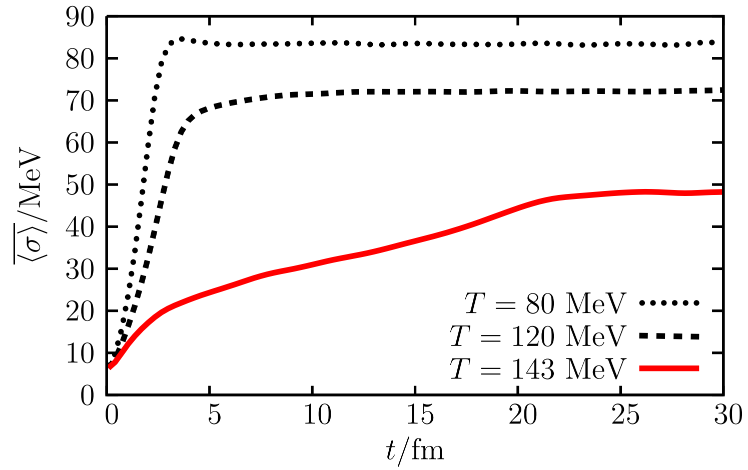
Effective potential for 1st order PT:
Tc=173 MeV, g=4.7

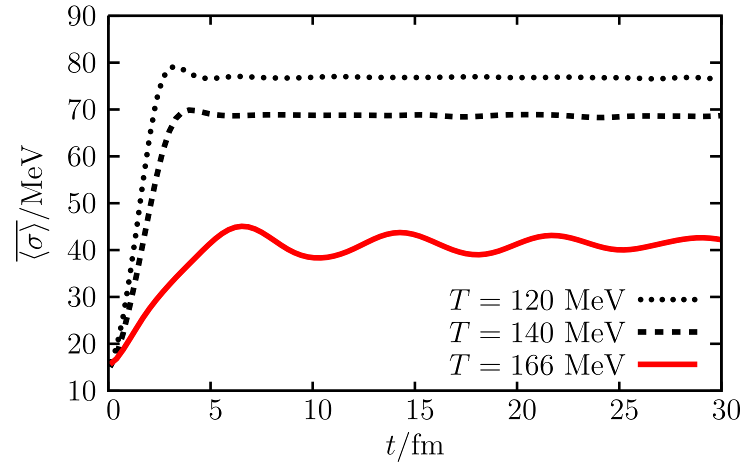
Effective potential for CP:
Tc=180 MeV, g=3.5

→first step: vanishing baryon chemical potential, second step: full calculation

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Box calculations: sigma equilibration





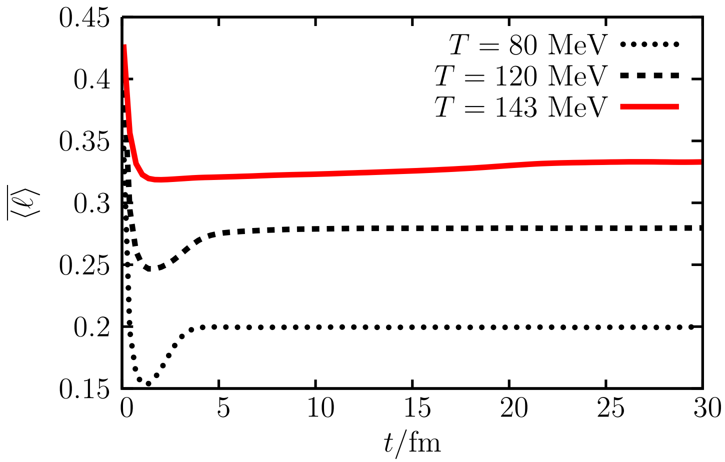
1st order PT, Tc=173 MeV:
Equilibration sigma-field for several temperature quenches (Tini=180 MeV)

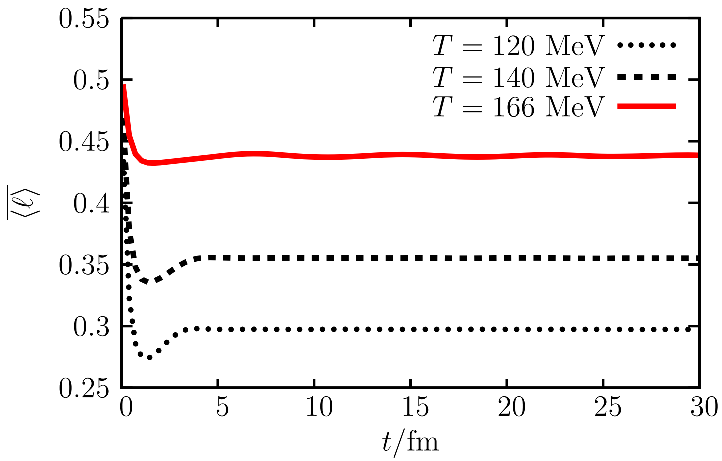
CP, Tc=180 MeV:
Equilibration sigma-field for several temperature quenches (Tini=186 MeV)

→ At CP: Critical slowing down delays equilibration and produced oszillations around the flat minimum

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Box calculations: Polyakov loop equilibration





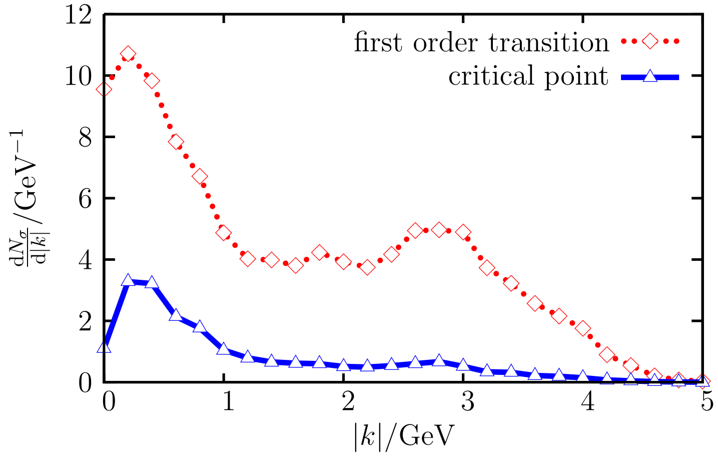
1st order PT, Tc=173 MeV:
Equilibration of Polyakov loop for several temperature quenches (Tini=180 MeV)

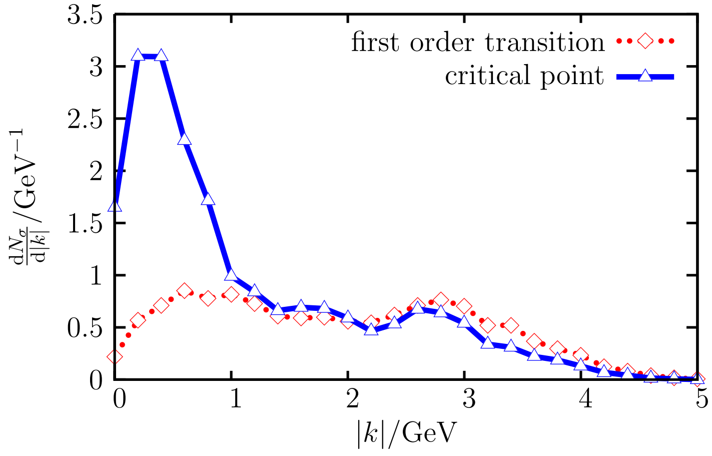
CP, Tc=180 MeV:
Equilibration Polyakov loop for several temperature quenches (Tini=186 MeV)

→ Polyakov loop equilibrates quickly

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Box calculations: intensities





Mode occupation during the transition: at 12 fm (1st order) and 3 fm (CP)

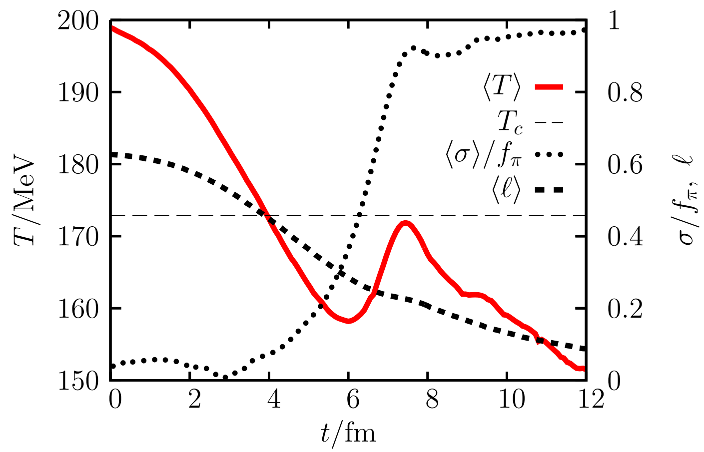
Mode occupation in equilibrium: at 24 fm/c

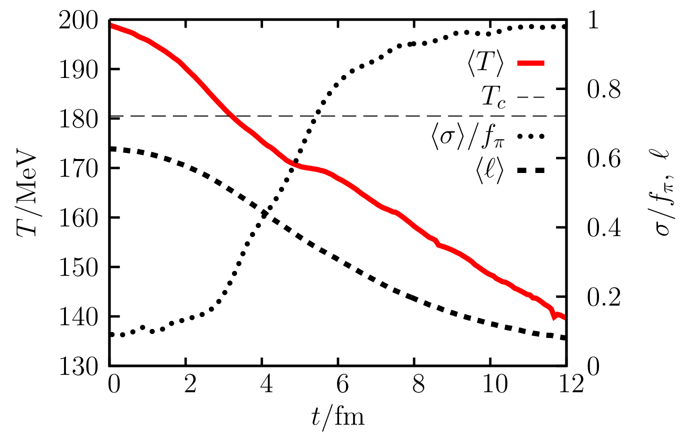
→ Damping of low frequency modes in case of 1st o. PT, strong enhancement of low freq. modes in case of CP

→ Do not trust the modes above 1 GeV!

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Time evolution in expanding system





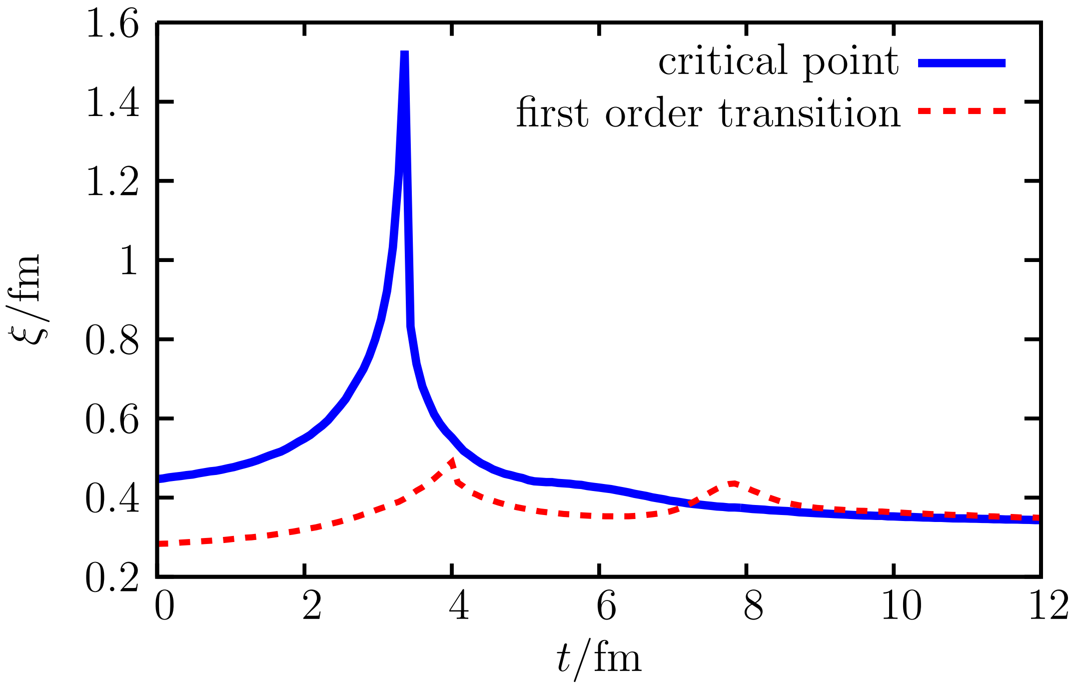
1st order PT, Tc=173 MeV:
Sigma field, Polyakov loop,
av. temperature

CP, Tc=180 MeV:
Sigma field, Polyakov loop,
av. temperature

→ At 1st order PT: super cooling by 10 MeV with reheating is observed

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Correlation length: dynamical system

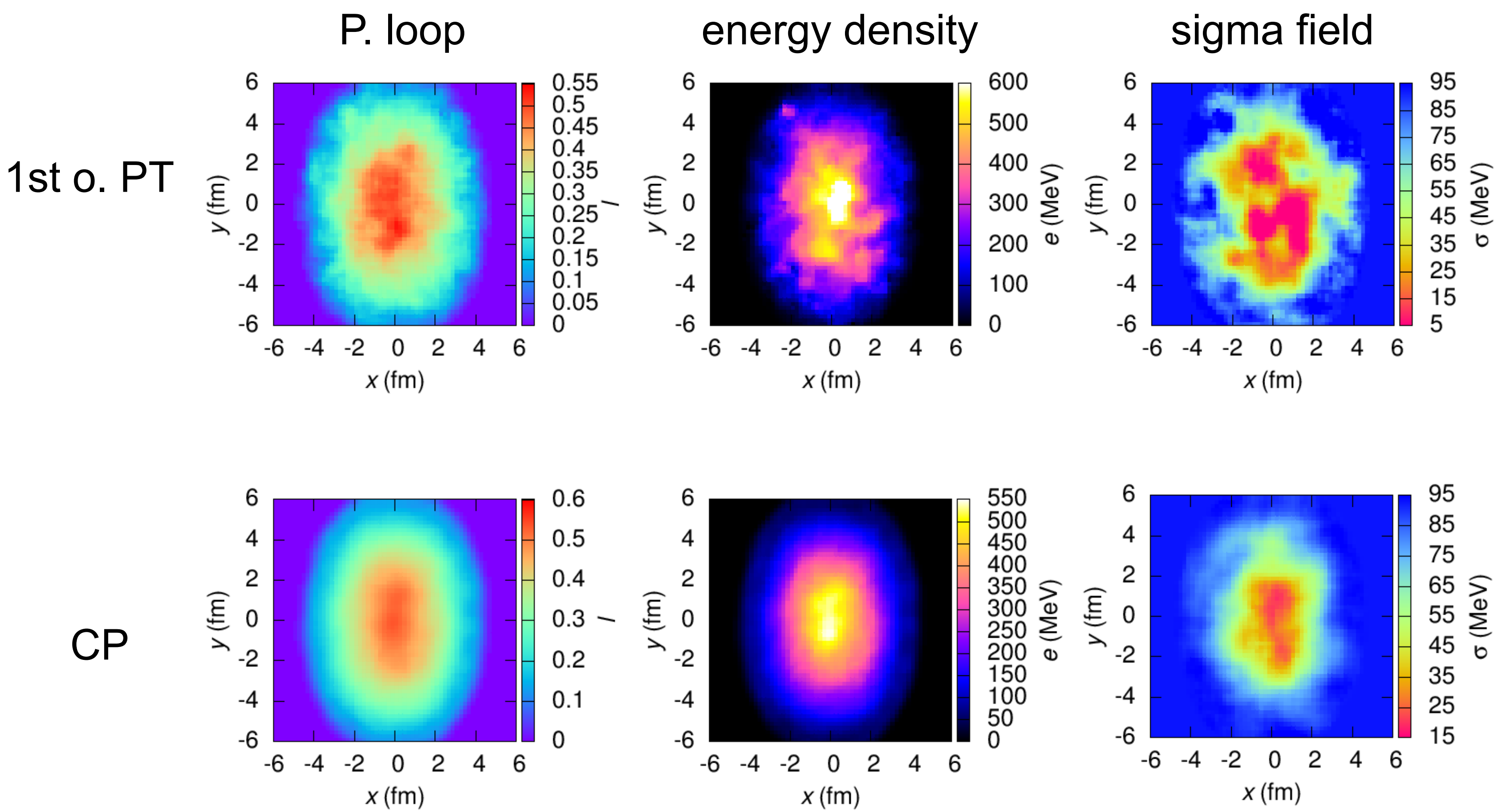


- Strongly increased correlation length at CP
- Reheating clearly visible at 1st o. PT
- However magnitude stays small (1-2 fm)

→ Correlations are restricted to small scales,
no large fluctuations emerge → difficult to observe!

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Domain formation



Profiles at the crossing of the transition line

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin, PRC 87 (2013)

Extension to finite baryon densities

Extension requires 3 steps:

1. Include μ-dependence in Polyakov loop potential, cf. Schäfer, Pawlowski, Wambach

$$\mathcal{U}(\ell, T, T_0)\ , \quad T_0 \to T_0(\mu)$$

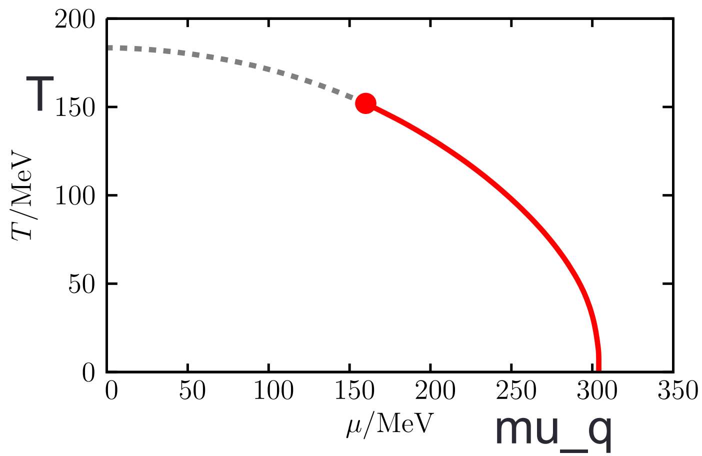
2. Calculate grand canonical potential for finite chemical potential

$$\Omega_{q\bar{q}} = -2N_f T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \{(\ln\left[1 + 3\ell \mathrm{e}^{-\beta(E-\mu)} + 3\ell \mathrm{e}^{-2\beta(E-\mu)} + \mathrm{e}^{-3\beta(E-\mu)}\right] + (\mu \to -\mu)\}$$

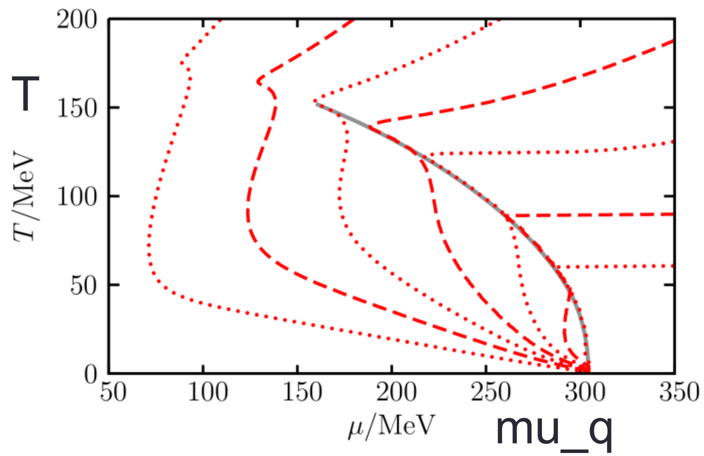
3. Propagate (net) baryon density in the hydro sector

$$\partial_\mu n^\mu = 0\ , \quad n^\mu = \rho u^\mu$$

Phase diagram at finite mu



Critical point

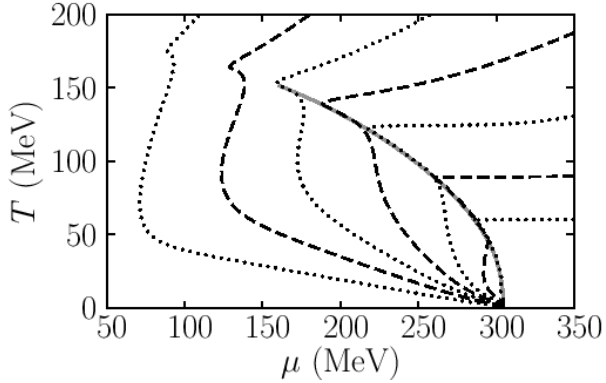


Isentropes

→ Bending of the isentropes due to dynamical mass increase of the quark fields

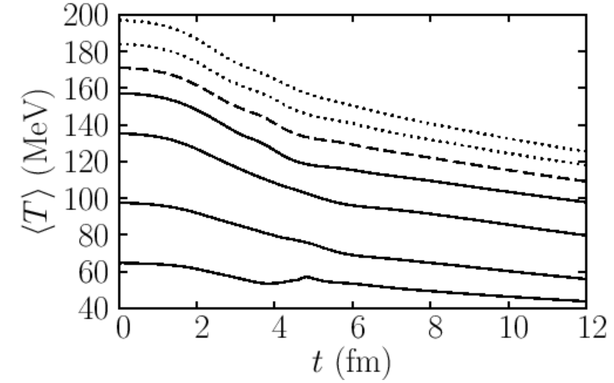
See also Wambach, Buballa;
C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin

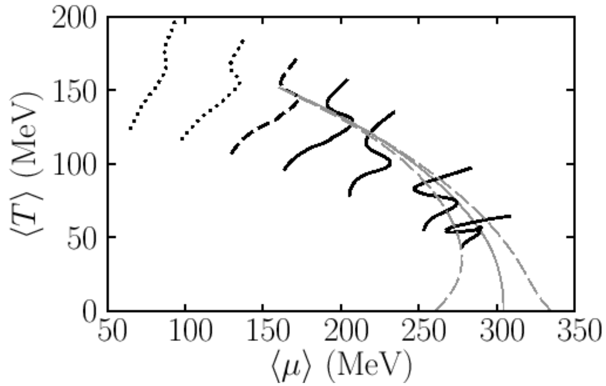
Trajectories in the phase diagram



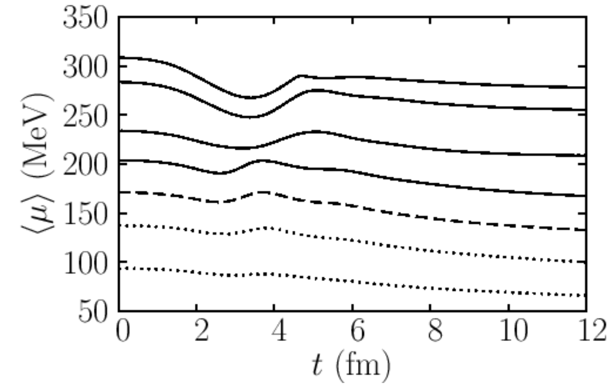
Isentropes

S/A=[24,16,12.5,10,8,6,4]





Nonequilibrium trajectories

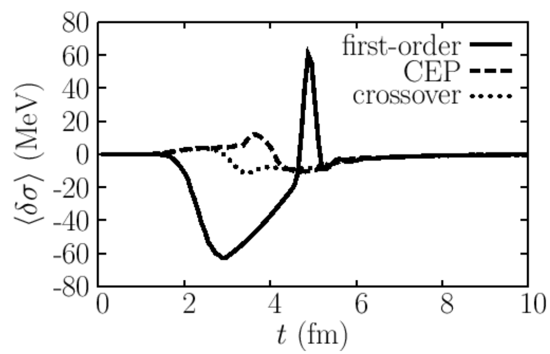


→ Reheating at high baryon densities, in and out of spinodal region

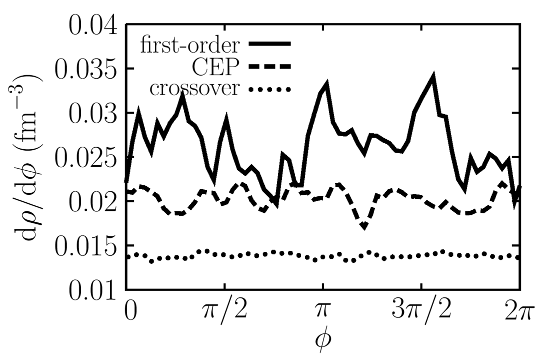
C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin

Fluctuations and quark densities

Nonequilibrium fluctuations, sigma field

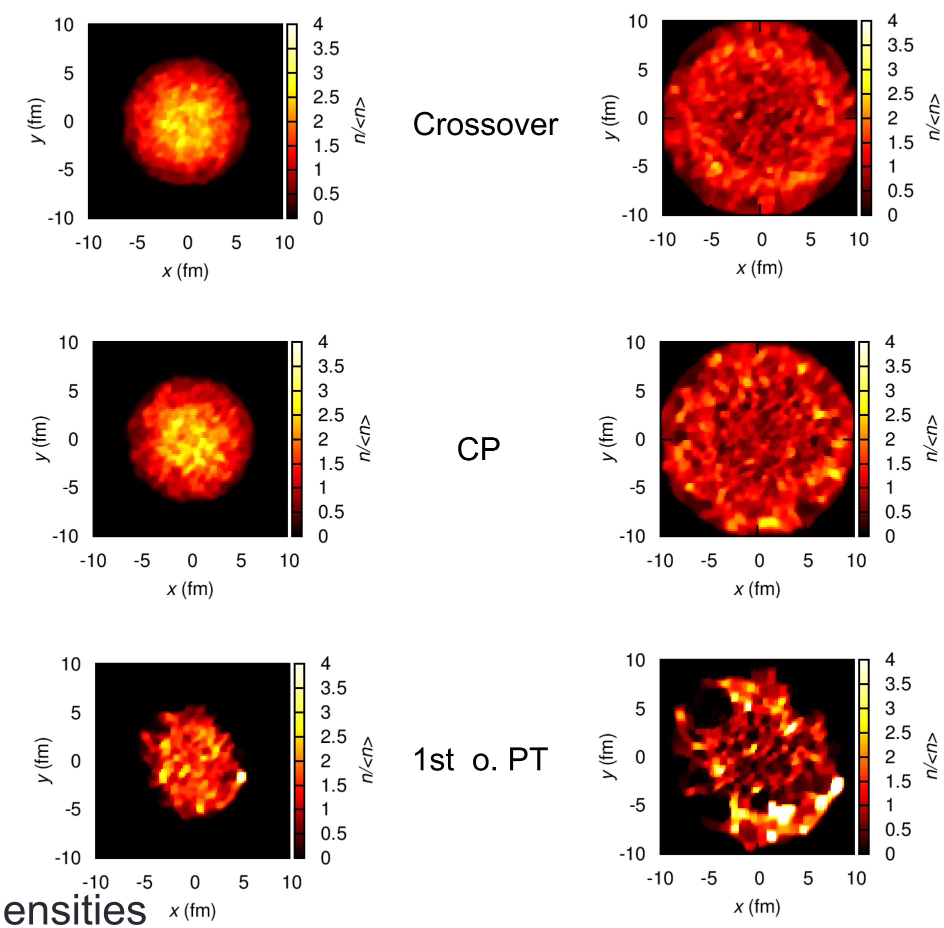


Angular distribution, 12 fm/c



6 fm/c

12 fm/c



→ Strong fluctuations,inhomogeneous quark densities

C. Herold, M. Nahrgang, M. Bleicher, I. Mishustin

Chiral Dynamics: Summary, Outlook

- Dynamical simulation of PQM model successful
- Extension to finite mu on the way
- Equilibration → Trajectories → reheating
- Fluctuations of the sigma field
- Quark density clusters (domain formation)
- Include pion fluctuations
- However, rather small correlation length

QUARK DYNAMICS

Hadronisation

- How to go from partonic matter to hadronic matter?
  - energy conservation?
  - free quarks in the end?
  - what to do with gluons?
  - decrease in entropy?
  - transition to fragmentation?
- Chromodielectric model
- Quark Molecular Dynamics
- ….

A model for QCD

Chromodielectric model (CDM) [G.Martens et.al., Phys.Rev. D70/D73]

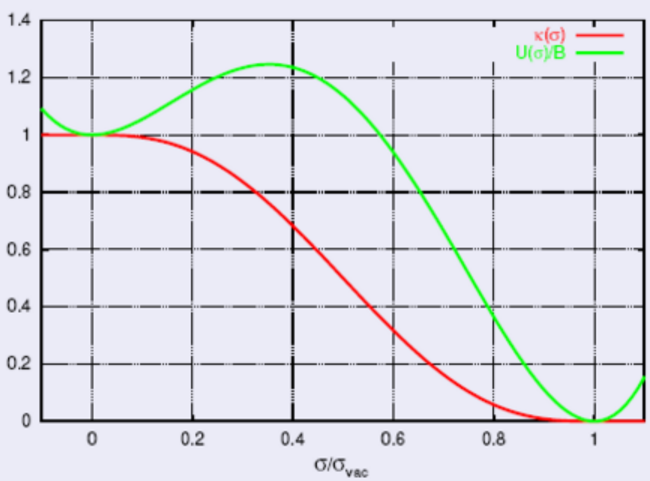
$$\mathcal{L}_{\rm cdm} = \frac{1}{4}\kappa(\sigma) F^a_{\mu\nu} F^{\mu\nu,a} \qquad \mathcal{L}_{glue}$$

$$-g\, j^a_\mu A^{\mu,a} \qquad \mathcal{L}_{q,g}$$

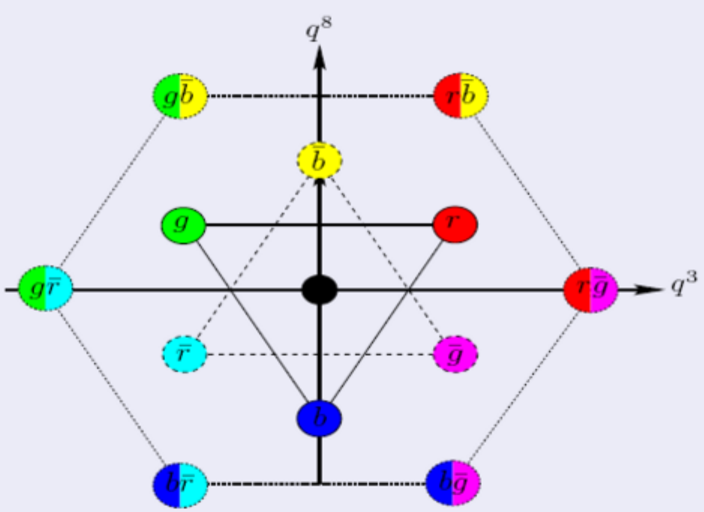
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) \qquad \mathcal{L}_\sigma$$

$$F^{\mu\nu,a} = \partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a} \qquad a \in \{3,8\}$$

Self interaction & Dielectric



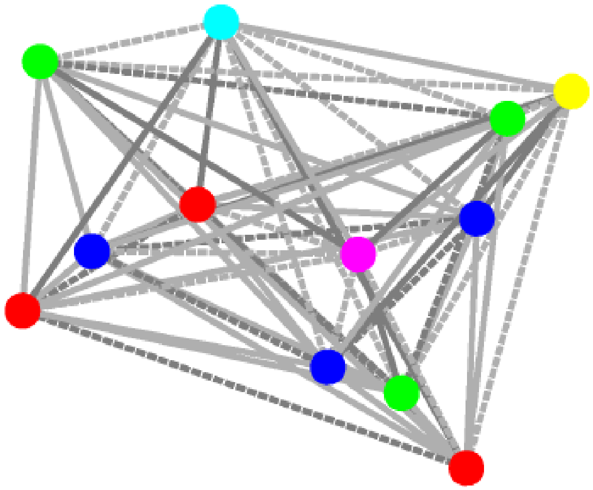
Color multipletts



Quark Molecular Dynamics

Hamiltonian of the model :

$$H = \sum_{i=1}^{N} \sqrt{\mathbf{p}_i^2 + m_i^2} + \frac{1}{2} \sum_{i \neq j} C_{ij} V(|\mathbf{r}_i - \mathbf{r}_j|)$$



- Potential :

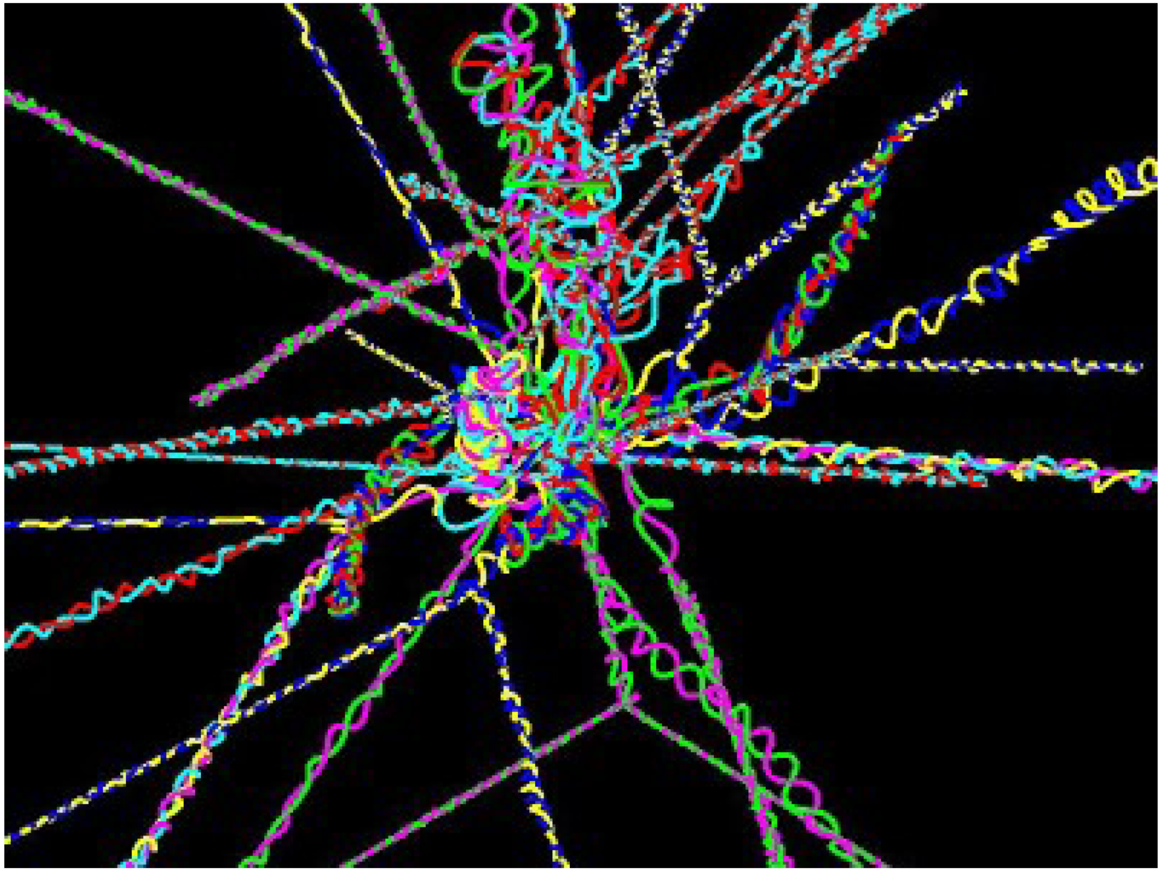
  linear potential $V(r) = \kappa r$
- Color factor C_{ij} :

  can be attractive or repulsive depending on the color of the quarks
- Quarks :

  classical point-particles with light masses $m_{u,d} = 5$ MeV, $m_s = 150$ MeV

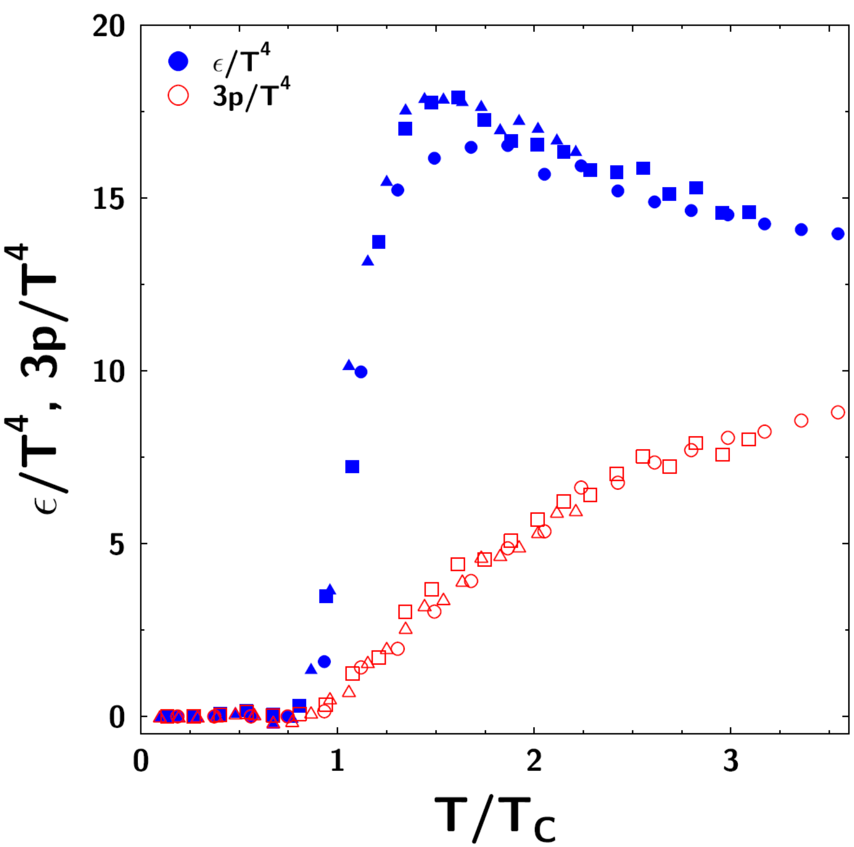
Some critical remarks on EbyE and susceptibilities

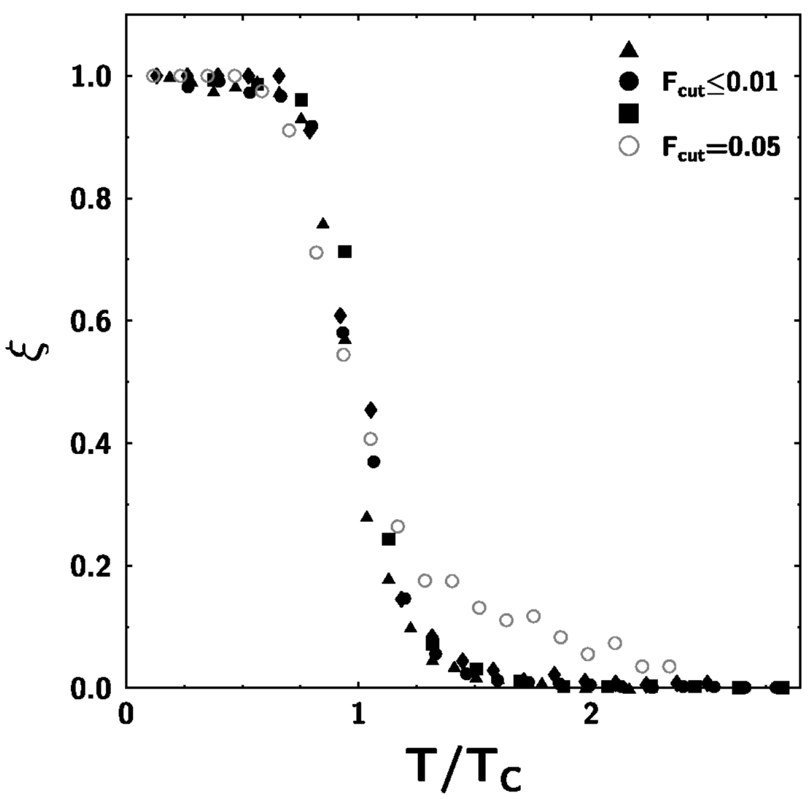
- Dynamics
- Quarks
- Mesons
- Baryons
- Confinement/ deconfinement



M. Hofmann, PhD thesis

Some properties: equilibrium

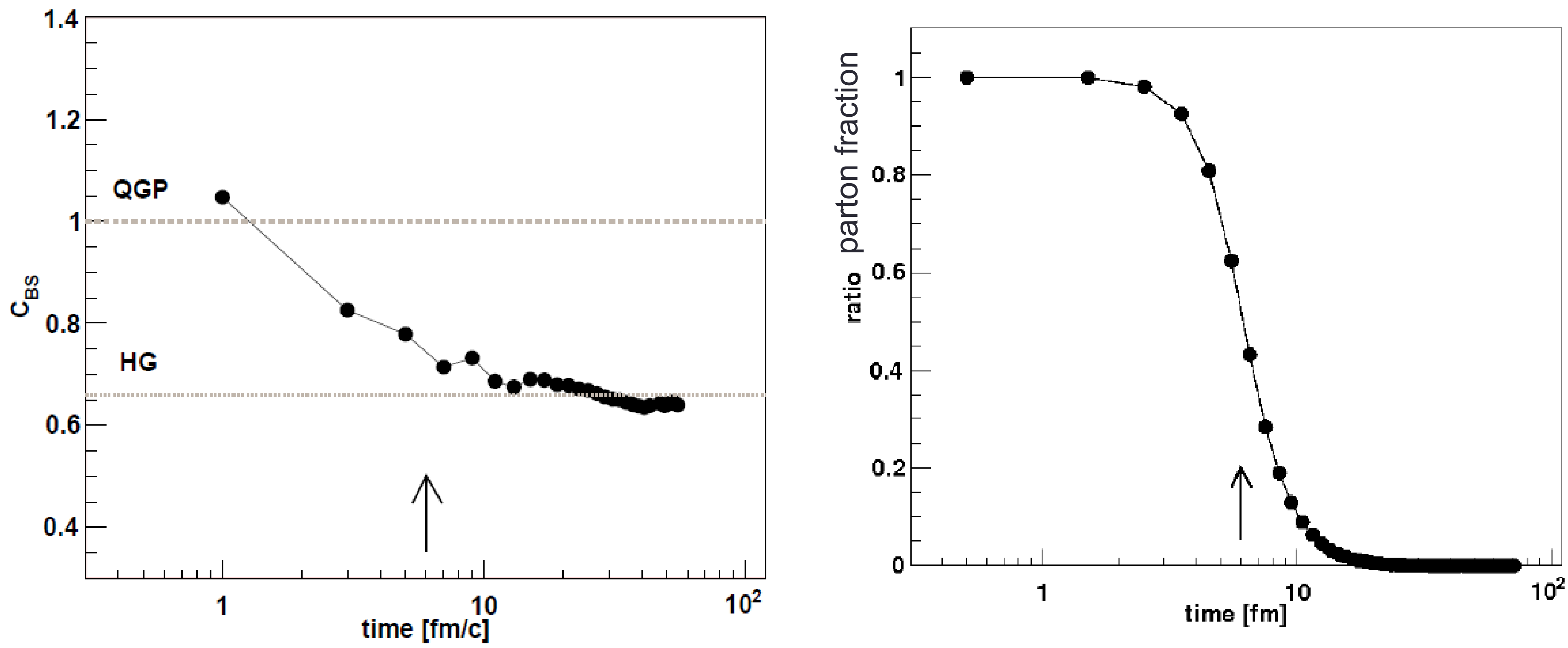




Tc ~ 140 MeV

$$\xi = N_{hadrons} / N_{all\ \ particles}$$

Time evolution



The signal vanishes at hadronization
for all observed quantities, i.e. susceptibilities

Haussler, Bleicher, Stoecker, PLB 2008

Fluctuations and susceptibilities

$$Z = \sum_i \exp[-\beta(E_i - \mu_Q Q_i - \mu_B B_i - \mu_S S_i)]$$

$$(X, Y) = (Q, B, S)$$

variances and correlations

$$\langle(\delta X)^2\rangle = T^2 \frac{\partial^2}{\partial\mu_X^2}\log(Z) = -T\frac{\partial^2}{\partial\mu_X^2}F$$

$$\langle(\delta X)(\delta Y)\rangle = T^2 \frac{\partial^2}{\partial\mu_X\mu_Y}\log(Z) = -T\frac{\partial^2}{\partial\mu_X^2\mu_Y^2}F$$

susceptibilities

$$\langle\delta X^2\rangle = -\frac{1}{V}\frac{\partial^2}{\partial\mu_X^2}F = V\ T\ \chi_X$$

$$\langle\delta X\delta Y\rangle = -\frac{1}{V}\frac{\partial^2}{\partial\mu_X\partial\mu_Y}F = V\ T\ \chi_{XY}$$

Haussler, Bleicher, arXiv:0803.2846: Susceptibilities and fluctuations in a Quark-Hadron System
and JPG (2008): Correlations and fluctuations of conserved charges in a dynamical recombination approach

Baryon-Strangeness Correlations

Koch, Majumder, Randrup. Phys.Rev.Lett.95:182301,2005.
S. H., Stoecker, Bleicher. Phys.Rev.C73:021901,2006.

In a QGP, strangeness is always carried together with baryon number
In a Hadron Gas, Strangeness can be carried without baryon number

$$C_{BS} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} \approx -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

expectation values :

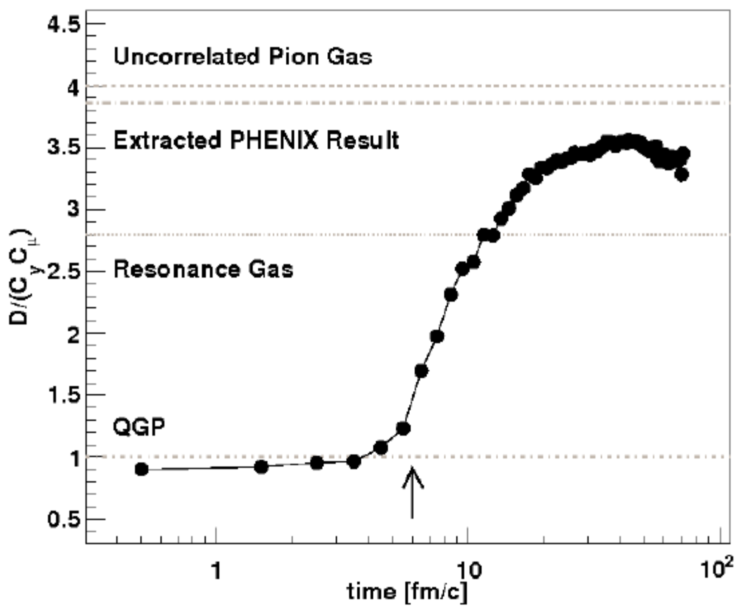
- $C_{BS} = 1$ in a QGP
- $C_{BS} = 0.66$ in a HG ($T = 170$ MeV, $\mu = 0$)

related quantities :

some particles are difficult to measure

- $C_{QS} = \frac{\langle QS\rangle - \langle Q\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} \approx \frac{3 - C_{BS}}{2}$
- $C_{MS} \approx C_{BS}$ with $M = B + 2I_3$
- take into account only strange charged particles

Recombination and fluctuation



$$\tilde{D} = \frac{\langle N_{\rm ch}\rangle_{\Delta y}\langle \delta R^2\rangle_{\Delta y}}{C_\mu C_y} = \begin{cases} 1 & \text{quark gluon gas} \\ 2.8 & \text{resonance gas} \\ 4 & \text{uncorrelated pion gas} \end{cases}$$

$$C_\mu = \tilde{R}^2_{\Delta y} = \frac{\langle N_+\rangle^2_{\Delta y}}{\langle N_-\rangle^2_{\Delta y}}$$

$$C_y = 1 - P = 1 - \frac{\langle N_{\rm ch}\rangle_{\Delta y}}{\langle N_{\rm ch}\rangle_{\rm total}}$$

Recombination kills the fluctuations

- $\tilde{D} = 1$ in the quark matter phase
- $\tilde{D}$ is compatible with the experiment result in the late stage
- Hadronization and the increase of $\tilde{D}$ occur at the same time

Summary

- No real dynamical model for the phase transition exisists
- Hybrid and hydro approaches circumvent the modeling of the PT
- Explicit quark dynamics can not capture feature like gauge invarince
- Major developments are still needed!