Analysis of the Scattering Amplitude of Proton on the Bounded Nucleons Basing on the Proton-Nucleus Scattering Data

Ibrahim A.M. **Abdulmagead**[1,2,3]

[1]Joint Institute for Nuclear Research, Dubna 141980, Russia
[2]Physics Department, Faculty of Science, Cairo University, Giza 12613, Egypt
[3]Academy of Scientific Research and Technology (ASRT), Cairo, Egypt

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Introduction

The **proton-nucleus scattering** is the traditional topic of investigations to obtain information on a **structure of nuclei**, including the nuclear mean-field optical potential (OP), nuclear radius, the nuclear density distribution.

At **low energies** of incident protons, one can construct the respective OP with accounting for the direct[1] and exchanged[2] contributions. In these studies the result was reduced to the non-local potential. In this case, many other constructions of nuclear potentials are made only of the real parts of potentials while their imaginary parts as usually are introduced in a phenomenological way.

In the case of **relativistic energies**, calculations of cross sections are based on the **high energy approximation (HEA)**.

Our studies are aimed onto analysis of the data at energies about **200-1000 MeV**, where the re-scattering processes and the non-locality effects do not play a decisive role but relativistic effects should be taken into account.

Below we construct **folding microscopic optical potentials** based on the **elementary nucleon-nucleon scattering amplitude** and on the **density distribution function of nucleons** in a nucleus.

[1] Doan Thi Loan et al., Phys.Rev. C **92** 034304 (2015)

[2] Doan Thi Loan et al., J. Phys G **47** 035106 (2020)

Our approach

We follow the theoretical approach previously developed and used for the pion-nucleus scattering analysis[3]. It is based on the **microscopic HEA-based 3-parameter model of OP** and using the distorted wave Born approximation for calculating observables with a help of the standard computer code **DWUCK4**[4].

This folding model[5] of OP depends on the **nuclear density distribution function** of a nucleus and on the **elementary nucleon-nucleon scattering amplitude** which itself depends on three parameters:

1. σ : the total nucleon-nucleon scattering cross section ,
2. α : the ratio of real to imaginary parts of the scattering amplitude at forward angles,
3. β : the slope parameter .

These three "**in-medium**" **parameters of the NN scattering amplitude are adjusted to the experimental data of elastic proton-nucleus scattering** and compared with the "free" ones known from analysis of proton-nucleon scattering data. Such analysis allows one to estimate effect of nuclear matter on the NN scattering amplitude.

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[3] V.K.Lukyanov et al., Nucl. Phys A **1010** (2021) 122190

[4] P.D.Kunz & E.Rost, Computational Nuclear Physics 2, Springer 1993

[5] V.K.Lukyanov et al., Phys. Atom. Nucl. **69** (2006) 240

Mathematical Framework (1/6)

An expression for the OP can be written as follows:

$$U(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2 k}\int e^{-i\mathbf{qr}}\rho(q)\,F_N(q)\,d^3q. \qquad (1)$$

Here ρ is the density distribution function, $\beta_c = v_{c.m.}/c = k_{lab}/[E_{lab} + m^2/M_A]$ is the ratio of the proton velocity at the c.m. system to the light velocity[6], which is expressed through its total energy in the lab system $E_{lab} = (k_{lab}^2 + m^2)^{1/2} = = T_{lab} + m$, where k_{lab} is the moment of the relative motion, T_{lab} is the proton kinetic energy at the lab system, and M_A is the target nucleus mass.

Then, for the nucleon-nucleon amplitude of scattering $F_N(q)$ one uses the expression

$$F_N(q) = \frac{k}{4\pi}(i+\alpha)\,\sigma\, f_N(q), \;\; f_N(q) = e^{-\beta q^2/2}. \qquad (2)$$

One should underline that, in the case of the pA scattering, this amplitude describes the proton scattering on the bounded (not free!) nucleons.

[6] In Eq.(1) we use the units MeV and fm, which follow to $\hbar$c = 197.327 MeV fm. In the other cases, the natural system of units is used where $\hbar$=c=1, and thus E, T, k, m have the same dimensions [MeV].

Mathematical Framework (2/6)

It is convenient to expand the **plane waves** $e^{\pm i\mathbf{q}.\mathbf{r}}$ in Eq. (1) into their **multipole components**

$$e^{\pm i\,\mathbf{q}.\mathbf{r}} = 4\pi \sum_{l,m} (\pm i)^l \, j_l(qr) \; Y_{l,m}(\hat{q}) \; Y^*_{l,m}(\hat{r}). \tag{3}$$

After substituting from Eqs. (2) and (3) into Eq.(1) and integrating over the angular variables using the **orthonormality** property of **spherical harmonics**, one can get the OP in the form

$$U(r) \,=\, V(r) + iW(r) \,=\, -\frac{(\hbar c)\beta_c}{(2\pi)^2}\sigma(\alpha + i) \int_0^\infty j_0(qr) \; \rho(q) \; f_N(q) \, q^2 dq. \tag{4}$$

In calculations, we used the nuclear density distribution in the form of the **symmetrized Fermi function**

$$\rho(r) \,=\, \rho_0 \frac{\sinh(R/a)}{\cosh(R/a) + \cosh(r/a)}, \;\; \rho_0 \,=\, \frac{3A}{4\pi R^3}[1 + (\frac{\pi a}{R})^2]^{-1} \tag{5}$$

with the corresponding Fourier transform

$$\rho(q) = -\rho_0 \frac{4\pi^2 aR}{q} \frac{\cos qR}{\sinh(\pi aq)}[1 - (\frac{\pi a}{R}) \coth(\pi aq) \tan qR], \; R \geq \pi a. \tag{6}$$

Transition Potential

To study **inelastic scattering** of proton on a nucleus which will be excited to a **collective excited state** λ, we construct the transition potentials for inelastic scattering starting from the above prescription. So, one needs to take into account explicitly the **multipole decomposition of the nuclear density distribution** $\rho(\mathbf{r})$ that enters the folding calculation, Eq. (1), as follows

$$\rho(\mathbf{r}) = \rho(r) + \rho_\lambda(r) \sum_\mu \alpha_{\mu\lambda} Y_{\mu\lambda}(\hat{r}), \quad \lambda = 2, 3, ... \tag{7}$$

where $\alpha_{\lambda\mu}$ are variables associated with collective motion of a nucleus (for example: rotations and vibrations) with $\rho_\lambda(r) = -r\ (\frac{r}{R})^{\lambda-2} \frac{d\rho(r)}{dr}$. Therefore, the nuclear density form factor becomes

Mathematical Framework (4/6)

$$\rho(\mathbf{q}) = \rho(q) + \rho_\lambda(q) \sum_\mu \alpha_{\mu\lambda} Y_{\mu\lambda}(\hat{q}), \tag{8}$$

where the form factor of the ground state density, $\rho(q)$, is given by Eq.(6) while $\rho_\lambda(q)$ has the expression

$$\rho_\lambda(q) = 4\pi \int_0^\infty j_\lambda(qr)\ \rho_\lambda(q)\ q^2 dq. \tag{9}$$

Finally, substituting from Eq.(8) in Eq.(1) we obtain the direct and transition potentials for calculations of elastic and inelastic scattering observables

$$U(\mathbf{r}) = U(r) + U_\lambda(r) \sum_\mu \alpha_{\mu\lambda} Y_{\mu\lambda}(\hat{r}), \tag{10}$$

where $U(r)$ is given by Eq.(4) while the **transition potential** $U_\lambda(r)$ reads as

$$U_\lambda(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2} \sigma(\alpha + i) \int_0^\infty j_\lambda(qr)\ \rho_\lambda(q)\ f_N(q)\ q^2 dq. \tag{11}$$

Differential cross sections of the **proton-nucleus scattering** are calculated via **numerical solving the wave equation**

Mathematical Framework (5/6)

$$(\nabla^2 + k^2)\,\psi(\vec{r}) = 2\mu\, U_{eff}(\vec{r})\,\psi(\vec{r}). \tag{12}$$

Here the relativization is taken into account by introducing the effective mass $\bar{m}$ in the effective potential

$$U_{eff}(\mathbf{r}) = \gamma^{(r)} \cdot U(\mathbf{r}), \quad U(\mathbf{r}) = U(\mathbf{r}) + U_c(\mathbf{r}), \quad \gamma^{(r)} = \frac{\bar{\mu}}{\mu}, \tag{13}$$

where U is the folding OP (10), μ and $\bar{\mu}$ are the reduced masses of the form:

$$\mu = \frac{mM}{m+M}, \quad \bar{\mu} = \frac{\bar{m}M}{\bar{m}+M}, \quad \bar{m} = \sqrt{k^2 + m^2}. \tag{14}$$

The relativistic momentum k in the center of mass system is equal to

$$k = \frac{M\,k_{lab}}{\sqrt{(m+M)^2 + 2MT_{lab}}} = \frac{M\sqrt{T_{lab}(T_{lab}+2m)}}{\sqrt{(m+M)^2 + 2MT_{lab}}}. \tag{15}$$

DWUCK4

The **wave equation, Eq. (12)**, is solved **numerically** with a help of the standard **computer code DWUCK4**. So, one takes into account the relativization and distortion effects in scattering of a nucleon in the target nucleus field.

Average free pN-amplitude parameters

In our study, we do not distinguish the cases when the incident proton scatters on the proton or on the neutron inside the target nucleus. Therefore we compare our calculations of “in-medium” proton-nucleon cross sections with the averaged “ “free” cross sections

$$\sigma = \frac{\sigma_{pp} \cdot Z + \sigma_{pn} \cdot N}{Z + N}, \tag{16}$$

where Z is number of protons and N is number of neutrons in the target nucleus, σ_{pp} and σ_{pn} are, respectively, the experimental proton-proton and proton-neutron cross sections[7] and are given by

$$\sigma_{pp} = 19.6 + 4253/E - 375/\sqrt{E} + 3.86 \cdot 10^{-2} E \ mb \tag{17}$$

and

$$\sigma_{pn} = 89.4 - 2025/\sqrt{E} + 19108/E - 43535/E^2 \ mb. \tag{18}$$

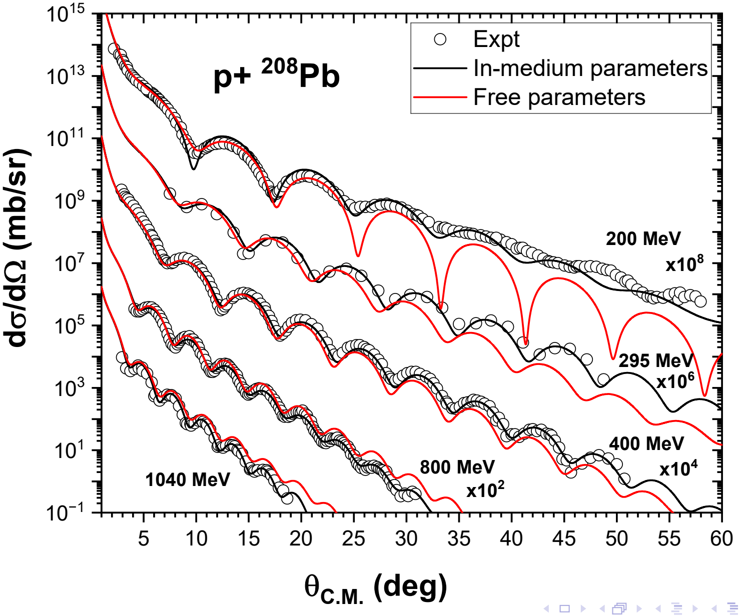
[7] C.A. Bertulani & C. De Conti, Phys. Rev. C **81** 064603 (2010)

Results

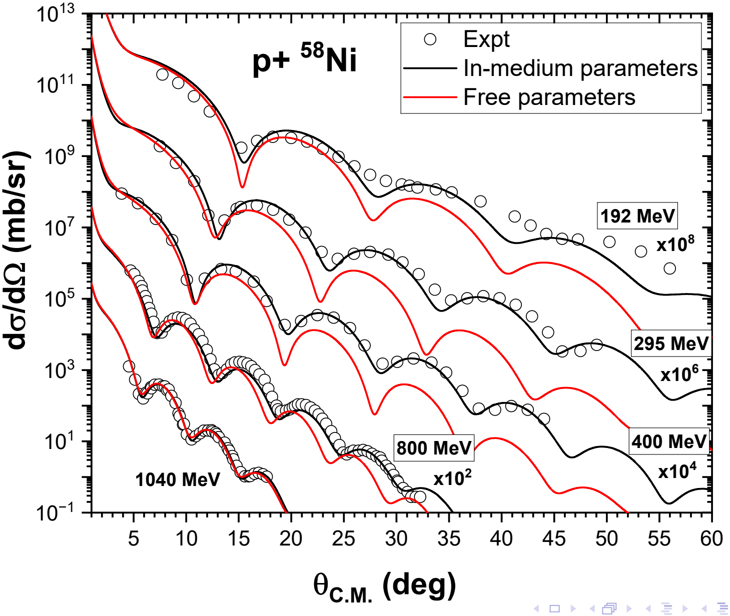
- In calculations of the elastic and inelastic scattering cross sections, basing on the constructed proton-nucleus OP, and then in comparison of them with the corresponding experimental data one can get the best fit parameters σ, α, β of the elementary amplitude of scattering of nucleons on the bounded nucleons, (in-medium parameters).

- Thus we have a possibility to compare the information on these typical characteristics of the proton scattering on the free nucleons with the same characteristics but for the proton scattering on the bounded nucleons ("in-medium" effect). It is natural one that the obtained sets of such parameters are different for different collision energies.

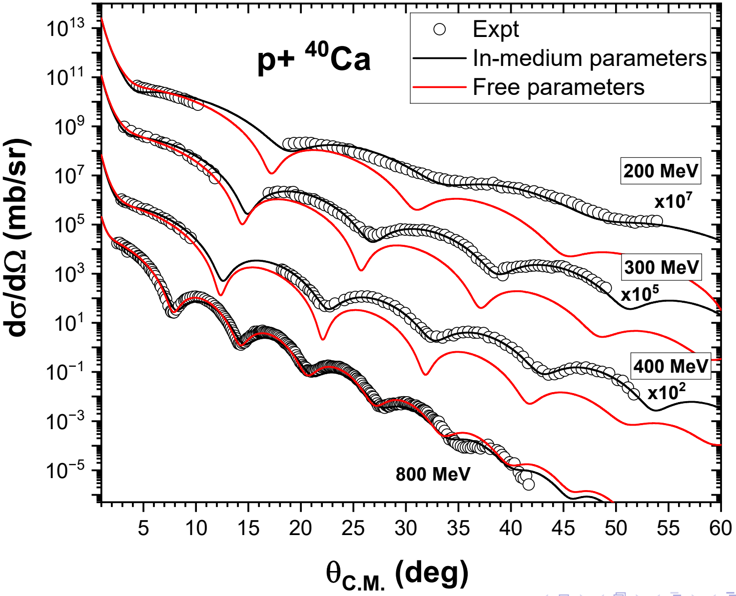
Elastic Angular Distributions



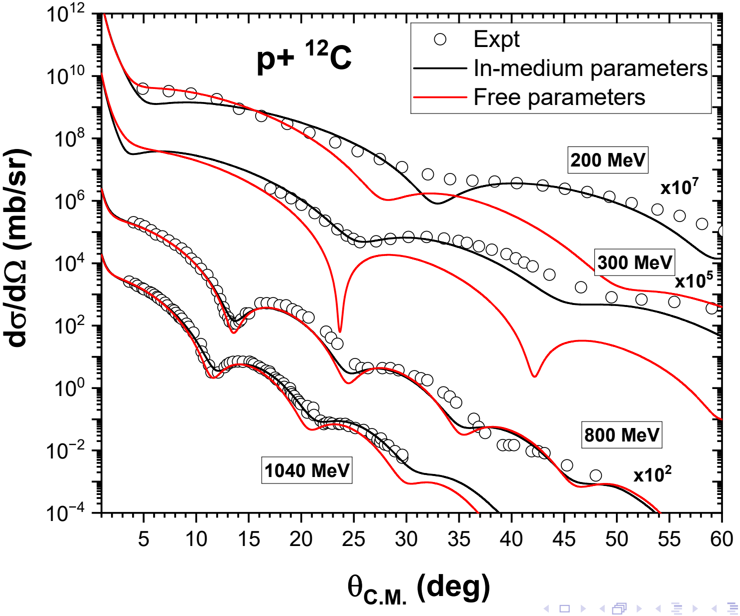
Elastic Angular Distributions



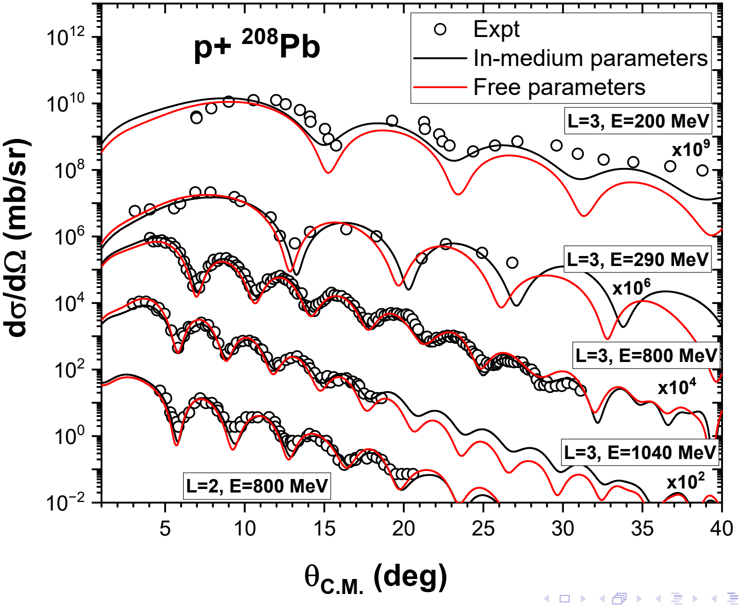
Elastic Angular Distributions



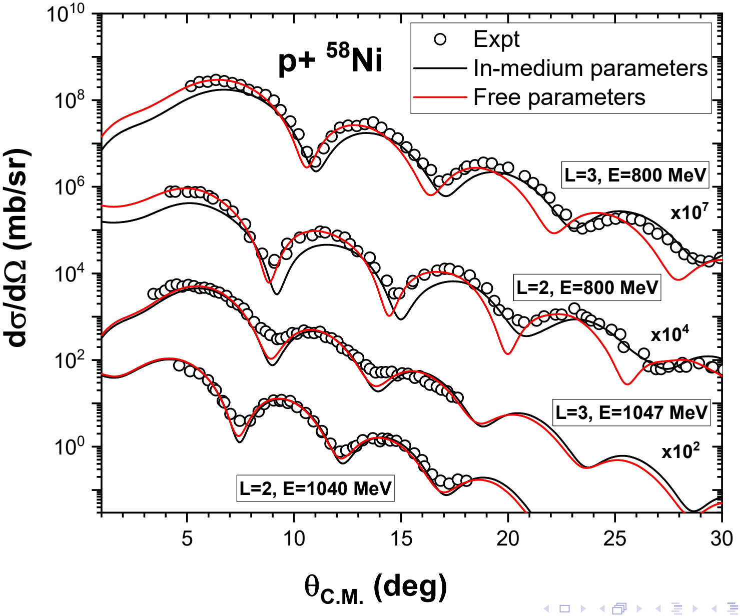
Elastic Angular Distributions



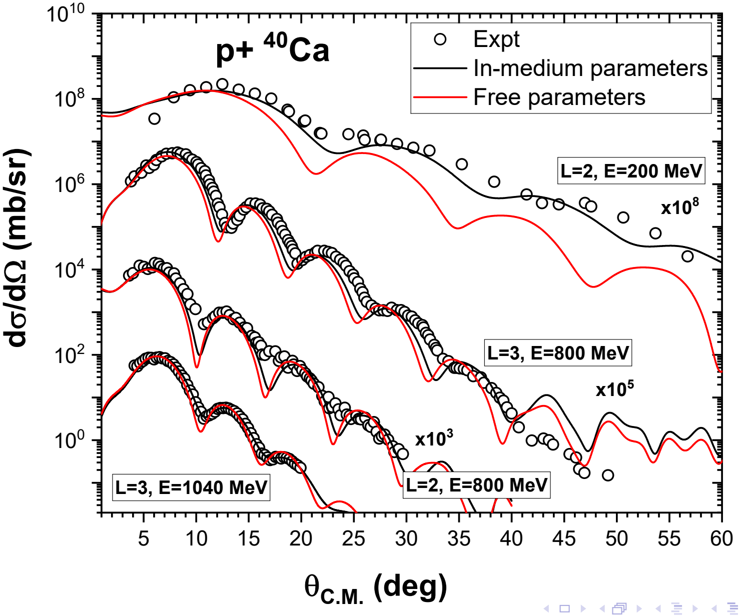
Inelastic Angular Distributions



Inelastic Angular Distributions



Inelastic Angular Distributions



Energy dependence of Potential Parameters; σ, α, β

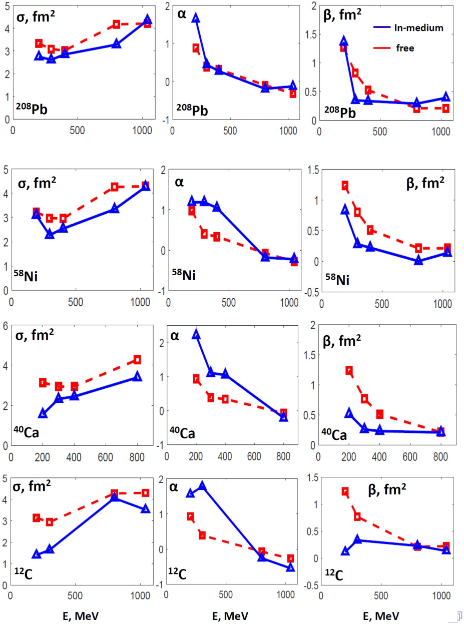


Table: Microscopic Optical Potential parameters and Deformation ones

	E, MeV	σ, fm^2		α		β, fm^2		α_2	α_3
		free	medium	free	medium	free	medium		
^{208}Pb	200	3.3332	2.7438	0.8835	1.6429	1.2654	1.3623		0.066
	295	3.0828	2.6153	0.3749	0.4440	0.8234	0.3463		0.079
	400	3.0243	2.8497	0.3180	0.2704	0.5239	0.3334		
	800	4.1706	3.2818	-0.0975	-0.1932	0.2067	0.2873	0.069	0.108
	1040	4.2079	4.3440	-0.3141	-0.1275	0.2067	0.3887		0.096
^{58}Ni	192	3.2185	3.0978	0.9765	1.1869	1.2364	0.8273		
	295	2.9717	2.2743	0.4026	1.1838	0.8014	0.2766		
	400	2.9566	2.5378	0.3375	1.0439	0.5115	0.2198		
	800	4.2571	3.3351	-0.0745	-0.1837	0.2110	0.0001	0.157	0.115
	1040	4.2835	4.2571	-0.2814	-0.2222	0.2110	0.1352	0.191	0.142
^{40}Ca	200	3.1366	1.5485	0.9300	2.2126	1.2400	0.5126	0.069	
	300	2.9365	2.3185	0.3879	1.1013	0.7663	0.2581		
	400	2.9434	2.4382	0.3412	1.0544	0.5091	0.2289		
	800	7.2740	3.3866	-0.0700	-0.2114	0.2100	0.2065	0.115	0.303
^{12}C	200	3.1366	1.3995	0.9300	1.5583	1.2400	0.1125		
	300	2.9365	1.6368	0.3879	1.7776	0.7663	0.3302		
	800	4.2740	4.0528	-0.0700	-0.2606	0.2100	0.2271		
	1040	4.2982	3.5156	-0.2750	-0.5450	0.2210	0.1328		

Conclusion

- The **HEA-based microscopic model of OP provides good agreement with experimental data** of **proton-scattering** on target nuclei ^{208}Pb, ^{58}Ni, ^{40}Ca and ^{12}C at energies between 200 and 1000 MeV.

- The peculiarity of this OP is that the target nucleon under consideration is not a free, but bounded, and therefore the fitting parameters obtained from proton-nucleus scattering data do not coincide with those obtained from proton scattering on free nucleons.

Nucleus-Nucleus scattering

Now, we are **developing our approach** to study **Nucleus-Nucleus** scattering. So, we **derived the microscopic optical potential of nucleus-nucleus interaction** which reads as

$$U(r) = V(r) + iW(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2}\sigma(\alpha + i)\int_0^\infty j_0(qr)\ \rho^p(q)\ \rho^t(q)\ f_N(q)\ q^2 dq. \quad (19)$$

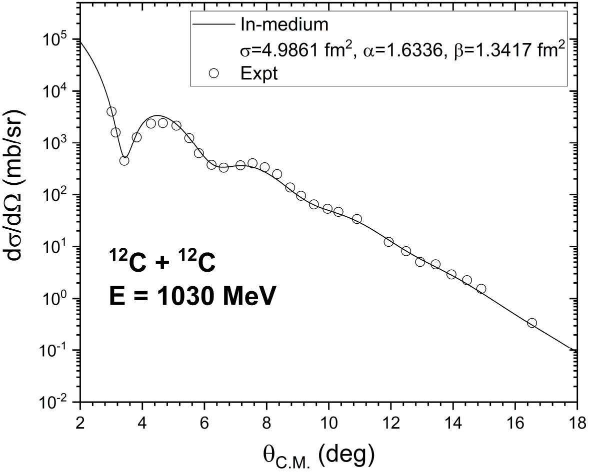
Here $\rho^p(q)$ and $\rho^t(q)$ are the ground state form factors of projectile and target nuclei, respectively. Also, the **transition potential** becomes

$$U_\lambda(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2}\sigma(\alpha + i)\int_0^\infty j_\lambda(qr)\ \rho^p(q)\ \rho^t_\lambda(q)\ f_N(q)\ q^2 dq. \quad (20)$$

Here we considered that the **target nucleus is only excited to a collective state**, λ, in this transition potential.

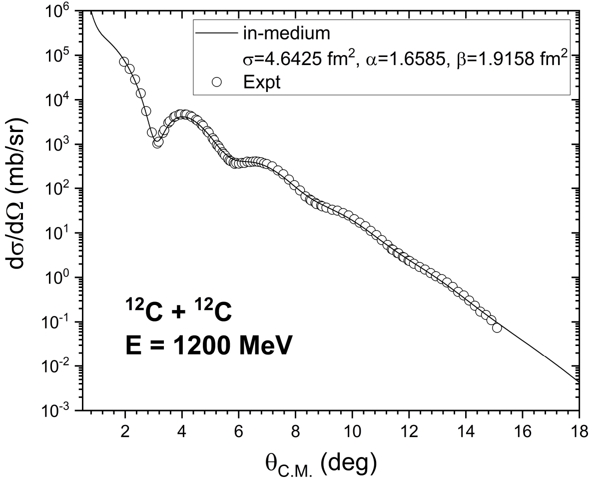
Results under consideration

Elastic Nucleus-Nucleus scattering

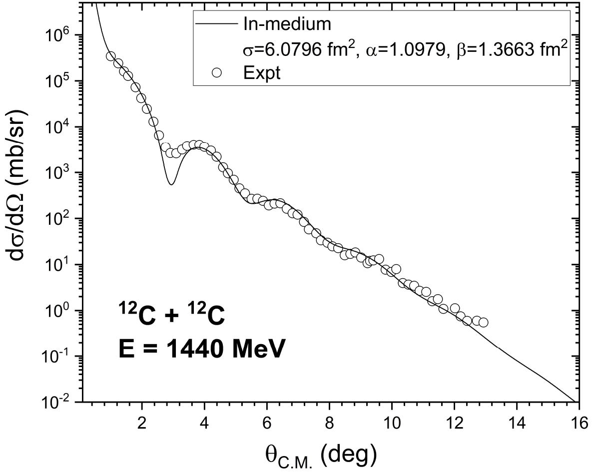


Results under consideration

Elastic Nucleus-Nucleus scattering

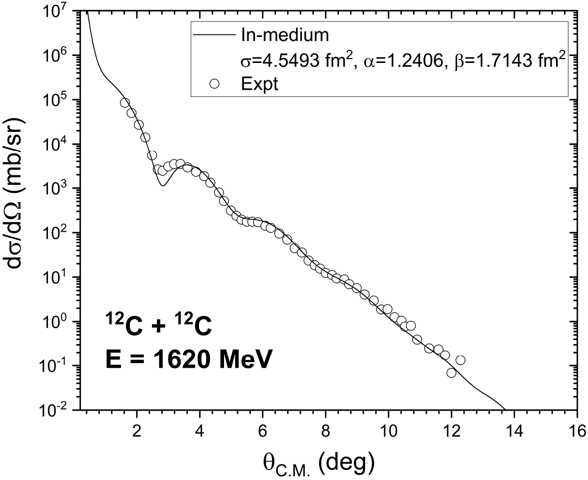


Results under consideration
Elastic Nucleus-Nucleus scattering

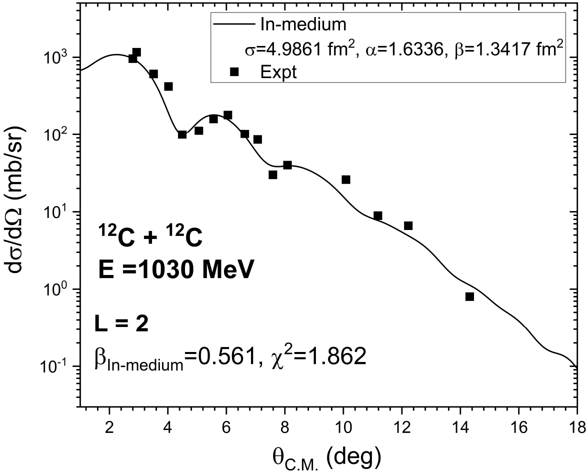


Results under consideration

Elastic Nucleus-Nucleus scattering

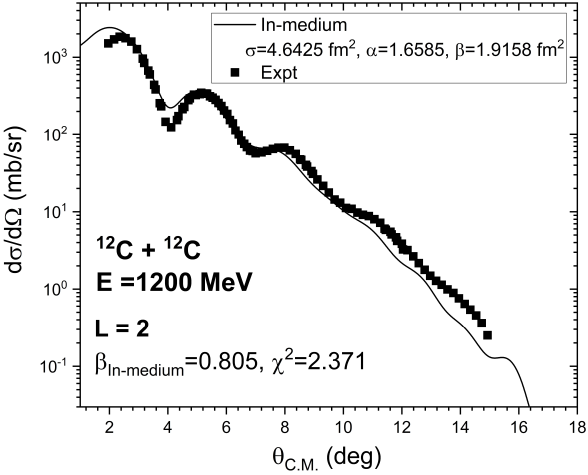


Results under consideration
Inelastic Nucleus-Nucleus scattering

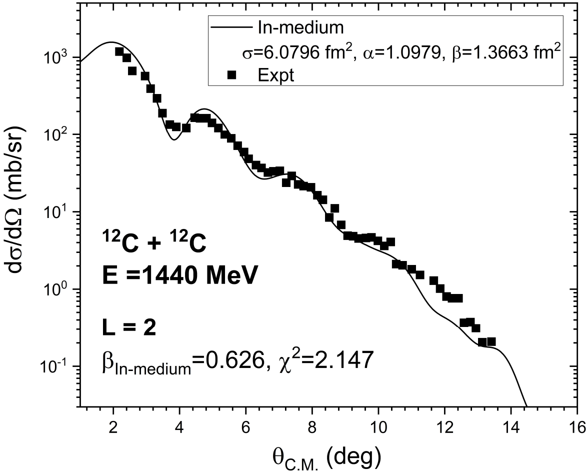


Results under consideration

Inelastic Nucleus-Nucleus scattering



Results under consideration
Inelastic Nucleus-Nucleus scattering



Main Conclusion

Our approach is successful in describing proton-nucleus and nucleus-nucleus scattering at high incident energies.

Acknowledgments

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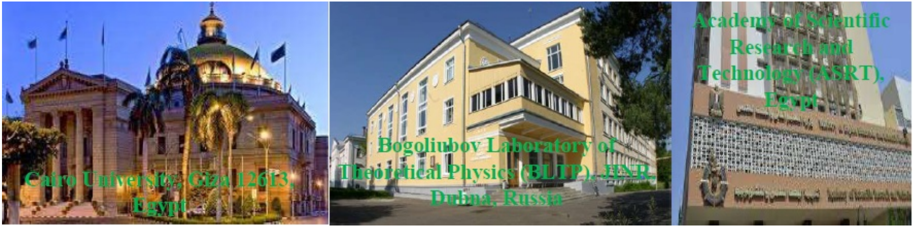

Thank you for your attention!