

Size, Shape and Deformation of Nuclei

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- The model: Strongly Correlated Quark Model (SCQM)
 - Nucleon Structure
 - Nuclear Structure: SCQM+FCC
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Motivation

Nuclear Size and Shape

Experimental Observations

- Compactness of ${}^4\text{He}$ and a hole inside it
- Halo nuclei: the radius of halo nucleus appreciably larger than that predicted by the liquid drop model
- Neutron skin
- Fluctuation of the central nuclear matter density distribution

Motivation

Nuclear Size and Shape

Experimental Observations

- Compactness of and a hole inside ${}^4\text{He}$

Point-nucleon charge distributions of ${}^3\text{He}$ and ${}^4\text{He}$
Hole inside ${}^3\text{He}$ and ${}^4\text{He}$

I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236

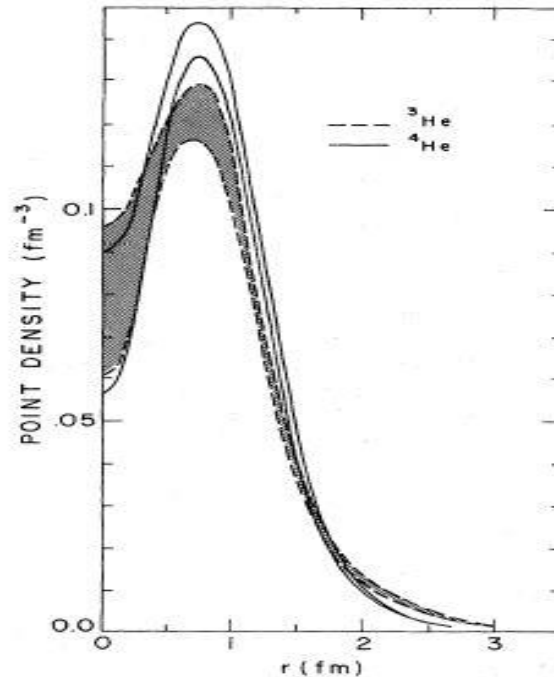


FIG. 15. Model-independent densities of pointlike protons in ${}^3,{}^4\text{He}$.

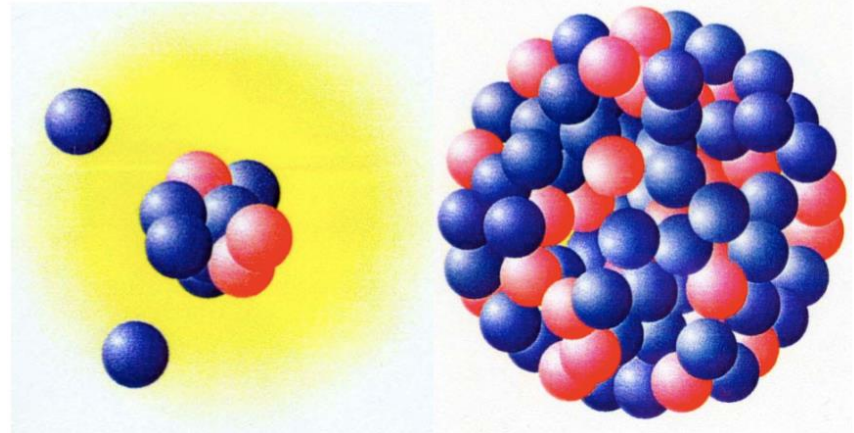
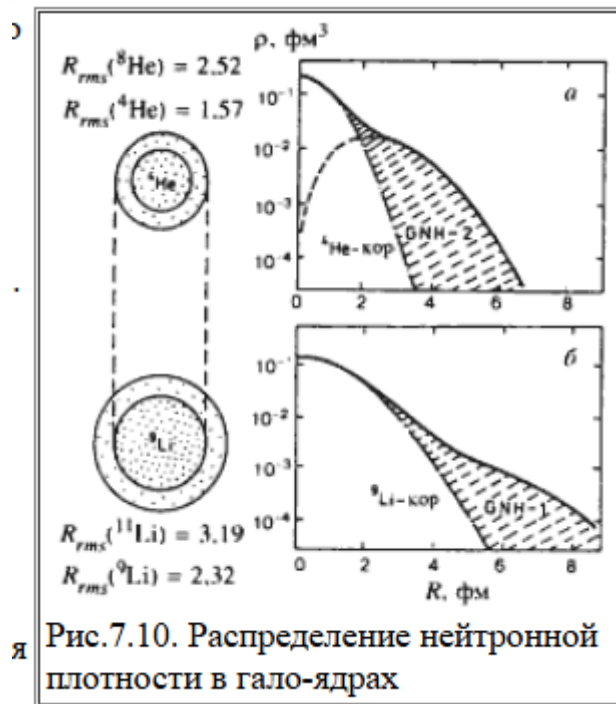
Motivation

Nuclear Size and Shape

Experimental Observations

- Halo nuclei: the radius of halo nucleus R_{halo} appreciably larger than that predicted by the liquid drop model
- Halo nuclei: ${}^6\text{He}$, ${}^8\text{He}$, ${}^{11}\text{Li}$, ...

$$R_{halo} \gg 1.3 A^{1/3}$$



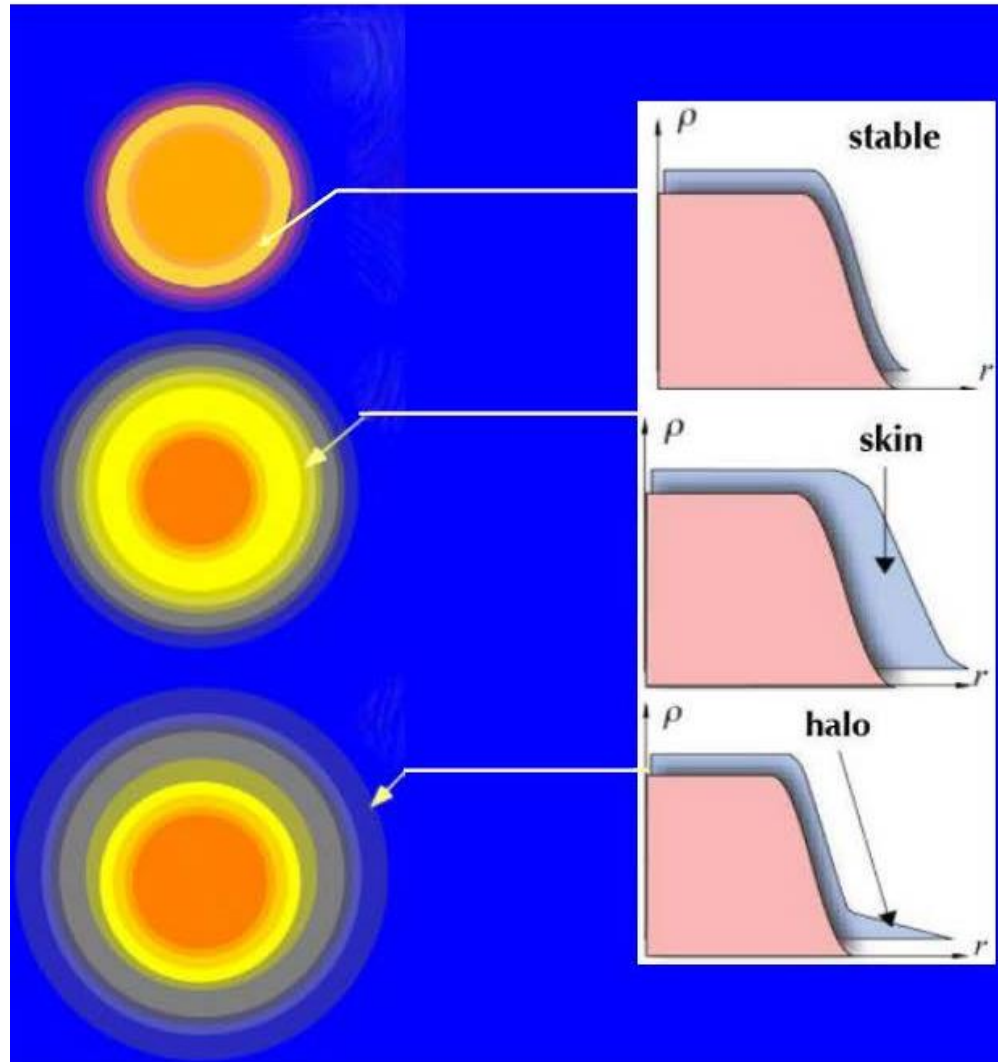
${}^{11}\text{Li}$

${}^{208}\text{Pb}$

Motivation

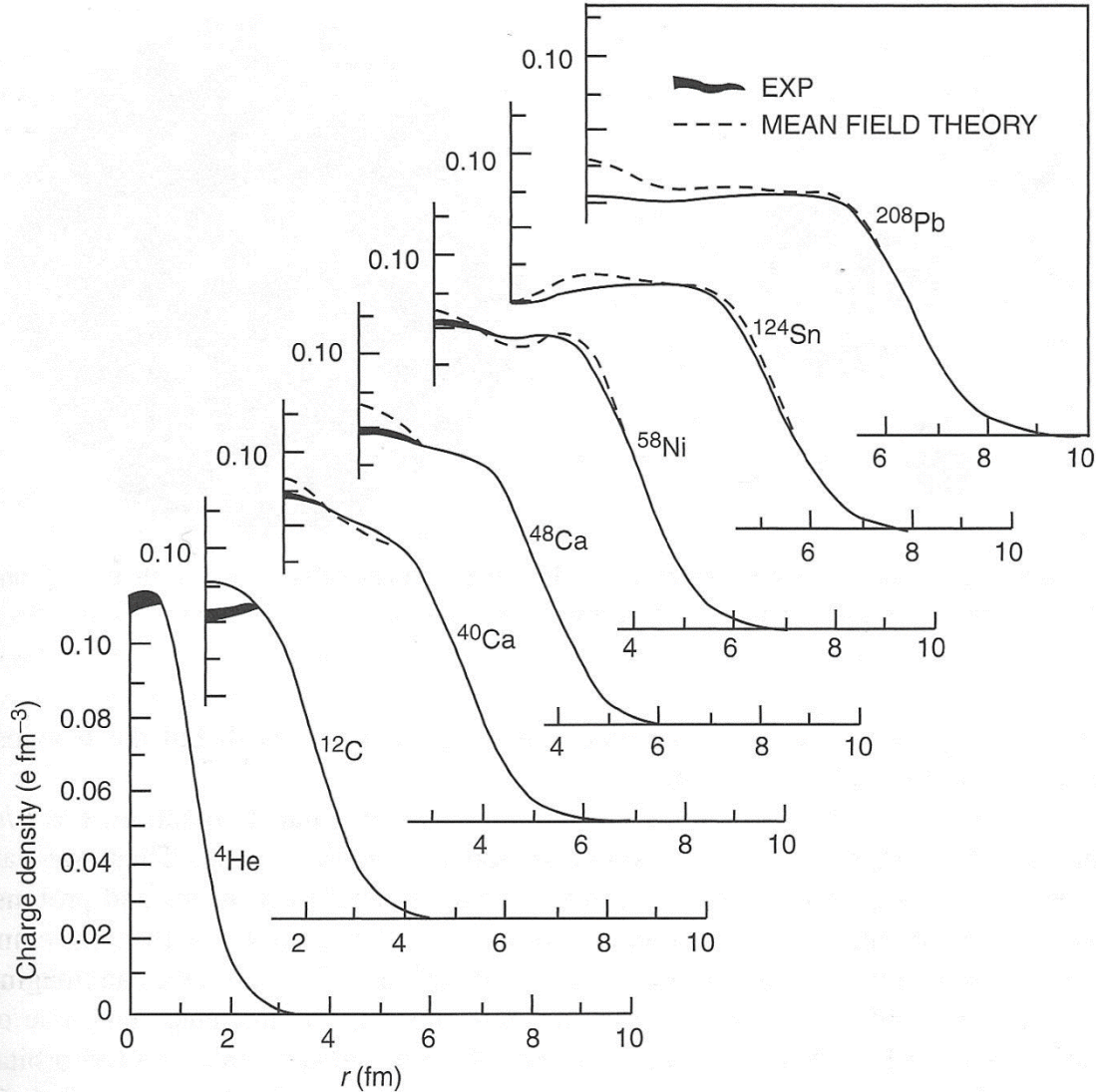
Nuclear Size and Shape

Neutron skin



Motivation

Fluctuation of central nuclear density



Motivation

Nuclear Deformation

Deformation of colliding nuclei leads to increasing fluctuations of many observables

- multiplicities,
- centrality estimation,
- reaction plane estimation
- direct, elliptic, ... flows,
- ...


For example, Multiplicity Fluctuations

Total multiplicity:
$$N = \sum_{i=1}^{N_s} m_i$$
 N_s – number of sources
 m_i - multiplicity from a single source

Mean multiplicity:
$$\langle N \rangle = \langle N_s \rangle \langle m \rangle$$

$$\frac{\sigma_N^2}{\langle N \rangle} = \frac{\sigma_m^2}{\langle m \rangle} + \langle m \rangle \frac{\sigma_{N_s}^2}{\langle N_s \rangle}$$

Shapes of nuclei
Geometry of collision



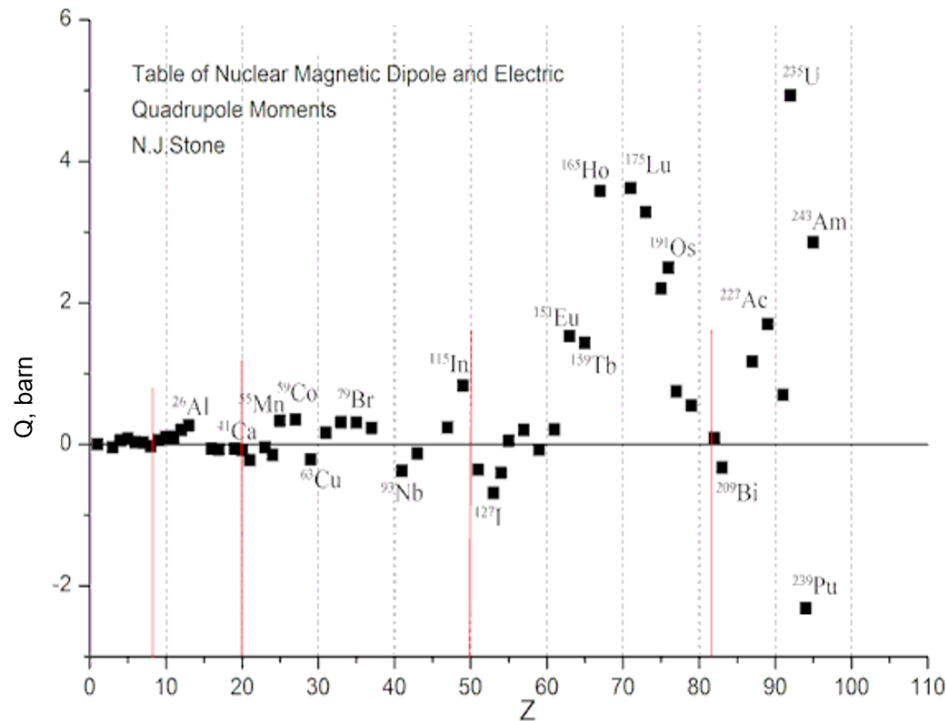
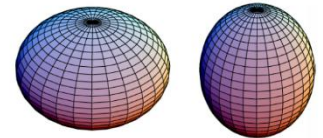
Motivation

Nuclear Deformation

Experiment

All nuclei are deformed

- The simplest deformation: **electric** quadrupole deformation



- Nuclear deformation is much more complicated: **multipole** deformations
- Nothing is known about deformation of the **neutron matter**

Diversity of models of Nuclear Structure

Nuclear Models in terms of nucleons and mesons

- Conventional models
 - Independent Particle Models (Shell Model, ...)
 - Collective models (Liquid Drop Model, ...)
 - Cluster models
 - Modifications of above models
- Non-conventional model
- There are more than 40 models ... (*W.Greiner et al.*)

Effective Field Theories, EFT

QCD \rightarrow CSB: quark, gluon fields \rightarrow meson fields

Diversity of models of Nuclear Structure

Is it possible to build a model composing the features of all conventional models?

Do quarks manifest themselves explicitly in nuclear structure?

Yes

It is possible!

Conventional models

- Shell Model.
- Liquid Drop Model
- Cluster models

FCC –Face-Centered Cubic Model
based on
SCQM

SCQM – Strongly Correlated Quark Model of Nucleon Structure

*G. Musulmanbekov, “Quarks as Vortices in Vacuum”
in book *Frontiers of Fundamental Physics*,
Kluwer Acad./Plenum Pub., 2001, p. 109-120.*

*G. Musulmanbekov, “Hadron Modifications in a Dense Baryonic Matter”
PEPAN Lett., Vol., № 5, p. 548-558*

SCQM

Motivation

proton-proton interactions

- soft elastic scattering
- hard elastic scattering
- single diffractive scattering
- double diffractive scattering
- inelastic non-diffractive scattering

QCD – fundamental theory of strong interactions

- **Constituents of hadrons – quarks** of different flavors carrying spin, charge, color.
 - **flavors: u, d, s, c, b, t**
 - **spin: $1/2$**
 - **charge: $1/3$, $2/3$**
 - **color: $SU(3)_{\text{Color}}$ - $R, G, B, \bar{R}, \bar{G}, \bar{B}$**
- **Fields – gluons** – perform interactions between quarks.
- **Nucleons** – 3–quark (**u/d**), color-singlet systems
- **Mesons** – quark-antiquark systems

QCD (cont.)

QCD is non-abelian theory

Hadronic processes with high Q^2

pQCD: $\alpha_s < 1$, $m_q \rightarrow 0$, **chiral symmetry**

Low energy hadron and nuclear physics

non-pQCD: $\alpha_s > 1$, $m_q \neq 0$, **chiral symmetry breaking**

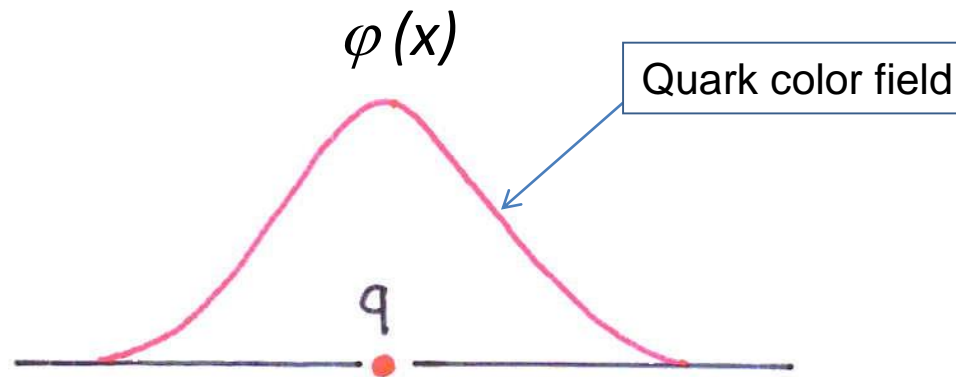
- Low energy approx. of QCD, effective theories., ...
- QCD–inspired phenomenology
 - NR constituent quark models
 - Bag models
 - Chiral quark models
 - Soliton models

“Elementary particles are
no more than holes in
vacuum.”

Henry Poincare

SCQM

Single Colored Quark inside Vacuum

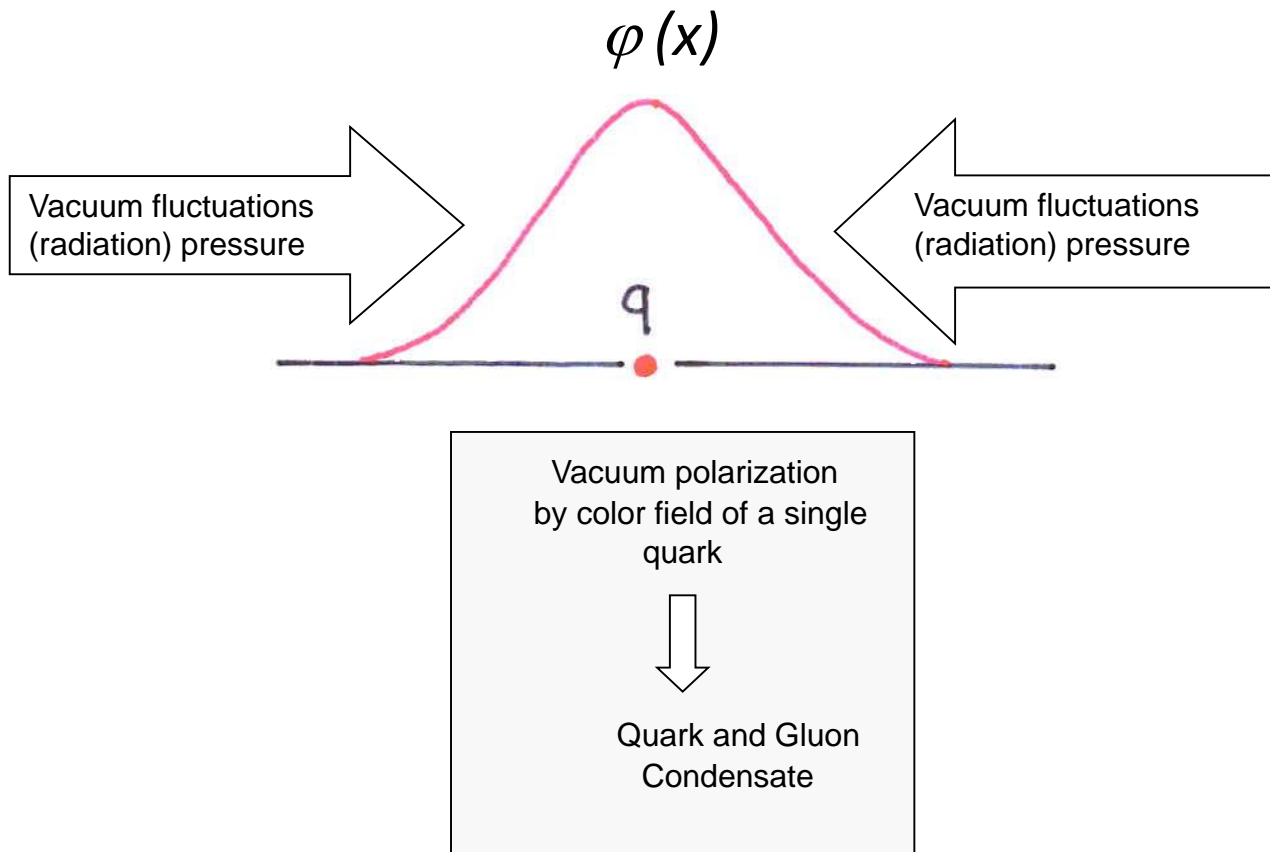


Vacuum polarization
by color field of a
single quark



Quark and Gluon
Condensate

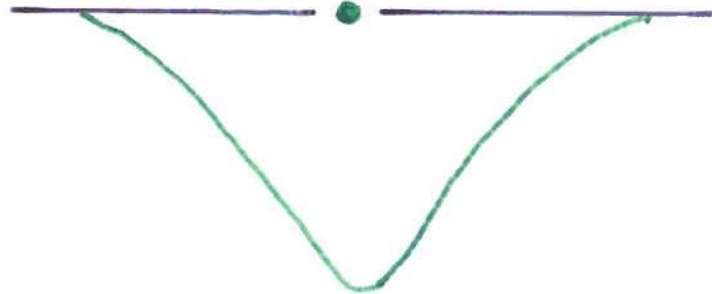
Strongly Correlated Quark Model (SCQM)



Strongly Correlated Quark Model (SCQM)

$\varphi(x)$

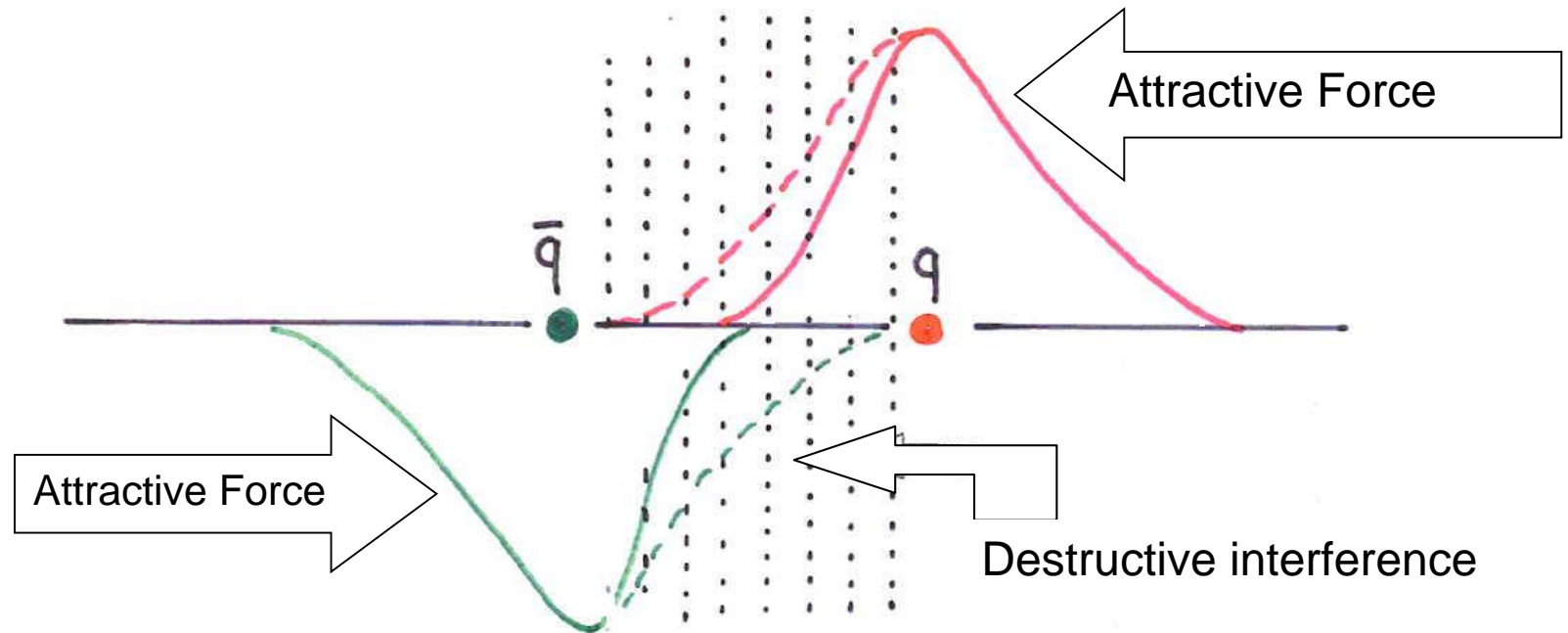
\bar{q}



Vacuum fluctuations
(radiation) pressure

Vacuum fluctuations
(radiation) pressure

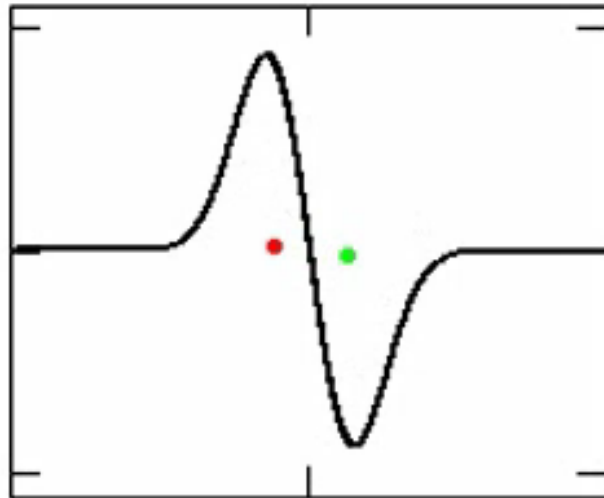
Strongly Correlated Quark Model (SCQM)



Overlap of opposite color fields \rightarrow attraction force between quark and antiquark
“Color Casimir” effect

quark – antiquark pair

$\varphi(x,t)$



Quarks – Solitons

SCQM \equiv Breather Solution of Sine-Gordon equation

$$\phi(x,t) = \alpha \cos\left(\frac{2\pi x}{\lambda} - \frac{2\pi ct}{\lambda}\right)$$

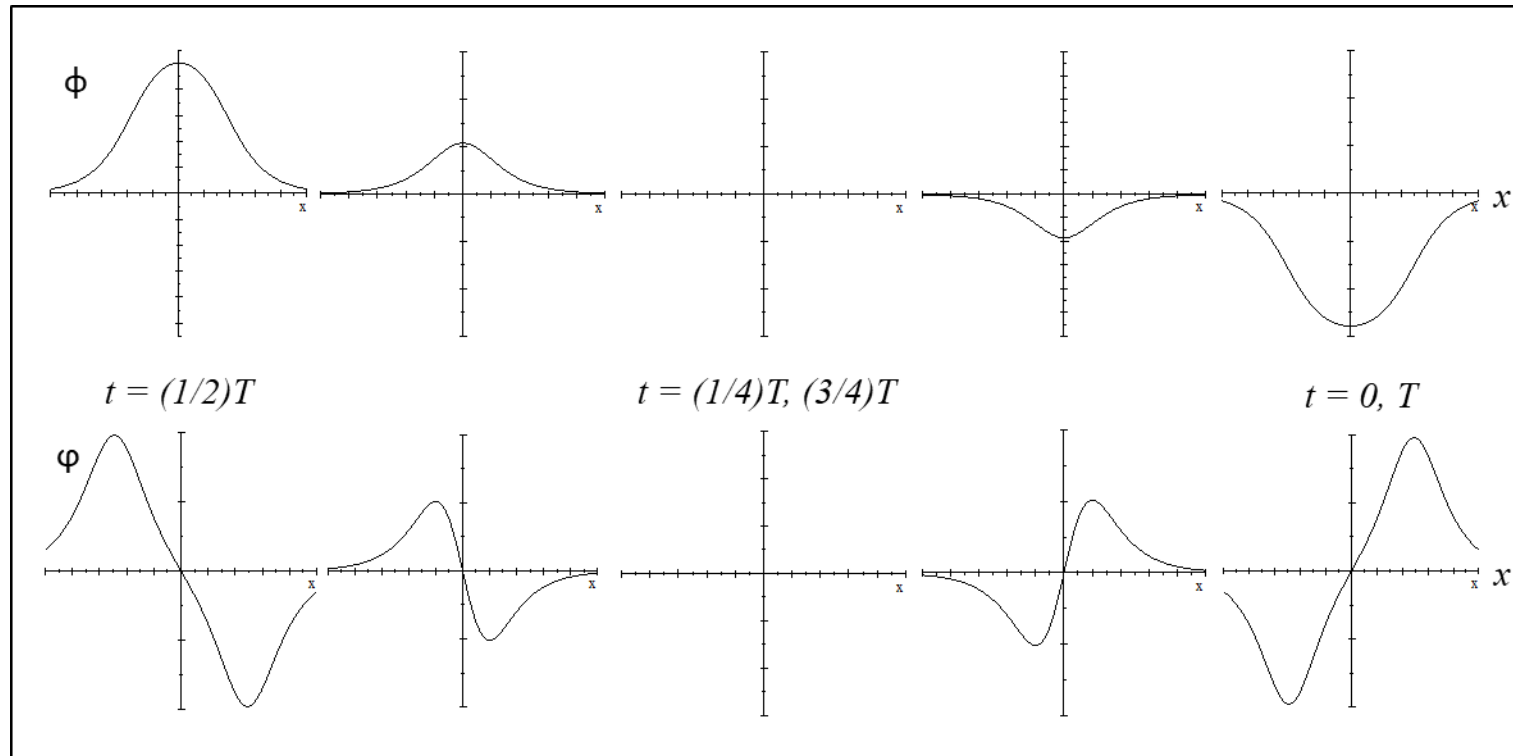
Breather – oscillating soliton-antisoliton pair, the periodic solution of SG:

$$\phi_{sol} = A \tan\left[\frac{S}{\alpha} \left(\frac{x - vt}{\lambda} \right) \right]$$

$$\phi_{sol} = \frac{\phi_{sa}}{\alpha}$$

is **identical** to our quark-antiquark system.

Breather, $\phi(x, t)$ non-linear “Standing wave”



The SCQM

Hamiltonian of the quark – antiquark system

$$H = \frac{m_{\bar{q}}}{\sqrt{1-\beta^2}} + \frac{m_q}{\sqrt{1-\beta^2}} + V_{\bar{q}q}$$

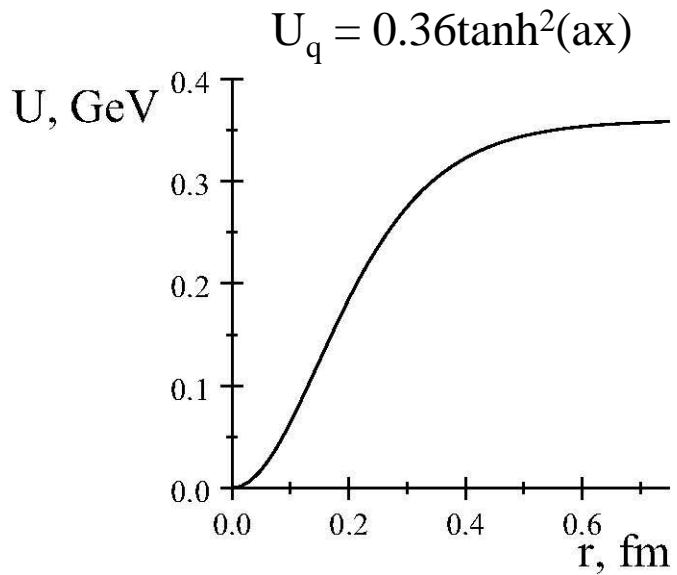
$m_{\bar{q}}, m_q$ - current masses of quarks,
 $\beta = \beta(\mathbf{x})$ - velocity of the quark (antiquark),
 $V_{\bar{q}q}$ - quark–antiquark potential.

$$U(x) = \frac{1}{2} V_{\bar{q}q}(2x)$$

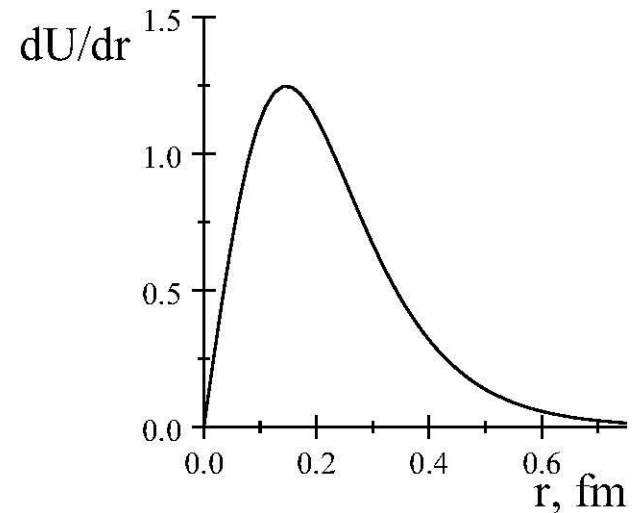
$U(x) = \frac{1}{2} V_{\bar{q}q}(2x)$ is the potential energy of a single quark/antiquark.

$$U(x) = \frac{1}{2} V_{\bar{q}q}(2x) = m \tanh^2(ax)$$

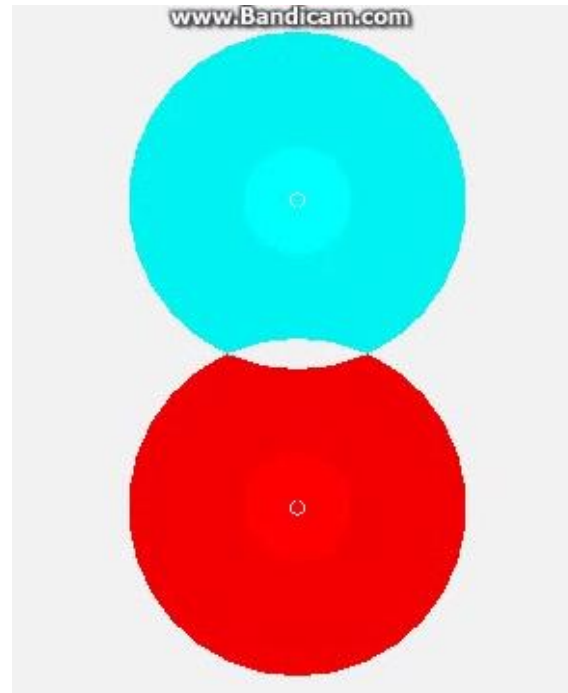
Quark Potential



Force of quark-antiquark interaction



quark–antiquark pair meson



QCD: Exchange by gluons $\frac{1}{\sqrt{2}}(R\bar{R}+B\bar{B})$

SCQM: Overlap of color fields

Generalization to the 3 – quark system (baryons)

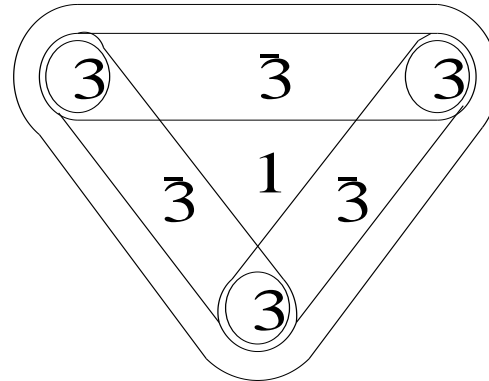
$SU(3)_{Color}$

$q \Rightarrow SU(3) \Leftrightarrow RGB \quad q \Rightarrow SU(3) \Leftrightarrow CMY$

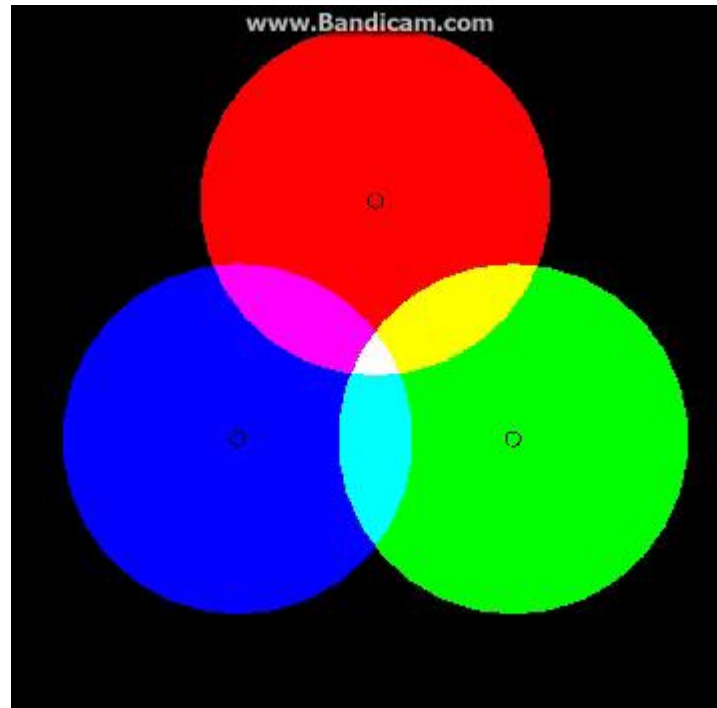


~~$q\bar{q}q\bar{q}$~~ = $\bar{q} \rightarrow qq$

$qqq \Rightarrow$

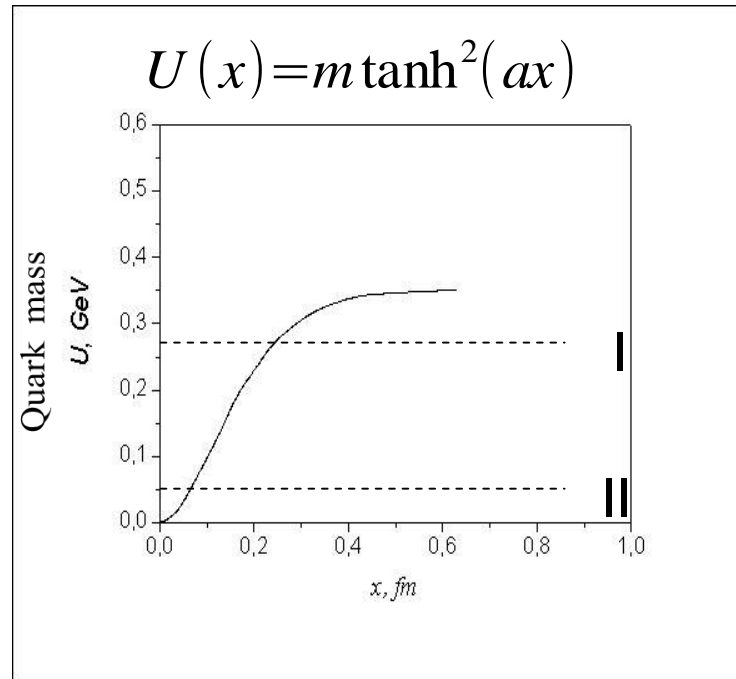


Nucleon as 3 oscillating color quarks



“The wave packet solution of time-dependent Schrodinger equation for harmonic oscillator moves in exactly the same way as corresponding classical oscillator”
E. Schrodinger, 1926

Dynamic Breaking-Restoration of Chiral Symmetry



$U(x) > I$ – constituent quarks

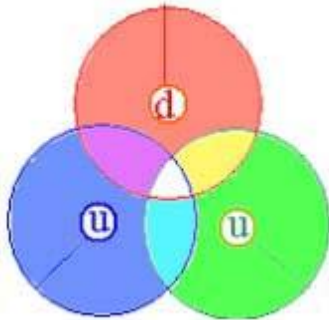
$U(x) < II$ – current (relativistic) quarks

Interplay between constituent and current quark states

Chiral Symmetry Breaking \longleftrightarrow

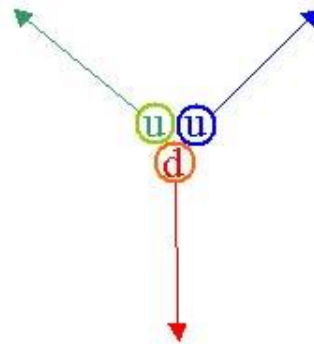
Restoration

$t = 0$
 $x = x_{max}$



Constituent quarks

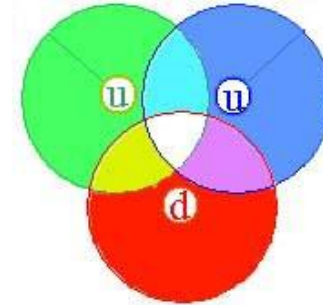
$t = T/4$
 $x = 0$



current quarks

Asymptotic freedom

$t = T/2$
 $x = -x_{max}$

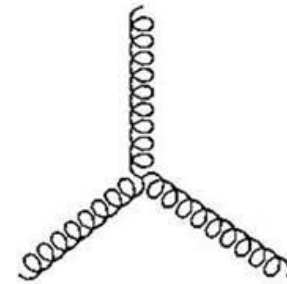
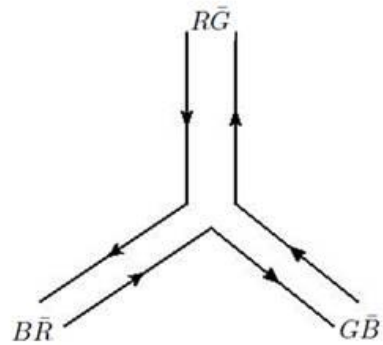
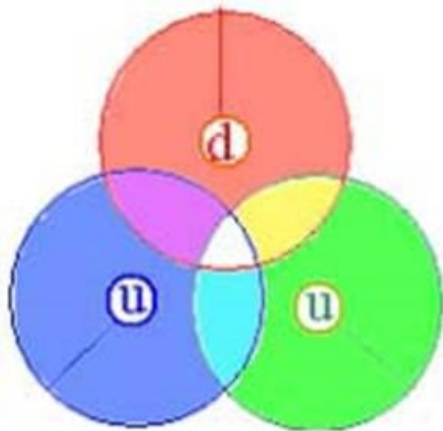


Constituent quarks

During the valence quarks oscillations:



SCQM vs QCD



Parameters of SCQM for the Nucleon

1. Mass of Constituent Quark



2. Amplitude of VQs oscillations : $x_{max}=0.64 fm$,

3. Constituent quark dimensions (parameters of gaussian distribution): $\sigma_{x,y}=0.24 fm$, $\sigma_z=0.12 fm$

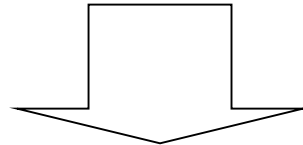
Parameters 2 and 3 are derived from comparison of **Inelastic Overlap Function (IOF)** and σ_{tot} in $p p$ and pp – collisions.

Nucleons are nonspherical!

They are three-colored objects!

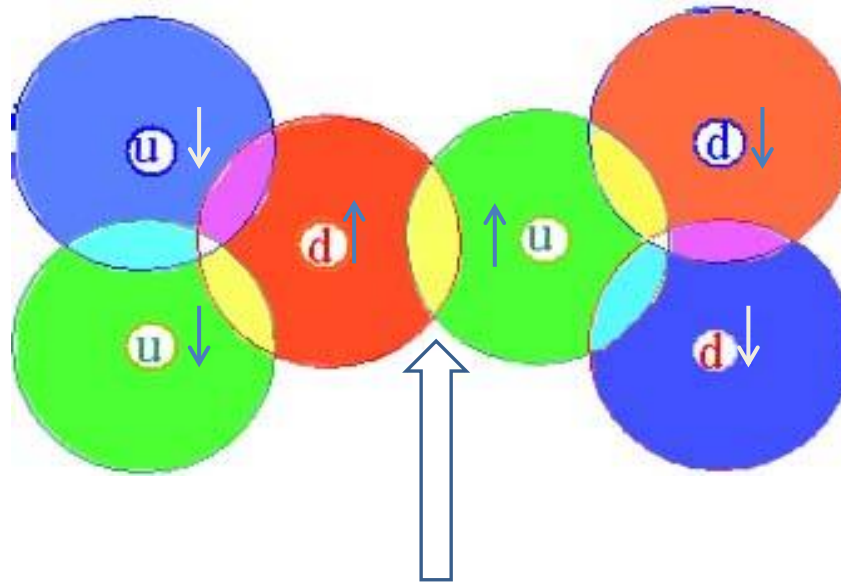
Quark Arrangements inside Nuclei

Strongly Correlated Quark Model

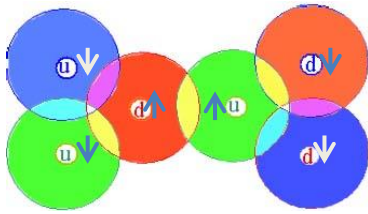


Lattice-like arrangement of Nuclear
Structure

Two Nucleon System in SCQM



Interaction between nucleons is due to **overlap** of their quark color fields



Antisymmetrization

We need to define isospins, spins and colors at junctions

${}^4\text{He}$: 4 nucleons = 12 quarks in s-state

Antisymmetrization

$$SU(12) \longrightarrow SU(2)_{\text{isospin}} \otimes SU(2)_{\text{spin}} \otimes SU(3)_{\text{color}}$$

But $\sim 90\%$ of 3-quark clusters are colored states (*Matveev, Sorba, 1978*)

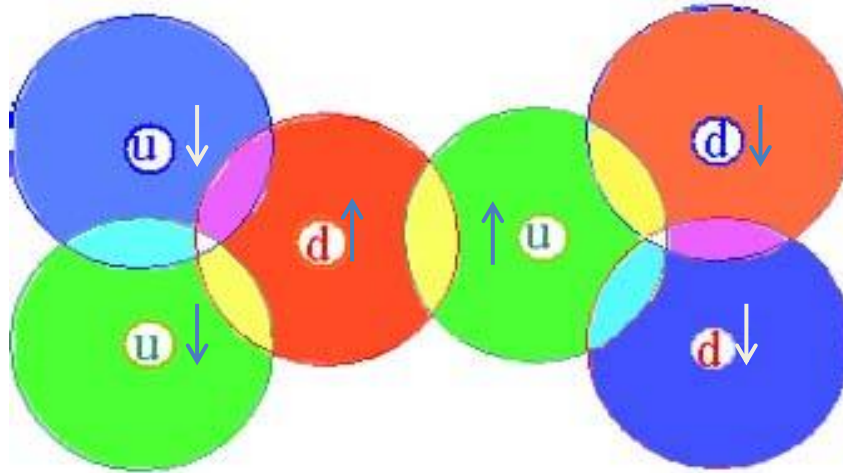
We select colorless 3-quark clusters by combinatorics imposing the following requirements to isospins, spins and colors at junctions:

$SU(2)_{\text{isospin}}$ – of different flavors (assumed)

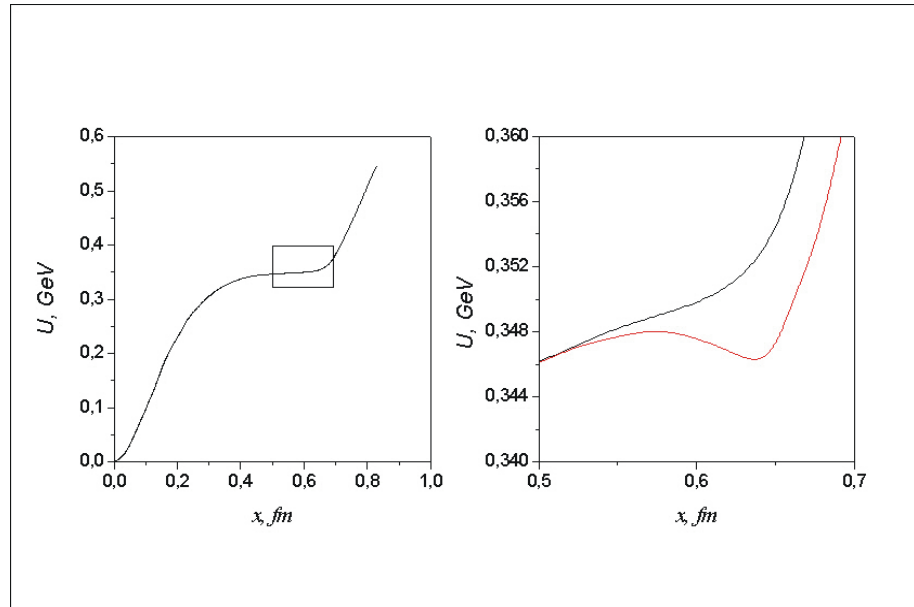
$SU(2)_{\text{spin}}$ – of parallel spins (calculated)

$SU(3)_{\text{color}}$ – of different colors (assumed)

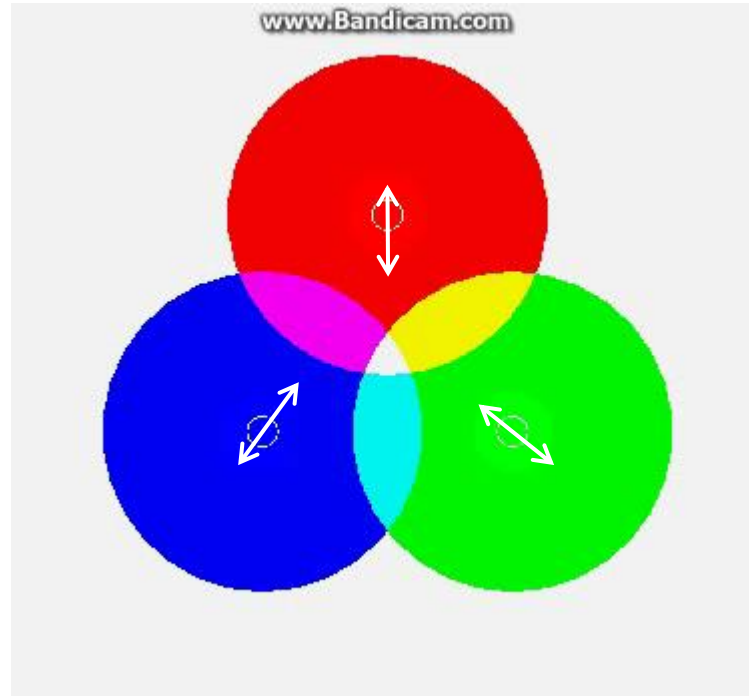
Two Nucleon System in SCQM



Quark Potential Inside Nuclei

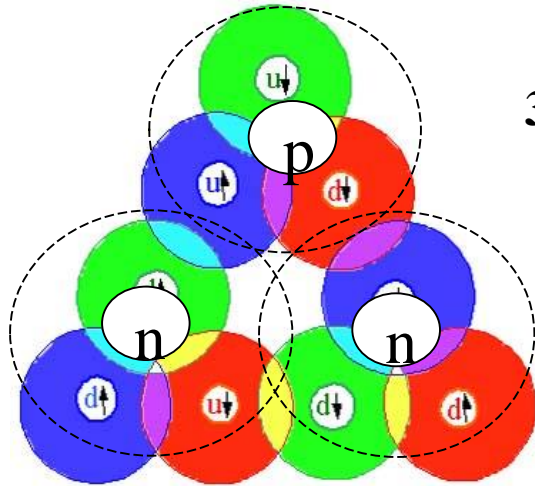


Quarks inside nucleus

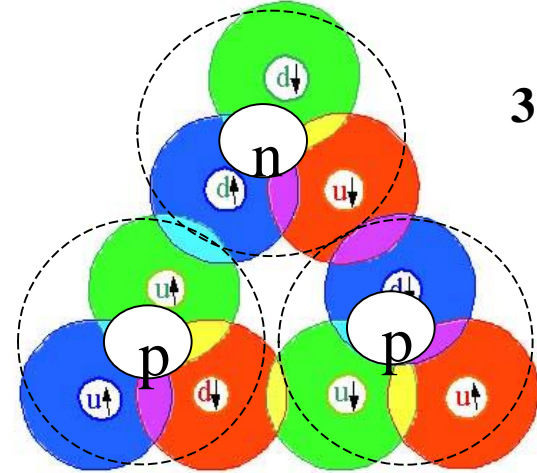


Quarks oscillate with small amplitudes
near maximal displacements

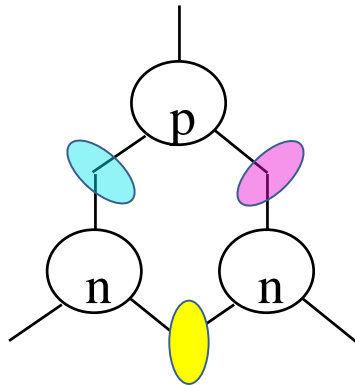
Three Nucleon Systems in SCQM



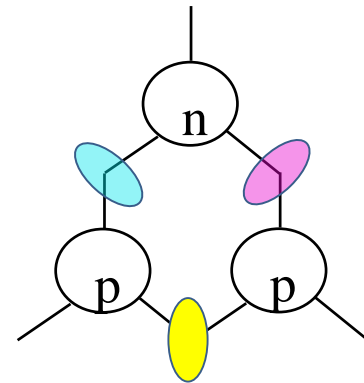
${}^3\text{H}$



${}^3\text{He}$



Summary color
of 3 junctions is white,
total color charge = zero!

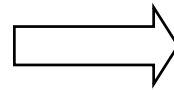


$$\overline{q}q\overline{q}q\overline{q}q = \overline{q} \rightarrow qq$$

Quark loop formed by 3 nucleons \rightarrow 3-body force

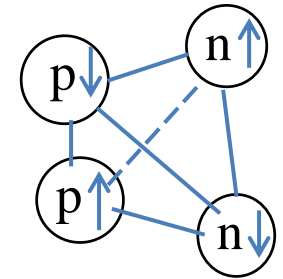
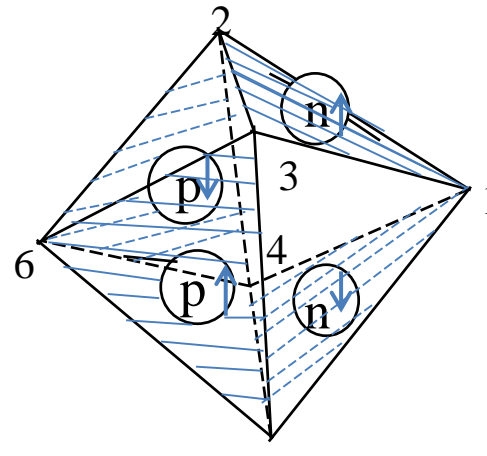
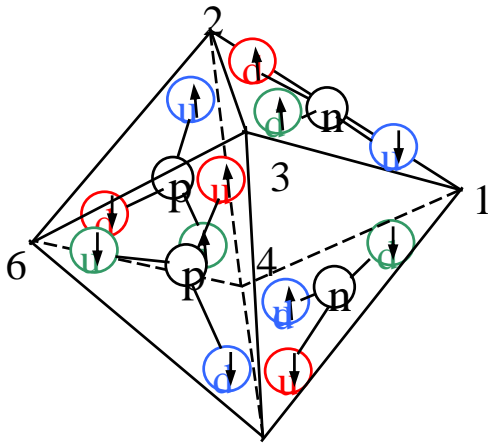
The closed shell $n = 0$, nucleus ${}^4\text{He}$

Antisymmetrisation of
12 quarks in $SU(12)$ state
 $SU(2)_I \times SU(2)_S \times SU(3)_C$



Totally antisymmetrized
4 nucleons in s -state

Shell Closure



Selection⁵ rules for binding two quarks of neighboring nucleons at a junction:

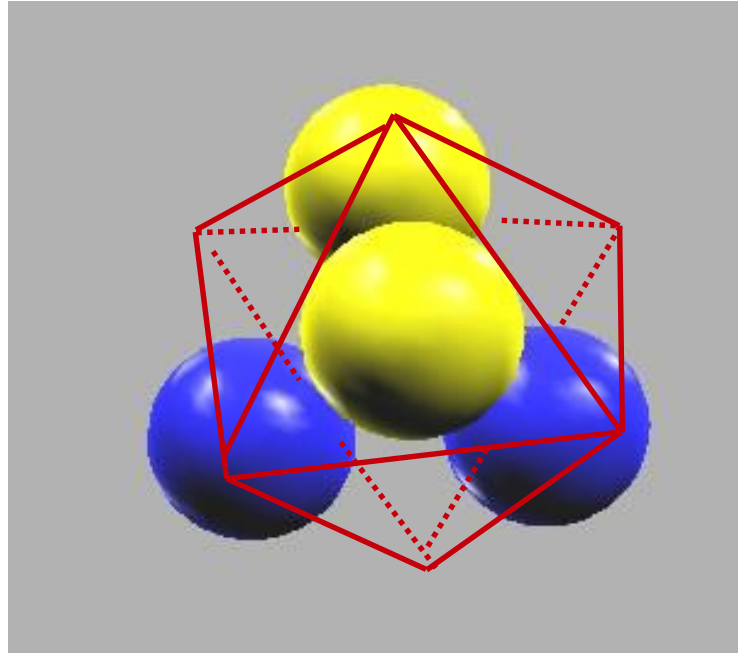
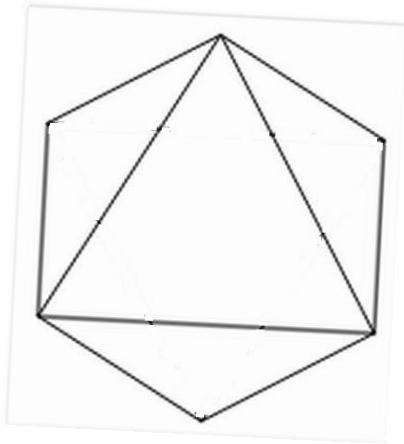
- $SU(2)_{\text{Isospin}}$ – of different flavors
- $SU(3)_{\text{Color}}$ – of different colors
- $SU(2)_{\text{Spin}}$ – of parallel spins

Experimental Binding Energy of Stable Nuclei and Quark Loops in SCQM

Nucleus	E_B , MeV/A Exp.	Number of quark loops	Free quark ends	Nuclear forces
d	1.1	0	4	2-body
^3H	2.83	1	3	3-body
^3He	2.57	1	3	3-body
^4He	7.07	4	0	4-body

The more quark loops, the more a binding energy!

The closed shell $n = 0$, nucleus ${}^4\text{He}$

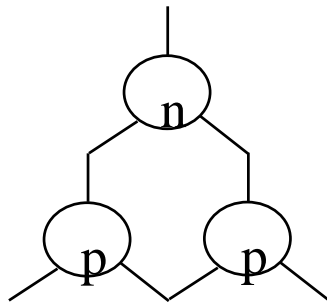
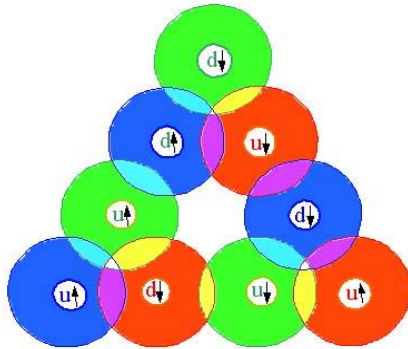


Yellow – protons are on opposite faces of upper piramid

Blue – neutrons are on another faces of below lower piramid

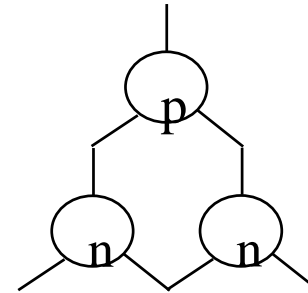
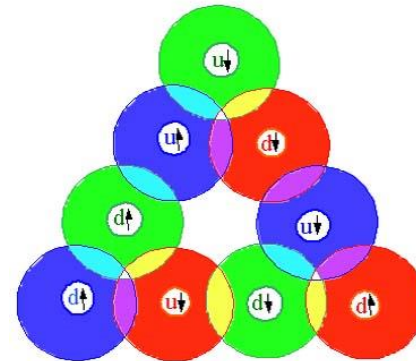
Building blocks in Shell Structure

${}^3\text{He}$



${}^3\text{He}$ – block

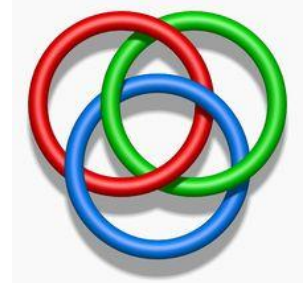
${}^3\text{H}$



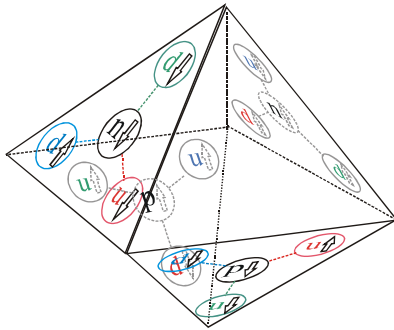
${}^3\text{H}$ – block

Forms Neutron Halo

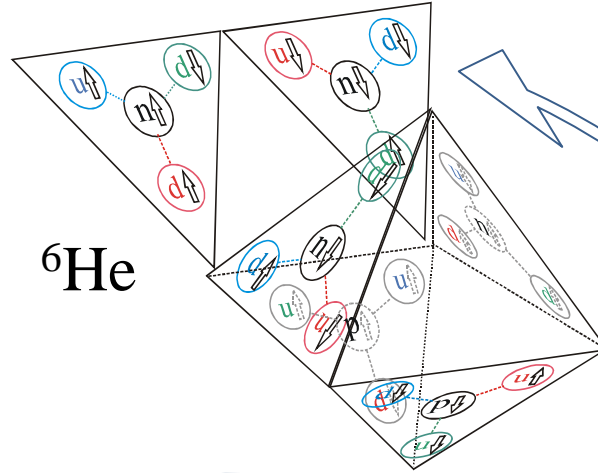
Helium Isotopes Borromean Nuclei



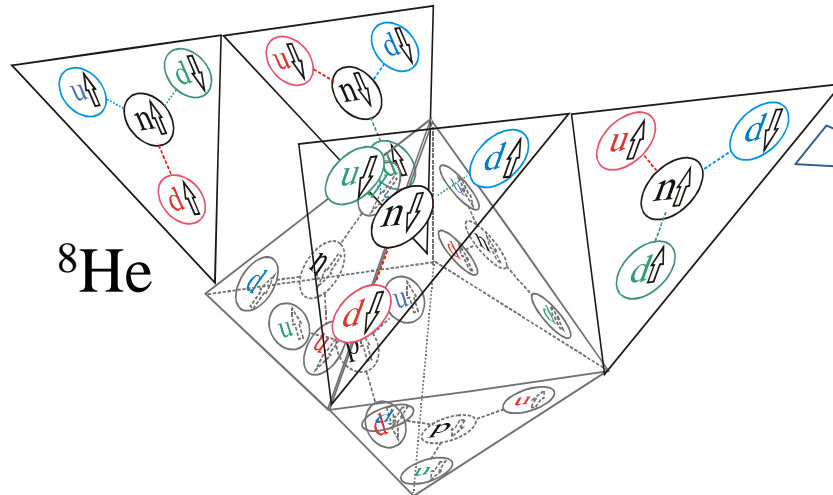
^4He
Core



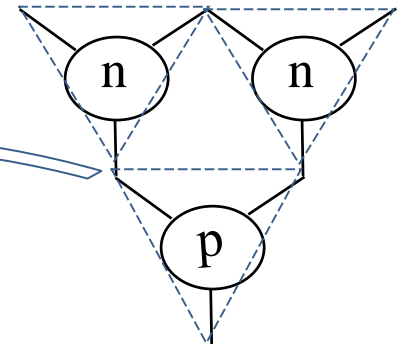
^6He



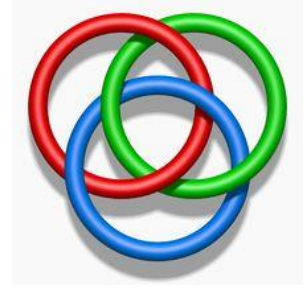
Quark loop



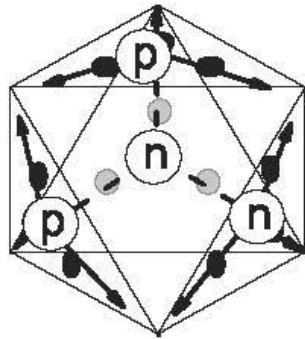
^8He



Helium Isotopes Borromean Nuclei

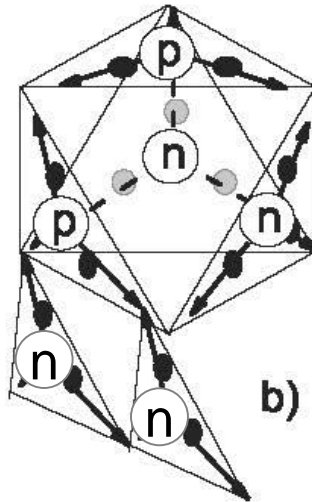


${}^4\text{He}$
Core



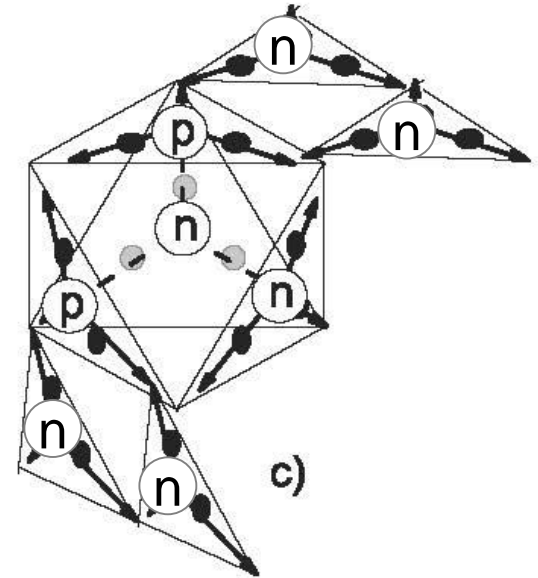
a)

${}^6\text{He}$



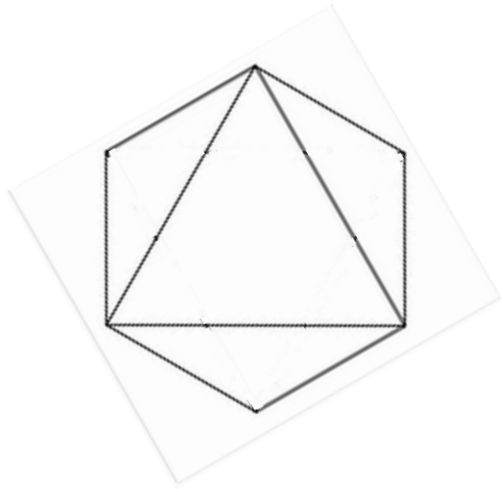
b)

${}^8\text{He}$

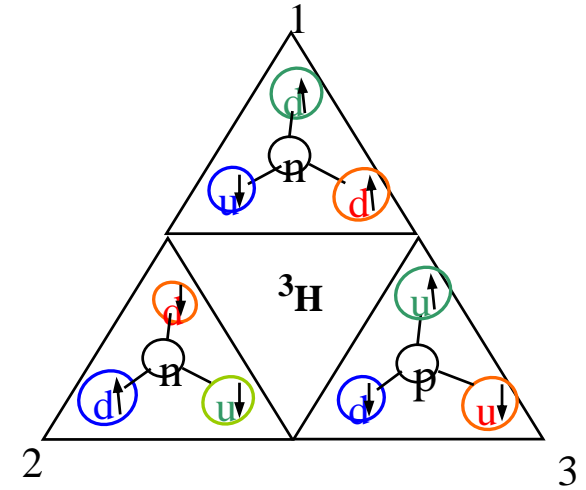
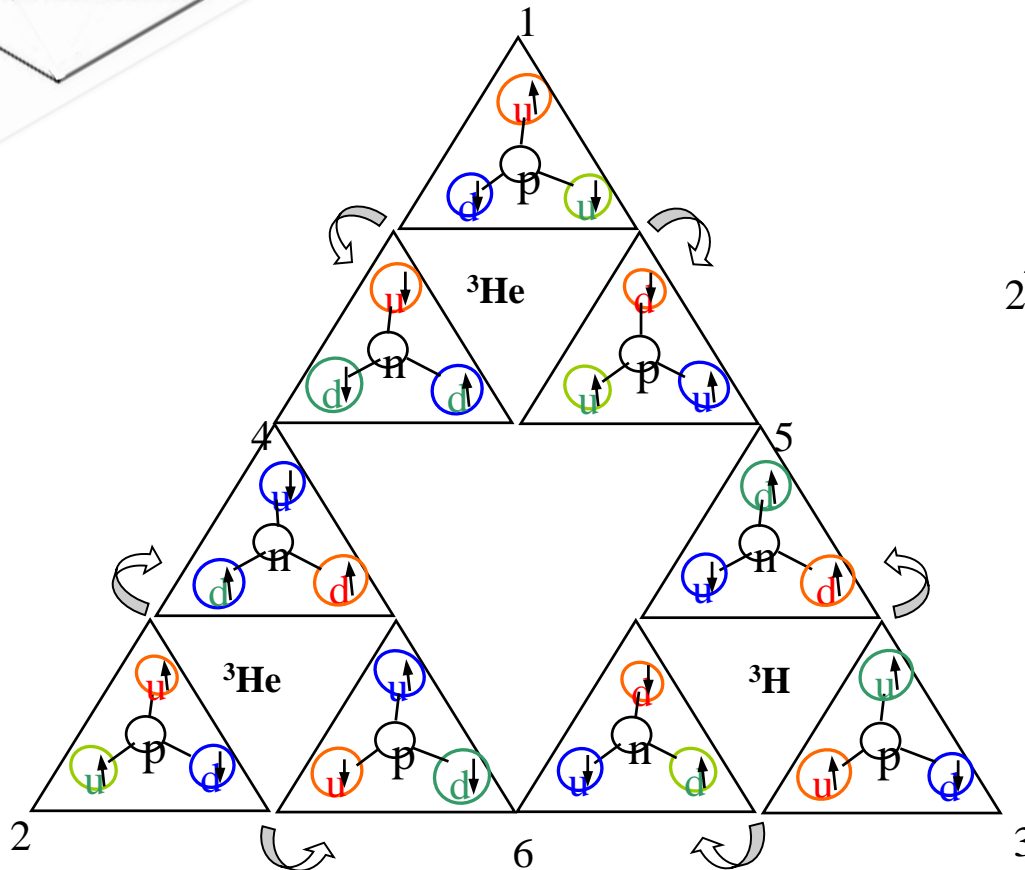


c)

Next closed shell $n = 1, {}^{16}\text{O}$

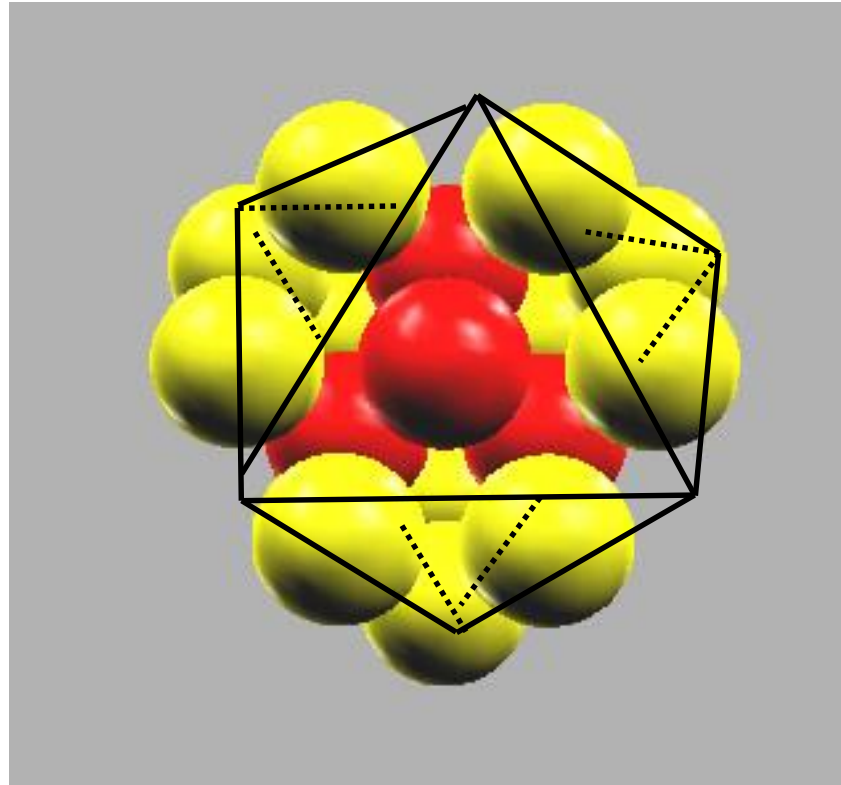


Face of ${}^{16}\text{O}$ octahedron



In analogy with ${}^4\text{He}$
 ${}^3\text{He}$ and ${}^3\text{H}$ as
 proton and neutron
 in ${}^4\text{He}$

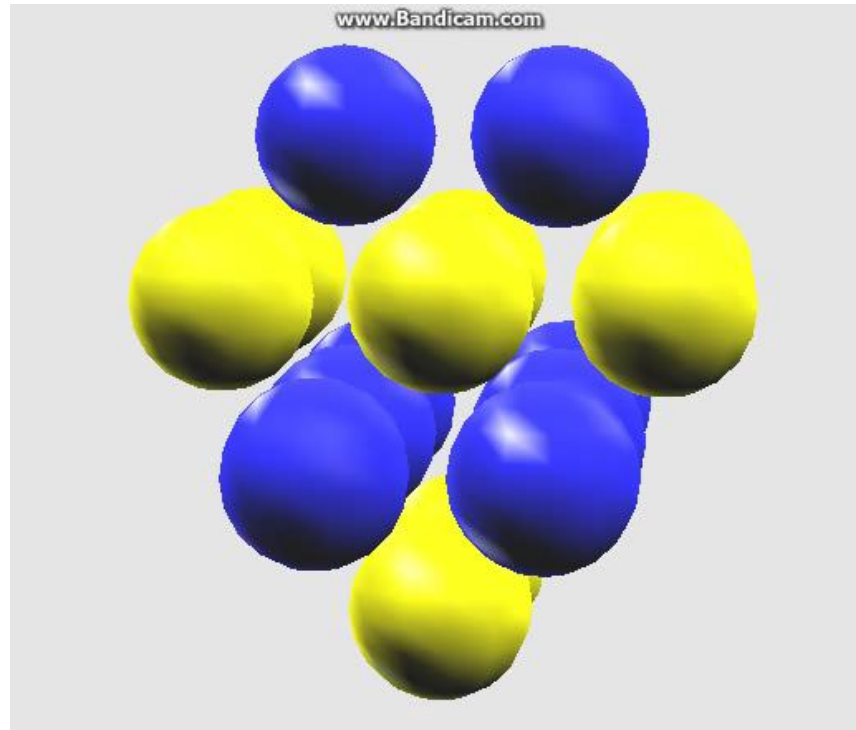
^{16}O



RED – s-shell

YELLOW – p-shell

^{16}O

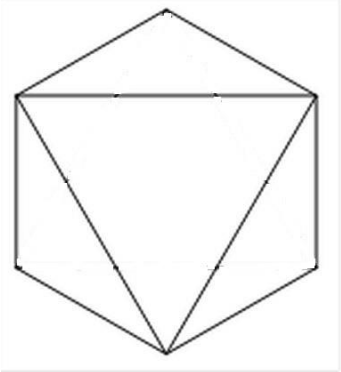


Yellow – protons

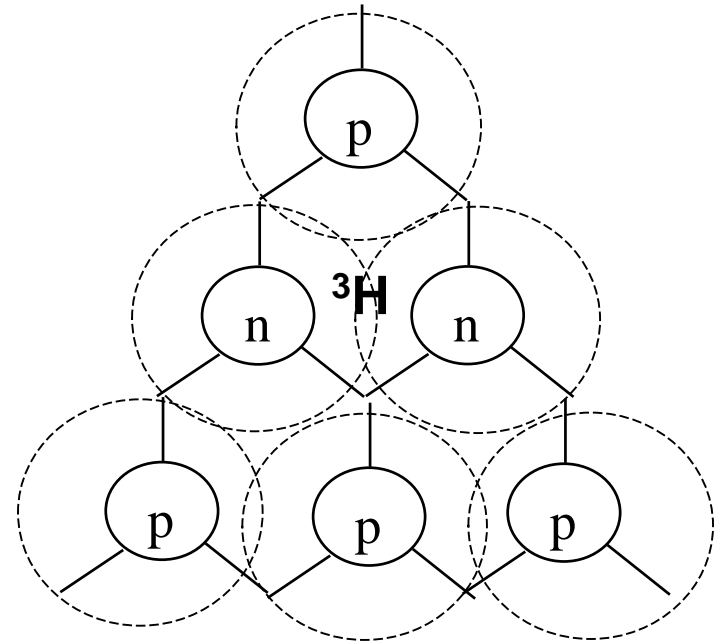
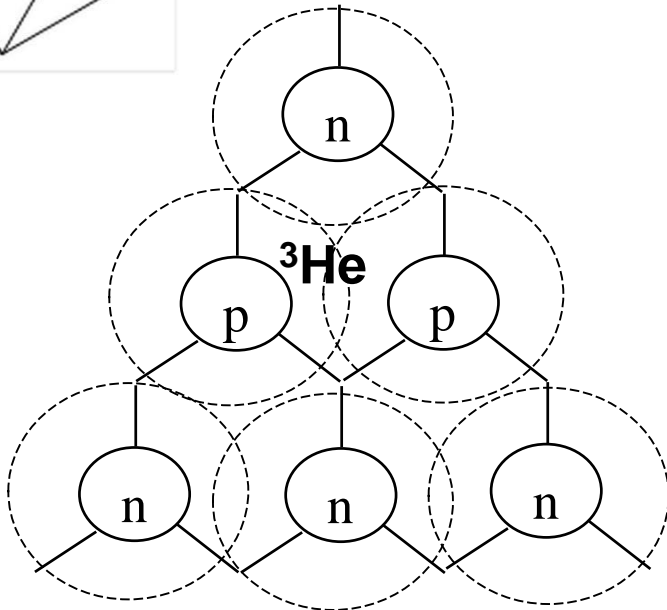
Blue – neutrons

The closed shell $n = 2$, ^{40}Ca

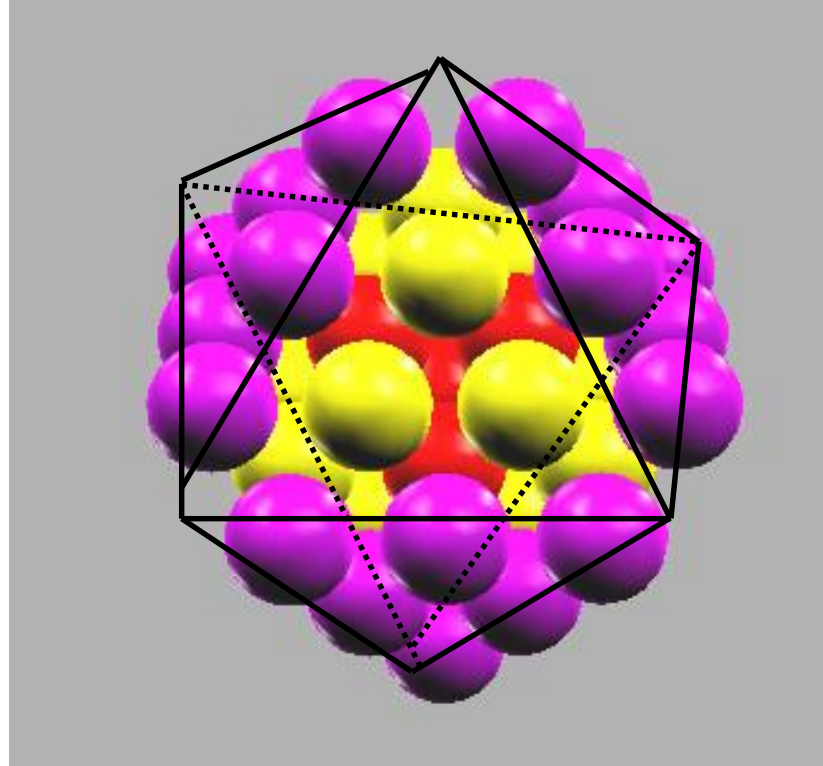
Shell Closure



Faces of ^{40}Ca octahedron



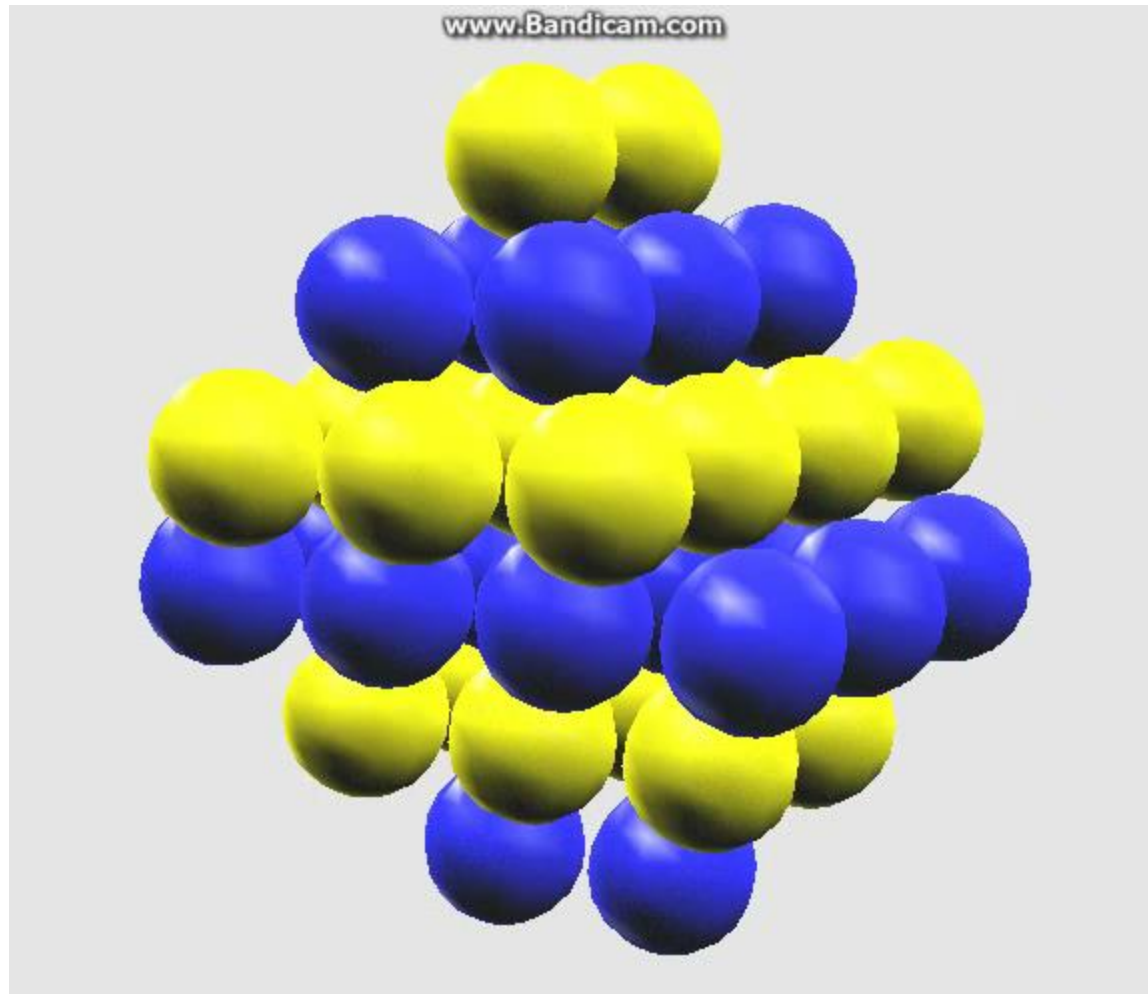
^{40}Ca



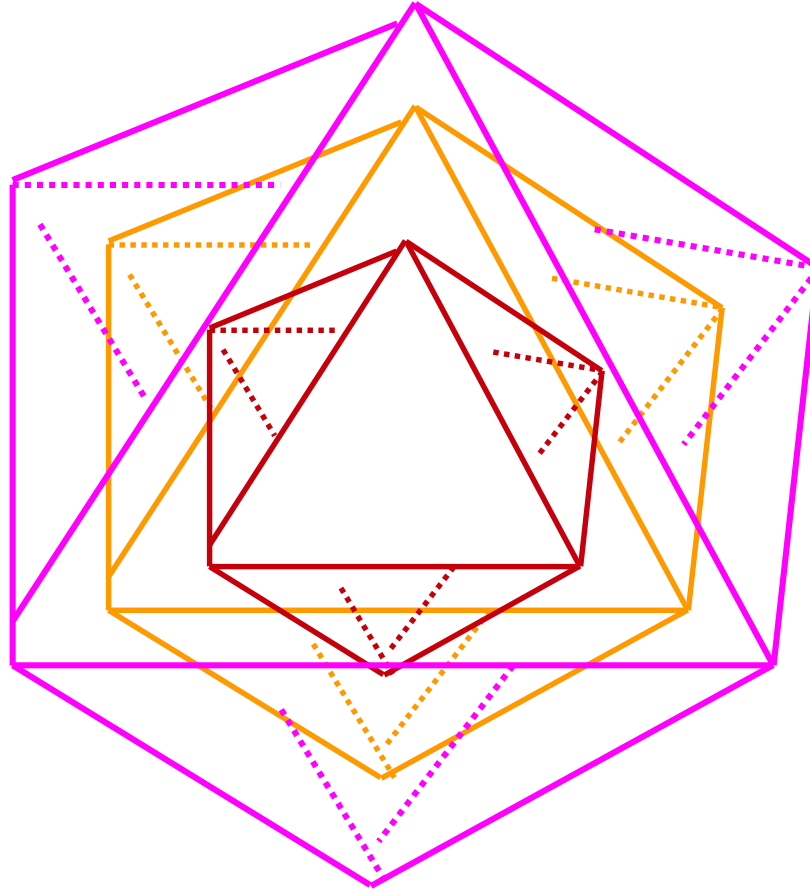
s-shell – red

p-shell – yellow

^{40}Ca



^{40}Ca



3 Nested Octahedra – s, p, d -shells

What's Further?

Nested Octahedra – $N = 0, 1, 2, \dots, \infty$

No!

Deviations from octahedral form:

- Peculiarities of Nuclearsynthesis
- Coulomb repulsion of protons

Restricting factor from infinity:

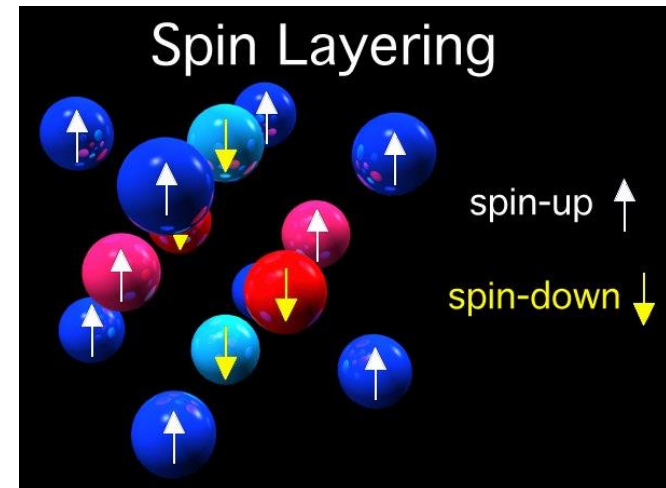
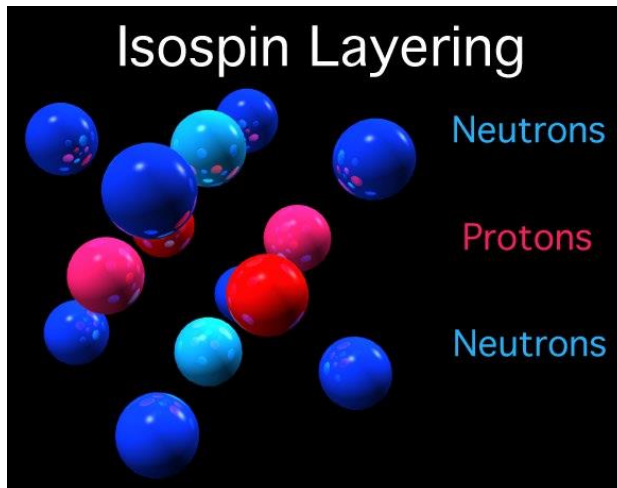
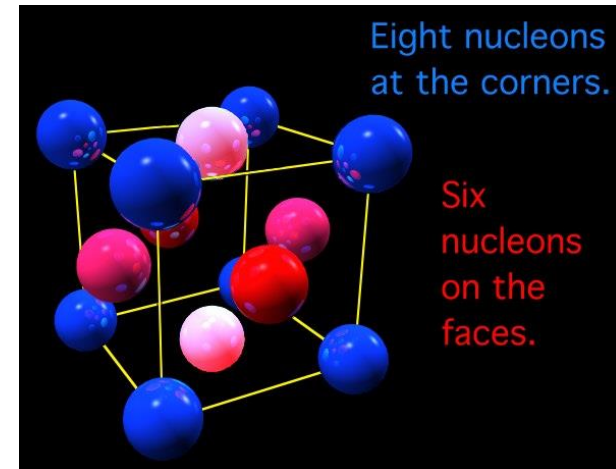
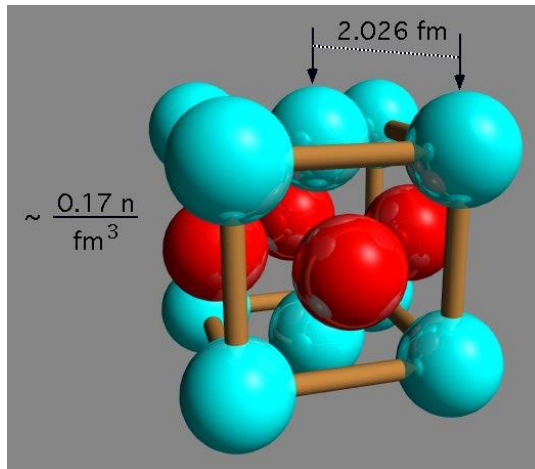
Coulomb repulsion of protons.

SCQM to FCC symmetry of Nuclear Structure

- Nuclear shells correspond to faces of nested octahedra
- Nucleons are arranged in alternating isospin and spin layers
- Protons and neutrons are **strongly correlated**
- **It turned out that** nucleons occupy the nodes of **Face Centered Cubic Lattice (FCC)**

SCQM → Face-Centered Cubic Lattice

Nucleons are arranged in face-centered cubic lattice



Lattice Models of Nuclear structure

In terms of nucleons

- Simple Cubic Lattice
- Body Centered Lattice
- Hexagonal Close Packing
- **Face Centered Cubic Lattice (FCC)**

E. Wigner, Phys. Rev. 51(1937)106

Cook N. and V. Dallacasa, Phys. Rev. C35(1987)1883

FCC Lattice Model

(*N. Cook, 1987*)

Particle in 3D box

$$-(\hbar/2m) d^2\Psi/dr^2 + V(r) \Psi(r) = E\Psi(r)$$

For harmonic oscillator

$$E = \hbar\omega_0(n_x + n_y + n_z + 3/2) = \hbar\omega_0(N + 3/2)$$

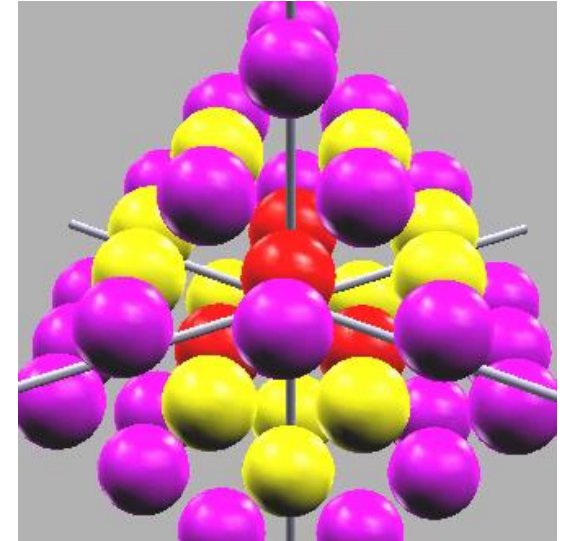
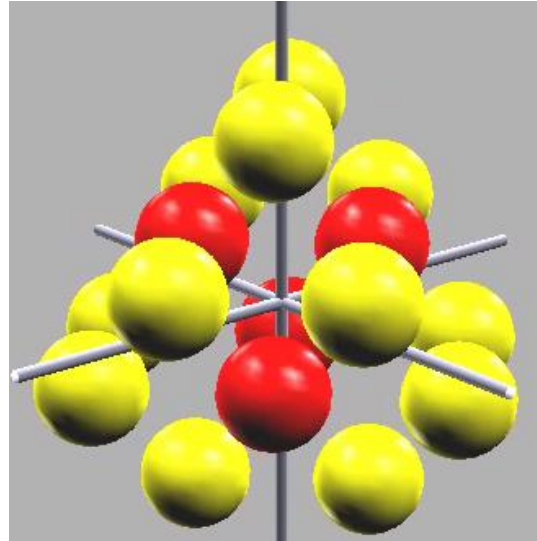
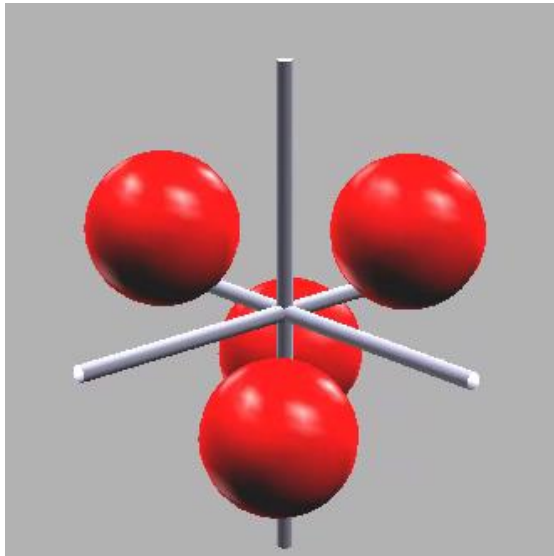
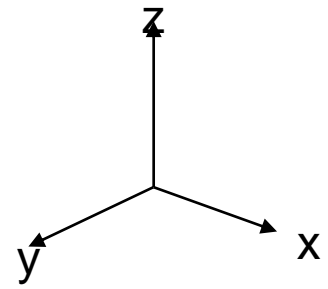
$$N = 0, 1, 2, 3, \dots$$

- Different combinations $\mathbf{n}_x, \mathbf{n}_y, \mathbf{n}_z$, giving the same total \mathbf{N} , denote the **number** of “degenerate” states with the same energy

FCC Lattice Model

(N. Cook, 1987)

s, p, d - shells



- Origin of the coordinate system - at the center of the central tetrahedron
- The closure of each consecutive, symmetrical ($x=y=z$) shell in the lattice composes **precisely** the numbers of nucleons in the shells derived from the three-dimensional Schrodinger equation

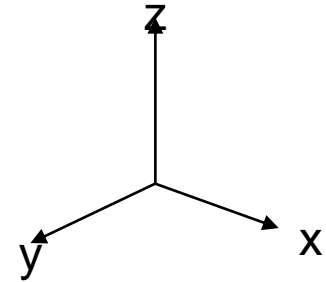
FCC Lattice Model

(N. Cook, 1987)

- Principal quantum number, \mathbf{N}

Assuming \mathbf{x} , \mathbf{y} and \mathbf{z} coordinates of nucleons are **odd** – integers,

$$\mathbf{N} = (|\mathbf{x}| + |\mathbf{y}| + |\mathbf{z}| - 3)/2$$



The first shell (**s**-shell, $\mathbf{N} = 0$) contains 4 nucleons with coordinates 111, -1-11, 1-1-1, -11-1.

The second shell (**p**-shell, $\mathbf{N} = 1$): 12 nucleons
31-1, 3-11, -311, -3-1-1, 1-31, -131, 13-1, -1-3-1, -113,
11-3, 1-13, -1-1-3

The **d**-shell ...

and so on ...

- Total angular momentum, \mathbf{j}

$$\mathbf{j} = (|\mathbf{x}| + |\mathbf{y}| - 1)/2$$

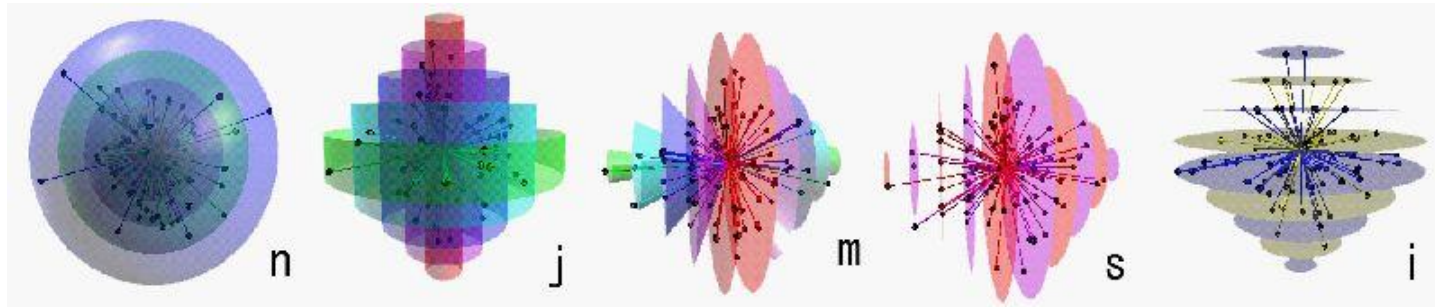
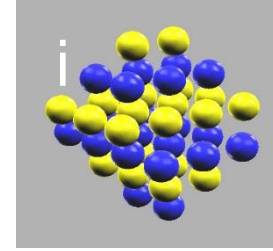
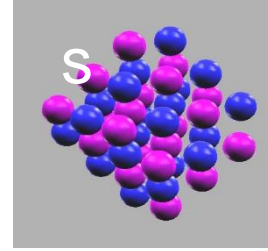
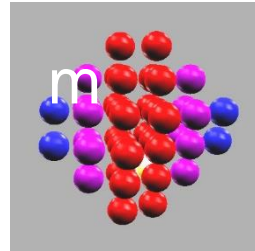
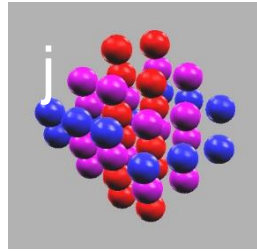
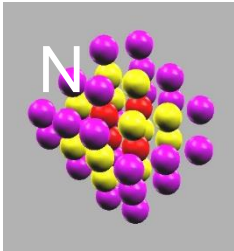
- Magnetic quantum number, \mathbf{m}

$$\mathbf{m} = |\mathbf{x}|/2$$

FCC Lattice Model

(N. Cook, 1987)

Different colors correspond to
different quantum numbers



FCC-SCQM vs Shell Model

Close relation between

nucleon location in FCC-SCQM and quantum numbers of SM

$$n = (|x| + |y| + |z| - 3)/2$$

$$j = |l + s| = (|x| + |y| - 1)/2$$

$$m = (|x|/2)(-1)^{(x-1)}$$

$$s = (-1)^{(x-1)}/2$$

$$i = (-1)^{(z-1)}/2$$

and reversely

$$x = |2m|(-1)^{(m + 1/2)}$$

$$y = (2j + 1 - |x|)(-1)^{(i+j+m+1/2)}$$

$$z = (2n + 3 - |x| - |y|)(-1)^{(i+n-j-1)}$$

What is the role of Quarks in FCC

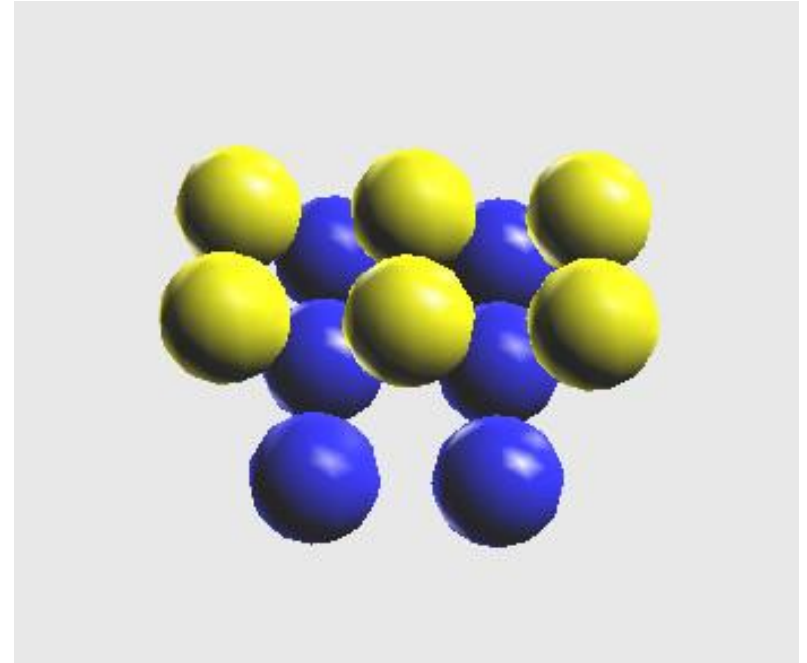
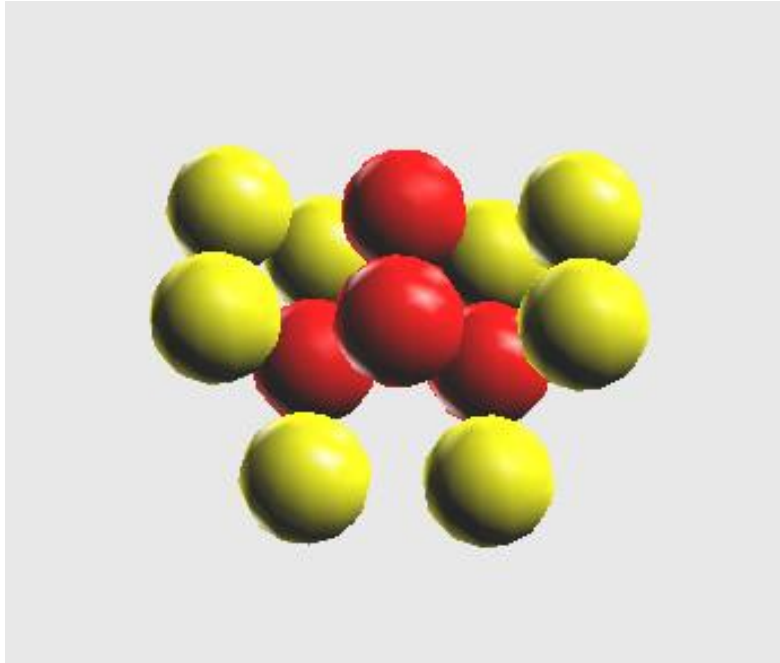
- Color fields of Quarks, responsible for strong interactions, arrange nuclear nucleons in FCC Lattice structure.
- Strong interactions are **tensorial**
- Quark loops form **virtual** 3- and 4-nucleon clusters inside bound nuclei
- Evidence of quark loops is **big separation energy** in even-even nuclei
- Halo nuclei are formed by core and virtual 3-nucleon clusters (${}^3\text{H}$ -type)
- Ground state nuclei are formed by virtual ${}^3\text{H}$ - and ${}^4\text{He}$ -type clusters.
- There are no real ${}^4\text{He}$ cluster in ground state nuclei

^{12}C

6 protons, 6 neutrons

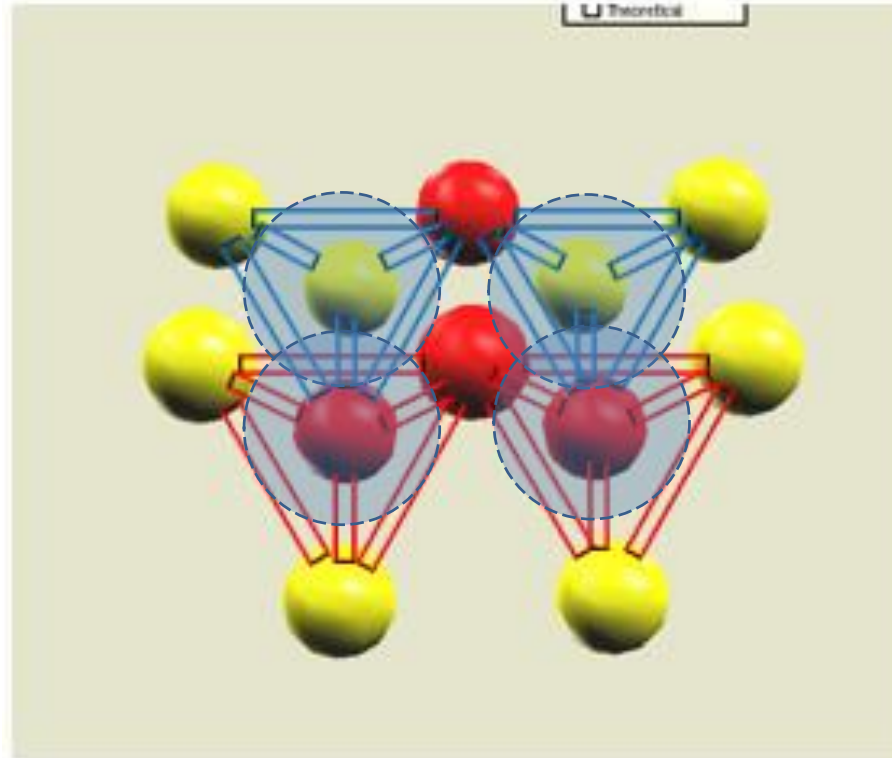
n, principal number
n=0, red; n=1, yellow

i, isospin
yellow – protons
blue - neutrons



Problem for SM:
Why ^{12}C is so stable?

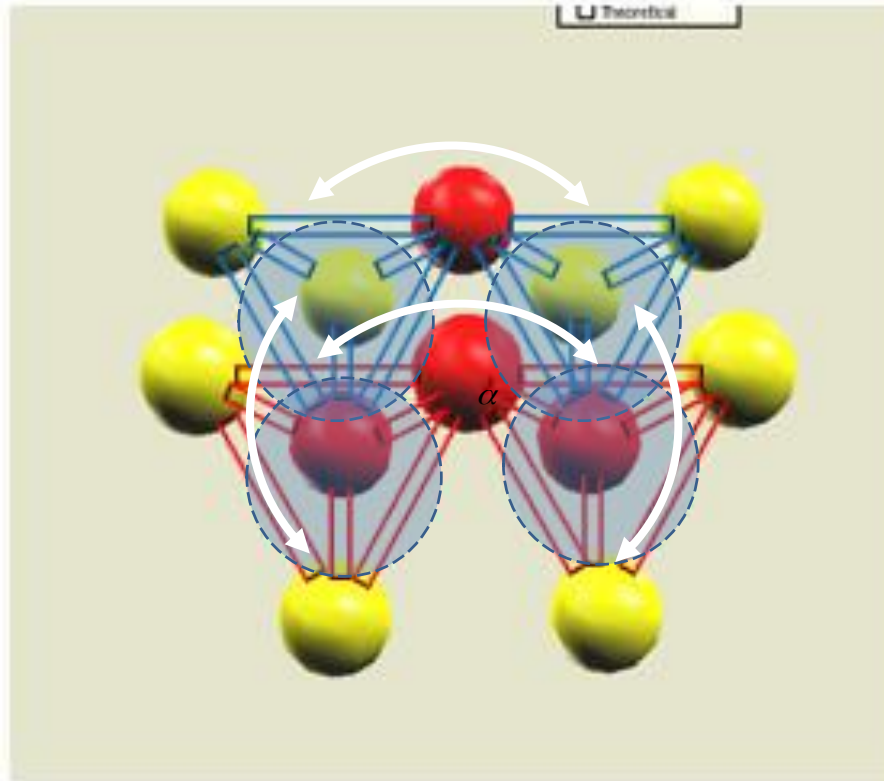
^{12}C - 4 virtual α -clusters



- 4 nucleons of s -shell (red) form with 6 nucleons of p -shell (yellow) 4 virtual α -clusters.
- s -shell nucleons are exchange particles

^{12}C

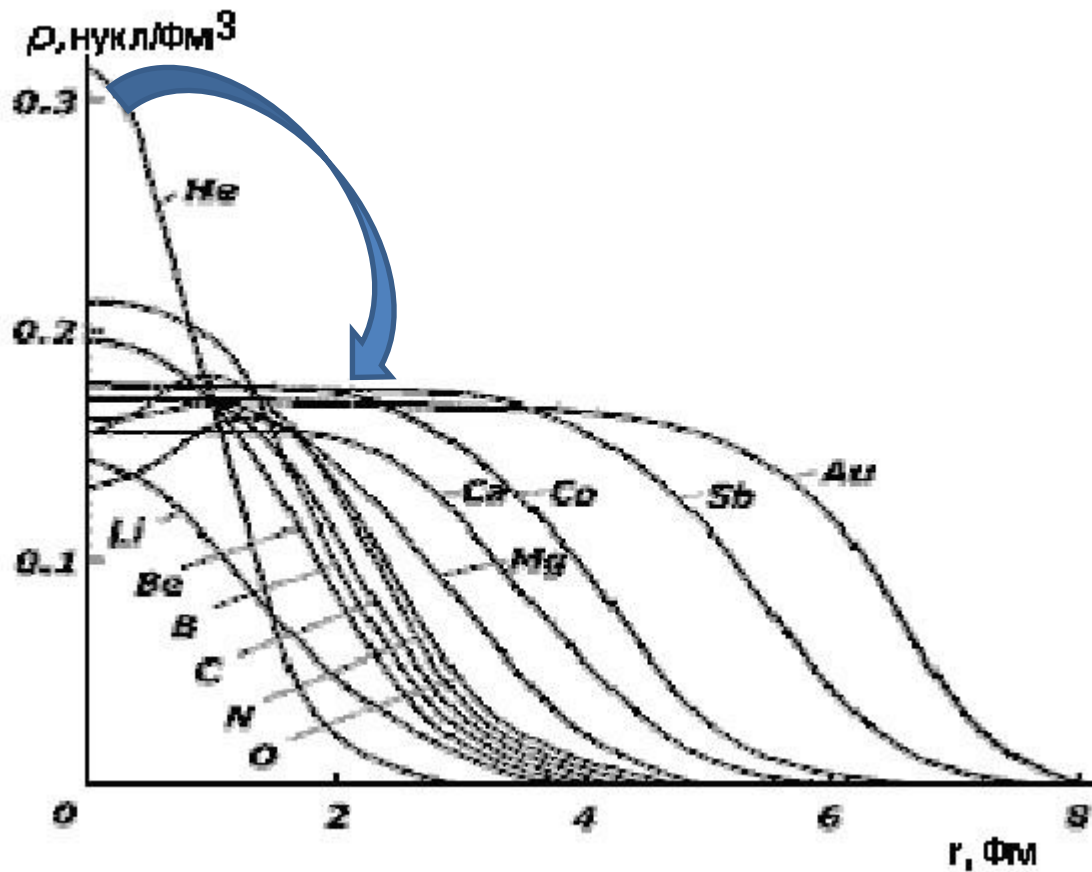
Crosswise bindings of 4 virtual α -clusters
by exchange (red) nucleons of s-shell



- exchange nucleons acquire larger binding energy as belonging simultaneously to 2 alpha clusters
- s-shell core is rearranged and disappears

Nuclear density

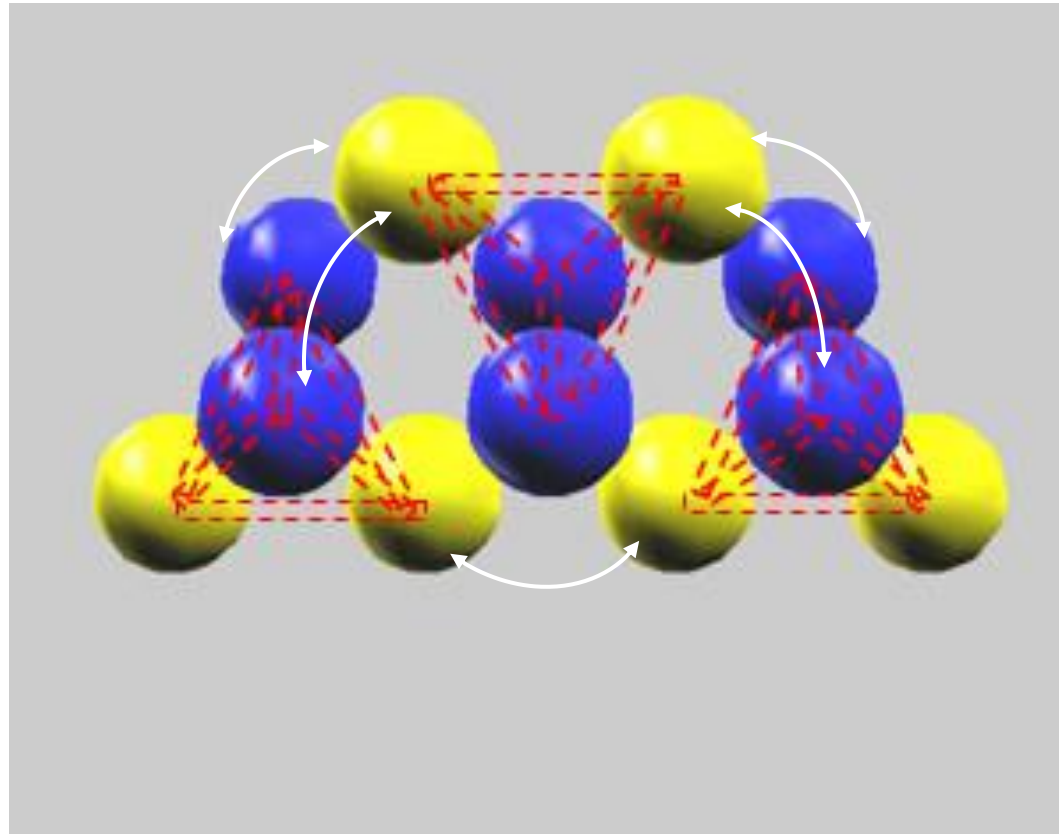
Result of rearrangement – of s -shell
No s -core structure for $A \geq 12$



^{12}C Hoyle state

Borromean nucleus

Loosely bound 3 **real** α - cluster nucleus



Frames of α -clusters are depicted as tetrahedrons. Neutrons of left and right α -clusters are bound with protons of central α -cluster (like in ^8He), and their 2 nearest protons are bound together.

FCC-SCQM vs SM

What about spin-orbital coupling (SOC)?

SOC

- Splitting of nuclear levels
- Lowering of levels with higher J
- Description of observed magic numbers of protons and neutrons

2, 8, 20, 28, 50, 82, 126

Is it possible get the same numbers in FCC-SCQM?

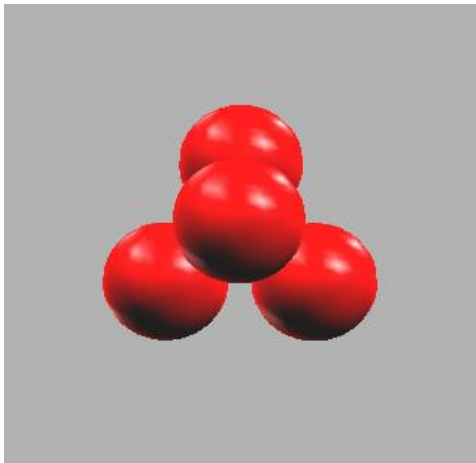
YES !

FCC-SCQM vs SM

Spin-orbital coupling

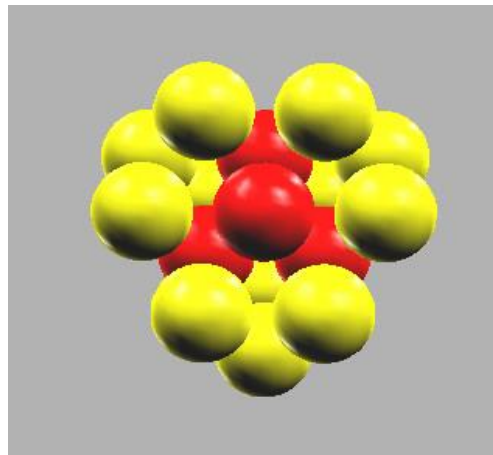
In SCQM Increasing number of exchange nucleons leads to **Lowering** of levels with higher **J**

$$J = 1/2$$



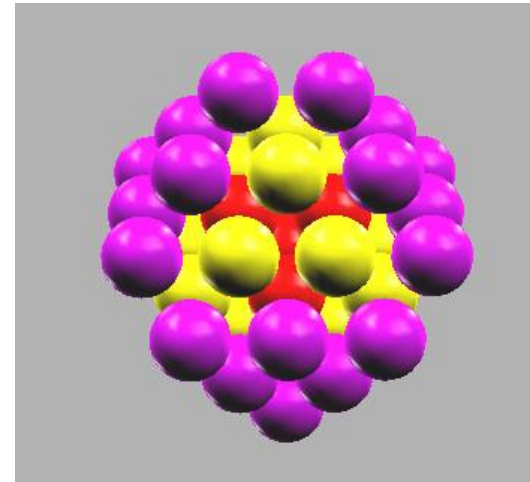
s: $n=0, l=0$
 1 alpha
 $1/2 - 0$ exchange
 nucleons

$$J = 1/2, 3/2$$



s: $n=1, l=1$
 6 virtual alpha
 $1/2 - 0$ exch. nucl.
 $3/2 - 2$ exch. nucl.

$$J = 1/2, 3/2, 5/2$$



s: $n=2, l=0, 2$
 22 virtual alpha
 $1/2 - 0$ exch. nucl
 $3/2 - 2$ exch. nucl.
 $5/2 - 4$ exch. nucl.

FCC-SCQM vs SM

Source of spin-orbital coupling in FCC-SCQM

Increasing number of exchange nucleons, belonging to adjacent virtual alpha clusters with increasing J-value of sub-shells.



Lowering of levels with higher J

Splitting of nuclear levels

FCC-SCQM vs SM

What about magic numbers?

SM

Describes observed magic numbers of protons and neutrons

2, 8, 20, 28, 50, 82, 126

FCC-SCQM

Closed Shells – Octahedra with filled faces

2, 8, 20, 40, 70, 112, ... as given by HO potential

FCC-SCQM vs SM

What about magic numbers?

SM: 2, 8, 20, 28, 50, 82, 126

FCC-SCQM: 2, 8, 20, 40, 70, 112, ...

But, in FCC-SCQM the more preferable to start filling the next shell by the subshell with highest J (from the base of octahedron).

If these subsells are filled, we get the following magic numbers:

2, 6, 8, 14, 20, 28, 40, 50, 70, 82, 112, 126, ...

Red numbers arise from adding to filled faces (shell) of octahedra the subshells with highest value J.

However, takes place only if both protons and neutrons fill this subshells forming virtual alpha clusters.

Summary

Nuclei possess **crystal-like** structure:

- Quarks-quark interactions in nuclei lead to strong proton-neutron correlations.
- Nucleon centers are arranged according to FCC lattice
- All bound nuclei are composed of **virtual triton-like** and **^4He -like clusters**
- Closed Shells = Octahedral Faces
- All nuclei are deformed
- **Symmetry energy** is a consequence of strong quark correlations \rightarrow strong correlations of protons and neutrons.
- The **pairing effect** is a consequence lattice structure

Nuclear Size and Shape

Experimental Observations

- Compactness of and a hole inside ${}^4\text{He}$

Point-nucleon charge distributions of ${}^3\text{He}$ and ${}^4\text{He}$
 Hole inside ${}^3\text{He}$ and ${}^4\text{He}$

I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236

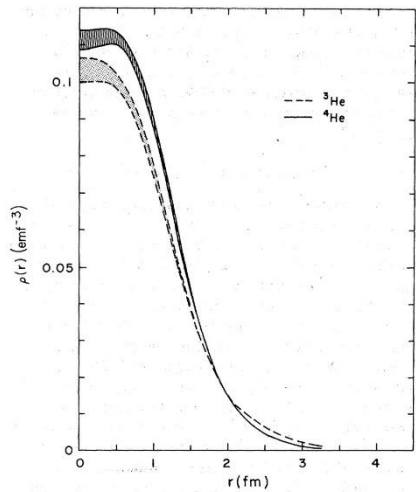
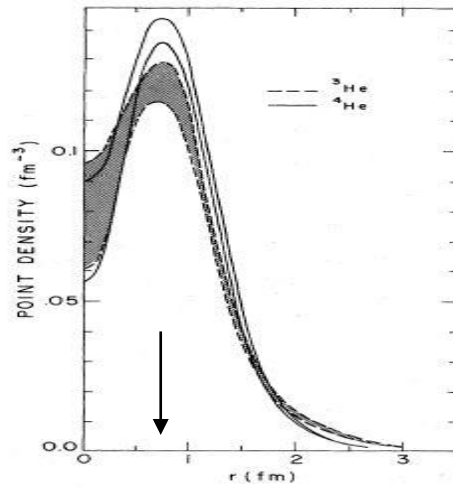


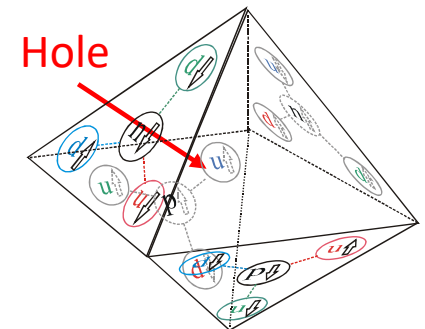
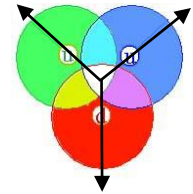
FIG. 11. ${}^3,{}^4\text{He}$ model-independent charge densities.



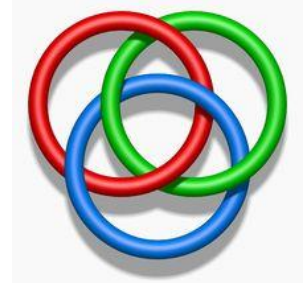
G. 15. Model-independent densities of pointlike nucleons in ${}^3,{}^4\text{He}$.

$$R_\alpha = 1.65 \text{ fm}$$

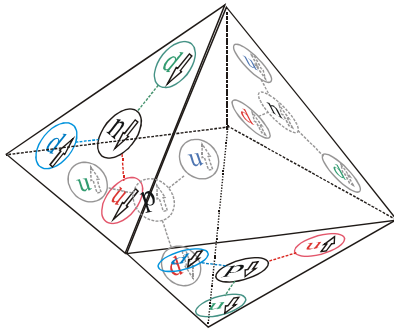
Non-spherical
nucleon



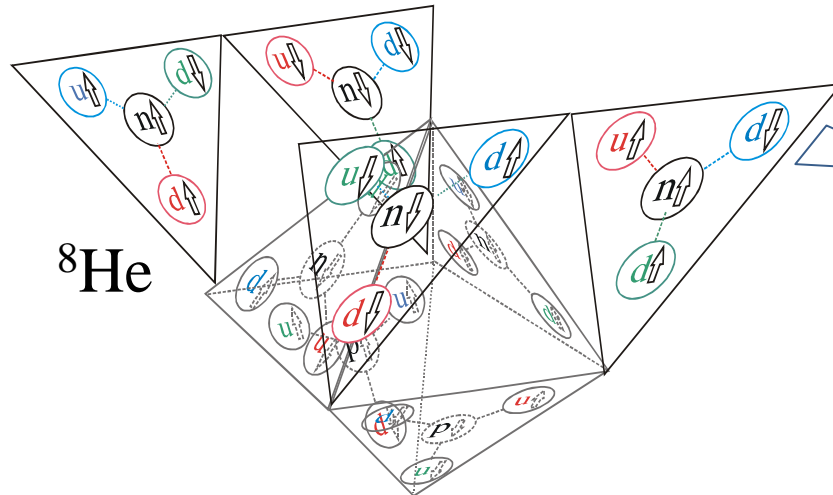
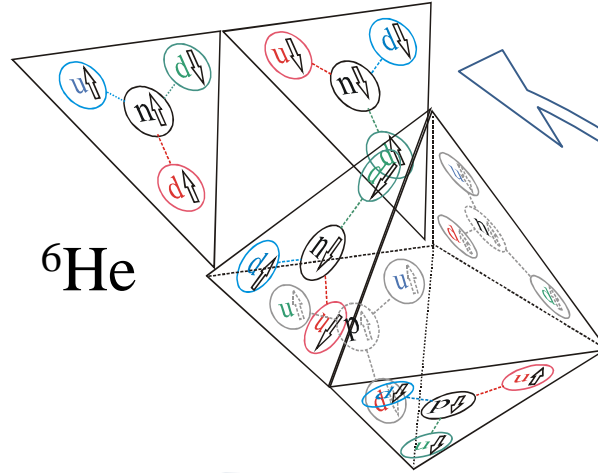
Helium Isotopes Borromean Nuclei



^4He
Core

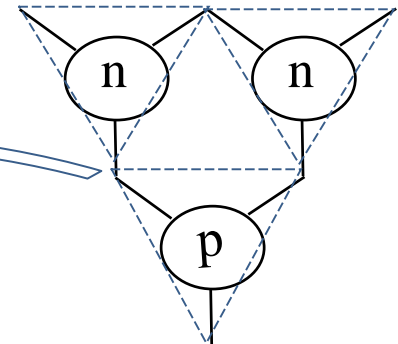


^6He

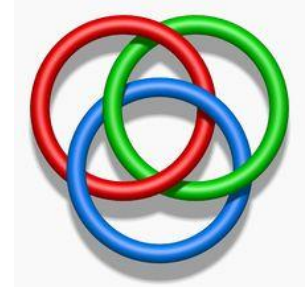


^8He

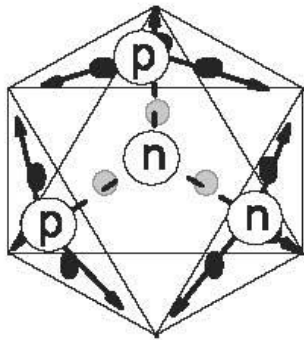
Quark loop



Helium Isotopes Borromean Nuclei



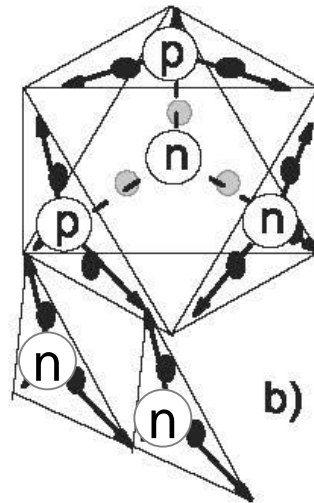
${}^4\text{He}$
Core



a)

$R = 1.57 \text{ fm}$
 $R_{\text{exp}} = 1.6 \text{ fm}$

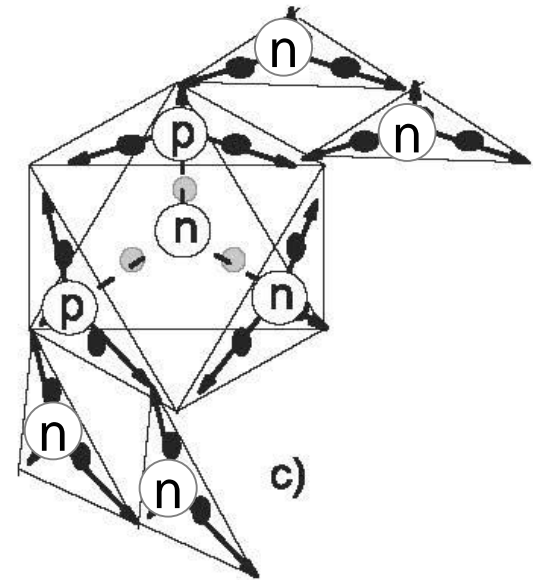
${}^6\text{He}$



b)

$R = 2.2 \text{ fm}$
 $R_{\text{exp}} = 2.45 \text{ fm}$

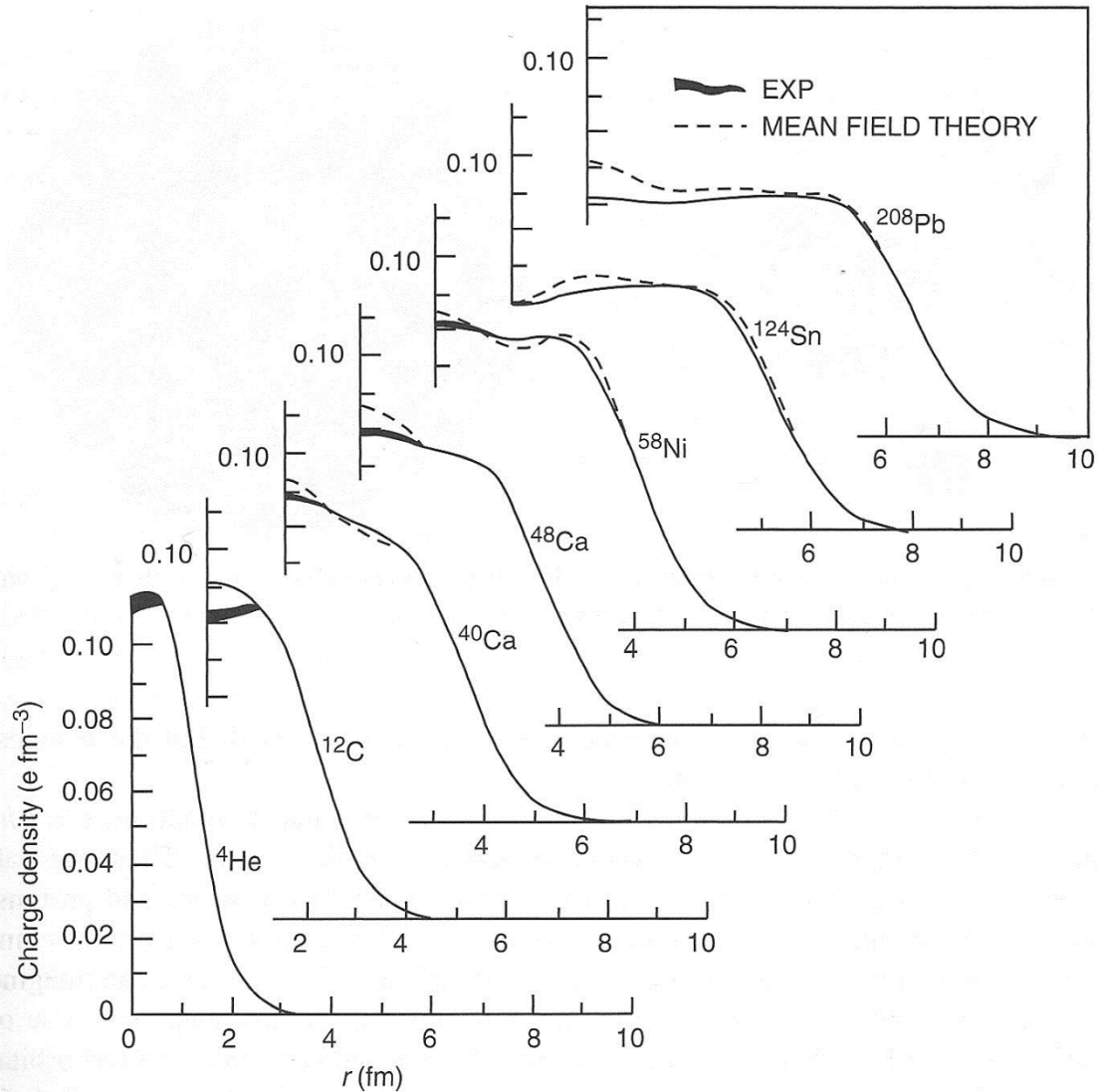
${}^8\text{He}$



c)

$R = 2.4 \text{ fm}$
 $R_{\text{exp}} = 2.53 \text{ fm}$

Fluctuation of central nuclear density



Fluctuation of central nuclear density

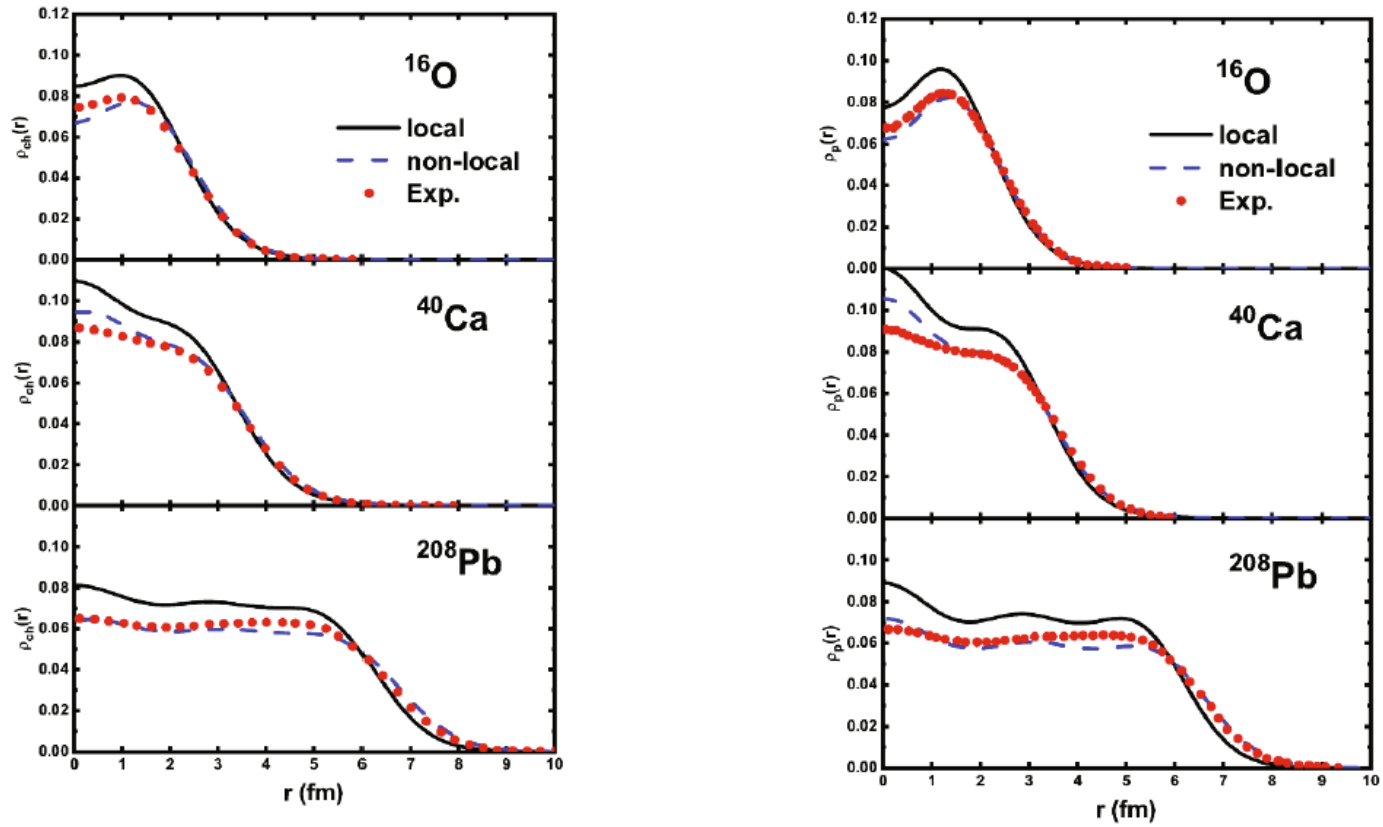
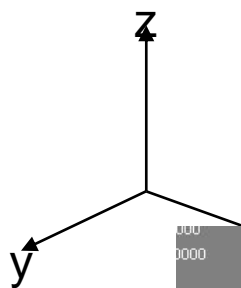


Fig. 3 (Color online) Same as Fig. 1 but for the charge

Fluctuation of central nuclear density



$J = 1/2$

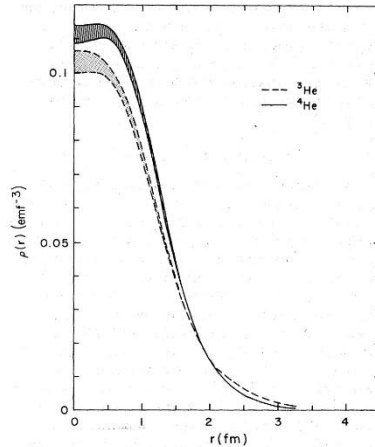
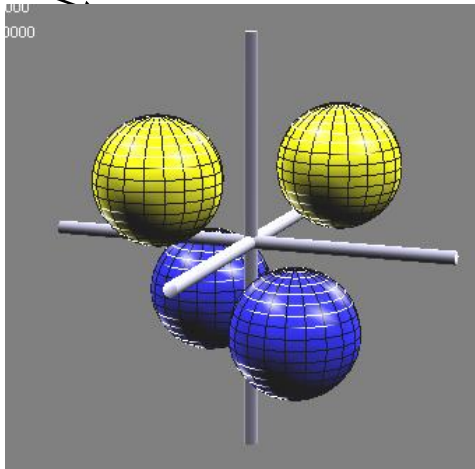
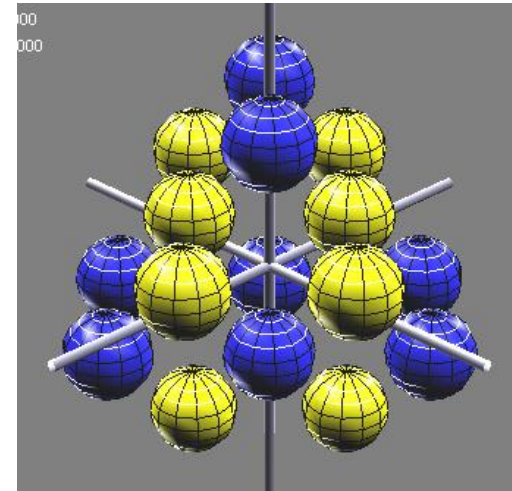


FIG. 11. ${}^3\text{He}$ model-independent charge densities.

$J = 1/2, 3/2$



$J = 1/2, 3/2, 5/2$

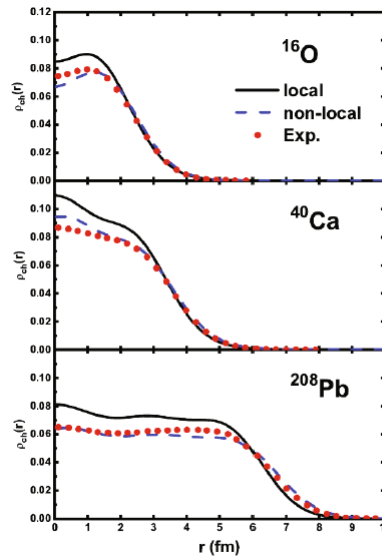
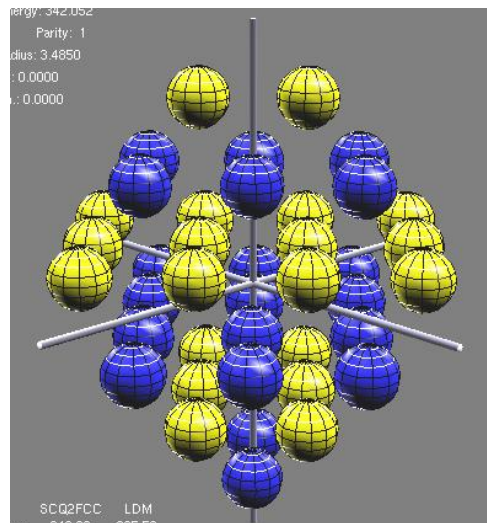
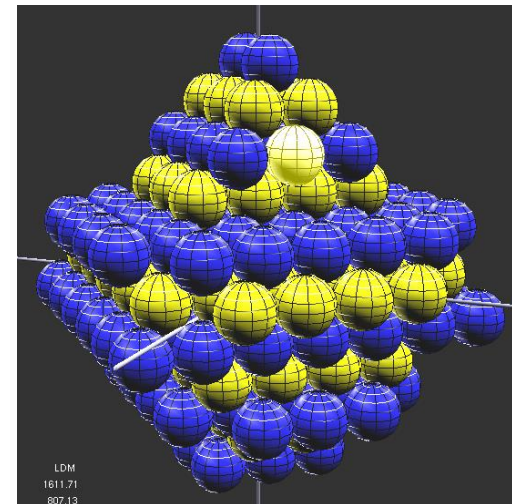
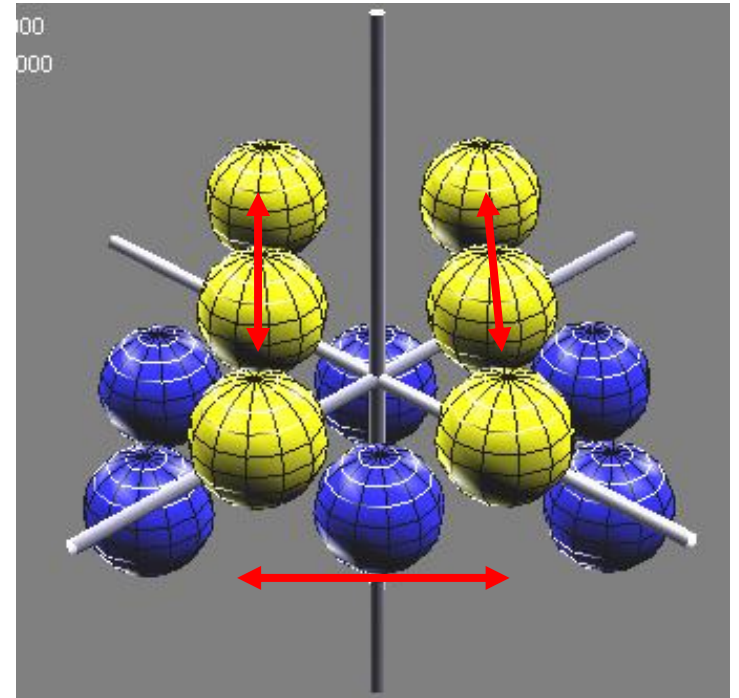
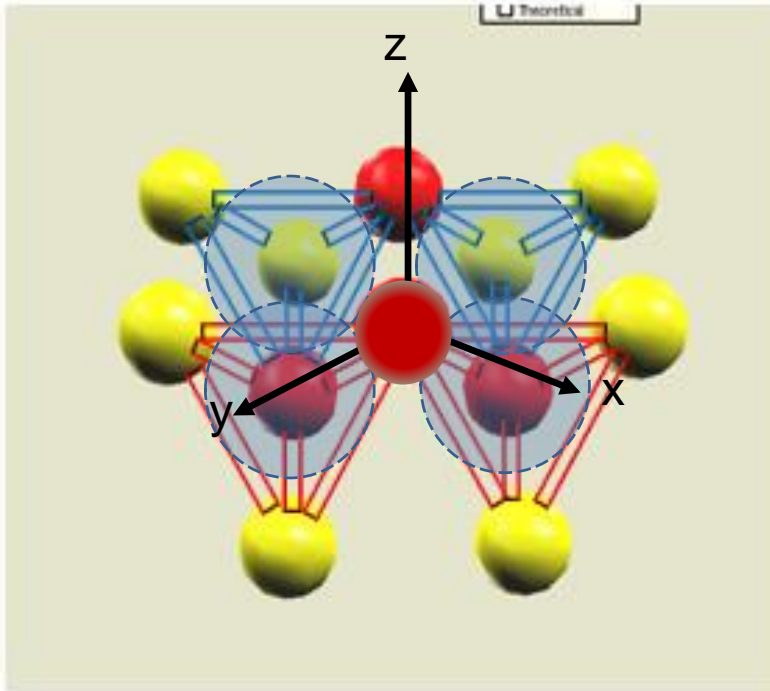


Fig. 3 (Color online) Same as Fig. 1 but for the charge

$J = 1/2, 3/2, 5/2, 7/2, 9/2$

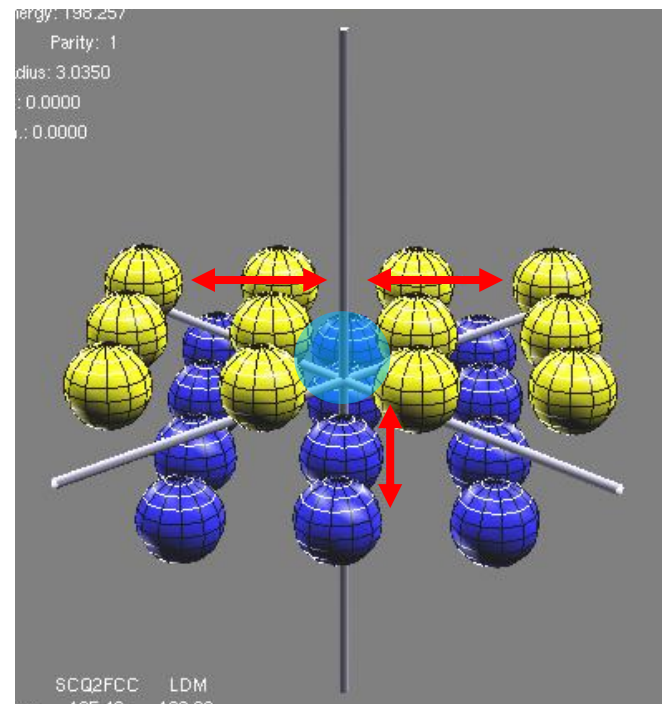
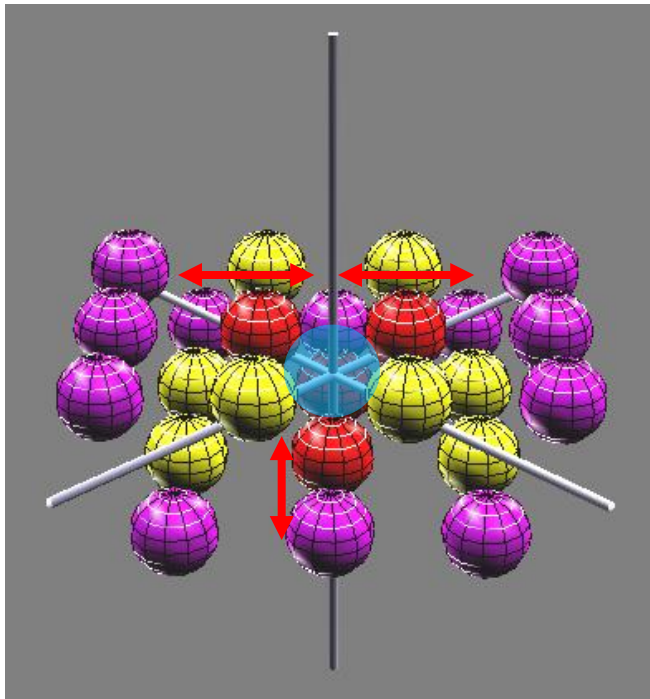


$$^{16}\text{O}$$
$$J = 1/2, 3/2$$



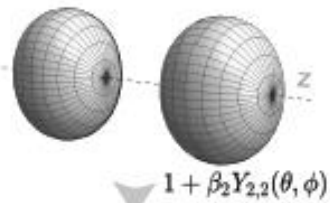
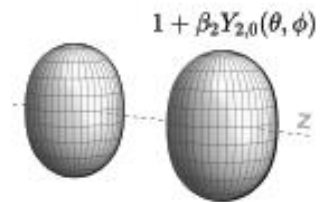
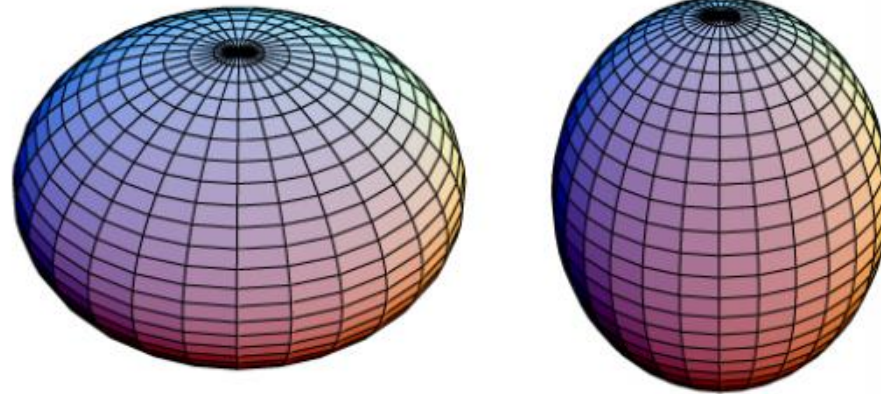
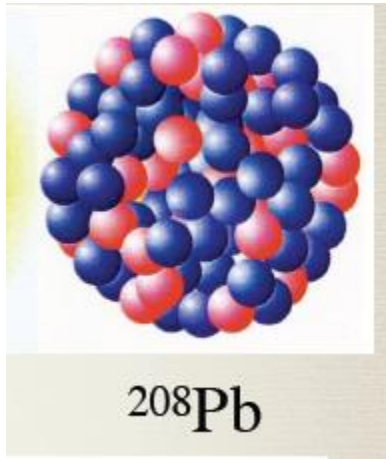
Fluctuation of central nuclear density

$${}^{40}\text{Ca}$$
$$J = 1/2, 3/2, 5/2$$

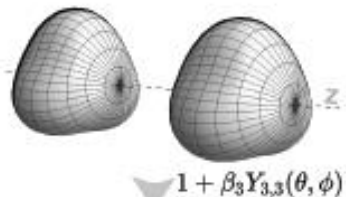
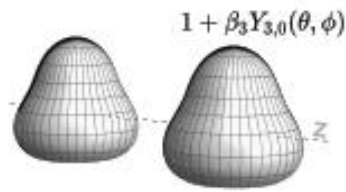


Nuclear Deformation

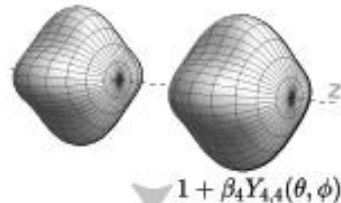
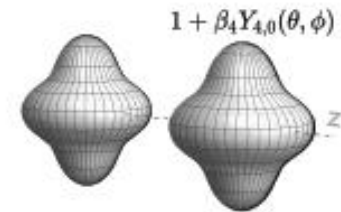
Nuclei are not spherically symmetric



quadrupole



octupole



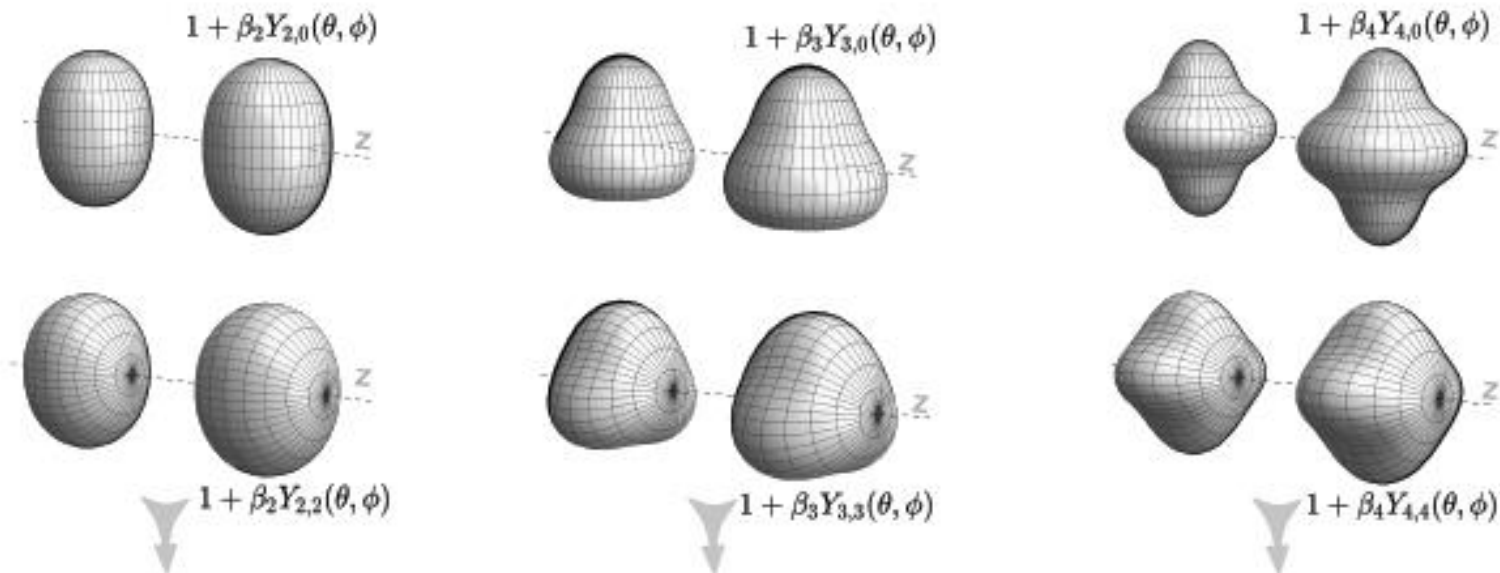
hexadecupole

Nuclear Deformation Theory

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{[r - R(\theta, \phi)/a_0]}} \quad - \text{ Nuclear density}$$

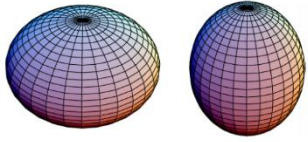
$$R(\theta, \phi) = C(\alpha_{\lambda\mu}) R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda}^{\mu}(\theta, \phi) \right] \quad - \text{ Nuclear radius}$$

$$R(\theta, \phi) = R_0 \left(1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right),$$



SCQM+FCC vs Experiment

Electric Quadrupole Moment

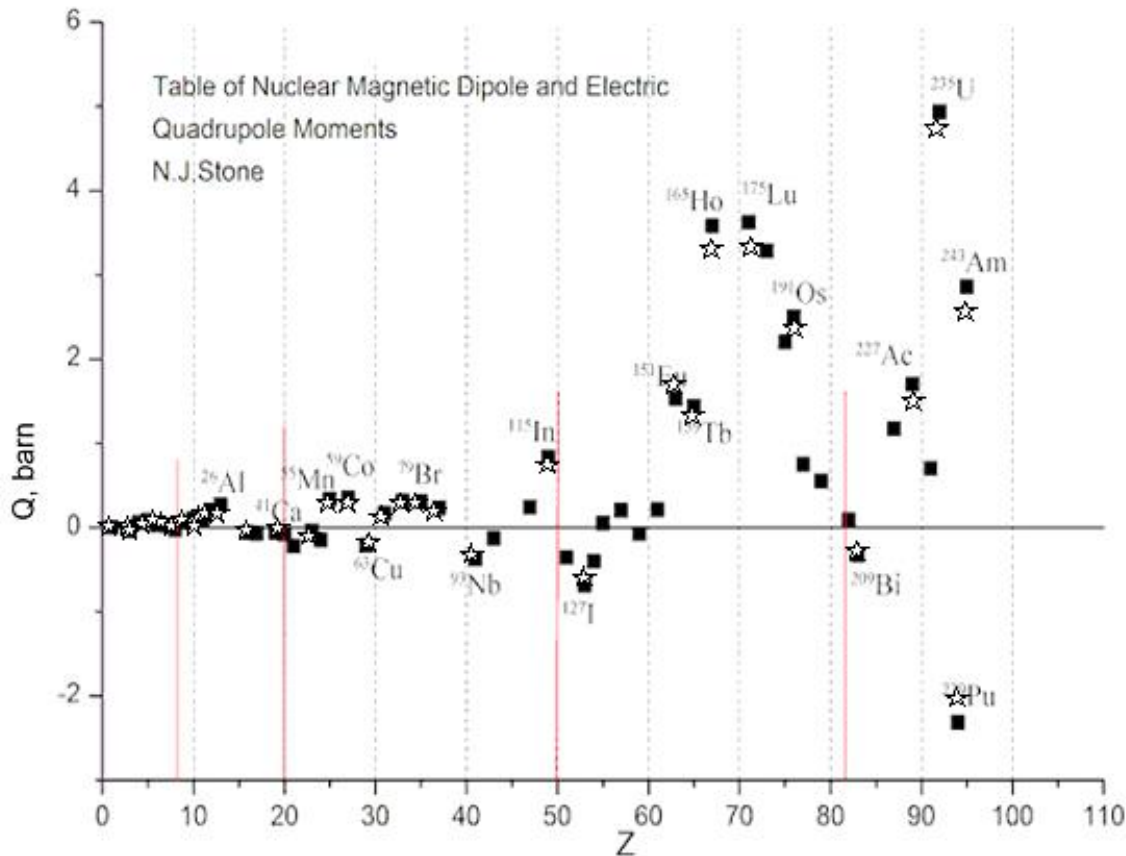


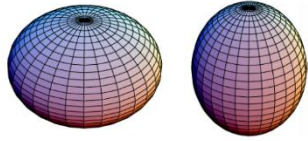
Model

$$Q = \frac{J(2J-1)}{(J+1)(2J+3)} Q_0$$

Q_0 – Intrinsic Quadrupole Moment

■ - Exp , ☆ Model





Nuclear Deformation

Model vs Experiment

Charged(proton) Quadrupole Moments

Neutron Quadrupole Moments

Nuclear Matter Quadrupole Moments

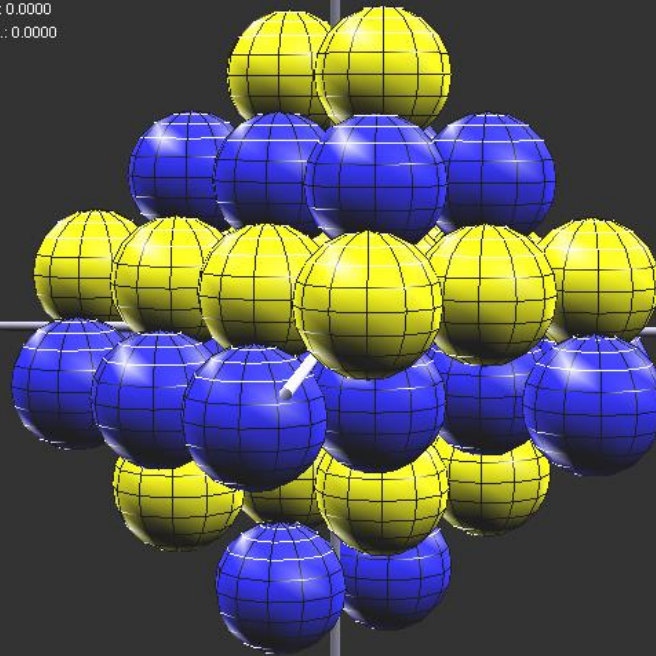
$$Q_0 = \sum_{k=1}^Z \langle 2z_k^2 - x_k^2 - y_k^2 \rangle \quad \text{Intrinsic Quadrupole Moment}$$

Nucleus		C	Al	Ar	Cu	¹¹⁵ In	¹¹⁸ Sn	¹³¹ Xe	¹⁹⁷ Au	²⁰⁸ Pb	²⁰⁹ Bi	²³⁵ U
Charged Q	Exp.	0	0.15	0	-0.21	0.8	0	-0.12	0.54	0	-0.37	4.9
	Model		0.18	0	-0.02	0.7	0	-0.6	0.58	0	-0.26	4.7
Model												
Charged Q ₀ ,		-0.08	0.49	0.16	-0.1	1.28	0.32	-1.92	2.96	-0.34	-0.49	10.1
Neutron Q ₀		-0.08	0.	0.64	0	-2.56	-0.32	0.72	-1.28	-5.42	-3.96	2.3
Matter Q ₀		-0.16	0.49	0.80	-0.1	-1.28	0	-1.2	1.68	-5.76	-4.45	12.4

^{40}Ar

NUCLEUS: Argon-40 Z: 18 N: 22
Nucleon radius shown as: 0.934 fm
VIEWING MODE: FCC Lattice

GROUND-STATE DATA
Binding Energy: 343.810
Spin: 0 Parity: 1
Charge Radius: unknown
Mag.Mom.: 0.0000
Quad.Mom.: 0.0000



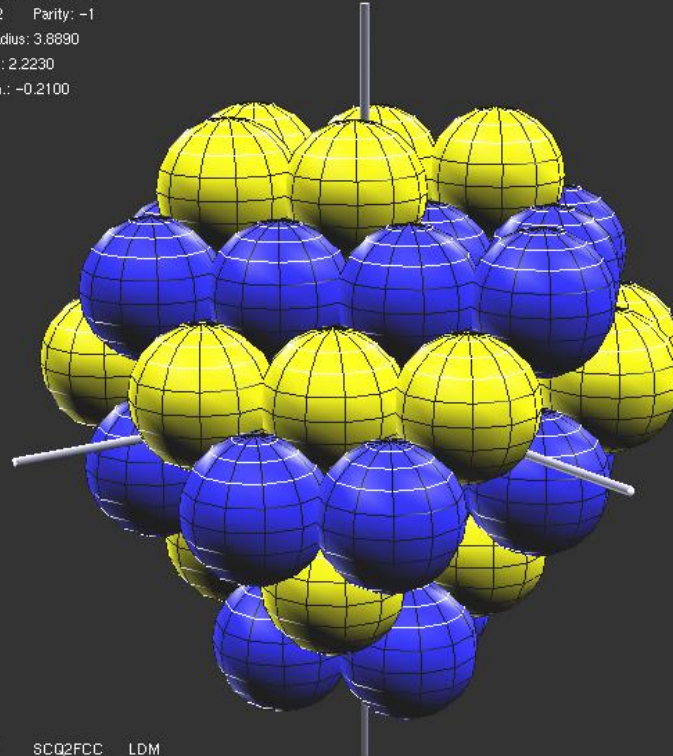
THEORY: SCQ2FCC LDM
Bind.Energy: 343.64 342.24
Coul.Energy: 60.87 64.42

***** SCQ2FCC *****
Spin 0+
Mag.Mom.: 0.00 0.00
Nuclear Radius: charged: 3.575 neutron: 3.723 mass: 3.656
Int. Quad.Mom.: charged: -0.160 neutron: -0.640 mass: -0.800
Lab. Quad.Mom.: charged: 0.000 mass: 0.000

^{63}Cu

NUCLEUS: Copper-63 Z: 29 N: 34
Nucleon radius shown as: 1.149 fm
VIEWING MODE: FCC Lattice

GROUND-STATE DATA
Binding Energy: 551.382
Spin: 3/2 Parity: -1
Charge Radius: 3.8690
Mag. Mom.: 2.2230
Quad. Mom.: -0.2100



THEORY: SCQ2FCC LDM
Bind. Energy: 553.04 544.27
Coul. Energy: 145.11 146.93

***** SCQ2FCC *****
Spin: 3/2-
Mag. Mom.: 0.00 0.00
Nuclear Radius: charged: 3.983 neutron: 4.125 mass: 4.060
Int. Quad. Mom.: charged: -0.080 neutron: 0.000 mass: -0.080
Lab. Quad. Mom.: charged: -0.016 mass: -0.016

^{131}Xe

Nucleon radius shown as: 0.934 fm
VIEWING MODE: FCC Lattice

GROUND-STATE DATA

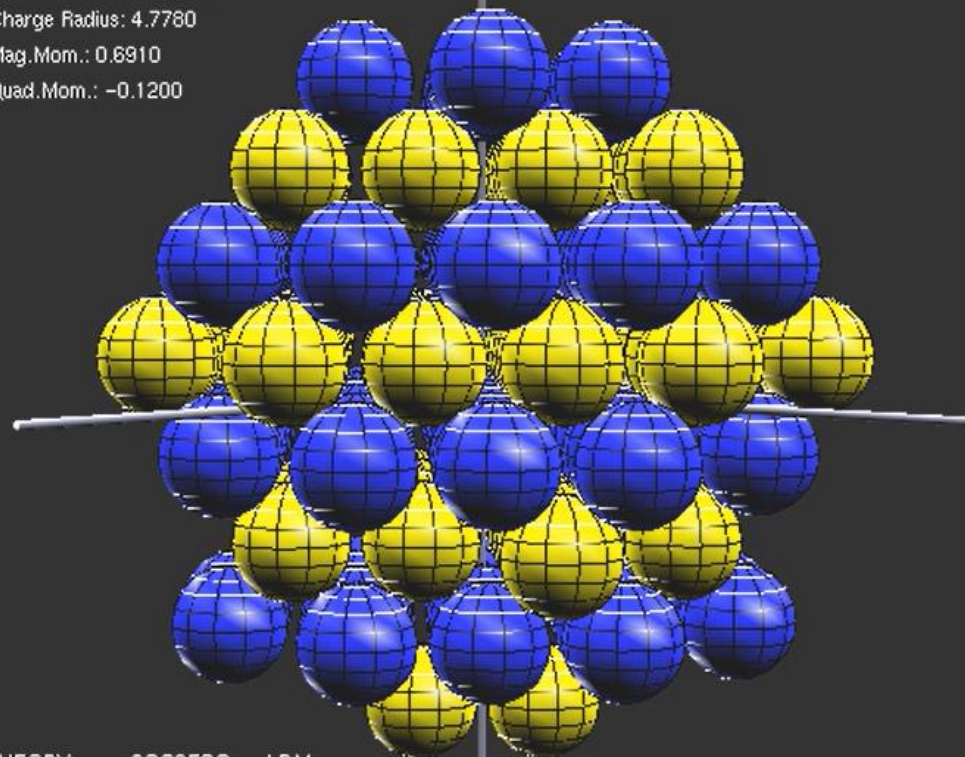
Binding Energy: 1103.511

Spin: 3/2 Parity: 1

Charge Radius: 4.7780

Mag. Mom.: 0.6910

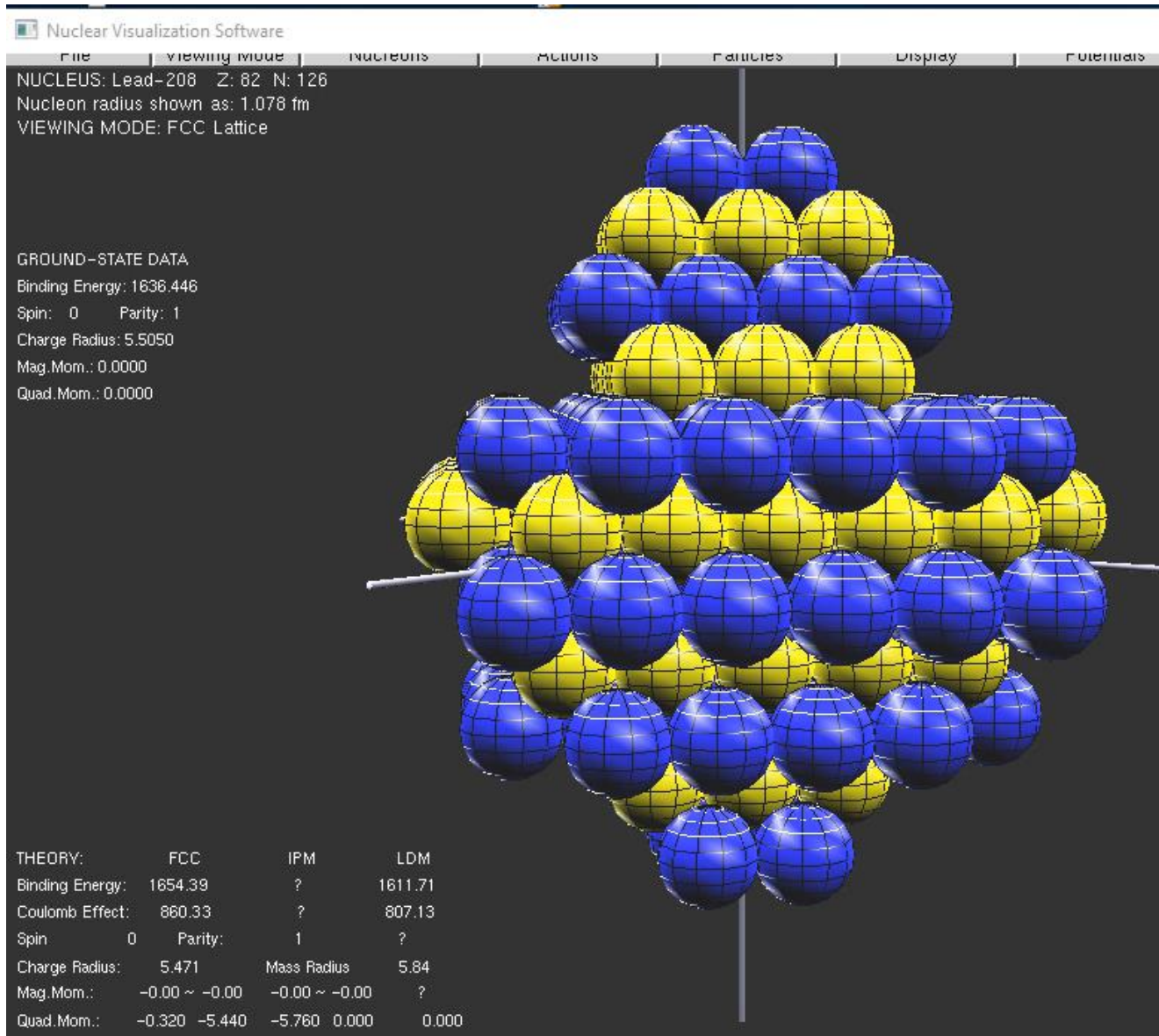
Quad. Mom.: -0.1200



THEORY:	SCQ2FCC	LDM
Bind. Energy:	1132.65	1090.70
Coul. Energy:	428.31	405.74

***** SCQ2FCC *****

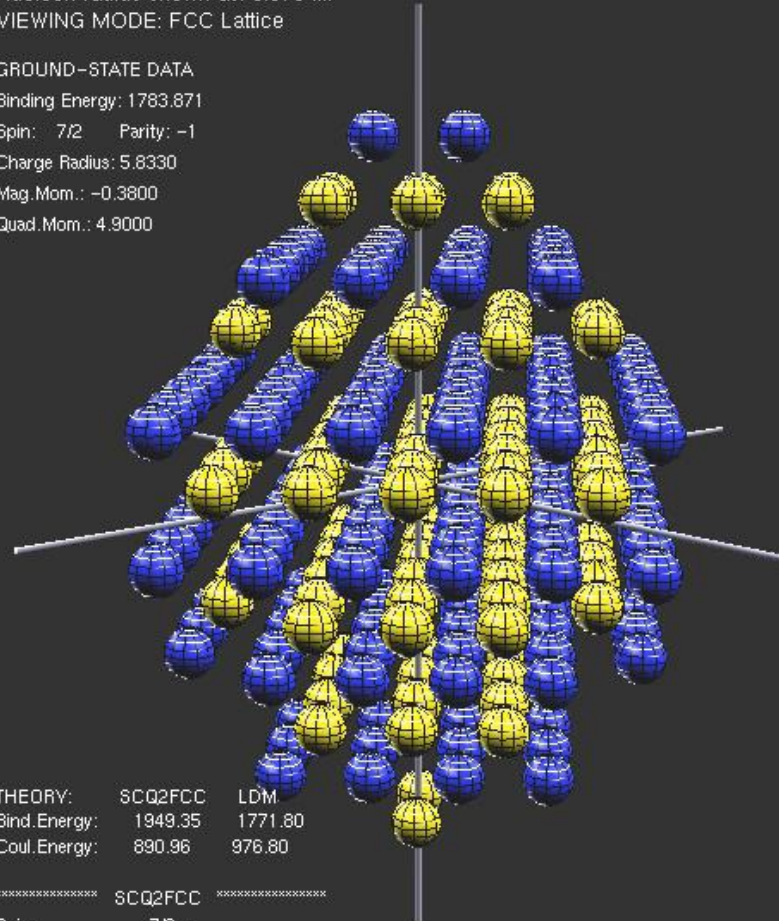
^{207}Pb



235U

NUCLEUS: Uranium-235 Z: 92 N: 143
Nucleon radius shown as: 0.575 fm
VIEWING MODE: FCC Lattice

GROUND-STATE DATA
Binding Energy: 1763.871
Spin: 7/2 Parity: -1
Charge Radius: 5.8330
Mag. Mom.: -0.3800
Quad. Mom.: 4.9000

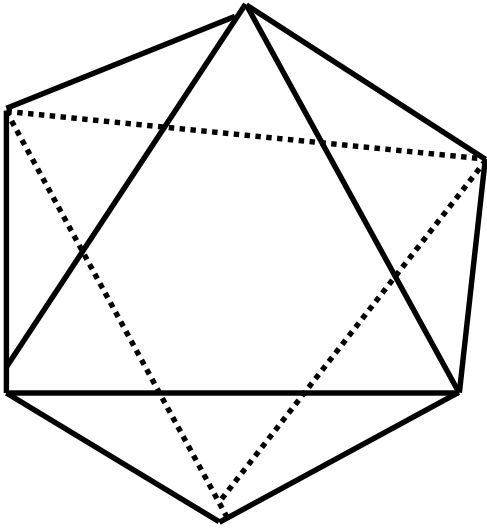


THEORY:	SCQ2FCC	LDM
Bind. Energy:	1949.35	1771.80
Coul. Energy:	890.96	976.80

***** SCQ2FCC *****

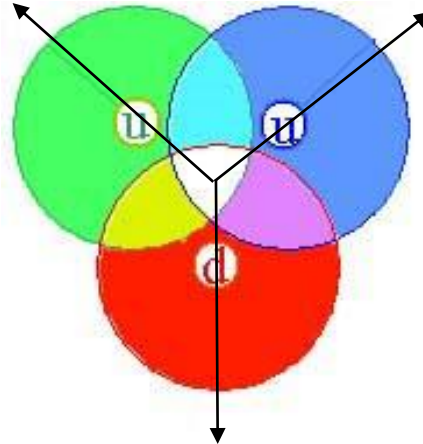
Spin:	7/2-		
Mag. Mom.:	722.50	6600468.00	
Nuclear Radius:	charged: 5.800	neutron: 6.689	mass: 6.341
Int. Quad. Mom.:	charged: 10.059	neutron: 2.339	mass: 12.398
Lab. Quad. Mom.:	charged: 4.694	mass: 5.766	

**Thank you for your
attention!**



Back Slides

Nucleon



Nucleon wave function composed of color quarks

$$\psi = \frac{1}{\sqrt{6}} \sum_{ijk} \epsilon_{ijk} |c_i\rangle |c_j\rangle |c_k\rangle$$

Where $|c_i\rangle$ are orthonormal states with $i,j,k \rightarrow R,G,B$

SCQM \implies The Local Gauge Invariance Principle

Destructive Interference of color fields \equiv Phase rotation of the quark w.f. in color space:

$$\psi(x)_{Color} \rightarrow e^{ig\theta(x)}\psi(x)$$

Phase rotation in color space \implies quark dressing (undressing) \equiv the gauge transformation

$$A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu\theta(x)$$

Therefore, during quark oscillation its

color charge

momentum

mass

are continuously varying function of time.

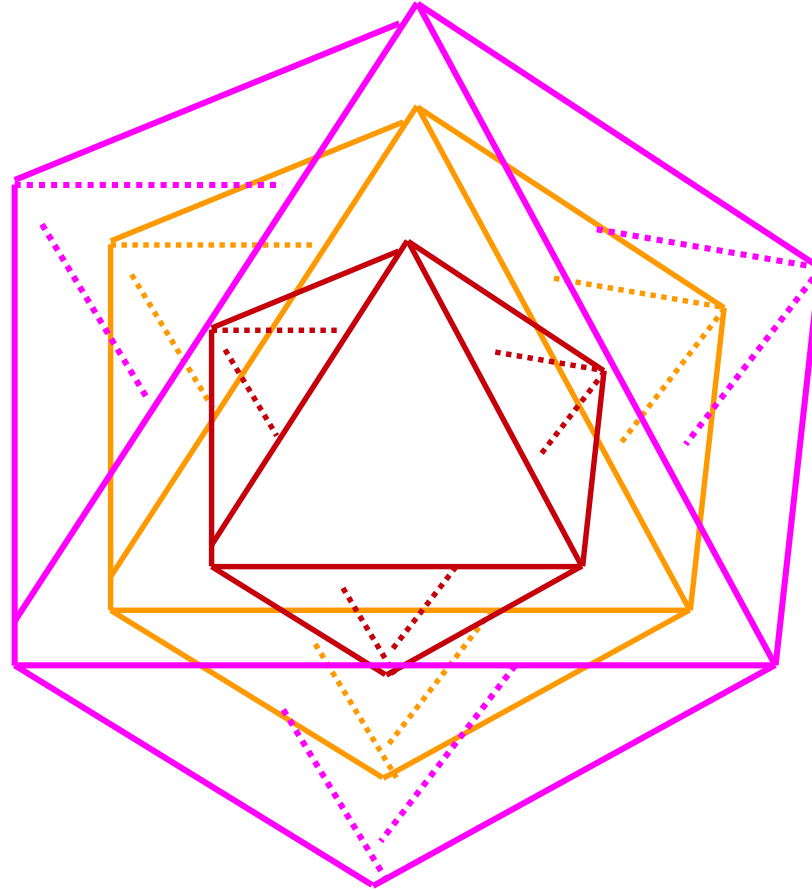
Relation SCQM to QCD

We reduce interaction of color quarks via **non-Abelian** fields to its **E-M** analog:

$$A_a^\mu(x) \rightarrow A^\mu(x)$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - \lambda f^{abc} A_b^\mu A_c^\nu \rightarrow F_{ch}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

^{40}Ca



3 Nested Octahedra – s, p, d -shells

Summary (cont.)

Quantization

Rigid body quantization

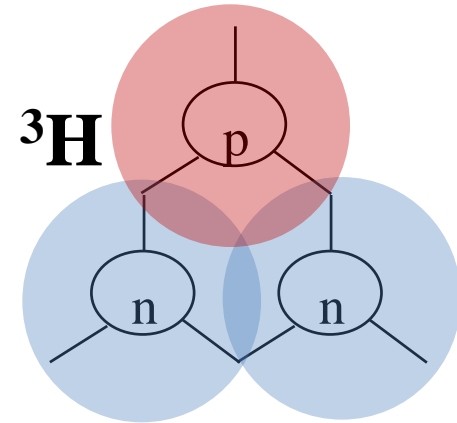
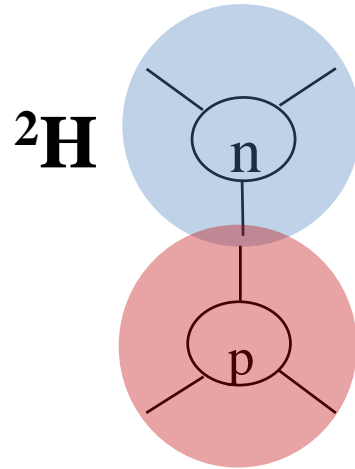
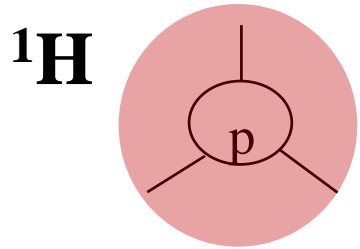
As a rigid body Nuclei can possess:

–particle – hole excitations

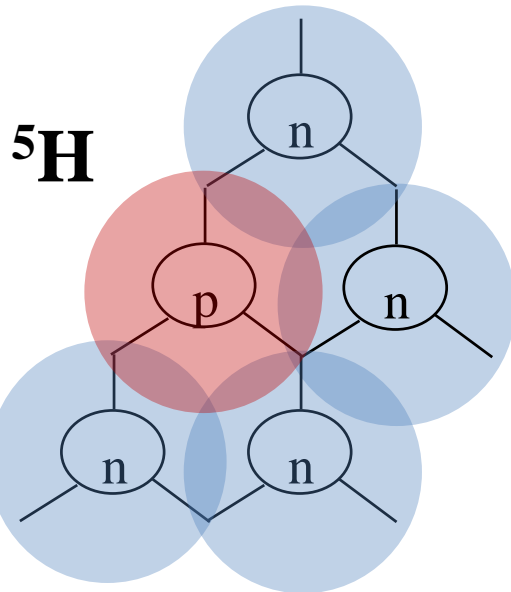
–collective modes of excitations

- Shape vibrations and fluctuations
- Rotations
- Isospin vibrations
- Sissor fluctuations

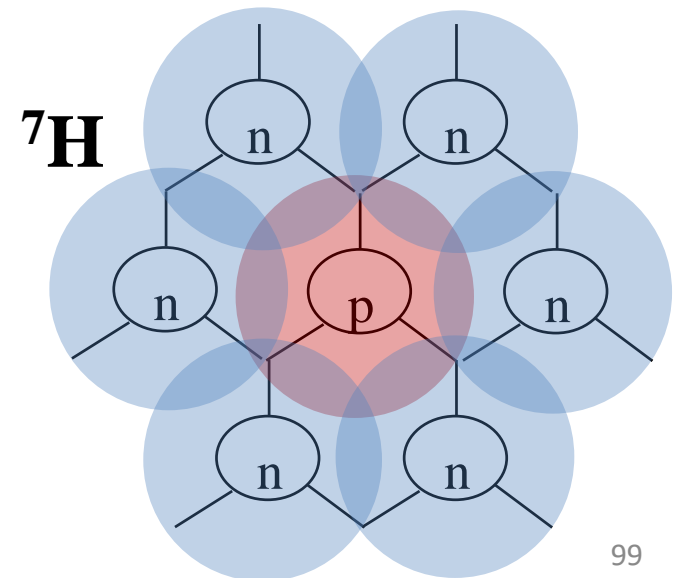
Bound Hydrogen Isotopes



~~${}^4\text{H}$~~



~~${}^6\text{H}$~~

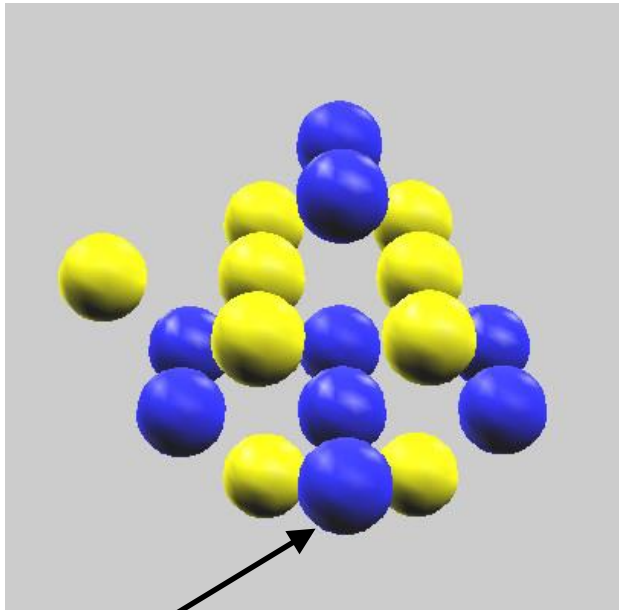


Summary

- Quarks play an explicit role in formation of the nuclear structure.
- Quark loops are building blocks of nuclear binding.
- Quarks and nucleons (protons and neutrons) inside nuclei are strongly correlated.
- ‘Halo’ nuclei – **fruits of quark-loop bindings**
- Effect of quark looping: $E_{\text{sep}} < E_{\text{bound}}/A$

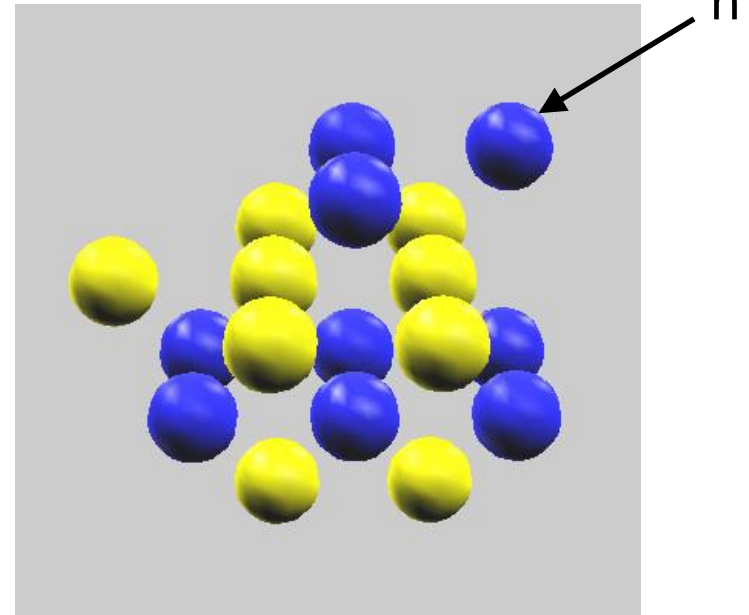
Fluorine Isomers

$^{18}\text{F}_m$
 $t_{1/2} \sim 200 \text{ ns}$



n 5^+

^{18}F
 $t_{1/2} \sim 110 \text{ min}$



1^+

protons – yellow
neutrons - blue