Small-p_T J/ψ production in the TMD parton model and NRQCD

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Outline

- NRQCD
- Soft gluon resummation approach
- Spectator model for gluon TMD PDF
- J/ψ production at low-p_T
- Polarized J/ψ production
- Conclusions

Hadronization model: NRQCD

- J/ψ wave function as a series with respect to relative constituent quarks velocity v:

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \\ + \mathcal{O}(v^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^1D_J^{(1,8)}]gg\rangle + \dots$$

- Hard cross section factorization:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X)\langle\mathcal{O}^{\mathcal{C}}[n]\rangle,$$

$\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$ – long distance matrix elements (LDME):

color singlet (CS) LDMEs — potential models,

color octet (CO) LDMEs — lattice QCD calculation or experimental data fitting

General remarks on our approximations in calculations of prompt J/ψ production

- Direct production: $g + g \to J/\psi + X$, feed-down contributions from $\psi(2S) \to J/\psi + X$ and $\chi_{cJ} \to J/\psi + \gamma$

  Prompt = Direct + Feed-down contributions

- We study here only the direct production. At the $\sqrt{s} = 200$ GeV (PHENIX data), feed-down contribution is about 30 %, at the $\sqrt{s} = 27$ GeV – about 20 % for the total cross sections (Our theoretical estimation).

- We study here only the gluon-gluon fusion taking in mind that quark-antiquark annihilation contribution may be about 30% for the total cross section [Saleev, Chernyshev, 2022]

- Our preliminary calculations were done in the LO approximation of the pQCD in α_S

Factorizations and initial parton transverse momenta

- **Collinear parton model (CPM)**: $q_T \ll k_T \sim \mu_F$,
  - parton distribution functions $f(x,\mu_F) \Rightarrow$ **DGLAP** equations,
$$q_1^\mu = x_1 p_1^\mu, \qquad q_2^\mu = x_2 p_2^\mu,$$
  - leading order (LO) processes are $2 \to 2$, the relevant for our study process is $g + g \to J/\psi + g$

- **Transverse Momentum Dependent (TMD) factorization**: $q_T, k_T \ll \mu_F \sim M$,
  - TMD parton distribution functions $F(x, \vec{q}_T, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations,
$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu,$$
  - preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:
$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2}\right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2}\right),$$
  - relevant processes only $2 \to 1$, $g + g \to J/\psi$ and intermediate $c\bar{c}$-state must be color-octet ${}^1S_0^{(8)}$ or ${}^3P_{0,2}^{(8)}$

TMD factorization and TMD PDFs

- General formula of TMD factorization [TMD Handbook, arXiv:2304.03302]:

$$d\sigma(J/\psi) \sim \int dx_1\, dx_2 \int d\boldsymbol{q_{1T}}\, d\boldsymbol{q_{2T}}\, f(x_1, \boldsymbol{q_{1T}}, \mu_F, \zeta_1)\, f(x_2, \boldsymbol{q_{1T}}, \mu_F, \zeta_2)\, \delta(\boldsymbol{q_{1T}} + \boldsymbol{q_{2T}} - \boldsymbol{k_T}) d\hat{\sigma}$$

- To implement **CS** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done:

$$d\sigma(J/\psi) \sim \int dx_1\, dx_2 \int d\boldsymbol{b_T}\, e^{i\boldsymbol{p_T}\boldsymbol{b_T}}\, \tilde{f}(x_1, \boldsymbol{b_T})\, \tilde{f}(x_2, \boldsymbol{b_T})\, d\hat{\sigma}(x_1, x_2, s)$$

$d\hat{\sigma}(x_1, x_2, s)$ is calulated as series in small α_S

Soft gluon resummation approach

- Soft and collinear gluon resummation approach by [J. Collins, D. Soper, 1981]:

$$d\sigma(J/\psi) \sim \int_0^\infty db_T \, b_T \, J_0(p_T b_T) \; e^{-S_P(b_T,\mu_F,Q)} \; e^{-S_{NP}(b_T)} \; \hat{F}(x_1, \mu'_{b^*}, b^*_T) \, \hat{F}(x_2, \mu'_{b^*}, b^*_T) \, d\hat{\sigma}$$

Note, $\hat{F}(x, \mu'_{b^*}, b^*_T)$ is not conventional TMD PDF !

- Sudakov factor in LL–LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_P(b_T, \mu_F, Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln \frac{Q^2}{\mu'^2} - \left(\frac{11 - 2N_f/C_A}{6} + 1 \right) \right] + \mathcal{O}(\alpha_s)$$

- Sudakov factor expression is valid only on region $b_0/Q \leqslant b_T \leqslant b_{T,\,\mathrm{max}}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu'_b = \frac{Q b_0}{Q b_T + b_0} \quad \text{and} \quad b^*_T(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\,\mathrm{max}})^2}}$$

Soft gluon resummation approach

- Master formula for soft gluon resummation [J. Collins (2011)]:

$$d\sigma(J/\psi) \sim \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T)}\; \hat{F}(x_1, \mu'_{b^*}, b_T^*)\, \hat{F}(x_2, \mu'_{b^*}, b_T^*)\, d\hat{\sigma}$$

- **Nonperturbative** quark factor obtained in SIDIS data fitting should be scaled by C_A/C_F for gluons [S. Aybat, T. Rogers (2011)]:

$$S_{NP}(b_T, Q) = \frac{C_A}{C_F}\left[g_1 \ln\frac{Q}{2Q_{NP}} + g_2\left(1 + 2g_3 \ln\frac{10xx_0}{x_0 + x}\right)\right] b_T^2$$

- In the leading order in α_S, the perturbative tail of TMD PDF is expressed with collinear PDF :

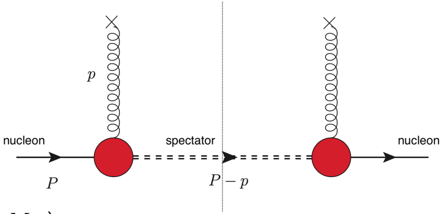
$$\hat{F}(x, b_T^*) = f(x, \mu'_{b^*}) + \mathcal{O}(\alpha_s)$$

Soft gluon resummation approach

- C. D. Davies, W.J. Stirling, Nucl. Phys. B244, 337 (1984) – **DY**
- P. Arnold, R. Kauffman, Nucl. Phys. B349, 381 (1991) – **W, Z**
- S. Catani et all. , Phys. Lett. B211, 335 (1988) – **H**
- T. Han, R. Meng, J. Ohnemus, Nucl. Phys. B384, 59 (1992) – **ZZ**
- E.L. Berger and R.B. Meng, Phys. Rev. D49, 3248 (1994) – $Q\bar{Q}$
- ..... .

Spectator model for gluon TMD PDF

- Spectator Model for gluon PDF by
  [A. Bacchetta, F. Conti, M. Radici (2008);
  A. Bacchetta, F. Celiberto, M. Radici, P. Taels (2020)]
- Expressions for unpolarized TMD f_1, helicity g_L, worm-gear g_T and Boer–Mulders $h_{\perp g}$ functions



- TMD PDF as a superposition of spectator's PDFs (at the fixed M_X) weighed on the spectral function:

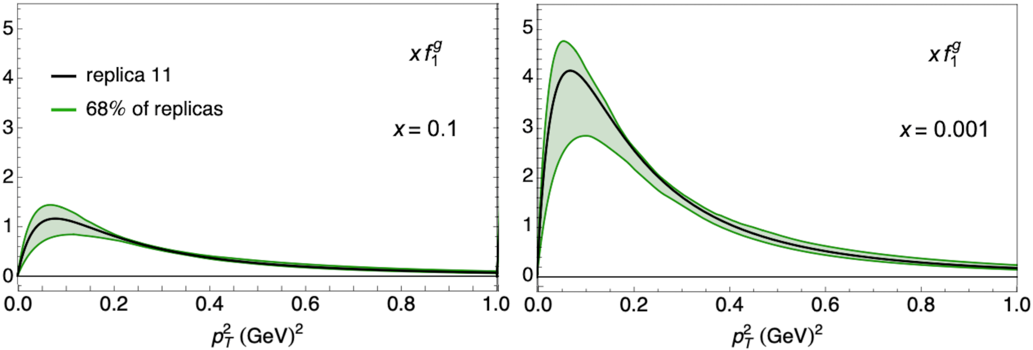
$$f_g(x, \boldsymbol{p_T^2}) = \int_M^\infty dM_X \rho_X(M_X) \widetilde{f}_g(x, \boldsymbol{p_T^2}, M_X), \qquad \rho_X(M_X) = \mu^{2a}\left[\frac{A}{B+\mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}}\right]$$

- Formula for Spectator Model TMD calculations, neglecting TMD evolution of PDF:

$$d\sigma(J/\psi) \sim \int dx_1 dx_2 \int d\boldsymbol{p_{1T}}\, d\boldsymbol{p_{2T}}\; f_g(x_1, \boldsymbol{p_{1T}^2})\, f_g(x_2, \boldsymbol{p_{2T}^2})\, \delta(\boldsymbol{p_{1T}} + \boldsymbol{p_{1T}} - \boldsymbol{k_T})\, d\hat{\sigma},$$

  We take $f_g(x, \boldsymbol{p_T^2})$ at the initial scale $\mu_F = \zeta = 1.6$ GeV.

Spectator model for gluon TMD PDF



The figures are from [A. Bacchetta, F. Celiberto, M. Radici, P. Taels (2020)]. There are 100 replicas, we are waiting another ones from F.C.

Matching of low-p_T and large-p_T regions with Inverse-Error Weighting Scheme

- Matched cross-section as a weighed sum of CPM and TMD terms [M. Echevarria, T. Kasemets, J.-P. Lansberg, C. Pisano, A. Signori (2018)]:

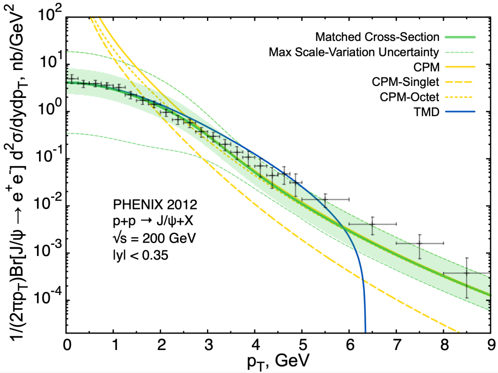
$$d\sigma = \mathcal{W}\, d\sigma^{\mathsf{TMD}} + \mathcal{Z}\, d\sigma^{\mathsf{CPM}}$$

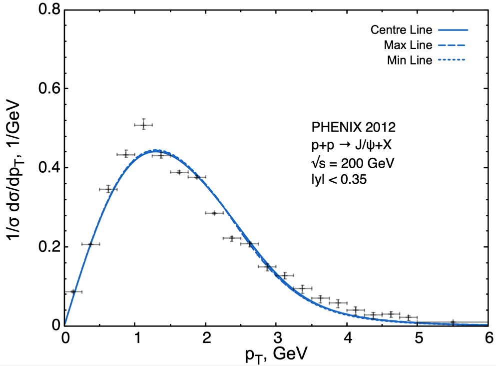
- Normalized weights for each of the two terms are

$$\mathcal{W} = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}, \qquad \mathcal{Z} = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}},$$

$$\Delta\mathcal{W} = \left(\frac{p_T}{Q}\right)^2, \qquad \Delta\mathcal{Z} = \left(\frac{M}{p_T}\right)^2$$

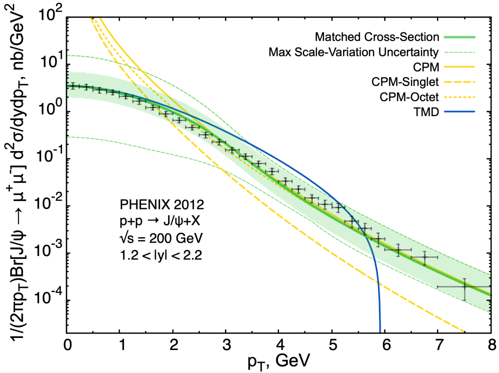
Extraction of CO LDME in the CPM (at $p_T > M$) and TMD (at $p_T < 1$ GeV) using the soft gluon resummation approach

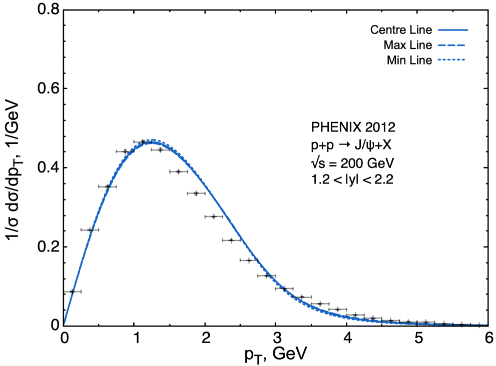




We may reduce scale uncertainties of theoretical calculations in study of normalized distributions !

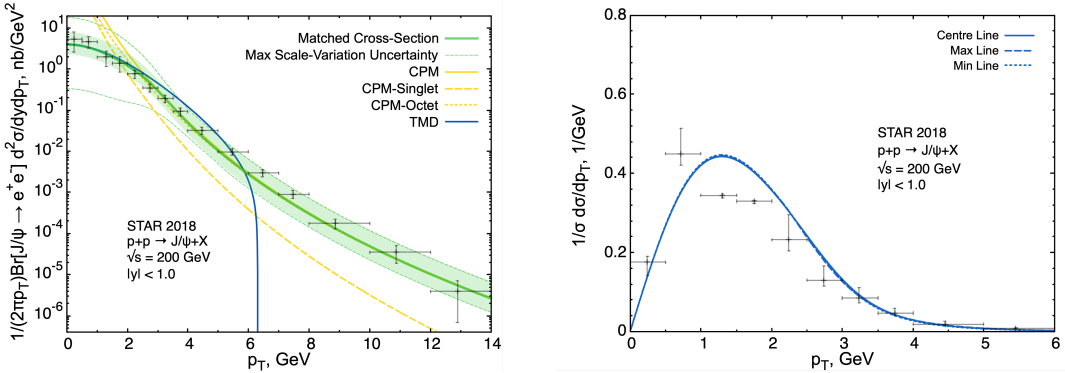
Extraction of CO NME in the CPM (at $p_T > M$) and TMD (at $p_T < 1$ GeV) using the soft gluon resummation approach





In the study of normalized distributions we may neglect dependence from unknown CO LDMEs !

Extraction of CO LDME in the CPM (at $p_T > M$) and TMD (at $p_T < 1$ GeV) using the soft gluon resummation approach



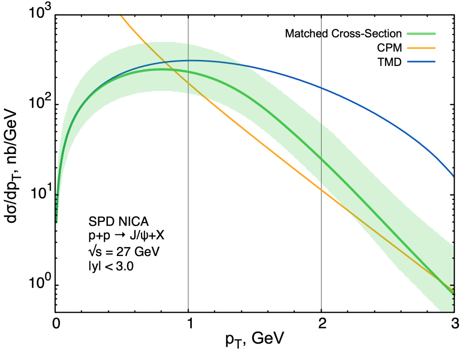
In the study of normalized distributions we may neglect dependence from feed-down contributions !

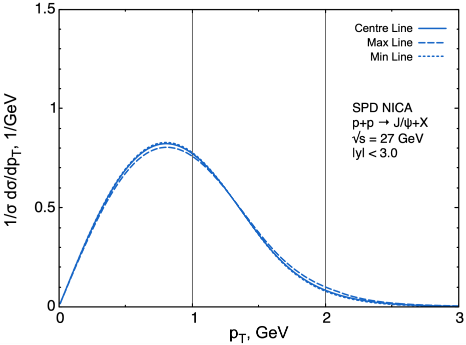
Extraction of CO LDME in the CPM (at $p_T > M$) and TMD (at $p_T < 1$ GeV) using the soft gluon resummation approach

- TMD: $M_7^{J/\psi} = \langle\mathcal{O}[^1S_0^{(8)}]\rangle + 7\cdot\langle\mathcal{O}[^3P_0^{(8)}]\rangle/m_c^2 = 1.7\cdot 10^{-1}$ GeV^3,
  $\chi^2/\text{d.o.f.} = 2.61/9$
- CPM: $M_3^{J/\psi} = \langle\mathcal{O}[^1S_0^{(8)}]\rangle + 3\cdot\langle\mathcal{O}[^3P_0^{(8)}]\rangle/m_c^2 = 7.7\cdot 10^{-2}$ GeV^3, $\langle\mathcal{O}[^3S_1^{(8)}]\rangle = 6.5\cdot 10^{-3}$ GeV^3,
  $\chi^2/\text{d.o.f.} = 118.15/33$

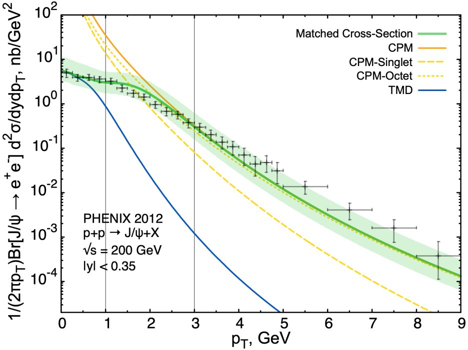
CO LDME	LO CPM [Cho, Leibovich (1996)]	NLO CPM [Butenschön, Kniehl (2011)]	NLO CPM [Ma, Wang, Chao (2011)]
$M_3^{J/\psi}$	$(6.6\pm1.5)\cdot10^{-2}$ GeV^3	$(1.83\pm0.56)\cdot10^{-2}$ GeV^3	$(-1.18\pm2.94)\cdot10^{-2}$ GeV^3
$\langle\mathcal{O}[^3S_1^{(8)}]\rangle$	$(6.6\pm2.1)\cdot10^{-3}$ GeV^3	$(1.68\pm0.46)\cdot10^{-3}$ GeV^3	$(8.86\pm3.91)\cdot10^{-2}$ GeV^3

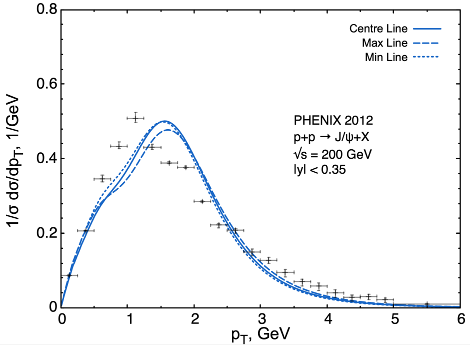
Predictions for SPD NICA using the soft gluon resummation approach





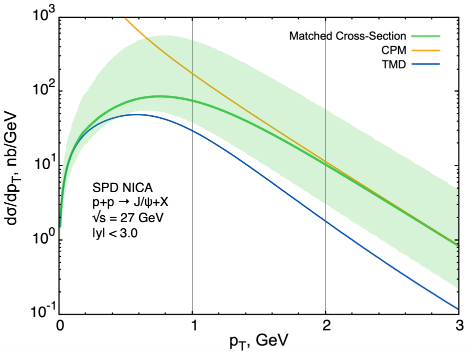
Calculations using the Spectator Model for gluon TMD

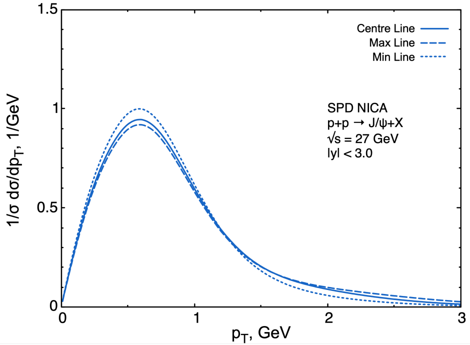




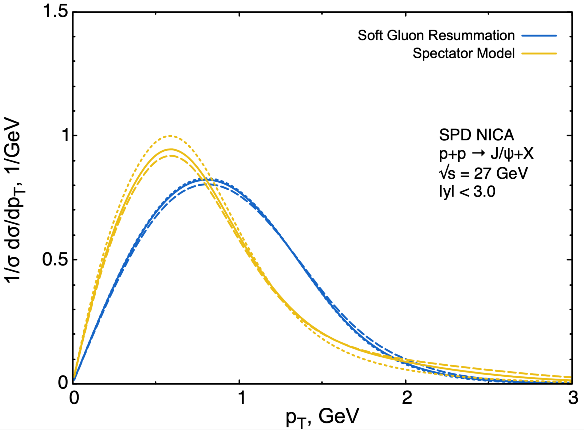
CO LDME	Soft Gluon Resummation	Spectator Model
$M_7^{J/\psi}$	$1.7 \cdot 10^{-1}$ GeV^3	$2.1 \cdot 10^{-2}$ GeV^3

Predictions for SPD NICA using the Spectator Model for gluon TMD





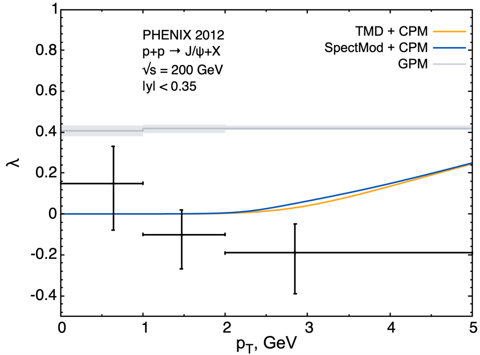
Comparison of predictions for J/ψ p_T−spectra at SPD NICA energy

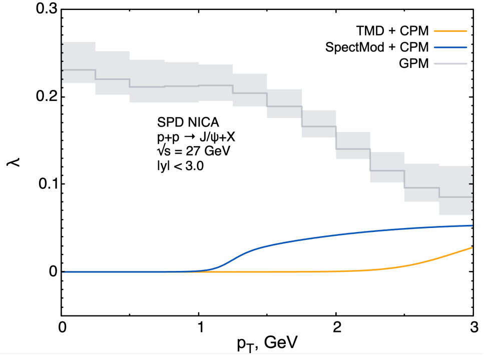


Polarized J/ψ production in TMD factorization

$$\frac{d\sigma}{d\Omega} \sim 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \, \cos 2\varphi + \lambda_{\theta\varphi} \cos\varphi$$

$$\lambda_\theta = \frac{\sigma_\mathsf{T} - 2\,\sigma_\mathsf{L}}{\sigma_\mathsf{T} + 2\,\sigma_\mathsf{L}} = \frac{\sigma - 3\,\sigma_\mathsf{L}}{\sigma + \sigma_\mathsf{L}}$$





Conclusions

- We have analyzed two different approaches to calculate low-p_T J/ψ production in the TMD factorization
- CO LDMEs of NRQCD are necessary to describe J/ψ production using the CPM and especially using the TMD factorization, where they are given main contributions
- Both approaches, the soft gluon resummation and the Spectator Model for gluon TMD PDF, satisfyingly describe experimental data of unpolarized J/ψ production at the $\sqrt{s} = 200$ GeV in regions of TMD and CPM factorizations, and in the intermediate region as well
- Description of J/ψ polarization using TMD and NRQCD apparently doesn't controvert the experimental data of PHENIX, as opposed to our previous calculations based on Generalized Parton Model [A. Karpishkov, V. Saleev, K. Shilyaev, Physics of Atomic Nuclei, 4 (2024)]
- We estimate the perspective region for the extraction of gluon TMD PDF in the J/ψ production as $p_T \leqslant 1$ GeV

Plans

- To take into account factorization scale evolution in the Spectator Model
- To repeat calculations for all replicas in the Spectator Model
- To take correctly into account feed-down contributions
- To estimate quark-antiquark contribution in the J/ψ production
- To do calculations with different matching schemes

THANK YOU FOR YOUR ATTENTION!