Direct photon production in the Parton Reggeization Approach: from high to low energy[1]

A. Chernyshev[†,⋆,2] & V. Saleev[†,∗]

[†]Bogoliubov Laboratory of Theoretical Physics, JINR

[⋆]Moscow State U., Dubna Branch

[∗]Samara National Research U.

VIII SPD Collaboration meeting

5–8$^{\text{Nov}}$, 2024

Supported by:

Foundation for the Advancement of Theoretical Physics and Mathematics BASIS, grant No. 24–1–1–16–5.

[1]Based on: `arXiv:2410.06644`

[2]Email: chernyshev@theor.jinr.ru

Motivation: direct photon production

Consider *inclusive process* of **direct photon** production in the region $\mu_F \gg \Lambda$:

$$p\,(P_1)\;+\;p\,(P_2)\;\to\;\gamma\,(p^{\pm},\mathbf{p}_T)\;+\;X,$$

where $P_{1,2} = \left(P^{\pm}/2\right) n_{\mp}$ with $P^{\pm} = \sqrt{S}$. We use Sudakov light–cone basis $n_{\pm}$: $(n_{\pm}, n_{\mp}) = 2$, $p^{\pm} = (p, n_{\pm})$, so $y(p) = (1/2)\ln(p^{+}/p^{-})$.

Properties:

- Single–scale process: $\mu_F \sim p_T \Rightarrow$ DGLAP logs $\alpha_s^n \ln^n\left(\mu_F^2/\Lambda^2\right)$ supposed to be dominant;
- Fully perturbative process at parton level;
- Good probe to study PDFs / asymmetries / etc.

Disadvantages:

- Higher order corrections are found to be large $\Rightarrow$ *resummation is needed*;
- Direct photon production is studied at high p_T $\Rightarrow$ Collins–Soper–Sterman (CSS) approach is not suitable for studying TMD PDFs.

One possible solution is the **Parton Reggeization Approach**[Nefedov, Saleev, Shipilova '13].

Motivation: direct photon production in the PRA

Introduce $x_\pm = p^\pm / P^\pm$, then cross section for the direct photon production in the *Collinear Parton Model (CPM)* reads:

$$\sigma^{(\mathrm{CPM})} = \int\limits_{x_+}^{1} \frac{dz_+}{z_+} F\left(\frac{x_+}{z_+}, \mu_F^2\right) \int\limits_{x_-}^{1} \frac{dz_-}{z_-} F\left(\frac{x_-}{z_-}, \mu_F^2\right) \times H\left(z_\pm, \alpha_s(\mu_R^2)\right) + \mathcal{O}\left(\frac{\Lambda^\#}{\mu_F^\#}\right),$$

here $F(x, \mu_F^2) = x f(x, \mu_F^2)$. We assume $\mu_F \simeq \mu_R \simeq \mu$.

- **Motivation:** resum already in LO large radiative corrections keeping $\boxed{z_\pm \sim 1}$.
- **Resummation formalism:** Parton Reggeization Approach (PRA), uses properties of the *Modified MRK (mMRK) approximation* correct up to $z_\pm \sim 1$. Based on the High–Energy Factorization *or* k_T*–factorization* [Gribov, Levin, Ryskin '84; Collins, Ellis '91, 94; Catani, Hautman '94].

The purpose of this study is to calculate direct photon production cross section in the NLO* approximation of the PRA from $\sqrt{S} = 13$ TeV down to SPD energies $\sqrt{S} \sim 20 - 30$ GeV.

PRA: I

Cross section in the PRA has a k_T*–factorization form*:

$$\sigma^{(\mathrm{PRA})}(pp\to\gamma X) = \int\frac{dx_+}{x_+}\int\frac{d^2\mathbf{q}_{T_1}}{\pi}\,\Phi_q(x_+,\mathbf{q}_{T_1}^2,\mu_F^2)\int\frac{dx_-}{x_-}\int\frac{d^2\mathbf{q}_{T_2}}{\pi}\,\Phi_{\bar q}(x_-,\mathbf{q}_{T_2}^2,\mu_F^2)$$
$$\times\ \mathcal{H}^{(\mathrm{mMRK})}\left(x_\pm,\mathbf{q}_{T_{1,2}},\alpha_s(\mu_R^2),\dots\right) + \mathcal{O}\left(\frac{\Lambda^\#}{\mu_F^\#},\frac{\mu_F^2}{S}\right).$$

PROCESS INDEPENDENT ***Unintegrated PDF (uPDF)*** calculated in the *modified KMRW (mKMRW)* model[KMR '01; MRW '03; Nefedov, Saleev '20]:

$$\Phi_i(x,\mathbf{q}_T^2,\mu_F^2) = \frac{\alpha_s\left(\mathbf{q}_T^2\right)}{2\pi}\frac{T_i\left(x,\mathbf{q}_T^2,\mu_F^2\right)}{\mathbf{q}_T^2}\sum_j\int_x^1 dz\,P_{ij}(z)\,F_j\left(\frac{x}{z},\mathbf{q}_T^2\right)\theta\left(\Delta\left(\mathbf{q}_T^2,\mu^2\right)-z\right).$$

- *Exact normalization condition*:

$$\int_0^{\mu_F^2} d\mathbf{q}_T^2\,\Phi_i(x,\mathbf{q}_T^2,\mu_F^2) = F_i(x,\mu_F^2).$$

- In the limit $\mathbf{q}_T^2 \ll \mu_F^2$: $1-\Delta\left(\mathbf{q}_T^2,\mu_F^2\right)\simeq|\mathbf{q}_T|/\mu_F$ and Sudakov form factor becomes:

$$T_q\left(x,\mathbf{q}_T^2,\mu_F^2\right) \simeq \exp\left[-\frac{\alpha_s}{2\pi}C_F\left(\frac{1}{2}\ln^2\frac{\mu_F^2}{\mathbf{q}_T^2}-\frac{3}{2}\ln\frac{\mu_F^2}{\mathbf{q}_T^2}\right)\right]$$

$\Rightarrow$ *consistency with the Collins–Soper–Sterman approach*[CSS '85; C '11].

PRA: II

Cross section in the PRA has a k_T*–factorization form*:

$$\sigma^{(\mathrm{PRA})}(pp \to \gamma X) = \int \frac{dx_+}{x_+} \int \frac{d^2\mathbf{q}_{T_1}}{\pi}\, \Phi_q(x_+, \mathbf{q}_{T_1}^2, \mu_F^2) \int \frac{dx_-}{x_-} \int \frac{d^2\mathbf{q}_{T_2}}{\pi}\, \Phi_{\bar q}(x_-, \mathbf{q}_{T_2}^2, \mu_F^2)$$
$$\times \quad \mathcal{H}^{(\mathrm{mMRK})}\left(x_\pm, \mathbf{q}_{T_{1,2}}, \alpha_s(\mu_R^2), \ldots\right) + \mathcal{O}\left(\frac{\Lambda^\#}{\mu_F^\#}, \frac{\mu_F^2}{S}\right).$$

PROCESS DEPENDENT ***Hard Scattering Coefficient (HSC)*** $\mathcal{H}^{(\mathrm{mMRK})}$ calculated in the *modified MRK (mMRK)* approximation for QCD amplitudes, so it is gauge invariant due to *gauge invariance of the effective vertices*, f.e., in the LO:

$$\boxed{(q_1+q_2)^\mu \mathcal{A}_\mu^{(\mathrm{MRK})}(Q(q_1)\, \bar{Q}(q_2) \to \gamma) = 0,}$$

unlike to case of the TMD factorization where:

$$(q_1+q_2)^\mu \mathcal{A}_\mu^{(\mathrm{TMD})}(q(q_1)\, \bar{q}(q_2) \to \gamma) = \mathcal{O}\left(\mathbf{q}_{T_{1,2}}^2/\mu_F^2\right).$$

Together with mKMRW uPDFs properties, PRA may be considered as a gauge–invariant TMD.

Direct photon production in the PRA

The tree–level partonic subprocesses of the orders $\mathcal{O}(\alpha^n \alpha_s^m)$ with $n+m \leq 2$:

$$
\begin{aligned}
\text{LO, IR finite:} \quad & Q\,(q_1) + \bar{Q}\,(q_2) \to \gamma\,(q_3), \\
\text{NLO}^\star, \text{ IR finite:} \quad & Q\,(q_1) + R\,(q_2) \to \gamma\,(q_3) + q\,(q_4), \\
\text{NLO}^\star, \text{ IR diverge:} \quad & Q\,(q_1) + \bar{Q}\,(q_2) \to \gamma\,(q_3) + g\,(q_4).
\end{aligned}
$$

- There is **no collinear divergences** when $\mathbf{q}_3 \parallel \mathbf{q}_4$ since the experimental definition of the cross section requires no hadrons in the *isolation cone of the photon:*

$$
\boxed{\sigma^{(\text{iso})} = \sigma\left(r \geq R^{(\text{iso})}\right),} \qquad r = \sqrt{\Delta y^2 + \Delta\phi^2}.
$$

- To estimate the fragmentation photons contribution, one may use the Frixione modification of the cone condition:

$$
E_T \leq E_T^{(\text{iso})}\,\chi(r;n), \qquad \chi(r;n) = \left(\frac{1-\cos r}{1-\cos R^{(\text{iso})}}\right)^n .
$$

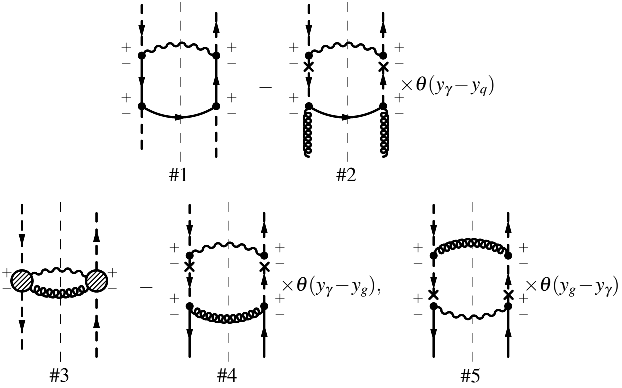
We found the impact of the Frixione cone condition is negligibly small.

mMRK double counting subtraction scheme: I

Matching of the LO contribution with the NLO* tree–level corrections:

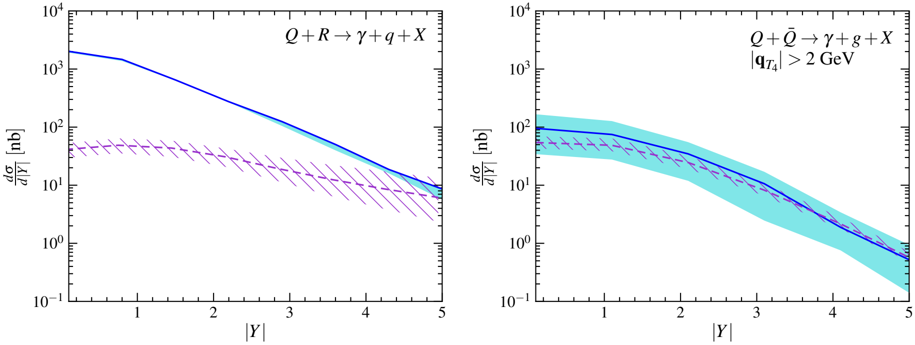
$$\boxed{\sigma \;=\; \sigma^{(\text{LO, PRA})} + \sigma^{(\text{NLO}^\star\text{, PRA})} - \sigma^{(\text{mMRK, sub.})},}$$

subtraction term may be obtained using the exact normalization of the uPDF, SUCH PROCEDURE IS IR SAFE SINCE INITIAL STATE REGGEONS IN THE PRA HAVE FACTORS $q^\pm/(2|\mathbf{q}_{T_{1,2}}|) \Rightarrow$ CORRECT COLLINEAR LIMIT.



NOTE THAT $\hat{s}$– AND $\hat{u}$–CHANNEL DIAGRAMS ARE VANISH IN THE REGGE LIMIT AND DON'T CONTRIBUTE TO THE SUBTRACTION TERM.

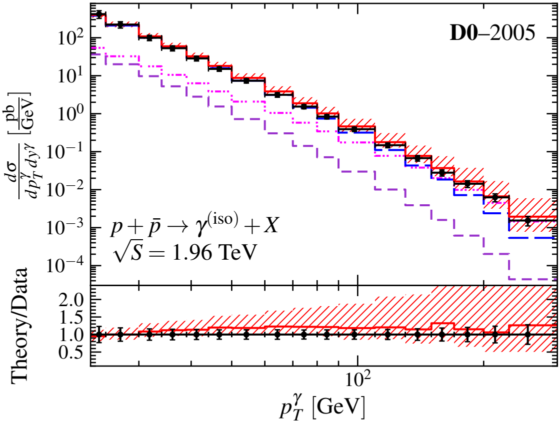
mMRK double counting subtraction scheme: II



NLO* vs. mMRK subtraction term

- In both cases NLO* coincides with the relevant mMRK subtraction at large $Y = y^\gamma - y^{q,g}$;
- In case of the second NLO* subprocess, the subtraction is large even at intermediate values of Y, so the resulting contribution is small $\Rightarrow$ **self–consistency of the PRA**.

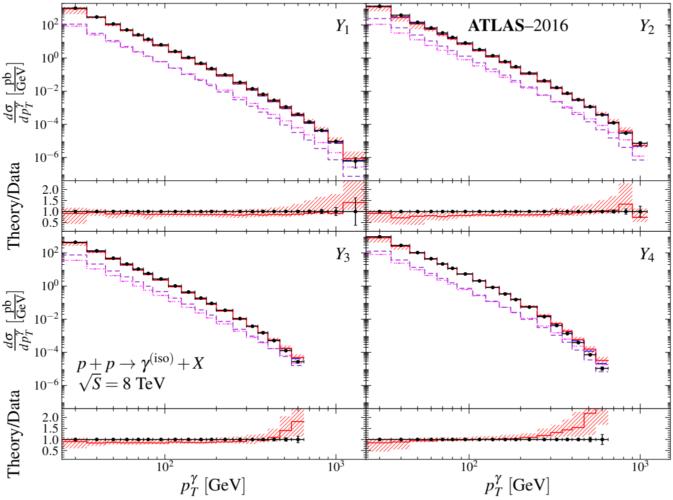
Results @ DØ '05, $\sqrt{S} = 1.96$ TeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub.

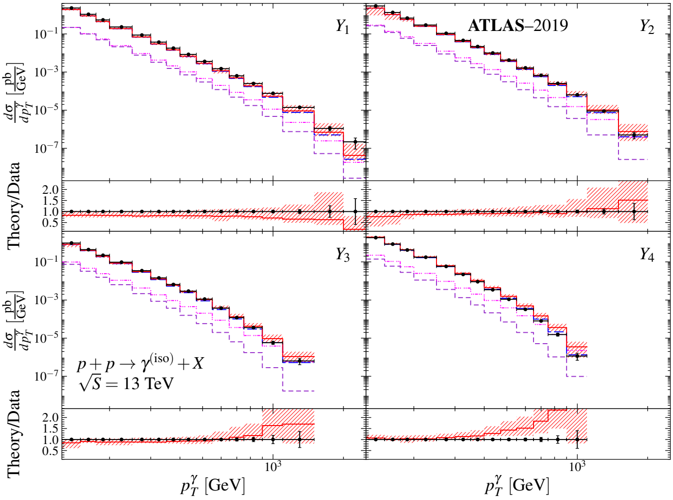
Results @ ATLAS '16, $\sqrt{S} = 8$ TeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub.

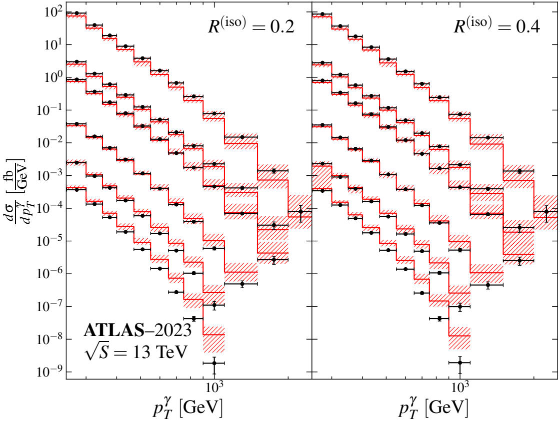
Results @ ATLAS '19, $\sqrt{S} = 13$ TeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub.

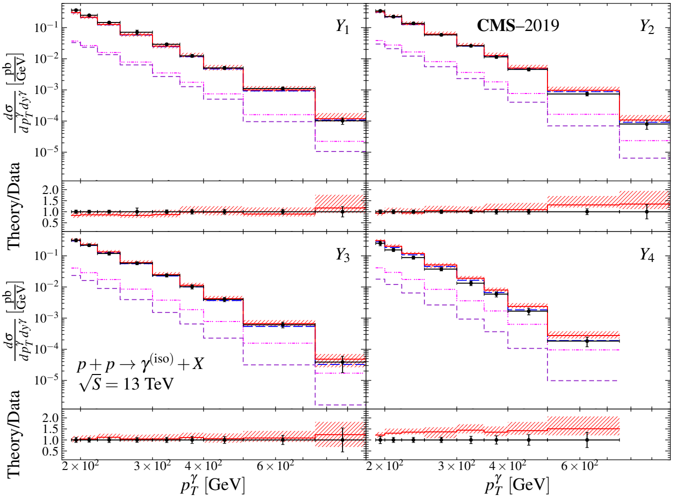
Results @ ATLAS '23, $\sqrt{S} = 13$ TeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub.

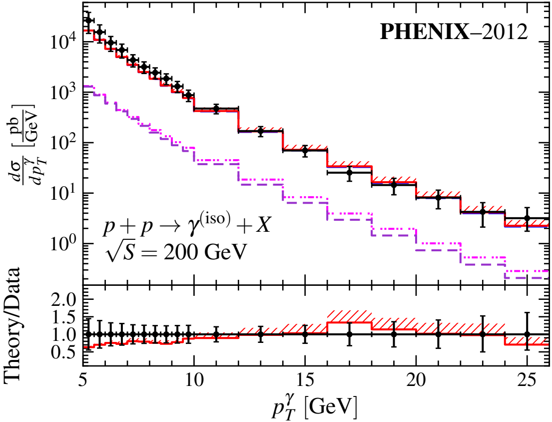
Results @ CMS '19, $\sqrt{S} = 13$ TeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub.

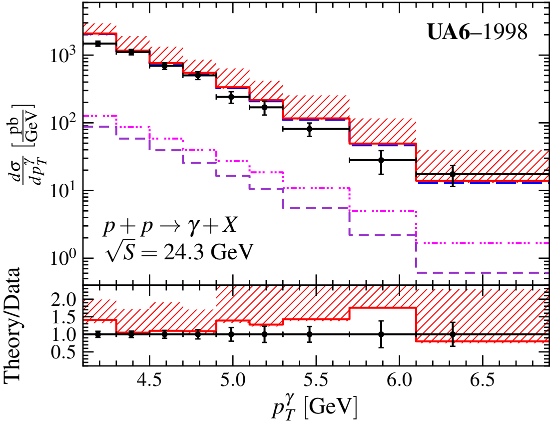
Results @ PHENIX '12, $\sqrt{S} = 200$ GeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub.

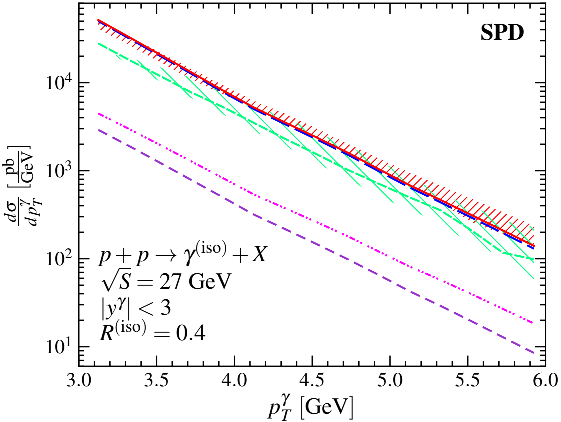
Results @ UA6 ’98, $\sqrt{S} = 24.3$ GeV



Contributions:

NLO⋆ + LO - mMRK sub. / NLO⋆ / LO / mMRK sub.

Results @ SPD, $\sqrt{S} = 27$ GeV



Contributions:

NLO* + LO - mMRK sub. / NLO* / LO / mMRK sub. / LO CPM

Conclusions

- The production of the single isolated photon within the framework of the NLO* approximation of the Parton Reggeization Approach is studied;
- The new modified Multi–Regge Kinematics double counting subtraction scheme is proposed;
- We have obtained a quite satisfactory description of the transverse momentum spectra at wide energy range up to $p_T^\gamma/\sqrt{S} \simeq 0.2 - 0.3$;
- The study of the direct photon production at low energies was performed, predictions for the SPD NICA experiment were made;
- We demonstrated that NLO tree–level correction $Q\bar{Q} \to \gamma g$ to the already existed LO subprocesses $Q\bar{Q} \to \gamma$ is small after subtraction procedure, this shows the self–consistency of the Parton Reggeization Approach.

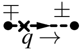
Acknowledgments: we are grateful to M. Nefedov for a fruitful discussion on the direct photon production in the High–Energy Factorization approach.

Thank you for your attention!

Lipatov's Effective Action approach

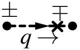
In Ref.[Antonov, Cherednikov, Kuraev, Lipatov '05], the Feynman rules of the EFT for *Multi–Regge Kinematics (MRK)* processes in QCD[Lipatov '95; Lipatov, Vyazovsky '02] were constructed. FOR AN OVERVIEW SEE [Nefedov '19; Hentschinski '20].

Some relevant for the study Feynman rules (implemented in the `ReggeQCD`[Nefedov]):



$$= {}_{\times}D^{(\pm)}(q) = \frac{i\not{q}}{q^2}\hat{P}^{\pm},$$

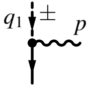
#1



$$= D^{(\pm)}_{\times}(q) = \hat{P}^{\pm}\frac{i\not{q}}{q^2};$$

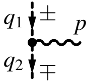
#2

#3



$$= \gamma^{\mu}_{(\pm)}(q_1, p) = \gamma^{\mu} + \not{q}_1 \frac{n^{\mu}_{\mp}}{p^{\mp}},$$

#4



$$= \Gamma^{\mu}_{(\pm\mp)}(q_1, q_2) = \gamma^{\mu} + \not{q}_1 \frac{n^{\mu}_{\mp}}{p^{\mp}} + \not{q}_2 \frac{n^{\mu}_{\pm}}{p^{\pm}}.$$

#1 and #2 are the Reggeized quark (Q) propagators;

#3 is a $Q\bar{q}\gamma$ effective vertex;

#4 is a $Q\bar{Q}\gamma$ effective vertex (known as *Fadin–Sherman*[FS '74] effective vertex).

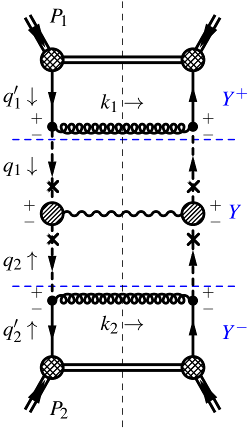
Parton Reggeization Approach @ mMRK

The PRA based on the *modified MRK (mMRK)* approximation for hard QCD amplitudes. This kind of approximation is actively used[Martin *et.al.* '03; Andersen *et.al.* '09; Nefedov, Saleev '20].

Using Feynman rules of the EFT, ME factorizes keeping $\boxed{z_\pm \sim 1}$:

$$\overline{|\mathcal{M}^{(\mathrm{mMRK})}(q\bar{q}\to g\gamma g)|^2} = \frac{4g^2}{q_1^2 q_2^2}\frac{P_{qq}(z_+)}{z_+}\,\theta\left(\Delta\left(\mathbf{q}_{T_2}^2,\mu^2\right)-z_+\right) \times \frac{P_{qq}(z_-)}{z_-}\,\theta\left(\Delta\left(\mathbf{q}_{T_2}^2,\mu^2\right)-z_-\right) \times \overline{|\mathcal{A}^{(\mathrm{MRK})}(Q(\mathbf{q}_{T_1})\bar{Q}(\mathbf{q}_{T_2})\to\gamma)|^2},$$

where $\boxed{z_+ = q_1^+/q_1'^+}$ and $\boxed{z_- = q_2^-/q_2'^-}$, and $\Delta(\mathbf{q}_T^2,\mu^2) = \mu/(\mu+|\mathbf{q}_T|)$ is a KMR cutoff ensures rapidity ordering: $y(k_1) > y(q_3)$ and $y(q_3) > y(k_2)$.



The MRK matrix element is known for a time[Saleev '08]:

$$\overline{|\mathcal{A}^{(\mathrm{MRK})}(Q(\mathbf{q}_{T_1})\bar{Q}(\mathbf{q}_{T_2})\to\gamma)|^2} = 4\pi\alpha\frac{C_A}{N_c^2}\times\left(\mathbf{q}_{T_1}^2+\mathbf{q}_{T_2}^2\right).$$

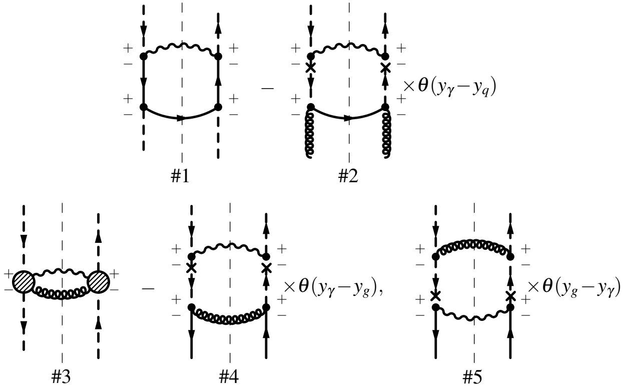
mMRK double counting subtraction scheme

Three main requirements to the double counting subtraction scheme:

i. **Gauge–invariance** of the subtraction terms;

ii. The **number of the rapidity** regions must leave three ($Y^{\pm}, Y$), SINCE WE ARE INTERESTING IN THE CASE OF DOUBLE COUNTING BETWEEN HSC AND UPDF, IT'S NOT ENOUGH ONLY TO PUT $\hat{t}$–CHANNEL PROPAGATOR REGGEIZED;

iii. The photon and additional parton must be strongly **divided by the rapidities**.



NOTE THAT $\hat{s}$– AND $\hat{u}$–CHANNEL DIAGRAMS ARE VANISH IN THE REGGE LIMIT AND DON'T CONTRIBUTE TO THE SUBTRACTION TERM.

mMRK double counting subtraction scheme

The squared subtraction amplitudes are **correct in the Regge limit** $(-\hat{t})/\hat{s} \to 0$:

$$\begin{aligned}
\overline{|\,\mathcal{A}^{(\text{mMRK, sub.})}(Qg \to \gamma q)\,|^2} &= g^2 \frac{2C_F N_c}{C_A (N_c^2-1)} \frac{\hat{s}+\mathbf{q}_{T_1}^2}{\hat{s}+\hat{t}+\mathbf{q}_{T_1}^2} \frac{T_F^{-1} P_{qg}(z)}{z(-\hat{t})}\, \theta\,(y(q_3)-y(q_4)) \\
&\times \overline{|\,\mathcal{A}^{(\text{MRK})}(Q(\mathbf{q}_{T_1})\,\bar{Q}(\mathbf{q}_T) \to \gamma)\,|^2}, \\
\overline{|\,\mathcal{A}^{(\text{mMRK, sub.})}(Q\bar{q} \to \gamma g)\,|^2} &= g^2 \frac{2C_F}{C_A} \frac{\hat{s}+\mathbf{q}_{T_1}^2}{\hat{s}+\hat{t}+\mathbf{q}_{T_1}^2} \frac{C_F^{-1} P_{qq}(z)}{z(-\hat{t})}\, \theta\,(y(q_3)-y(q_4)) \\
&\times \overline{|\,\mathcal{A}^{(\text{MRK})}(Q(\mathbf{q}_{T_1})\,\bar{Q}(\mathbf{q}_T) \to \gamma)\,|^2},
\end{aligned}$$

here $q = q_1 - q_3$, momenta fraction $z = q_4^- / q_2^-$, and $\boxed{q_2 \simeq \left(q_2^-/2\right) n_+}$.