Prompt charmonium production at small p_T in the Soft Gluon Resummation approach

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Outline

- Hadronisation model: NRQCD
- Soft Gluon Resummation approach
- InEW, matching of factorisation theorems
- J/ψ production at small p_T
- J/ψ polarisation
- Summary

Hadronisation model: NRQCD

- J/ψ wave function as a series with respect to relative constituent quarks velocity v:

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \\ + \mathcal{O}(v^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^1D_J^{(1,8)}]gg\rangle + \dots$$

- Approximate v-scaling due to $v^2 \approx 0.2$
- Hard cross section factorisation:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X)\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$$

- Nonperturbative (hadronisation) factors:

$\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$ – long-distance matrix elements (LDME):

color singlet LDMEs — potential models, data for leptonic decay

color octet LDMEs — lattice QCD calculation or experimental data fitting

General remarks on our approximations in calculations of prompt J/ψ production

- Direct production: $g + g \to J/\psi + X$, feed-down contributions from $\psi(2S) \to J/\psi + X$ and $\chi_{cJ} \to J/\psi + \gamma$

  Prompt = Direct + Feed-down contributions

- We study the direct production & the P-wave feed-down. At $\sqrt{s} = 200$ GeV (PHENIX data), feed-down contribution is estimated at 30%

- We study gluon-gluon fusion & quark-antiquark annihilation, quark-antiquark subprocesses may contribute about 10% to the total cross section at $\sqrt{s} = 200$ GeV (within the Improved Color Evaporation model) [Saleev, Chernyshev, 2022]

- Our preliminary calculations were done in the LO approximation of the pQCD in α_S

TMD factorisation and initial parton transverse momenta

- **Transverse Momentum Dependent (TMD) factorization**: $q_T, k_T \ll \mu_F \sim M$,
- TMD parton distribution functions $F(x, \boldsymbol{q_T}, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations,

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu,$$

- Preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2}\right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2}\right),$$

- Relevant processes only $2 \to 1$, intermediate $c\bar{c}$-states can be
  - color-octet ${}^1S_0^{(8)}$, ${}^3P_{0,2}^{(8)}$ and ${}^3S_1^{(8)}$ for J/ψ
  - color-sinlget ${}^3P_{0,2}^{(1)}$ and color-octet ${}^3S_1^{(8)}$ for χ_{cJ}

TMD factorisation and TMD PDFs

- General formula of TMD factorization [TMD Handbook, arXiv:2304.03302]:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int d\boldsymbol{q_{1T}}\, d\boldsymbol{q_{2T}}\, F(x_1, \boldsymbol{q_{1T}}, \mu_F, \zeta_1)\, F(x_2, \boldsymbol{q_{1T}}, \mu_F, \zeta_2)\, \delta(\boldsymbol{q_{1T}} + \boldsymbol{q_{2T}} - \boldsymbol{p_T})$$

- To implement **CS** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int \frac{d\boldsymbol{b_T}}{(2\pi)^2}\, e^{i\boldsymbol{p_T}\boldsymbol{b_T}}\, \hat{F}(x_1, \boldsymbol{b_T}, \mu_F, \zeta_1)\, \hat{F}(x_2, \boldsymbol{b_T}, \mu_F, \zeta_2)$$

  - σ_0 is calculated as series in small α_S

Soft Gluon Resummation approach

- Soft and collinear gluon resummation approach by [J. Collins, D. Soper, 1981]:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T)}\; \hat{F}(x_1, \mu'_{b^*}, b_T^*)\, \hat{F}(x_2, \mu'_{b^*}, b_T^*)$$

- Sudakov factor in LL–LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_P(b_T, \mu_F, Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln \frac{Q^2}{\mu'^2} - \left(\frac{11 - 2N_f/C_A}{6} \right) \right] + \mathcal{O}(\alpha_s)$$

- Sudakov factor expression is valid only on region $b_0/Q \leqslant b_T \leqslant b_{T,\,\mathsf{max}}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu'_b = \frac{Qb_0}{Qb_T + b_0} \quad \text{and} \quad b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\,\mathsf{max}})^2}}$$

Soft Gluon Resummation approach

- Master formula for soft gluon resummation:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T)}\; \hat{F}(x_1, \mu'_{b^*}, b_T^*)\, \hat{F}(x_2, \mu'_{b^*}, b_T^*)$$

- **Nonperturbative** quark factor obtained in SIDIS data fitting: [S. Aybat, T. Rogers (2011)]:

$$S_{NP}(b_T, Q) = \left[g_1 \ln \frac{Q}{2Q_{NP}} + g_2 \left(1 + 2g_3 \ln \frac{10 x x_0}{x_0 + x} \right) \right] b_T^2$$

  - it should be Casimir-scaled by C_A/C_F for gluons

- In the leading order of α_S, the perturbative tail of TMD PDF is expressed with collinear PDF:

$$\hat{F}(x, \mu'_{b^*}, b_T^*) = f(x, \mu'_{b^*}) + \mathcal{O}(\alpha_S) + \mathcal{O}(b_T \Lambda_{\mathsf{QCD}})$$

Matching of small-p_T and high-p_T regions within Inverse-Error Weighting Scheme

- Matched cross-section as a weighed sum of CPM and TMD terms [M. Echevarria, T. Kasemets, J.-P. Lansberg, C. Pisano, A. Signori (2018)]:

$$d\sigma = \mathcal{W}\, d\sigma^{\text{TMD}} + \mathcal{Z}\, d\sigma^{\text{CPM}}$$

- Normalised weights for each of the two terms:

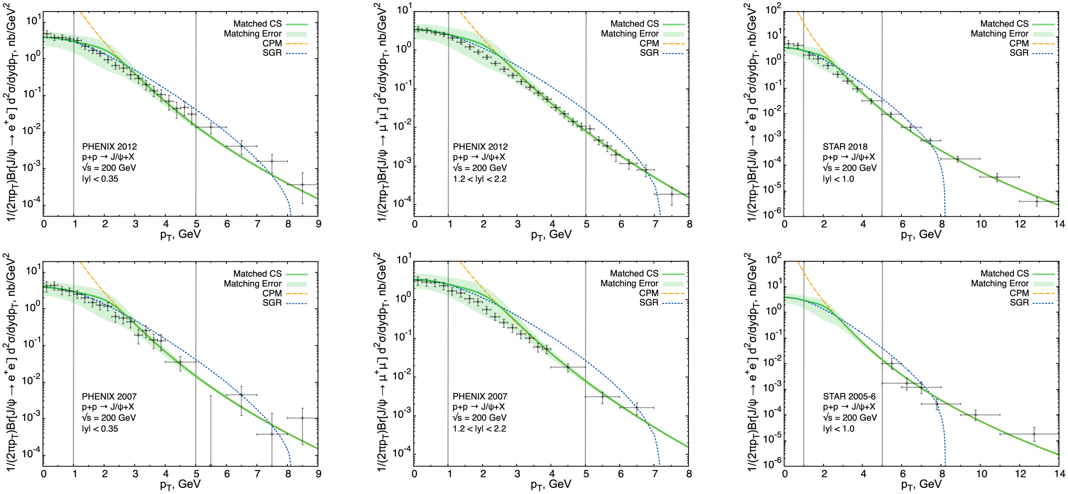
$$\mathcal{W} = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}, \qquad \mathcal{Z} = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}$$

$$\Delta\mathcal{W} = \left(\frac{p_T}{Q}\right)^2 + \left(\frac{m}{Q}\right)^2, \qquad \Delta\mathcal{Z} = \left(\frac{M}{p_T}\right)^2 \left(1 + \ln^2 \frac{Q_T}{p_T}\right)$$

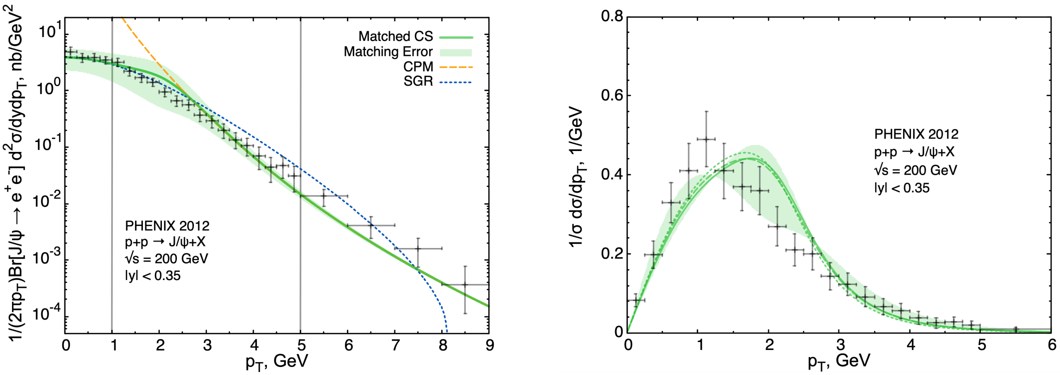
- Uncertainty due to the matching procedure:

$$\Delta d\sigma = \frac{d\sigma}{\sqrt{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}} = \frac{\Delta\mathcal{W} \cdot \Delta\mathcal{Z}}{\sqrt{\Delta\mathcal{W}^2 + \Delta\mathcal{Z}^2}} d\sigma$$

Extraction of CO LDME at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)



Extraction of CO LDME at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)



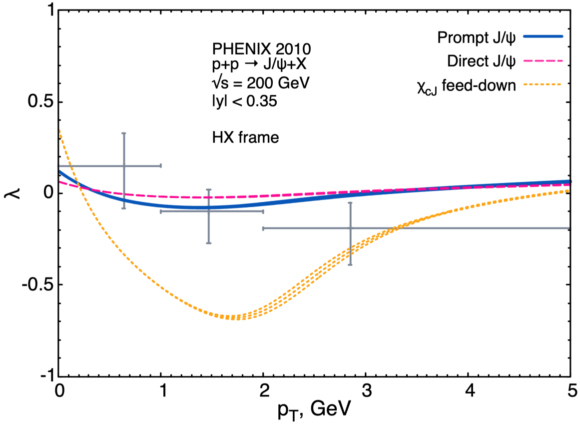
At $\sqrt{s} = 200$ GeV:

- quark-antiquark annihilation contribution $< 3\%$
- χ_{cJ} decays contribution $\approx 7\%$

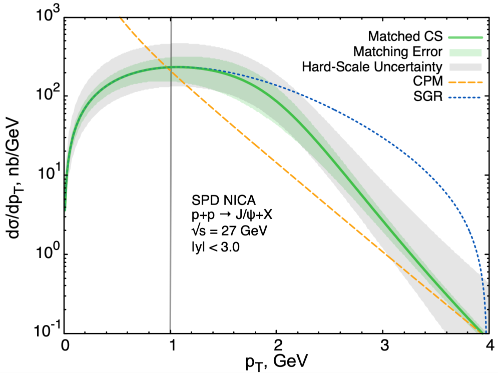
Extraction of CO LDME at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)

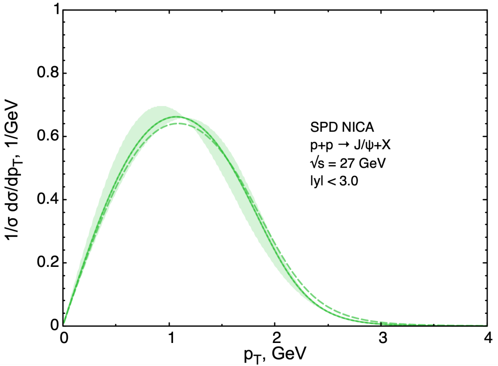
CO LDME	Our fit	LO CPM [Cho, Leibovich (1996)]	NLO CPM, global fit [Butenschön, Kniehl (2011)]
$\langle \mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle$, GeV^3	$(9.7 \pm 0.5) \cdot 10^{-2}$	–	$(3.0 \pm 0.4) \cdot 10^{-2}$
$\langle \mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle$, GeV^5	$(1.3 \pm 0.2) \cdot 10^{-2}$	–	$(-9.1 \pm 1.6) \cdot 10^{-3}$
$M_3^{J/\psi}$, GeV^3	$(1.1 \pm 0.1) \cdot 10^{-1}$	$(6.6 \pm 1.5) \cdot 10^{-2}$	$(1.8 \pm 0.6) \cdot 10^{-2}$
$\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle$, GeV^3	$(2.0 \pm 1.6) \cdot 10^{-3}$	$(6.6 \pm 2.1) \cdot 10^{-3}$	$(1.7 \pm 0.5) \cdot 10^{-3}$
$\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}]\rangle$, GeV^3	$(8.6 \pm 2.9) \cdot 10^{-3}$	$(3.3 \pm 0.5) \cdot 10^{-3}$	–
χ^2/n.d.f.	0.76	0.9	3.74

Polarisation test for CO LDME at PHENIX



Predictions for SPD NICA using the Soft Gluon Resummation approach





At $\sqrt{s} = 27$ GeV:

- quark-antiquark annihilation contribution $< 4\%$
- χ_{cJ} decays contribution $\approx 9\%$

Summary

- We have used the Soft Gluon Resummation approach to calculate small-p_T J/ψ production in the TMD factorisation
- CO LDMEs of NRQCD are necessary to describe J/ψ production using the TMD and CPM factorisations, where they are major contributions
- Soft Gluon Resummation approach for gluon and quark TMD PDF satisfyingly describe experimental data for unpolarised and polarised J/ψ production at $\sqrt{s} = 200$ GeV in the TMD domain of $p_T < 1$ GeV
- We have tested the factorisation matching scheme InEW for description of intermediate p_T domain, calculation agrees with experimental data within the uncertainty band
- We estimate the perspective region for the extraction of gluon TMD PDF in the J/ψ production as $p_T \leqslant 1$ GeV