Quantum Battery,Landau Levels Correlations

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What is a quantum battery?

Energy storing devices which exploits the quantum mechanical properties Entanglements and Coherence are called the quantum batteries.

A short history.

2013 [1] : Demonstrated that quantum entanglement enables a greater amount of extractable work compared to situations where entanglement is absent.

2013 [2] : Showed that entanglement is not related to the maximum extractable work, but, related to the rate at which the work is extracted (power).

[1] R. Alicki and M. Fannes, "Entanglement boost for extractable work from ensembles of quantum batteries", Physical Review E **87**, 042123 (2013).

[2] K. V. Hovhannisyan et al., "Entanglement generation is not necessary for optimal work extraction", Physical Review Letters **111**, 240401 (2013).

2015 [3] : Supported this hypothesis, demonstrating that N entangled spins possess the ability to charge N times faster than N spins that do not interact.

- The charging rate of quantum batteries is a superextensive property, as it grows exponentially with size: the charging per unit scales with N, meaning that quantum batteries with larger capacity actually take less time to charge, a counterintuitive and fascinating behaviour radically different from that of classical batteries.

2018 [4] : Showed experimental way to global entanglement by connecting all spins coherently to a single quantum energy source within a photonic cavity resonant to their transition energy, it becomes possible to achieve effective long-range interactions among all the spins.

[3] F. C. Binder et al., "Quantacell: powerful charging of quantum batteries", New Journal of Physics **17**, 075015 (2015).

[4] D. Ferraro et al., "High-power collective charging of a solid-state quantum battery", Physical Review Letters **120**, 117702 (2018).

2020 [5] : Through a geometric analysis of the associated Hilbert space they found that the charging power is bounded by the square root of the product of the quantum Fisher information and the variance of the quantum battery Hamiltonian.

[5] S. Julià-Farré et al., "Bounds on the capacity and power of quantum batteries", Physical Review Research **2**, 023113 (2020).

Unitary charging and work extraction of quantum battery

- Quantum battery is a d dimensional system with Hamiltonian:

$$H_0 = \sum_{k=1}^{d} \epsilon_k \left|k\right\rangle \left\langle k\right|$$

- The unitary charging process is:

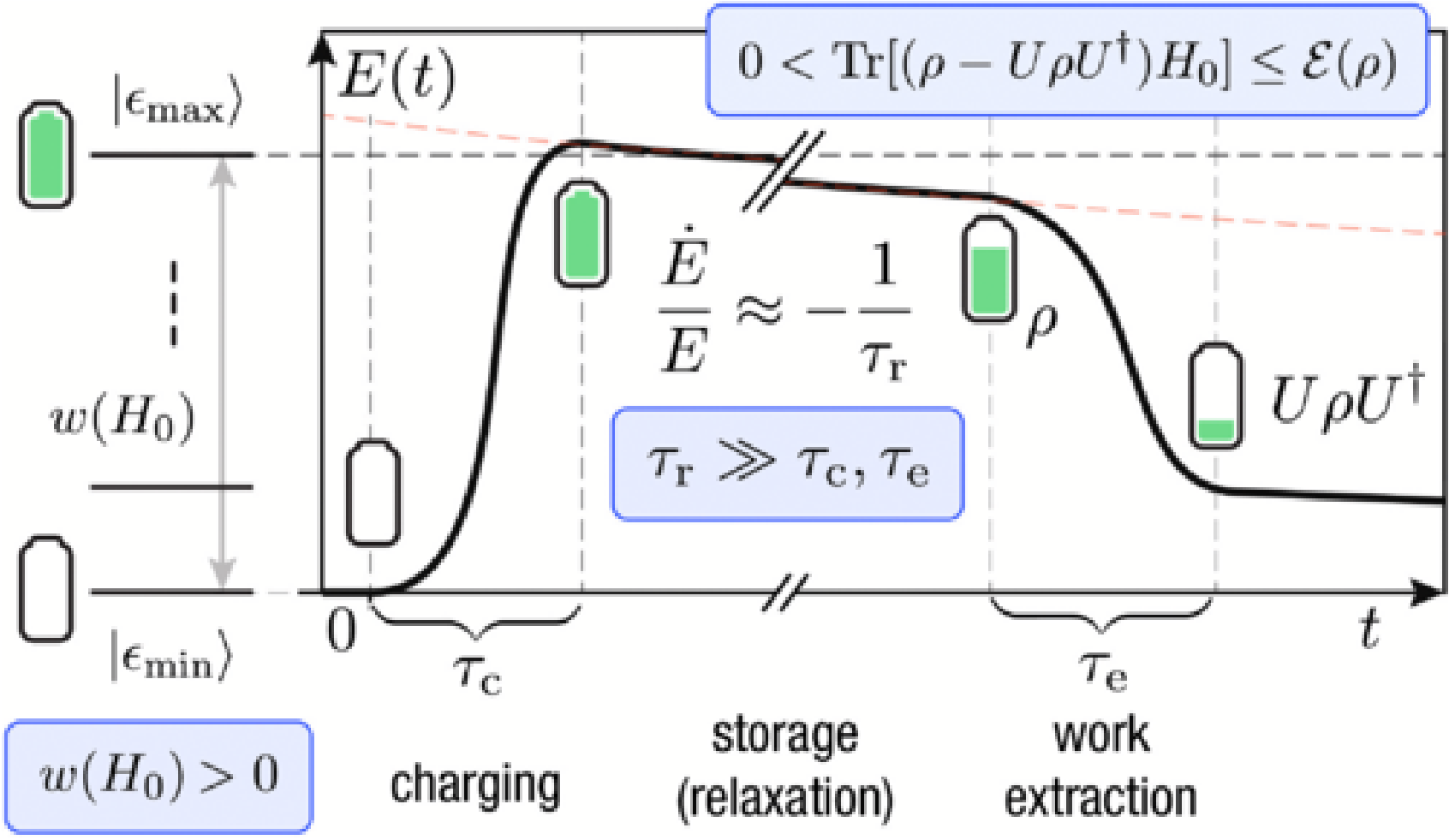
$$\dot{\rho} = \imath \left[H_0 + H_1(t), \rho(t)\right].$$

  H_1 is the interaction Hamiltonian, and $\rho(t) = U(t;0)\rho_0 U^{\dagger}(t:0)$ is the charge.

- Deposited energy:

$$W(\tau) = \mathrm{Tr}\left[H_0\rho(\tau)\right] - \mathrm{Tr}\left[H_0\rho_0\right].$$

Charging cycle

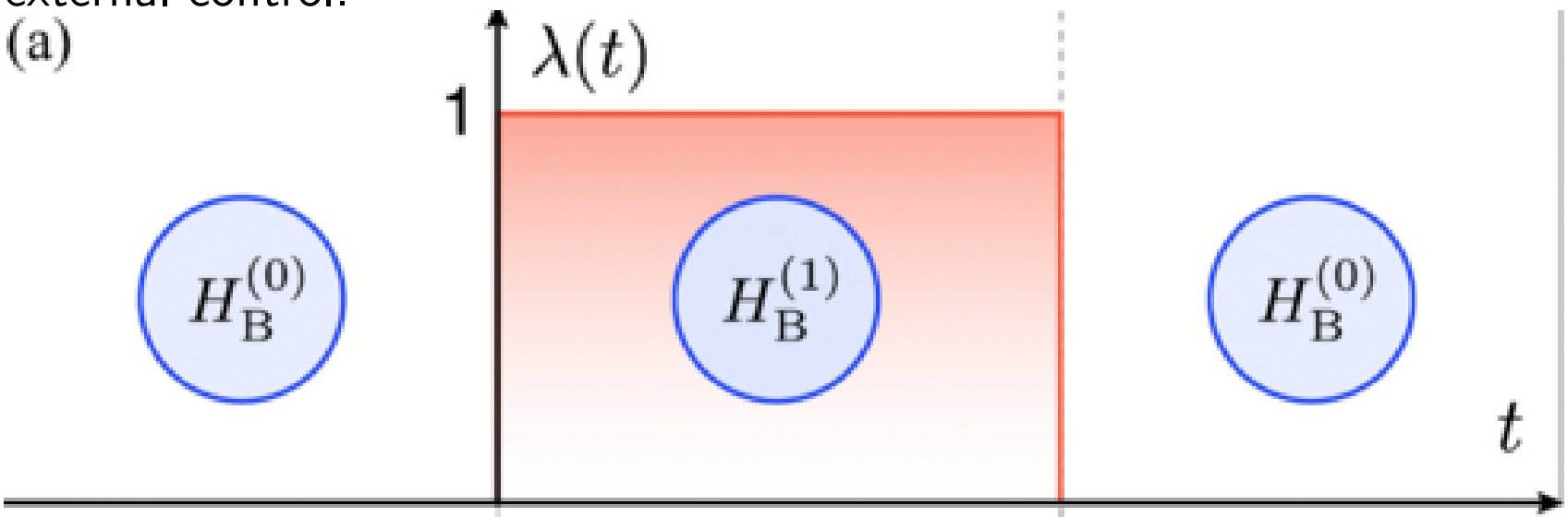


Charging protocol: Direct charging

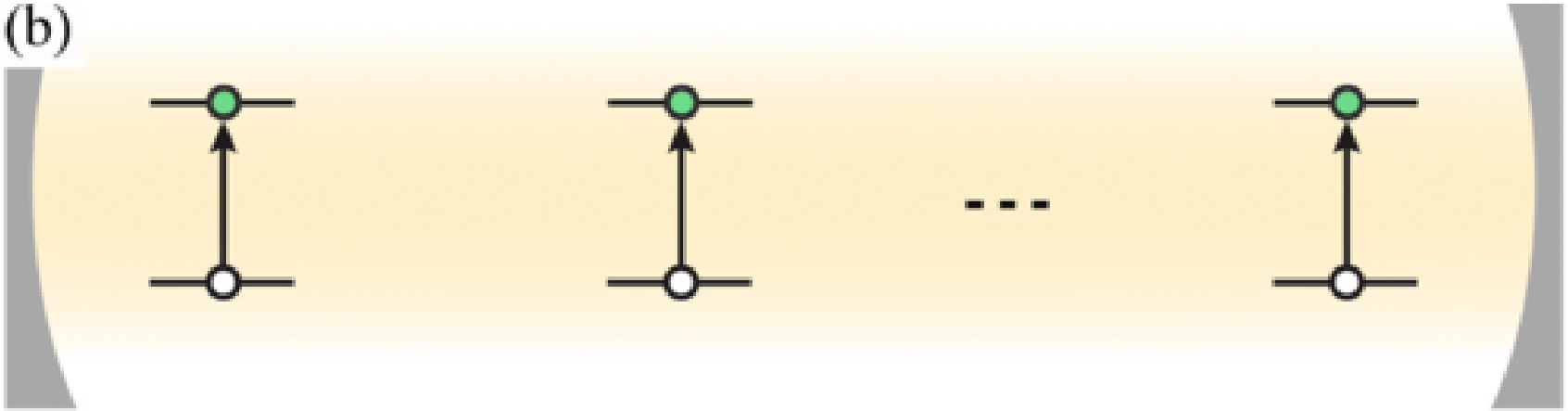
Hamiltonian during direct charging:

$$H_B(t) = H_B^{(0)} + \lambda(t)\left(H_B^{(1)} - H_B^{(0)}\right).$$

$H_B^{(1)}$: Charging Hamiltonian; $\lambda(t)$: Classical parameter representing external control.



The Dicke battery



$$H_{\text{Dicke}} = \omega_c a^\dagger a + \omega_0 \sum_i^N \sigma_i^z + g \sum_i^N \sigma_i^x \left(a^\dagger + a\right).$$

- $a(a^\dagger)$: Cavity photon anhilation (creation) opeators
- ω_c : Photon energy
- ω_0 : Cavity resonant energy
- g : Cavity coupling parameters

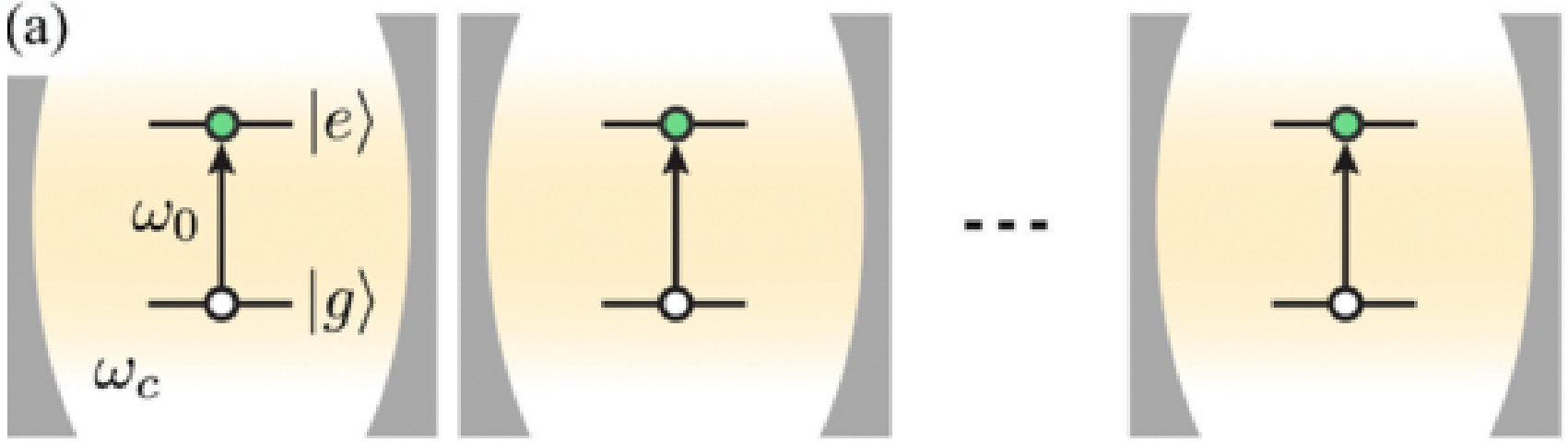
Charging of Dicke battery: Charger-mediated protocol

$$H_{\mathrm{C}} = \omega_{\mathrm{c}} \hat{a}^{\dagger} \hat{a},$$

$$H_{\mathrm{B}}^{(0)} = \frac{\omega_0}{2} \sum_{i=1}^{N} (\hat{\sigma}_i^z + \mathbb{1}),$$

$$H_1 = g \sum_{i=1}^{N} \hat{\sigma}_i^x (\hat{a}^{\dagger} + \hat{a}),$$

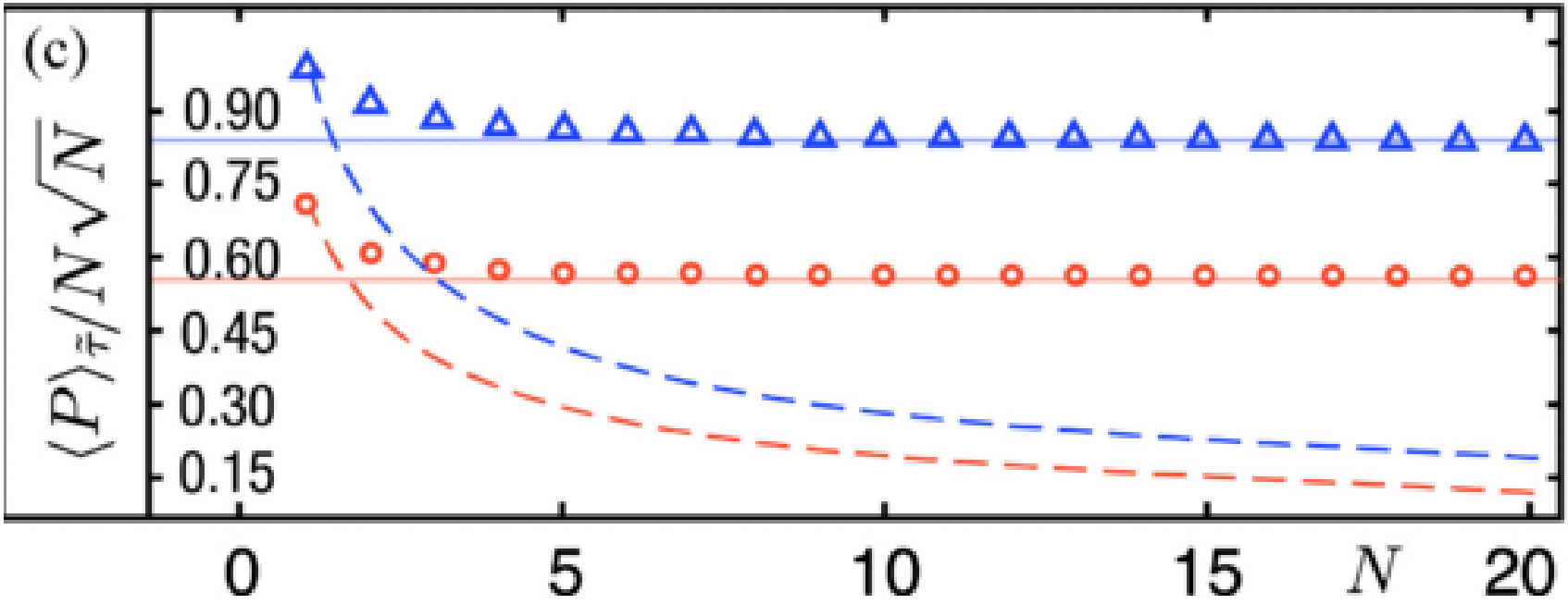
The Rabi battery



$$H_{\text{Dicke}} = \omega_c a^\dagger a + \omega_0 \sigma^z + g\sigma^x \left(a^\dagger + a\right).$$

- Rabi battery is charged with parallel charging protocol.

Charging speedup in Dicke battery compared to Rabi battery



- Solid (dashed) lines refer to the collective (parallel) protocol.
- The red lines correspond to the weak-coupling regime (i.e.,$g/\omega_0 = 0.05$), while the blue lines correspond to the strong-coupling regime (i.e., $g/\omega_0 = 0.5$).
- [6]

[6] D. Ferraro et al., "High-power collective charging of a solid-state quantum battery", Physical Review Letters **120**, 117702 (2018).

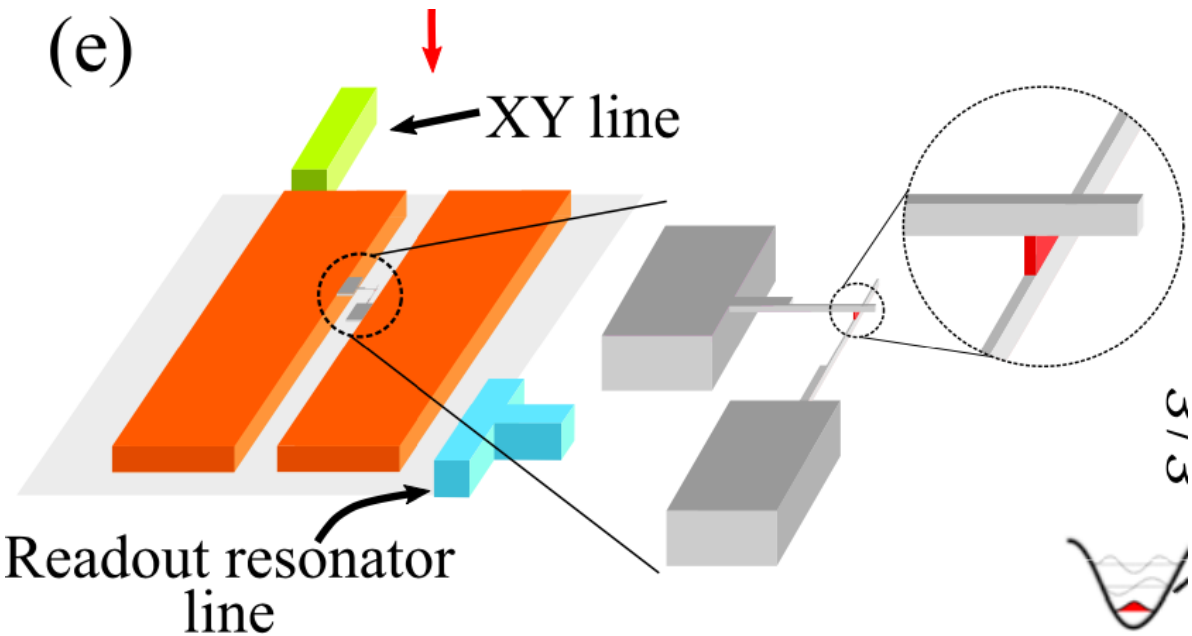
Experimental implementations

Two categories of systems are under investigation:

- Platforms operating at ultralow temperatures to be used in the quantum computers
- Platforms at room temperature ($\Delta E_{GE} > 25$ meV) to be used in usual energy storage

Superconducting qubits[7]

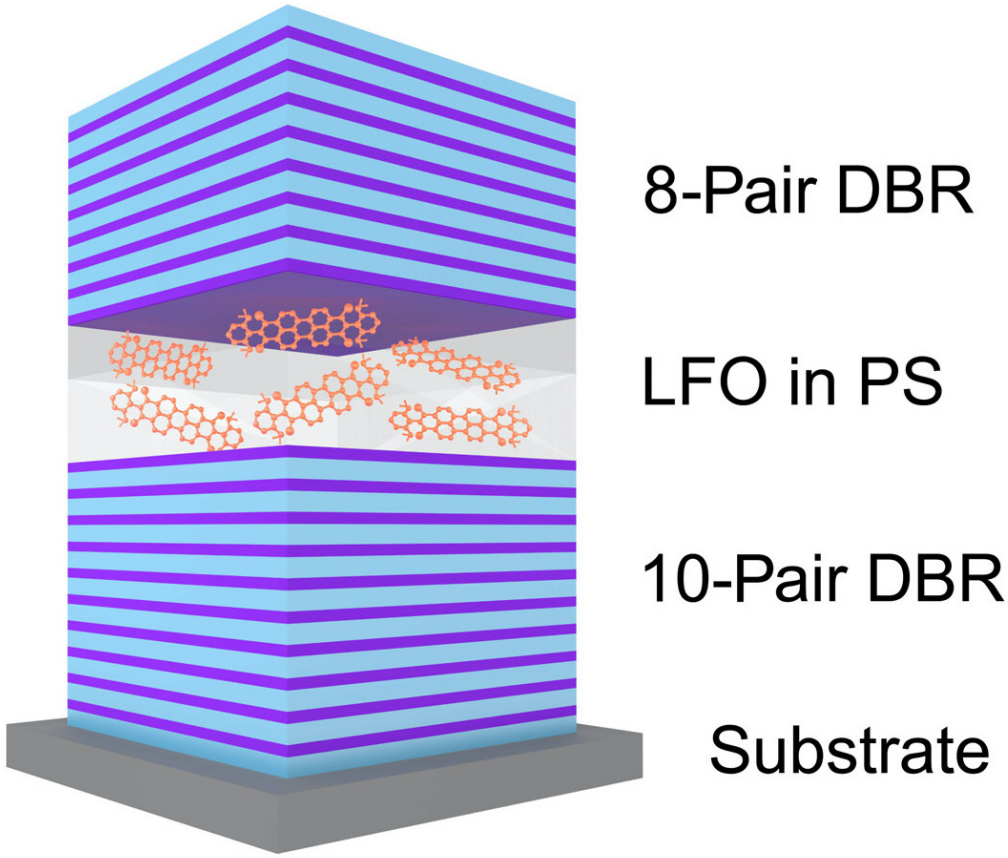
- Two level systems $\longmapsto$ Transmons (superconducting qubits)
- Cavity $\longmapsto$ Coplanar waveguide resonator
- The qubits are positioned at the antinodes of the first-harmonic standing wave electric field and have couplings that are nearly identical to the cavity mode.



[7] C.-K. Hu et al., "Optimal charging of a superconducting quantum battery", Quantum Science and Technology **7**, 045018 (2022).

Organic microcavities[8]

- Cavity $\longmapsto$ Dielectric mirrors (DBR)
- Two level system $\longmapsto$ low-mass molecular semiconductor [Lumogen F Orange (LFO)]

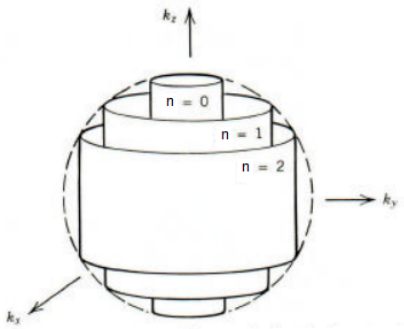


[8] J. Q. Quach et al., "Superabsorption in an organic microcavity: toward a quantum battery", Science Advances **8**, nil (2022).

Outlook from theoretical prospective

- Need to search for other models apart from (Dicke, SYK, Heisenber spin).
- Propose the work extraction protocols
- Finding the limits of energy density and increasing it
- ...

Landau Levels



In Landau gauge the energy of electrons:

$$E = k_z^2 + (\nu + \frac{1}{2})\hbar\omega_c.$$

- $\nu \in Z$: Landau levels
- $\omega_c = eH/mc$: Cyclotron frequency

N-body wave function for LLL

- In symmetric gauge single particle wave function:

$$\langle r|m\rangle = \sqrt{\frac{1}{2\pi l_B m!}} \left(\frac{r}{\sqrt{2}l_b}\right)^m \exp\left(-\frac{r^2}{4l_B^2} e^{im\theta}\right).$$

$l_B = \sqrt{\hbar/eB}$: magnetic length
$r = (r, \theta)$: polar coordinates
$m \in Z$: angular momentum

- First quantized N-body state in Lowest Landau levels (LLL):

$$|\Psi_{LLL}\rangle = \sum_{m_i} c_{m_1,m_2,\ldots,m_N} |m_1\rangle |m_2\rangle \ldots |m_N\rangle .$$

m_i : Single particle state
$c_{m_1,m_2,\ldots,m_N}$: Complex symmetric (anti-symmetric) coefficients for bosons (fermions)

Correlation function

- Number density:

$$\rho(r) = \sum_{i=1}^{N} \rho_i(r) = \sum_{i=1}^{N} \delta(r - r_i).$$

- Current density:

$$j(r) = \sum_{i}^{N} j_i(r) = \sum_{i}^{N} \frac{1}{2} \left\{ \frac{p_i - eA(r_i)}{M}, \rho_i(r) \right\}.$$

- The matrix elements of single particle wave function is:

$$\langle m_i | j_{\alpha,i} | m_i' \rangle = \frac{\hbar}{2M} \epsilon_{\alpha\beta} \frac{\partial}{\partial r_\beta} \langle m_i | \rho_{\alpha,i} | m_i' \rangle$$

- The correlation function derived is:

$$\langle \rho(r') j_\alpha(r) \rangle_{LLL} = \frac{\hbar}{2M} \epsilon_{\alpha\beta} \frac{\partial}{\partial r_\beta} \langle \rho(r') \rho(r) \rangle_{LLL}$$

- When the interaction with other Landau level is present then the deviation from the LLL correlation function gives the interaction parameters.
- For a system of bosons and weak contact interaction $g \ll \hbar\omega_c l_B^2$ (hence, perturbation theory) the deviation from LLL is:

$$\Delta_\alpha(r) = \epsilon_{\alpha\beta} \frac{\partial}{\partial r_\beta} \langle \rho(0)\phi(r) \rangle - \frac{2M}{r} \langle \rho(r) j_\alpha(r) \rangle .$$

- It was cross checked using two-body interaction bosons (Kohn's thereom), N-body interacting bosons (Schimdt decomposition), Laughling boson states.

Numerical Montecarlo simulation

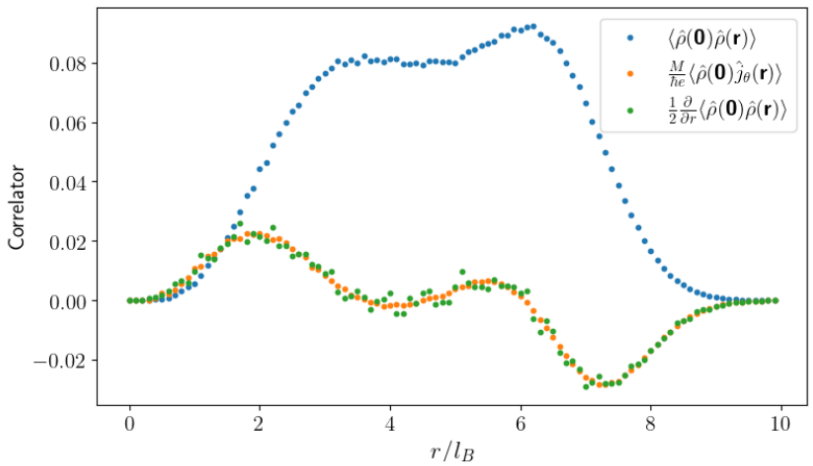


FIG. 2. Plot of current density-number density and number density-number density correlators obtained numerically using the Metropolis Monte Carlo method for the filling fraction $\nu = \frac{1}{2}$ Laughlin state in a disc geometry with particle number $N = 15$, after 500000 iterations and discarding the first 80000 "thermalisation" runs. The radial derivative of $\langle\hat{\rho}(\mathbf{0})\hat{\rho}(\mathbf{r})\rangle$ can be seen to coincide with the angular component of the current density-number density correlator, $\langle\hat{\rho}(\mathbf{0})\hat{j}_\theta(\mathbf{r})\rangle$, demonstrating the correspondence in Equation (6).