**NONLINEAR CNOIDAL WAVES AND FORMATION OF PATTERNS AND COHERENT STRUCTURES IN INTENSE CHARGED PARTICLE BEAMS**

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**Annotation**

The longitudinal dynamics of an intense high energy beam moving in a resonator cavity has been studied in some detail. Through the method of separation of variables and its obvious straightforward generalization, a solution of the Vlasov equation for the distribution function of an intense charged particle beam in the longitudinal direction has been obtained. The thus found Bernstein-Greene-Kruskal (BGK) equilibrium has been utilized to construct stationary wave patterns in the special case when the velocity distribution (energy error distribution) is Maxwellian. These are cnoidal wave patterns, showing rather intriguing and in a sense unexpected analogy between the equilibrium wave patterns in an intense charged particle beam and similar wave clusters originally observed in shallow water.

Based on the hydrodynamic model, fully equivalent to the coupled nonlinear system of the Vlasov equation for the distribution function of an intense beam in the longitudinal direction and the equation for the resonator cavity potential, an amplitude equation in the most general form has been derived. A very interesting and important property of the nonlinear amplitude equation is the fact that it is of hyperbolic type (nonlinear wave equation with complex coefficients) in the entire interval of admissible values for the wave number except for a single critical point, in which it is of parabolic type (non-linear Schrodinger equation with complex coefficients).