



Structure and fission properties of heavy and superheavy nuclei

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ScGrid of CNIC-CAS

CAS & ITP

□ CAS: Chinese Academy of Sciences

- >100 institutes & 3 universities in China; >40 in Beijing
- ~45,000 graduate students for Master's or PhD degrees

□ ITP: Institute of Theoretical Physics, Beijing

- Founded in 1978 & the smallest one in CAS
- 52 (assistant, associate & full) professors + ~50 postdocs + ~150 students
- Atomic physics; Nuclear physics; Particle physics; String theory; Cosmology; Condensed matter physics; Biophysics; Statistical physics; Quantum physics & quantum information; ...

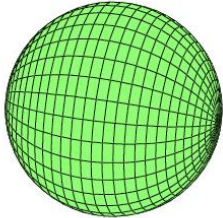
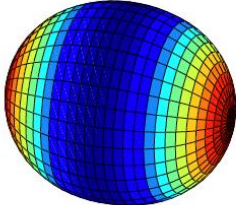
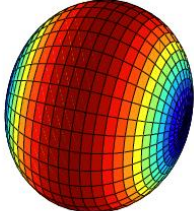
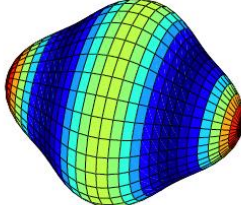
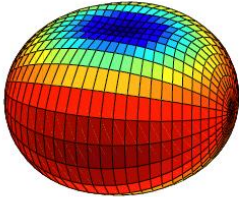
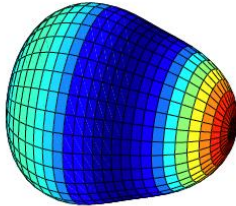
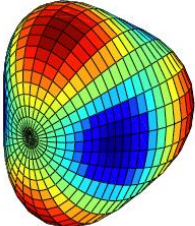
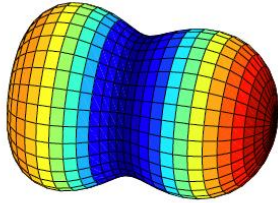
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Nuclear shapes

$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

2^λ -pole deformation (2^λ -极形变)

(a) $\beta_{\lambda\mu} = 0$	(b) $\beta_{20} > 0$	(c) $\beta_{20} < 0$	(d) $\beta_{40} > 0$
			
(e) $\beta_{22} \neq 0$	(f) $\beta_{30} \neq 0$	(g) $\beta_{32} \neq 0$	(h) $\beta_{20} \gg 0$
			

Courtesy of Bing-Nan Lu (吕炳楠)

Nonaxial quadrupole shape (β_{22} or γ)

J. Phys. G: Nucl. Part. Phys. **37** (2010) 064025

Meng_Zhang 2010_JPG37-064025

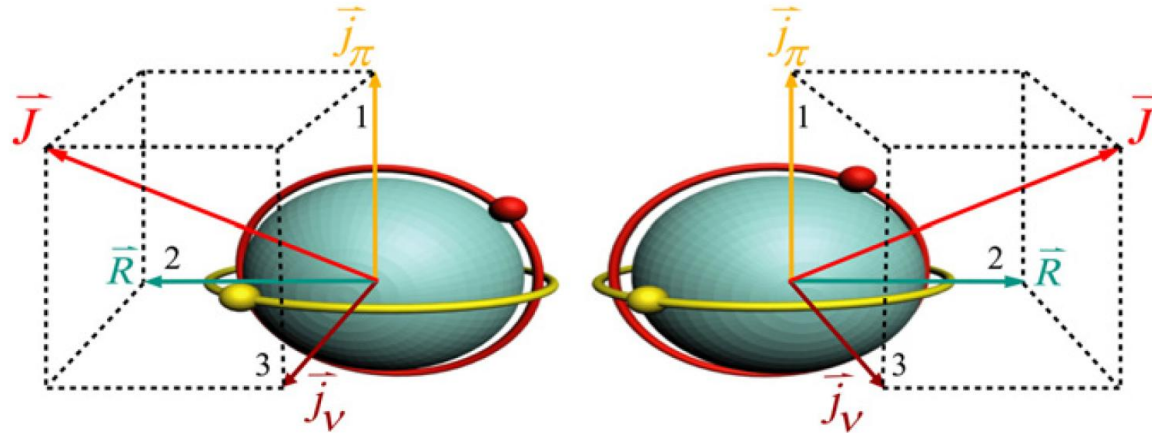


Figure 1. Left- and right-handed chiral systems for a triaxial odd-odd nucleus.

A static triaxial shape in atomic nuclei manifests itself by
the wobbling motion & **chiral doublet bands**

Bohr & Mottelson 1975
Odegard+2001_PRL86-5866

...

Frauendorf_Meng1997_NPA617-131
Starosta+2001_PRL86-971

...

Octupole shape (β_{30})

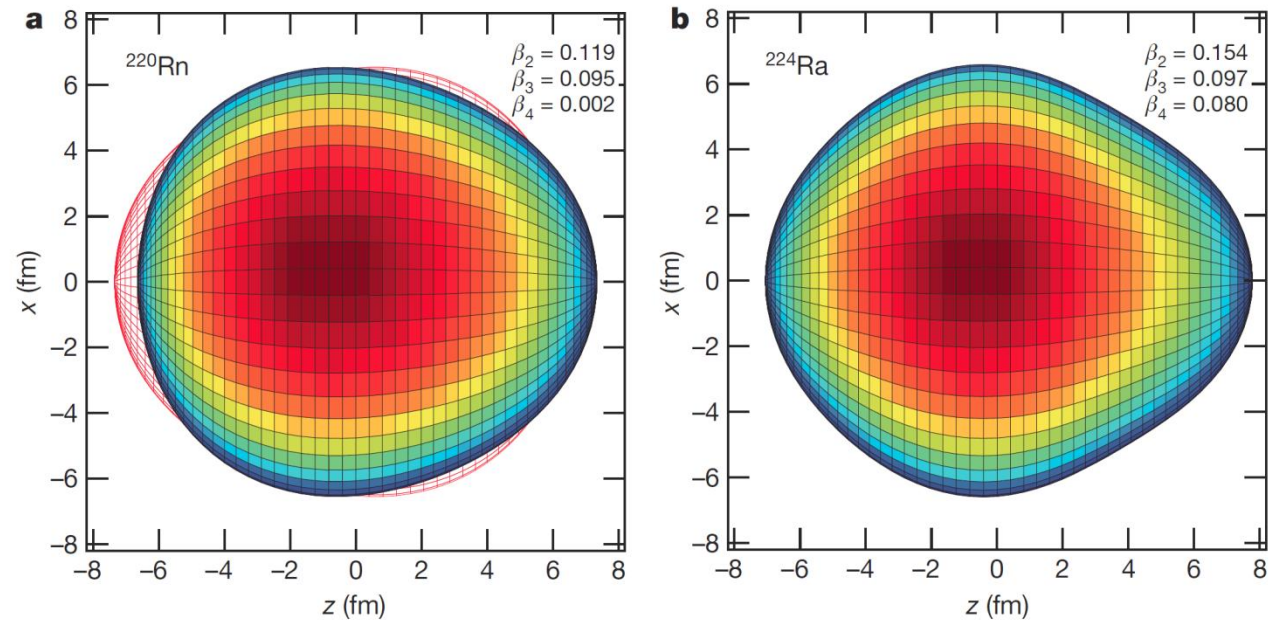
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Gaffney_Butler_Scheck+2013_Nature497-199

doi:10.1038/nature12073

Studies of pear-shaped nuclei using accelerated radioactive beams

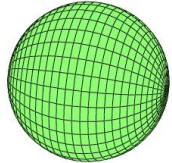
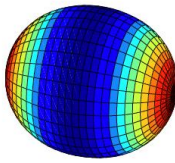
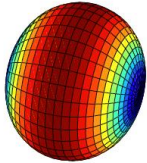
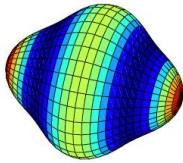
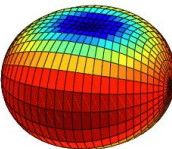
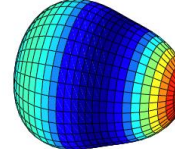
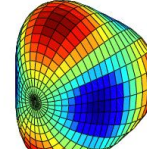
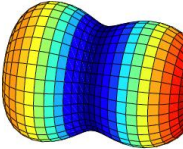
L. P. Gaffney¹, P. A. Butler¹, M. Scheck^{1,2}, A. B. Hayes³, F. Wenander⁴, M. N. Bree⁷, J. Cederkäll⁸, T. Chupp⁹, D. Cline³, T. E. Cocolios⁴, T. Davinson⁵, M. Huyse⁷, D. G. Jenkins¹³, D. T. Joss¹, N. Kesteloot^{7,11}, J. Konki¹², M. Kowa¹⁴, P. Napiorkowski¹⁴, J. Pakarinen^{4,12}, M. Pfeiffer⁵, D. Radeck⁵, P. Reiter⁵, K. S. Sambhi⁷, M. Seidlitz⁵, B. Siebeck⁵, T. Stora⁴, P. Thoele⁵, P. Van Duppen⁷, I. K. Wimmer¹⁸, K. Wrzosek-Lipska^{7,14}, C. Y. Wu¹⁵ & M. Zielinska^{14,19}

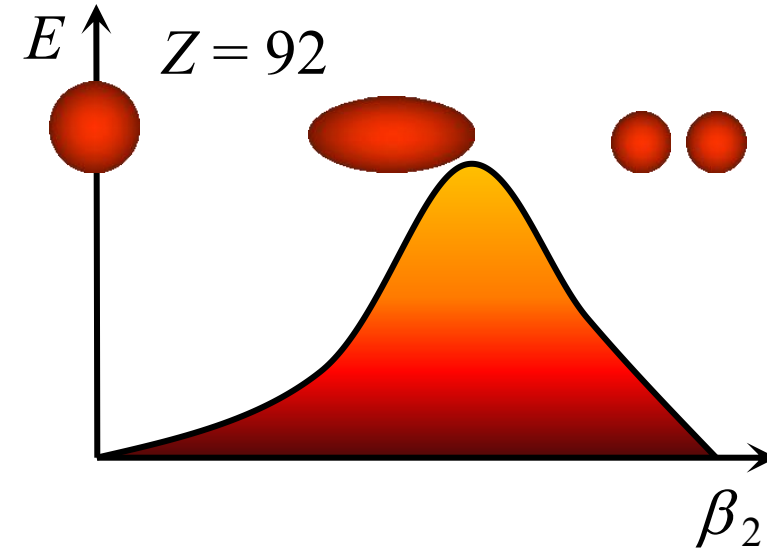


Nuclear fission

- ❑ Fission barrier is crucial for the description of fission
- ❑ Various shapes may appear during fission

$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

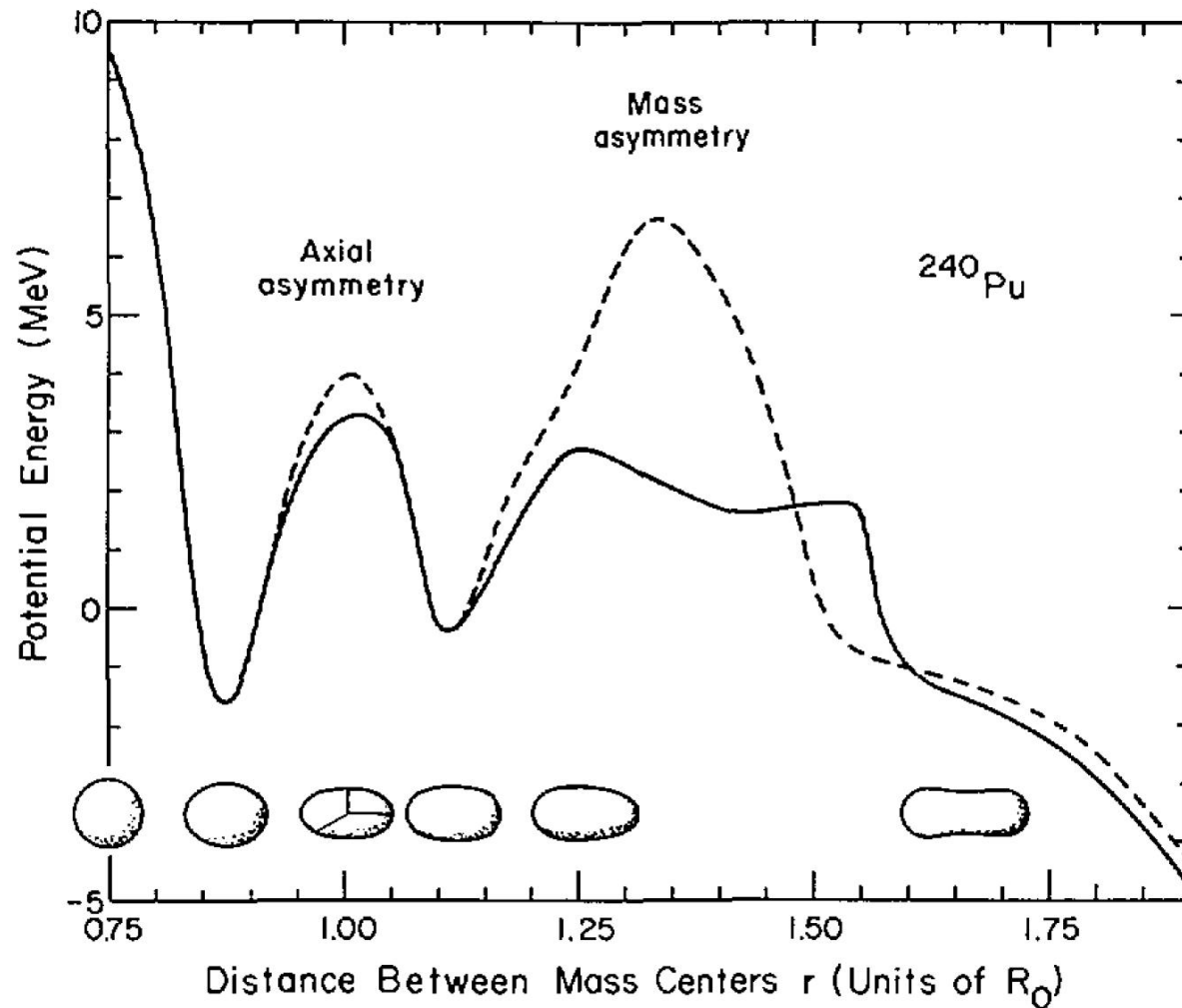
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(e) $\beta_{22} \neq 0$	(f) $\beta_{30} \neq 0$	(g) $\beta_{32} \neq 0$	(h) $\beta_{20} \gg 0$
			



$\{ \beta_2, \beta_4, \dots \}$

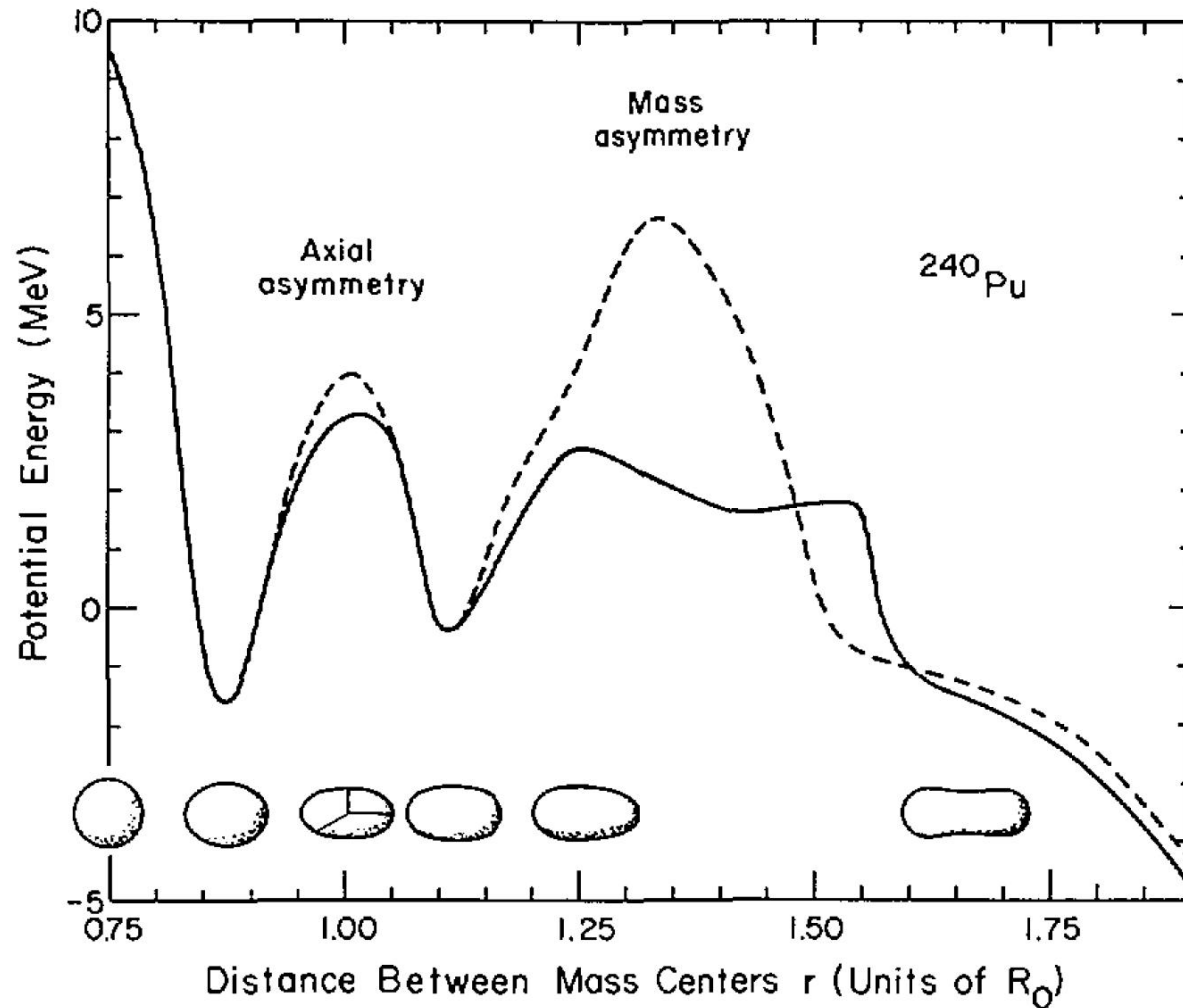
2^λ -pole deformation (2^λ -极形变)

Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Möller_Nix 1973
IAEA-SM-174/202

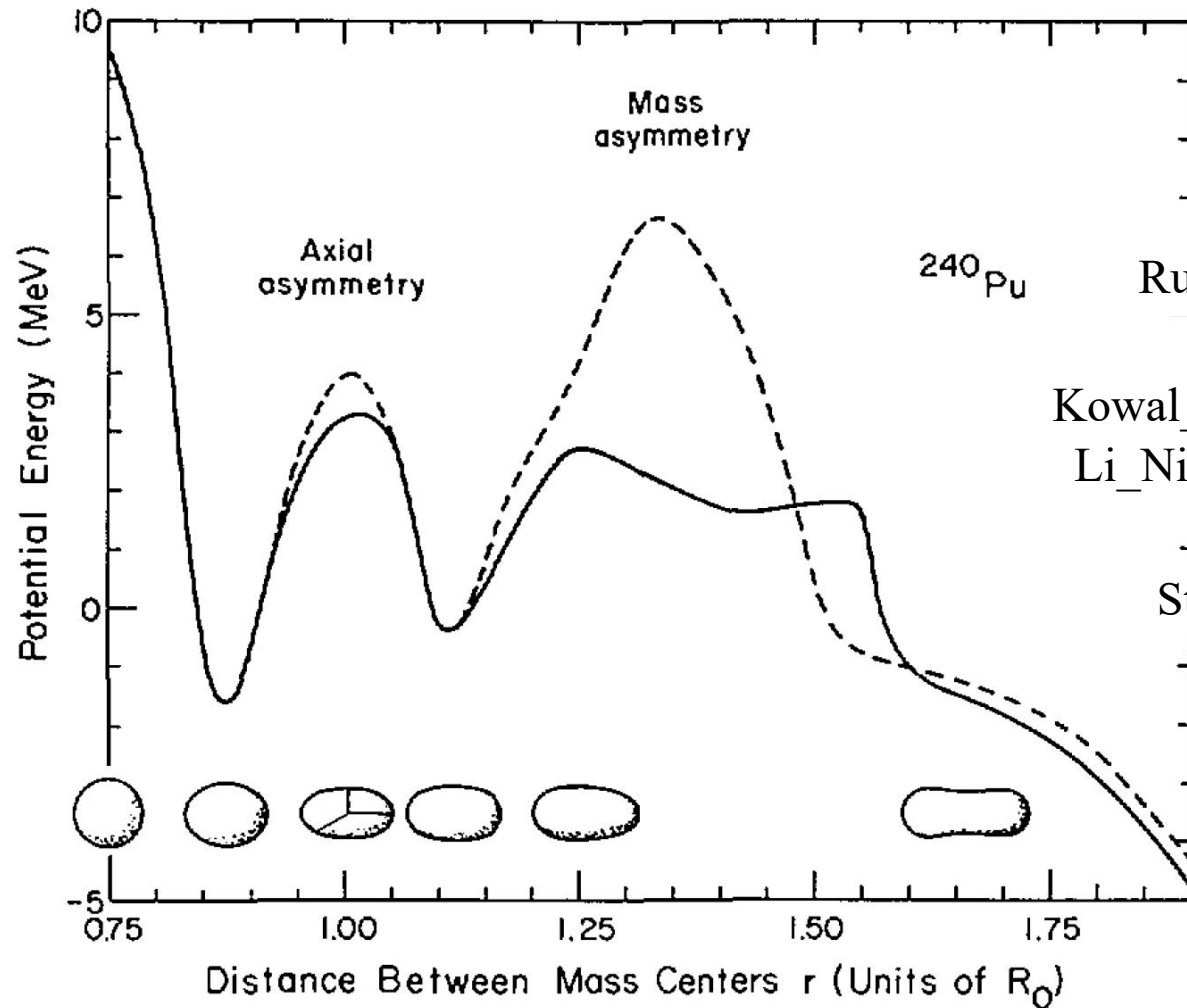
Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Möller_Nix 1973
IAEA-SM-174/202

Axial asymmetry plays important roles around the first barrier
Reflection asymmetry plays important roles around the second barrier

Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Möller_Nix 1973
IAEA-SM-174/202

Pashkevich1969_NPA133-400

Rutz_Maruhn_Reihard_Greiner1995_NPA590-680

Robledo_Warda2008_IJMPE17-204

Kowal_Jachikowicz_Sobiczewski2010_PRC82-014303

Li_Niksic_Vretenar_Ring_Meng2010_PRC81-064321

Abusara_Afanasjev_Ring2010_PRC82-044303

Staszczak_Baran_Nazarewicz2011_IJMPE20-552

Royer_Jaffre_Moreau2012_PRC86-044326

...

Axial asymmetry plays important roles around the first barrier

Reflection asymmetry plays important roles around the second barrier

Covariant Density Functional Theory (CDFT)

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_i (i\partial - M) \psi_i + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - g_\sigma \bar{\psi}_i \sigma \psi_i \\ & - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - g_\omega \bar{\psi}_i \psi \psi_i \\ & - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu - g_\rho \bar{\psi}_i \vec{\rho} \vec{\tau} \psi_i \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\psi}_i \frac{1 - \tau_3}{2} A \psi_i, \end{aligned}$$

Serot_Walecka1986_ANP16-1

Reinhard1989_RPP52-439

Ring1996_PPNP37-193

Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101

Meng_Toki_SGZ_Zhang_Long_Geng2006_PPNP57-470

Liang_Meng_SGZ2015_PR570-1

Meng_SGZ2015_JPG42-093101

Meng (ed.), Relativistic Density Functional for Nuclear structure (World Scientific, 2016)

$$(\alpha \cdot \mathbf{p} + \beta(M + S(\mathbf{r})) + V(\mathbf{r})) \psi_i = \epsilon_i \psi_i$$

$$(-\nabla^2 + m_\sigma^2) \sigma = -g_\sigma \rho_S - g_2 \sigma^2 - g_3 \sigma^3$$

$$(-\nabla^2 + m_\omega^2) \omega = g_\omega \rho_V - c_3 \omega^3$$

$$(-\nabla^2 + m_\rho^2) \rho = g_\rho \rho_3$$

$$-\nabla^2 A = e \rho_C$$

MDC-CDFTs ($\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}, \beta_{40}, \dots$)

□ Axially deformed harmonic oscillator (ADHO) basis

$$\left[-\frac{\hbar^2}{2M} \nabla^2 + V_B(z, \rho) \right] \Phi_\alpha(\mathbf{r}\sigma) = E_\alpha \Phi_\alpha(\mathbf{r}\sigma)$$

Ring_Gambhir_Lalazissis1997_CPC105-77

$$V_B(z, \rho) = \frac{1}{2} M (\omega_\rho^2 \rho^2 + \omega_z^2 z^2)$$

$$\Phi_\alpha(\mathbf{r}\sigma) = C_\alpha \phi_{n_z}(z) R_{n_\rho}^{m_l}(\rho) \frac{1}{\sqrt{2\pi}} e^{im_l \varphi} \chi_{s_z}(\sigma)$$

□ Fourier expansion for densities & potentials

$$f(\rho, \varphi, z) = f_0(\rho, z) \frac{1}{\sqrt{2\pi}} + \sum_{n=1}^{\infty} f_n(\rho, z) \frac{1}{\sqrt{\pi}} \cos(2n\varphi)$$

$f = V$ or ρ

□ A modified linear constraint method

$$E' = E_{\text{RMF}} + \sum_{\lambda\mu} \frac{1}{2} C_{\lambda\mu} Q_{\lambda\mu}$$

$$C_{\lambda\mu}^{(n+1)} = C_{\lambda\mu}^{(n)} + k_{\lambda\mu} \left(\beta_{\lambda\mu}^{(n)} - \beta_{\lambda\mu} \right)$$

Applications of MDC-CDFTs

□ Potential energy surface, ground state & fission properties

- $(\beta_{20}, \beta_{22}, \beta_{30})$: 1-, 2- & 3-dim PES of ^{240}Pu & B_f 's of actinides
- (β_{20}, β_{22}) : Shape polarization effect of Λ
- (β_{20}) : Superdeformed shapes in Λ hypernuclei
- (β_{20}) : Third barriers in light actinides
- (β_{20}, β_{30}) : Octupole correlations & shape transitions
- $(\beta_{20}, \beta_{22}, \beta_{30})$: Octupole correlations in $M_\chi D$
- (β_{20}, β_{32}) : Nuclear Tetrahedral shapes
- $(\beta_{20}, \beta_{22}, \beta_{30})$: 1-, 2-, & 3-dim PES of ^{270}Hs & B_f 's of even-even superheavies
- $(\beta_{\lambda\mu}, R)$: Clustering, bubble & toroidal structure; GMR

□ Fission dynamics based on PES from MDC-CDFTs

- Spontaneous fission
- Induced fission

□ Angular momentum & parity projected MDC-CDFTs

- Clustering & exotic shapes

MultiDimensionally-
Constrained
Covariant Density
Functional Theories

Collaborators

- ❑ Xiang-Quan Deng (邓祥泉) Univ. CAS
- ❑ Emiko Hiyama Kyushu Univ. & RIKEN
- ❑ Shivani Jain ITP/CAS
- ❑ Zhi-Pan Li (李志攀) Southwest Univ.
- ❑ Bing-Nan Lu (吕炳楠) Graduate School, China AEP
- ❑ Xiao Lu (陆晓) ITP/CAS
- ❑ Xu Meng (孟旭) Yanshan Univ.
- ❑ Tamara Niksic Univ. Zagreb
- ❑ Yu-Ting Rong (荣宇婷) Guangxi Normal Univ.
- ❑ Hiroyuki Sagawa RIKEN & Aizu Univ.
- ❑ Xiang-Xiang Sun (孙向向) Forschungszentrum Jülich
- ❑ Dario Vretenar Univ. Zagreb
- ❑ Kun Wang (王琨)
- ❑ Xiao-Qian Wang (王晓倩) ITP/CAS
- ❑ Jiang Xiang (向剑) Qiannan Normal Univ. Nationalities
- ❑ En-Guang Zhao (赵恩广) ITP/CAS
- ❑ Jie Zhao (赵杰) Pengcheng Lab

Applications of MDC-CDFTs

□ Potential energy surface, ground state & fission properties

➤ $(\beta_{20}, \beta_{22}, \beta_{30})$: 1-, 2- & 3-dim PES of ^{240}Pu & B_f 's of actinides

➤ (β_{20}, β_{22}) : Shape polarization effect of Λ

➤ (β_{20}) : Superdeformed shapes in Λ hypernuclei

➤ (β_{20}) : Third barriers in light actinides

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➤ (β_{20}, β_{32}) : Nuclear Tetrahedral shapes

➤ $(\beta_{20}, \beta_{22}, \beta_{30})$: 1-, 2-, & 3-dim PES of ^{270}Hs & B_f 's of even-even superheavies

➤ $(\beta_{\lambda\mu}, R)$: Clustering, bubble & toroidal structure; GMR

□ Fission dynamics based on PES from MDC-CDFTs

➤ Spontaneous fission

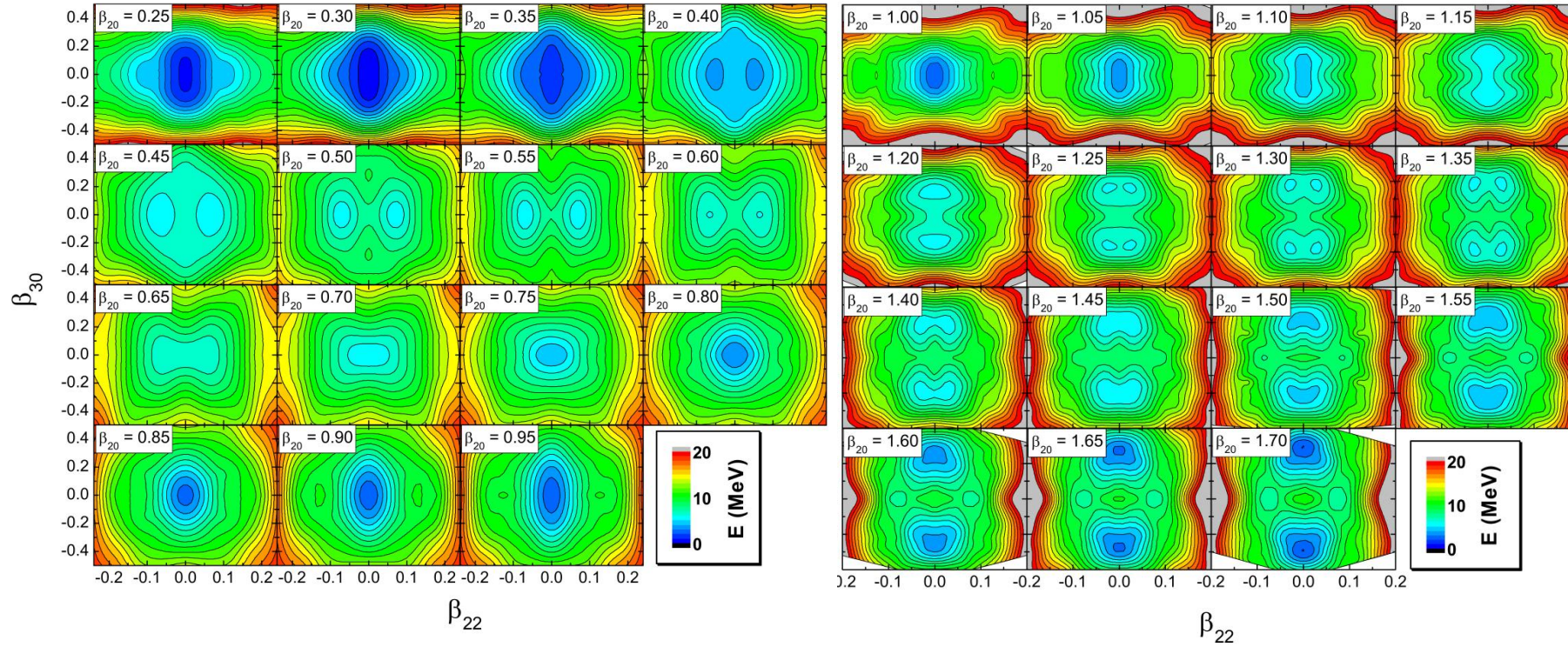
➤ Induced fission

□ Angular momentum & parity projected MDC-CDFTs

➤ Clustering & exotic shapes

MultiDimensionally-
Constrained
Covariant Density
Functional Theories

^{240}Pu : 3-dim. PES (β_{20} , β_{22} , β_{30})

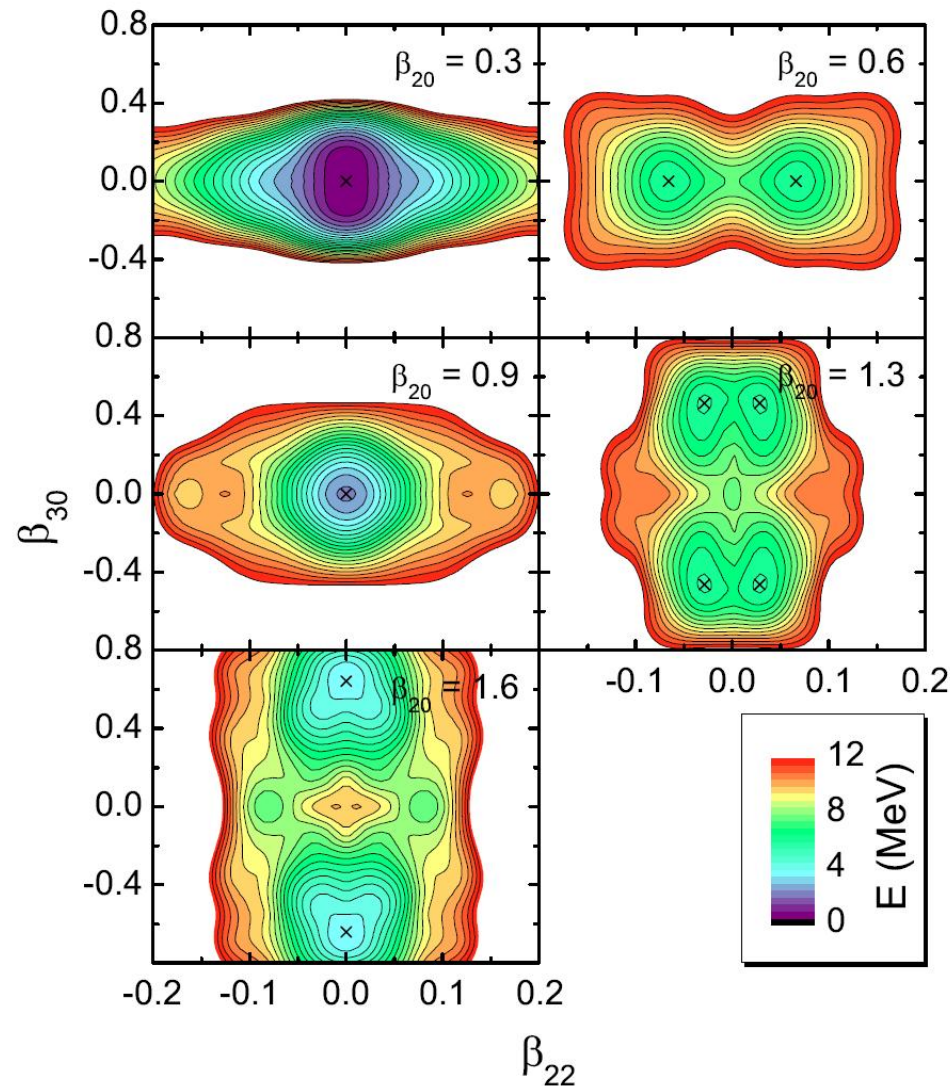


β_{20} : 0.25 to 1.70 w/ a step of 0.05
 β_{22} : 0.00 to 0.25 w/ a step of 0.01
 β_{30} : 0.00 to 0.50 w/ a step of 0.05

8580 points

Lu_Zhao_SGZ 2014_PRC89-014323

^{240}Pu : 3-dim. PES (β_{20} , β_{22} , β_{30})

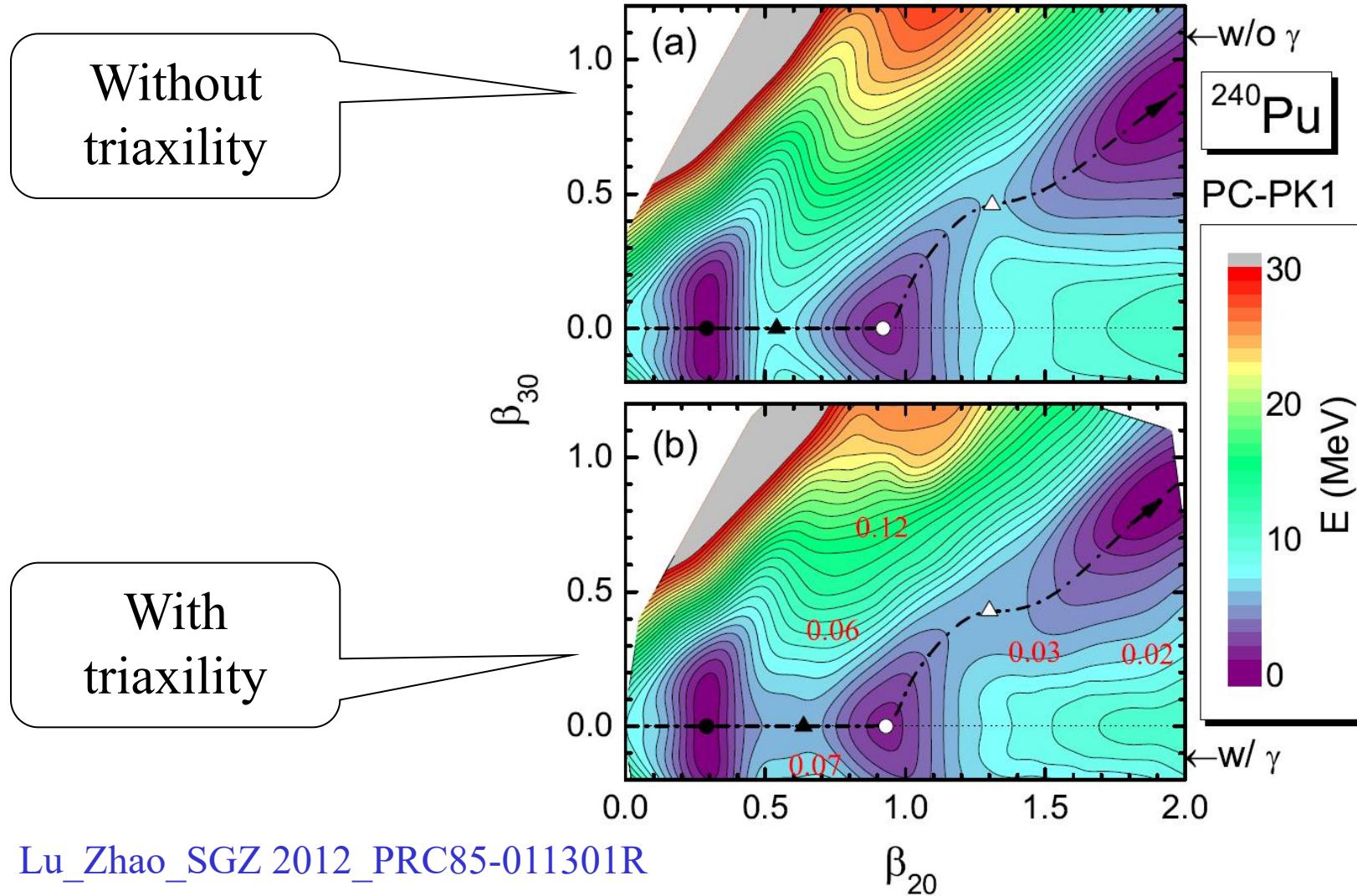


- AS & RS for g.s. & isomer, the latter is stiffer
- Triaxial & octupole shape around the outer barrier
- Triaxial deformation crucial around barriers

Lu_Zhao_SGZ 2012_PRC85-011301R

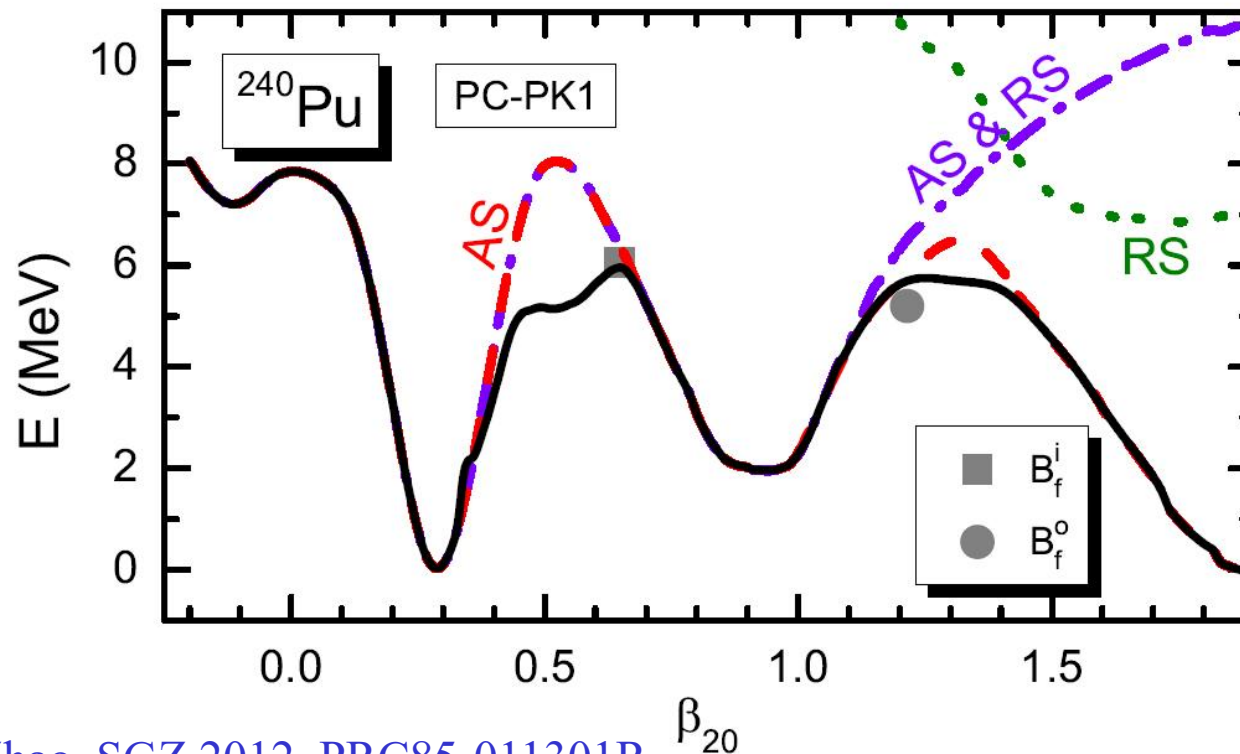
Lu_Zhao_SGZ 2014_PRC89-014323

^{240}Pu : 2-dim. PES (β_{20} , β_{30})



^{240}Pu : 1-dim. potential energy curve (β_{20})

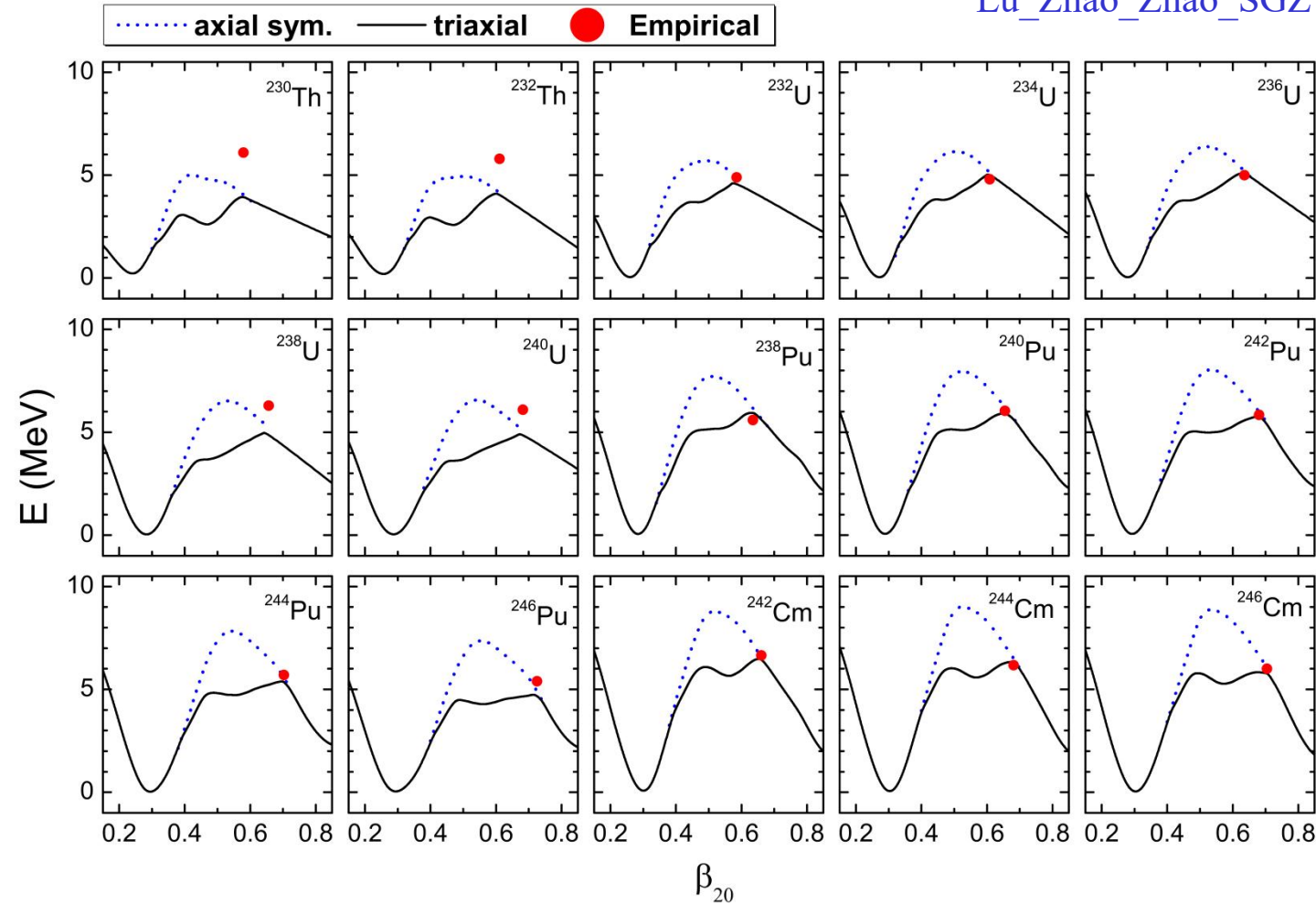
- Triaxiality lowers inner barrier height by more than 2 MeV
- Octupole deformation lowers outer barrier dramatically
- Triaxiality lowers outer barrier height by about 1 MeV



AS: Axially Sym.
RS: Reflection Sym.

B_f of actinide nuclei

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

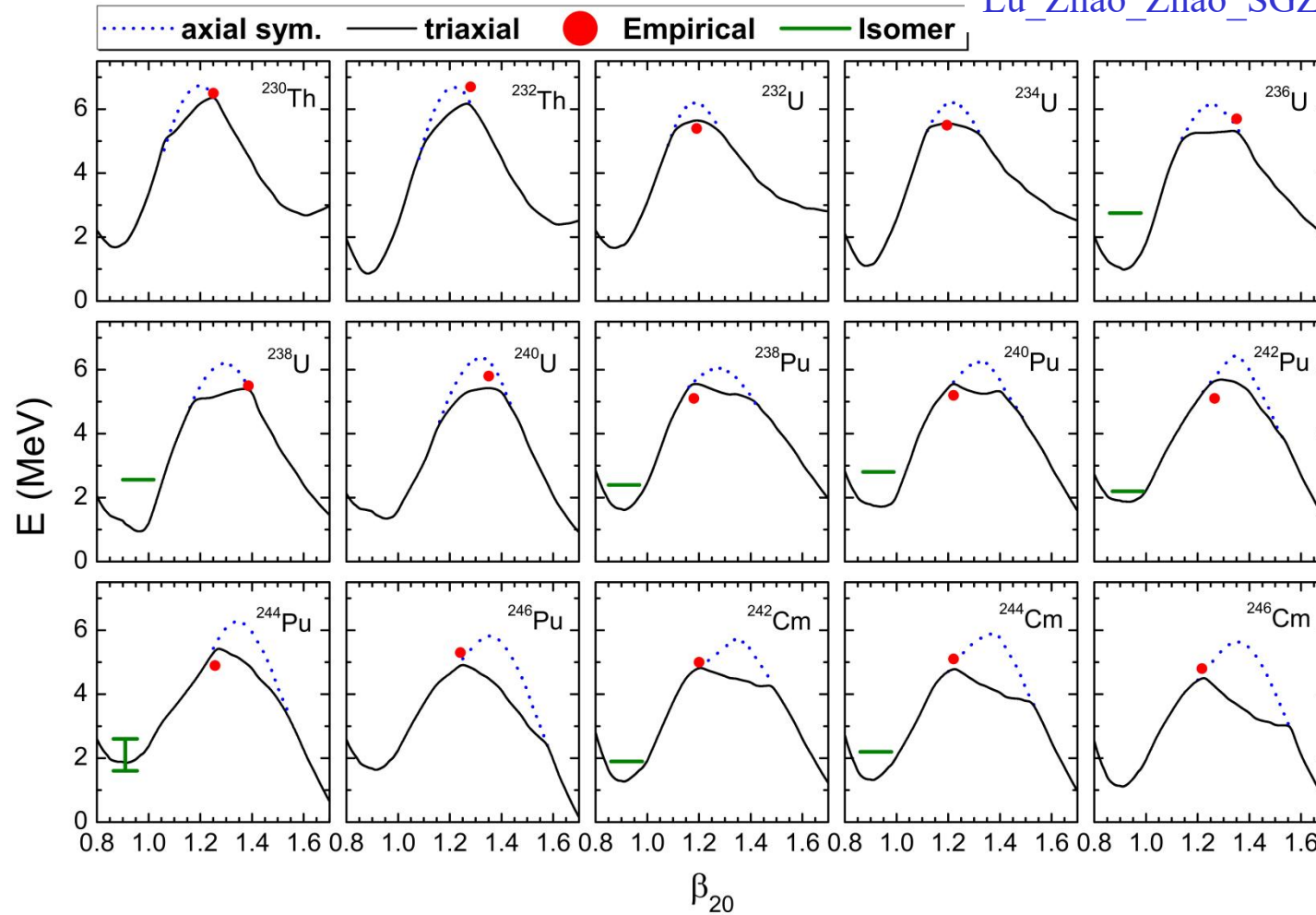


Around inner barriers

Empirical values: Capote...2009
NDC110-3107 (RIPL-3)

B_f of actinide nuclei

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

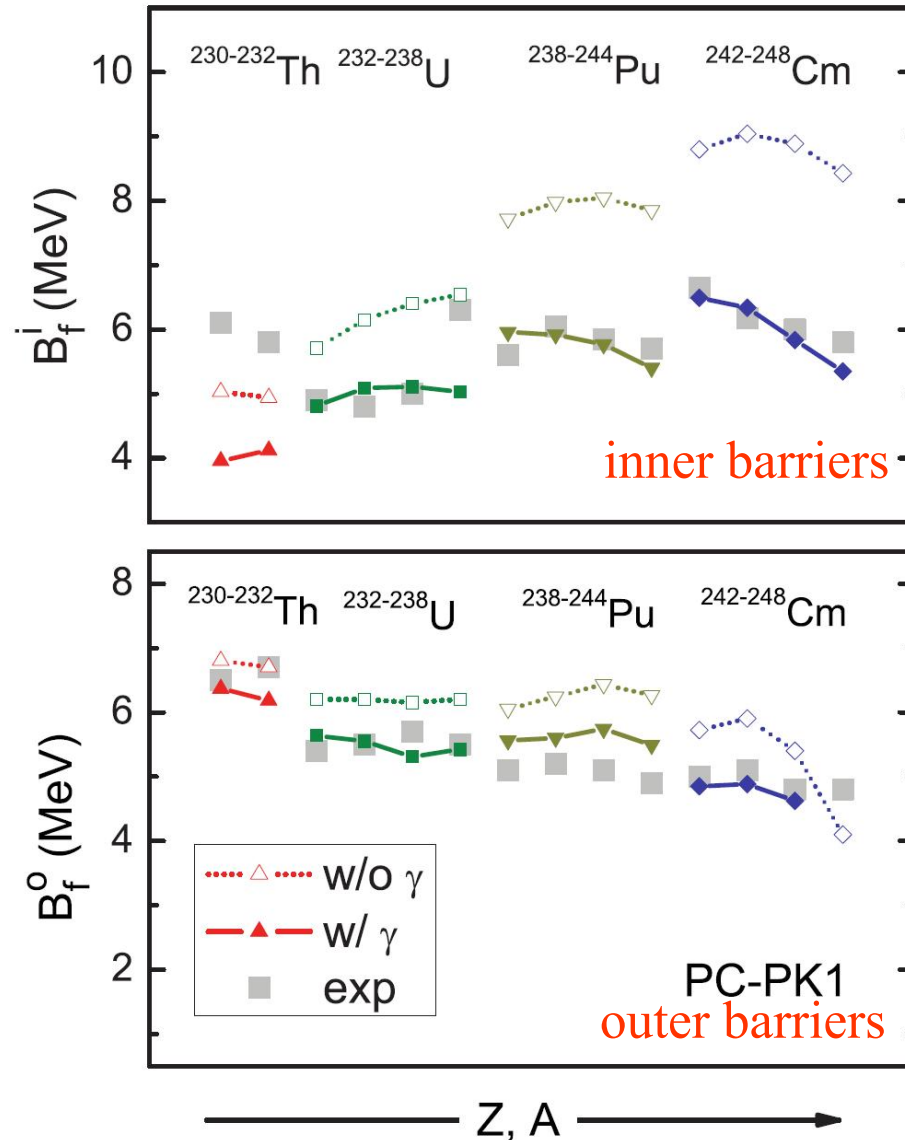


Around outer barriers

Empirical values: Capote...2009
NDC110-3107 (RIPL-3)

B_f of actinide nuclei

Lu_Zhao_SGZ 2012_PRC85-011301R



□ Influence of triaxiality

- Inner fission barriers lowered by 1~2 MeV
- Outer fission barriers lowered by 0.5~1 MeV

□ Problems

- $^{230-232}\text{Th}$: out barriers primary
- ^{238}U : ?
- ^{248}Cm : two fission paths

Empirical values: Capote...2009
NDC110-3107 (RIPL-3)

B_f of actinide nuclei

Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Nucleus	Z	N	A	First barrier			Second barrier		
				AS	TA	Emp	AS	TA	Emp
^{230}Th	90	140	230	5.03	3.96	6.10	6.80	6.37	6.50
^{232}Th	90	142	232	4.94	4.12	5.80	6.70	6.18	6.70
^{232}U	92	140	232	5.71	4.81	4.90	6.20	5.64	5.40
^{234}U	92	142	234	6.15	5.09	4.80	6.20	5.55	5.50
^{236}U	92	144	236	6.40	5.11	5.00	6.15	5.31	5.70
^{238}U	92	146	238	6.54	5.03	6.30	6.20	5.42	5.50
^{240}U	92	148	240	6.58	4.96	6.10	6.38	5.43	5.80
^{238}Pu	94	144	238	7.72	5.96	5.60	6.05	5.56	5.10
^{240}Pu	94	146	240	7.98	5.92	6.05	6.24	5.60	5.20
^{242}Pu	94	148	242	8.05	5.77	5.85	6.43	5.74	5.10
^{244}Pu	94	150	244	7.85	5.40	5.70	6.26	5.49	4.90
^{246}Pu	94	152	246	7.37	4.76	5.40	5.84	4.96	5.30
^{242}Cm	96	146	242	8.80	6.49	6.65	5.72	4.85	5.00
^{244}Cm	96	148	244	9.04	6.34	6.18	5.90	4.88	5.10
^{246}Cm	96	150	246	8.89	5.84	6.00	5.40	4.62	4.80
^{248}Cm	96	152	248	8.43	5.35	5.80	4.10	—	4.80
^{250}Cm	96	154	250	7.77	4.79	5.40	2.60	—	4.40
^{250}Cf	98	152	250	8.87	5.70	5.60	2.40	—	3.80
^{252}Cf	98	154	252	8.41	5.26	5.30	1.20	—	3.50

Empirical values: Capote...2009
NDC110-3107 (RIPL-3)

B_f of actinide nuclei

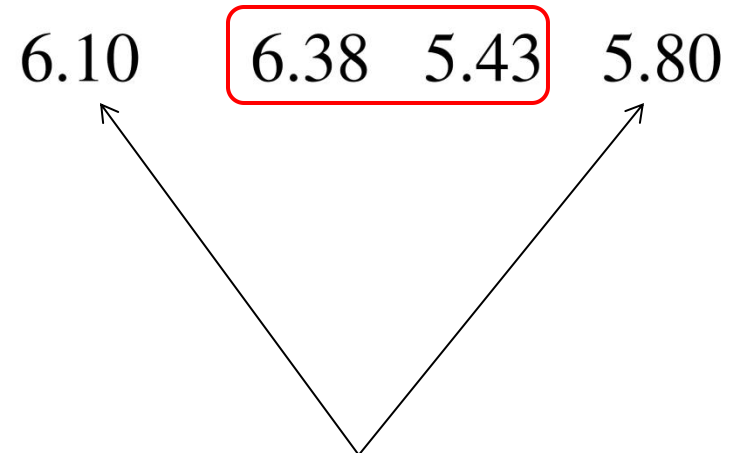
Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

PC-PK1

Nucleus	Z	N	A	First barrier			Second barrier		
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^{238}U	92	146	238	6.54	5.03	6.30	6.20	5.42	5.50
^{240}U	92	148	240	6.58	4.96	6.10	6.38	5.43	5.80
^{238}Pu	94	144	238	7.77	5.96	5.60	6.05	5.56	5.10
^{240}U	92	148	240	6.58	4.96				
^{244}Pu	94	150	244	7.85	5.40	5.70	6.26	5.49	4.90
^{246}Pu	94	152	246	7.37	4.76	5.40	5.84	4.96	5.30
^{242}Cm	96	146	242	8.80	6.49	6.65	5.72	4.85	5.00
^{244}Cm	96	148	244	9.04	6.34	6.18	5.90	4.88	5.10
^{246}Cm	96	150	246	8.89	5.84	6.00	5.40	4.62	4.80
^{248}Cm	96	152	248	8.43	5.35	5.80	4.10	—	4.80
^{250}Cm	96	154	250	7.77	4.79	5.40	2.60	—	4.40
^{250}Cf	98	152	250	8.87	5.70	5.60	2.40	—	3.80
^{252}Cf	98	154	252	8.41	5.26	5.30	1.20	—	3.50

Nishio (FUSION17):

$B_f = 5.5 \text{ MeV}$ for ^{240}U



Empirical values: Capote...2009
NDC110-3107 (RIPL-3)

^{270}Hs : A doubly magic deformed SHN

Möller_Nilsson_Nix1974_NPA229-292

Čwiok_Pashkevich_Dudek_Nazarewicz1983_NPA410-254

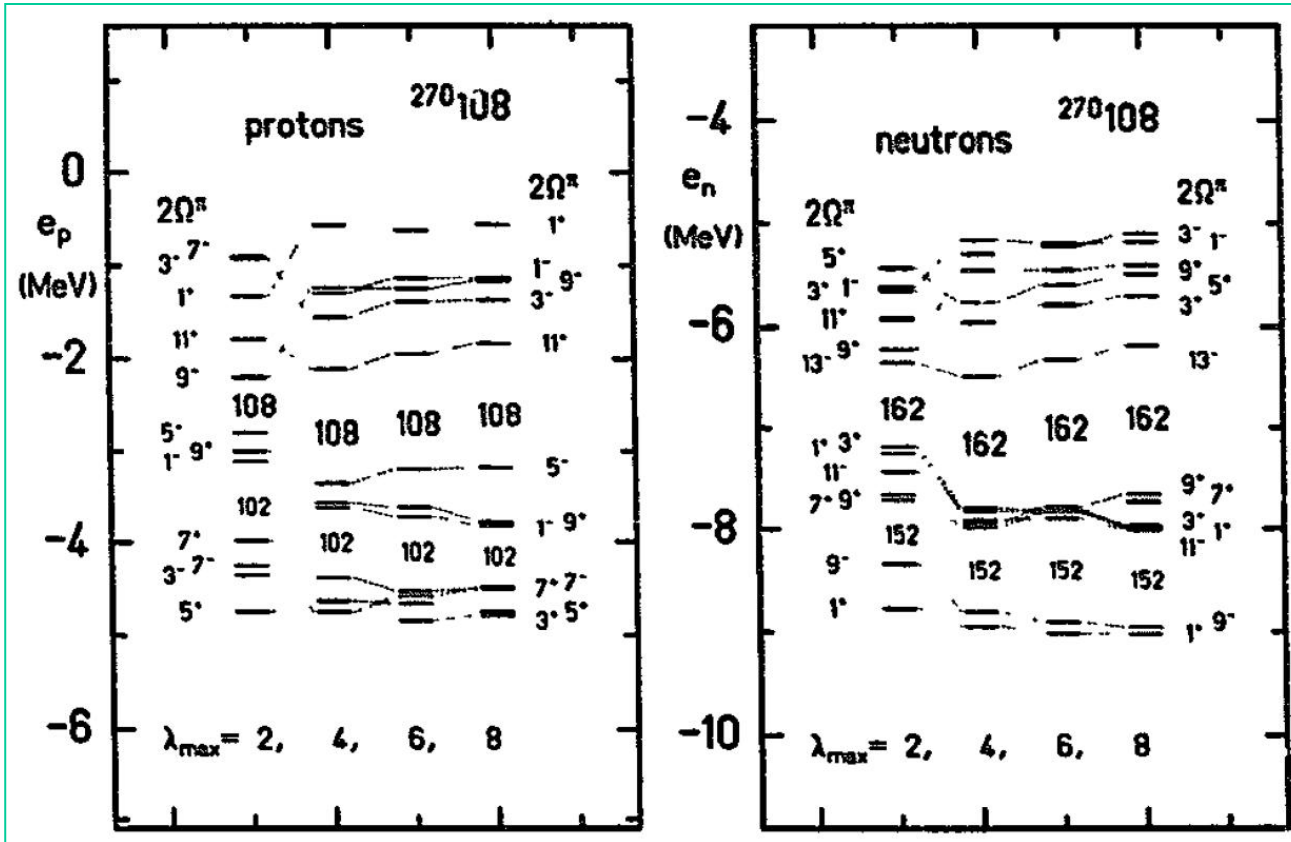
Möller_Leander_Nix1986_ZPA323-41

Sobiczewski_Patyk_Čwiok1987_PLB186-6

Patyk_Skalski_Sobiczewski_Cwiok1989_NPA502-591c

Patyk_Sobiczewski1991_NPA533-132

^{270}Hs : A doubly magic deformed SHN



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Patyk_Skalski_Sobiczewski_Cwiok1989_NPA502-591c

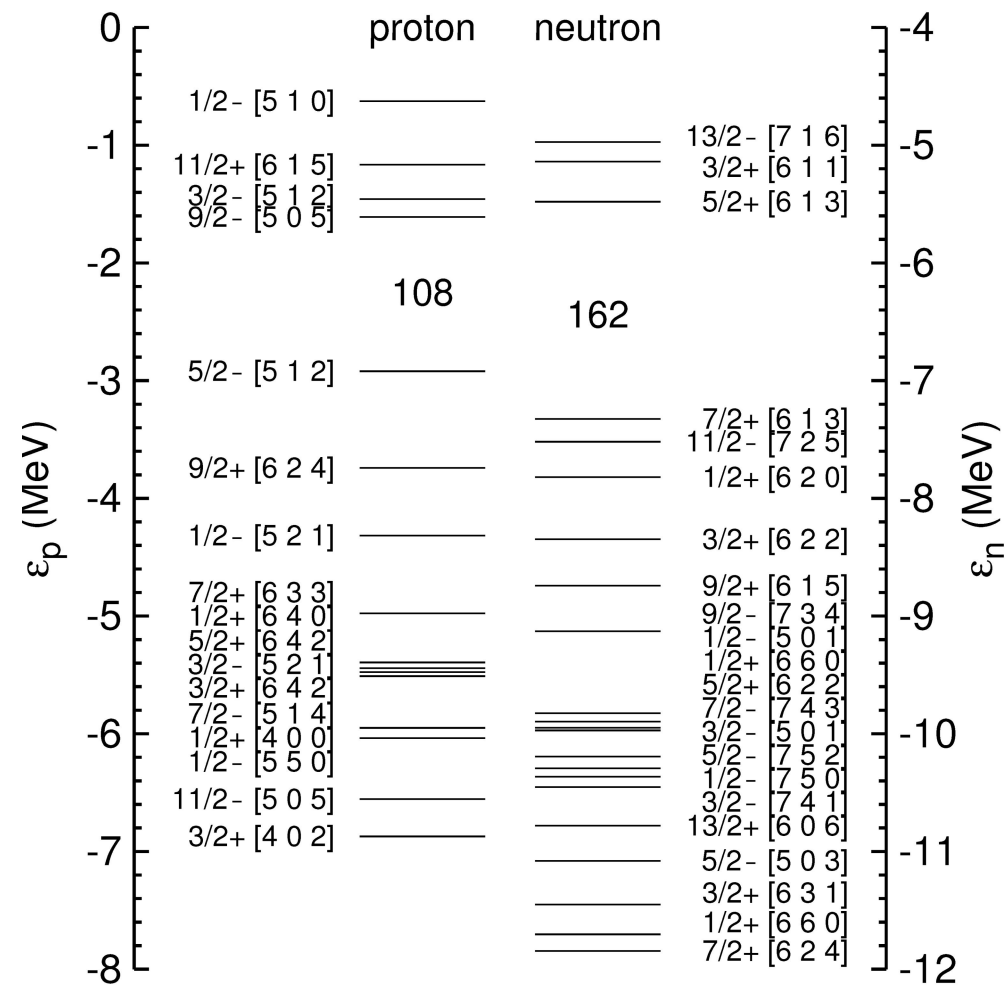
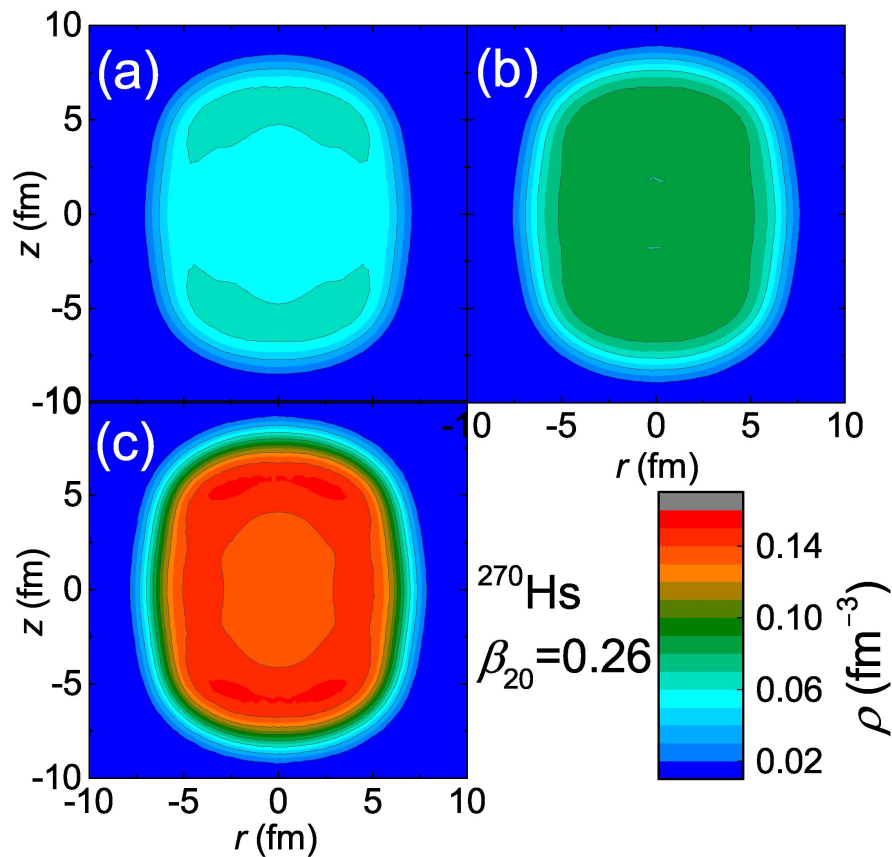
Patyk_Sobiczewski1991_NPA533-132

Patyk_Sobiczewski1991_NPA533-132

^{270}Hs : ground state properties

	E_B (MeV)	β_2	R_c (fm)	R_m (fm)	
MDC-RMF (PC-PK1)	1967.40	0.261	6.167	6.23	Meng_Lu_Zhou2020 Sci. China-Phys. Mech. Astron. 63, 212011
AME2016 [140–142]	1969.65				
MMM [40]	1969.20	0.229			Patyk_Sobiczewski1991
RMF (TMA) [143, 144]	1971.80	0.22	6.152	6.209	Ren...2002
RMF (NLZ2) [143, 144]	1969.22	0.274	6.251	6.333	Wu_Xu 2004
MMM [51]	1968.5	0.22			
RMF (TMA) [145, 146]	1971.93	0.222	6.142	6.199	Geng... 2005; Geng2006
HFB-24 [148]	1968.45	0.26			
RMF (NL3) [147]	1974	0.26			Zhang... 2012
WS4 [152]	1970.27	0.217			
FRDM (2012) [153]	1971.48	0.222			
RCHB (PC-PK1) [122] ¹⁾	1952.65		6.132	6.180	Xia... 2018
RCHB (PC-PK1) + RFB [162] ²⁾	1969.20		6.132		Shi... 2019

^{270}Hs : ground state from MDC-RMF calc.



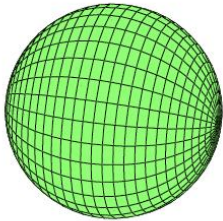
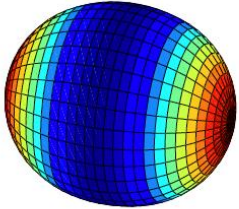
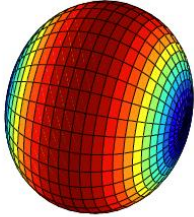
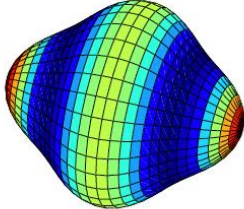
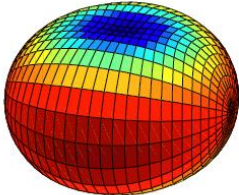
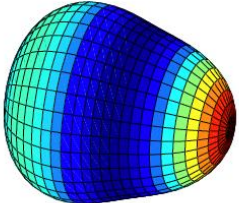
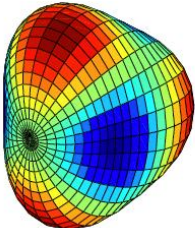
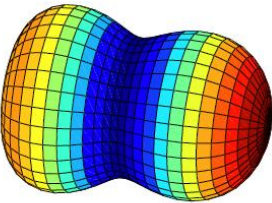
Xu Meng (孟旭), PhD thesis (2019)

Meng_Lu_Zhou2020
Sci. China-Phys. Mech. Astron. 63, 212011

Nuclear shapes

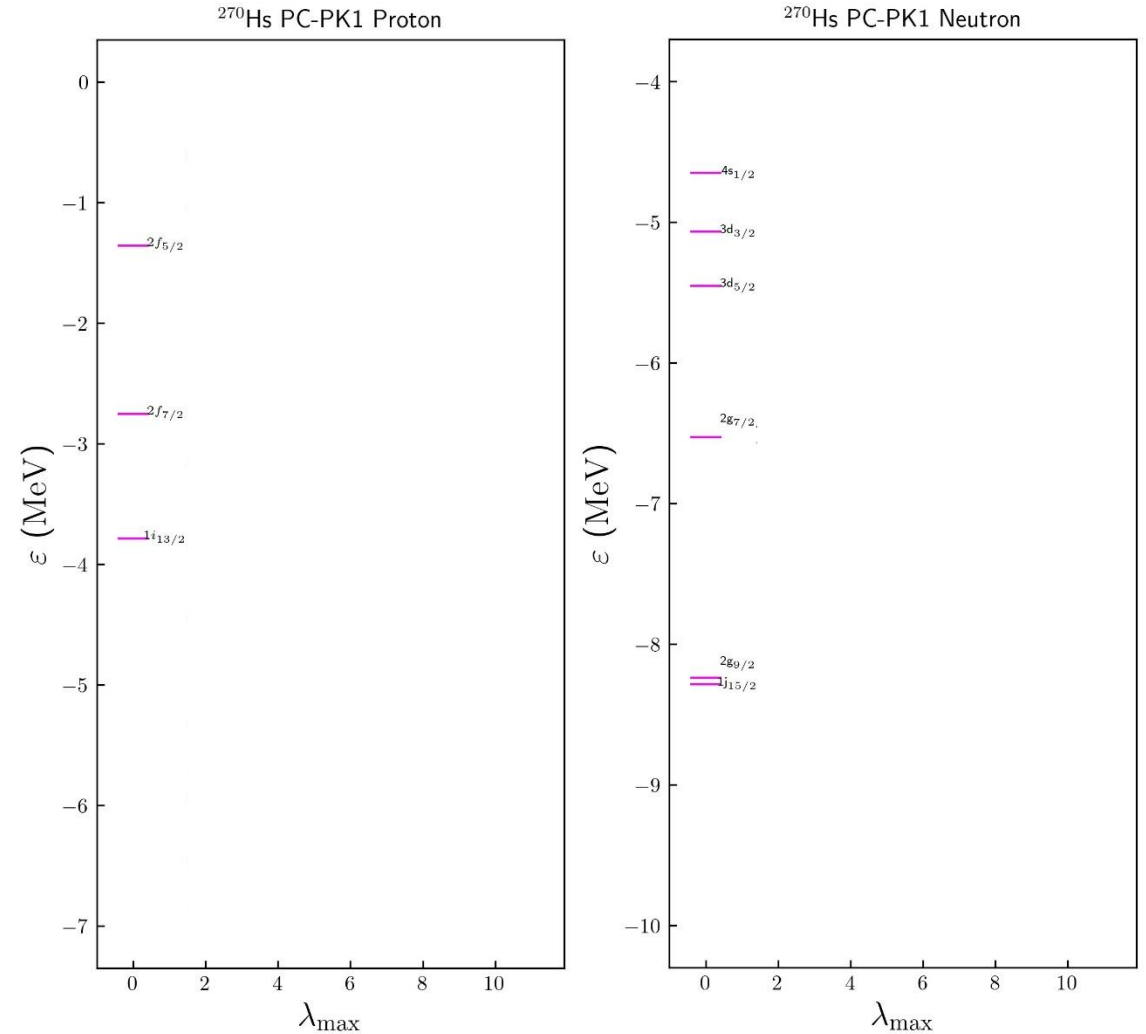
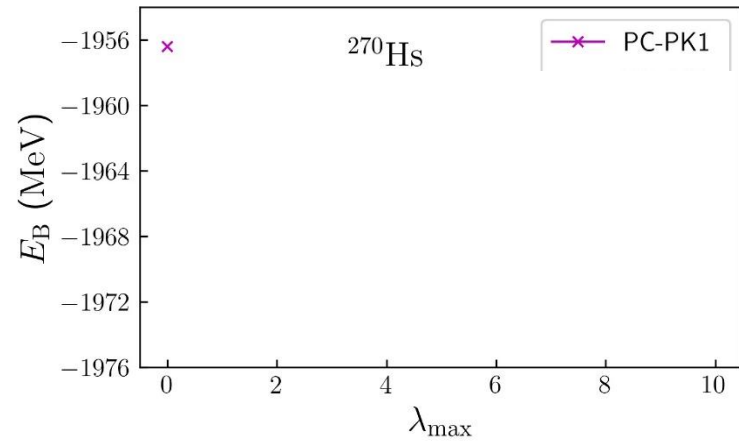
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

2^λ -pole deformation (2^λ -极形变)

(a) $\beta_{\lambda\mu} = 0$	(b) $\beta_{20} > 0$	(c) $\beta_{20} < 0$	(d) $\beta_{40} > 0$
			
(e) $\beta_{22} \neq 0$	(f) $\beta_{30} \neq 0$	(g) $\beta_{32} \neq 0$	(h) $\beta_{20} \gg 0$
			

Courtesy of Bing-Nan Lu (吕炳楠)

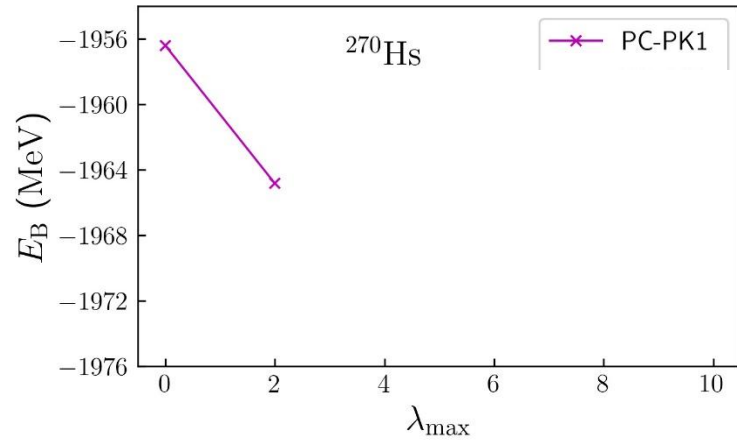
Effects of higher-order deformations



Courtesy of Xiao-Qian Wang (王晓倩)

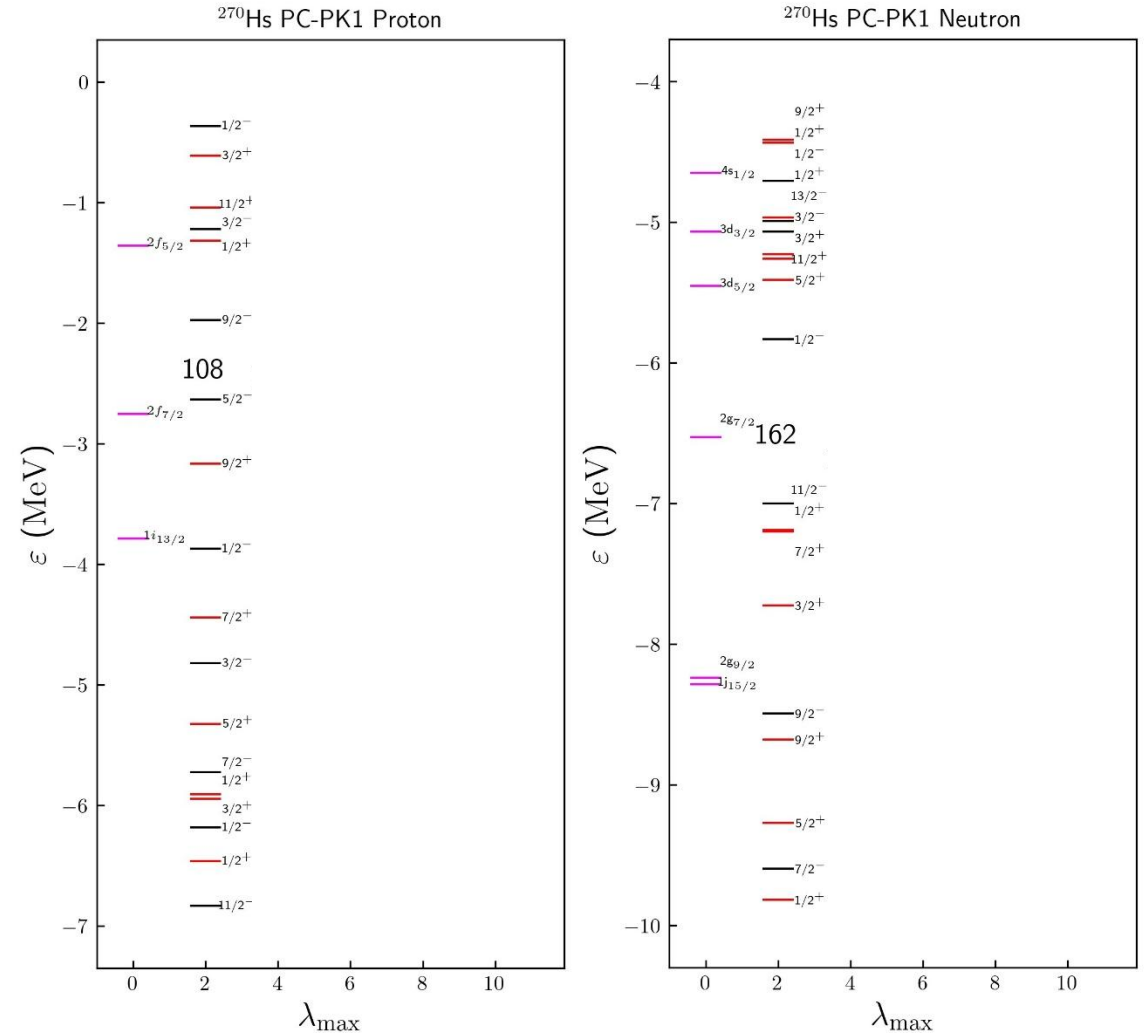
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

Effects of higher-order deformations



Changes in E_B & s.p. shell gap (in MeV)

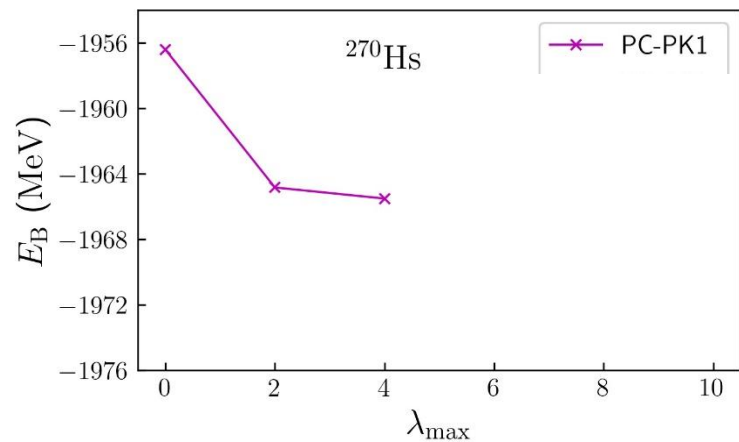
λ_{\max}	2	4	6	8	10
ΔE_B	8.43				
Δ^p_{sh}	0.66				
Δ^n_{sh}	1.17				



Courtesy of Xiao-Qian Wang (王晓倩)

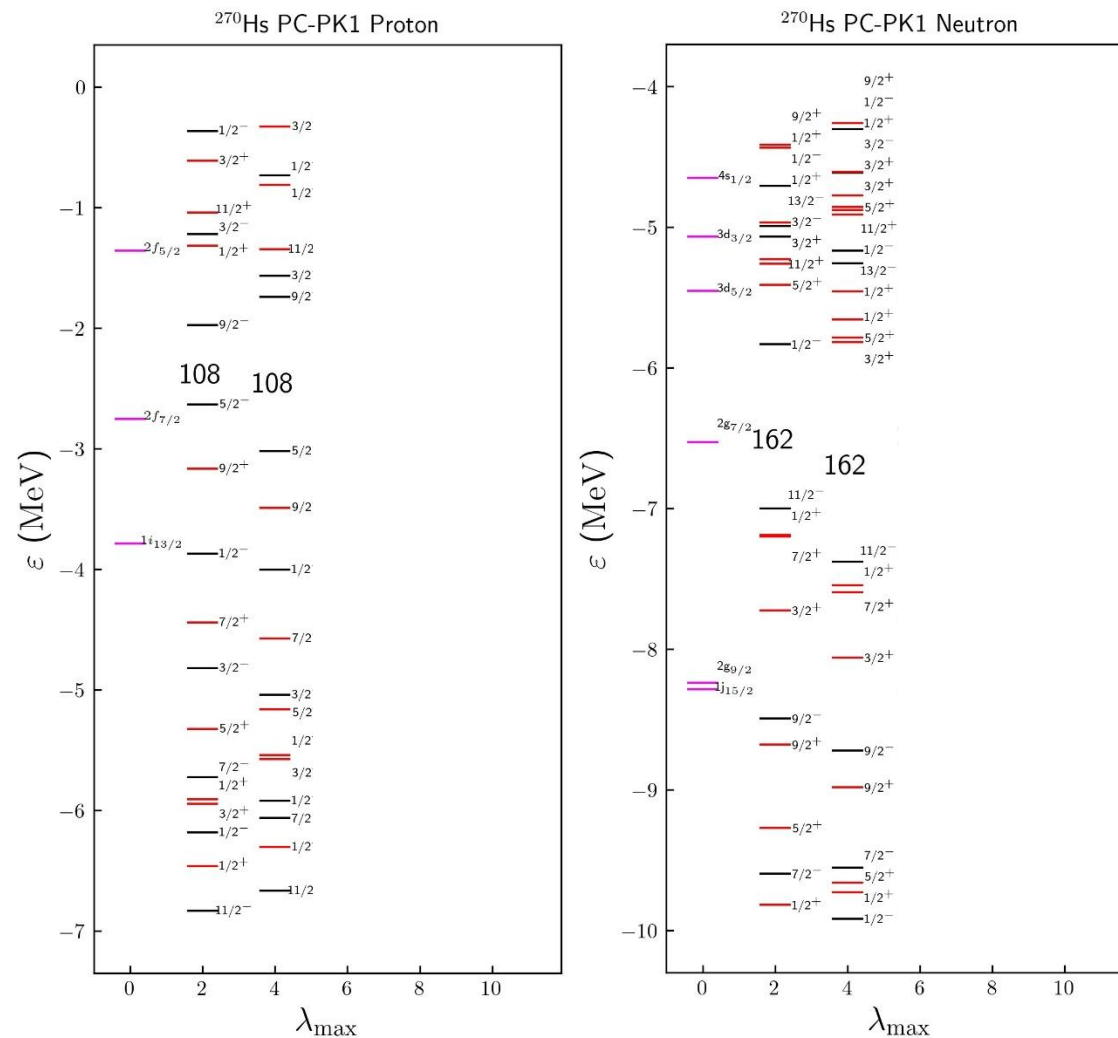
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

Effects of higher-order deformations



Changes in E_B & s.p. shell gap (in MeV)

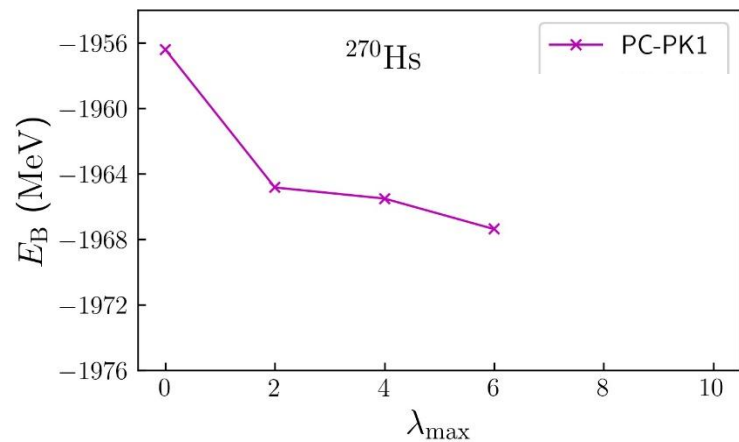
λ_{\max}	2	4	6	8	10
ΔE_B	8.43	0.68			
Δ^p_{sh}	0.66	1.28			
Δ^n_{sh}	1.17	1.56			



Courtesy of Xiao-Qian Wang (王晓倩)

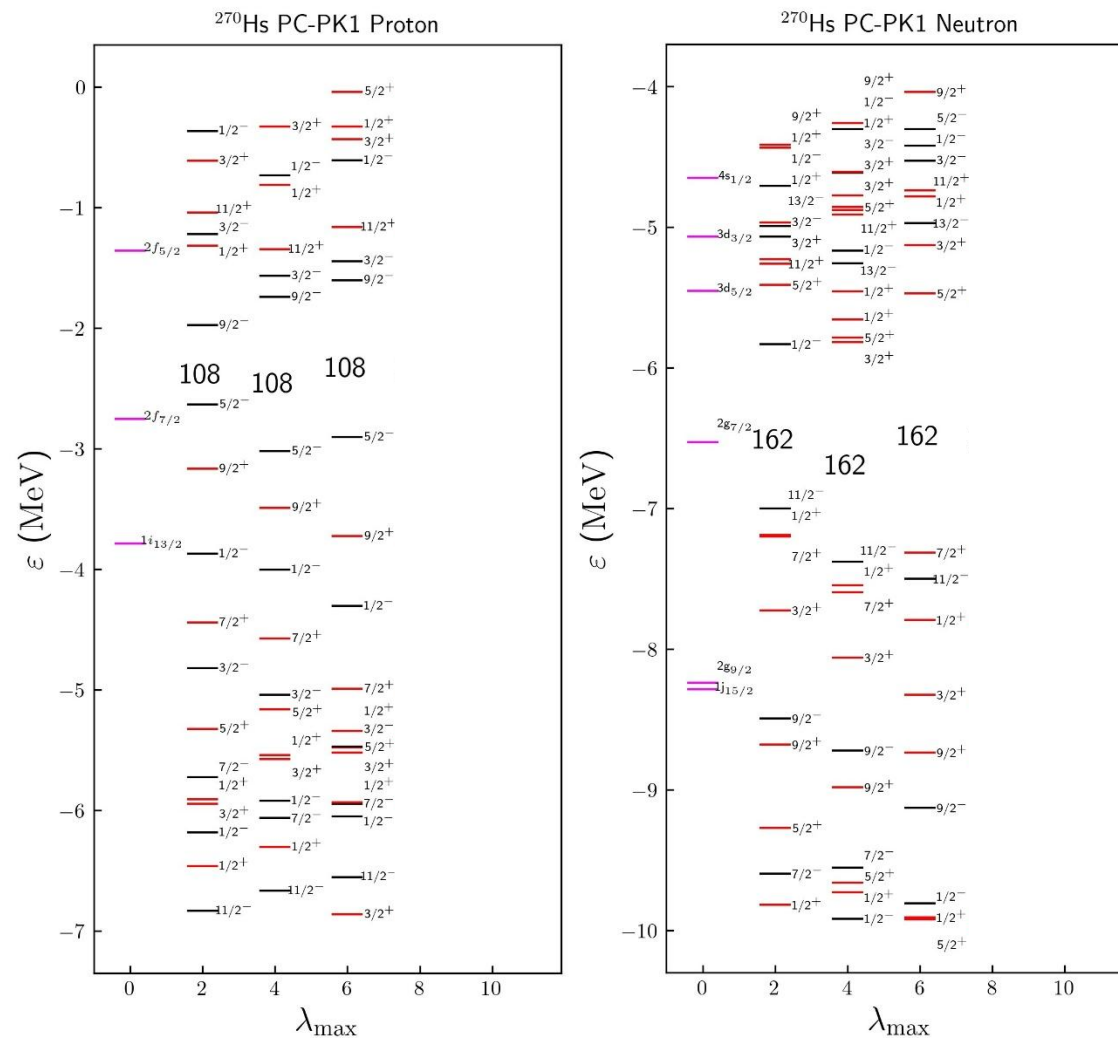
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

Effects of higher-order deformations



Changes in E_B & s.p. shell gap (in MeV)

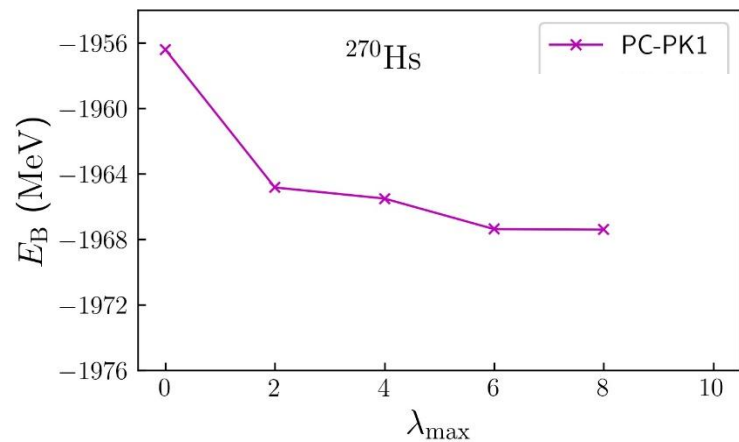
λ_{\max}	2	4	6	8	10
ΔE_B	8.43	0.68	1.87		
Δ^p_{sh}	0.66	1.28	1.30		
Δ^n_{sh}	1.17	1.56	1.84		



Courtesy of Xiao-Qian Wang (王晓倩)

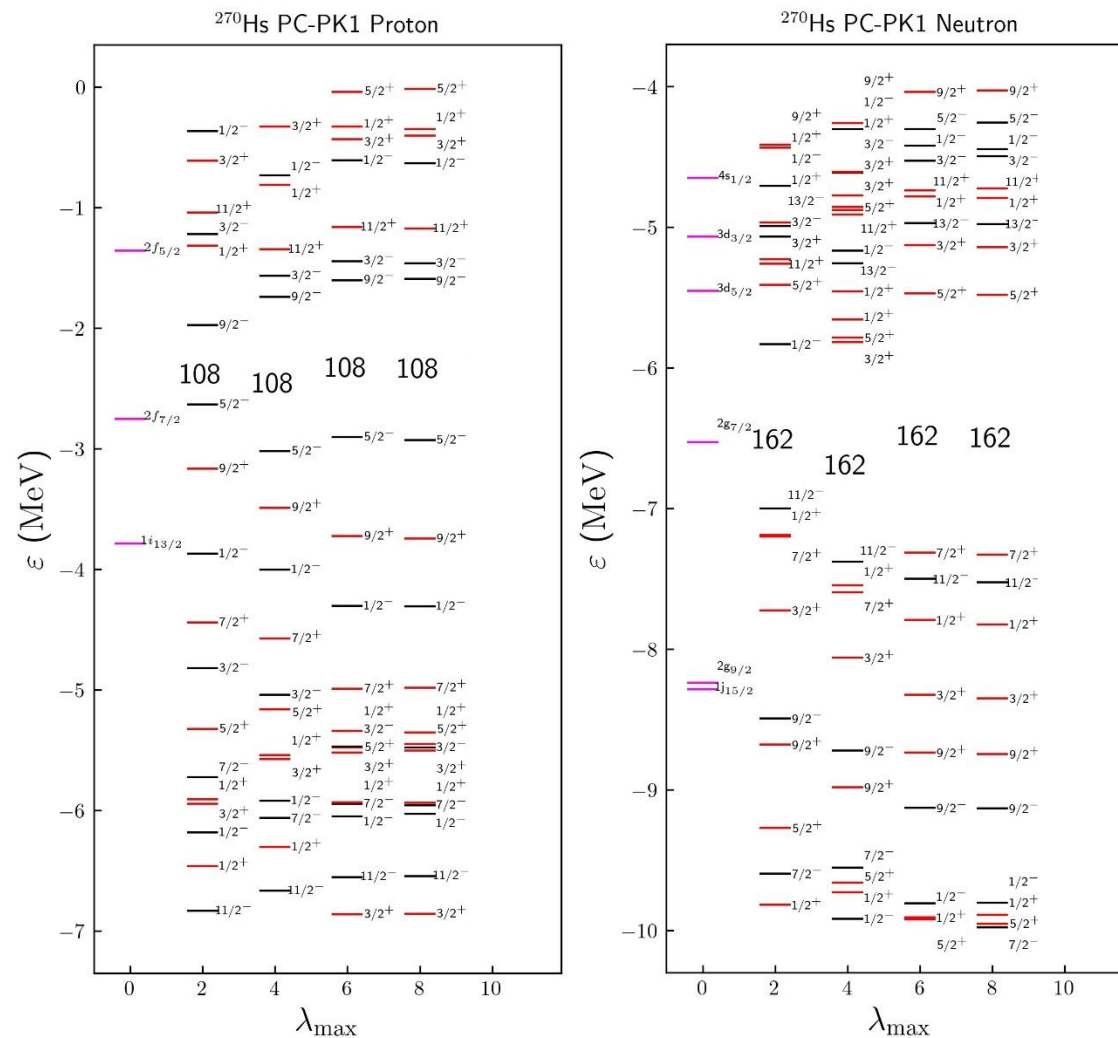
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

Effects of higher-order deformations



Changes in E_B & s.p. shell gap (in MeV)

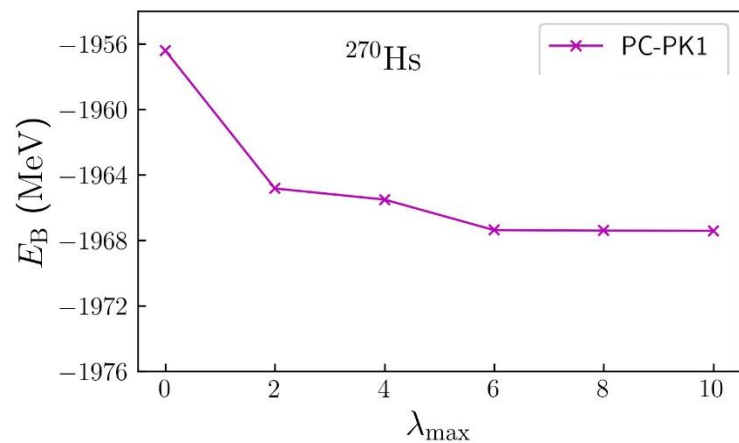
λ_{\max}	2	4	6	8	10
ΔE_B	8.43	0.68	1.87	0.03	
Δ^p_{sh}	0.66	1.28	1.30	1.34	
Δ^n_{sh}	1.17	1.56	1.84	1.85	



Courtesy of Xiao-Qian Wang (王晓倩)

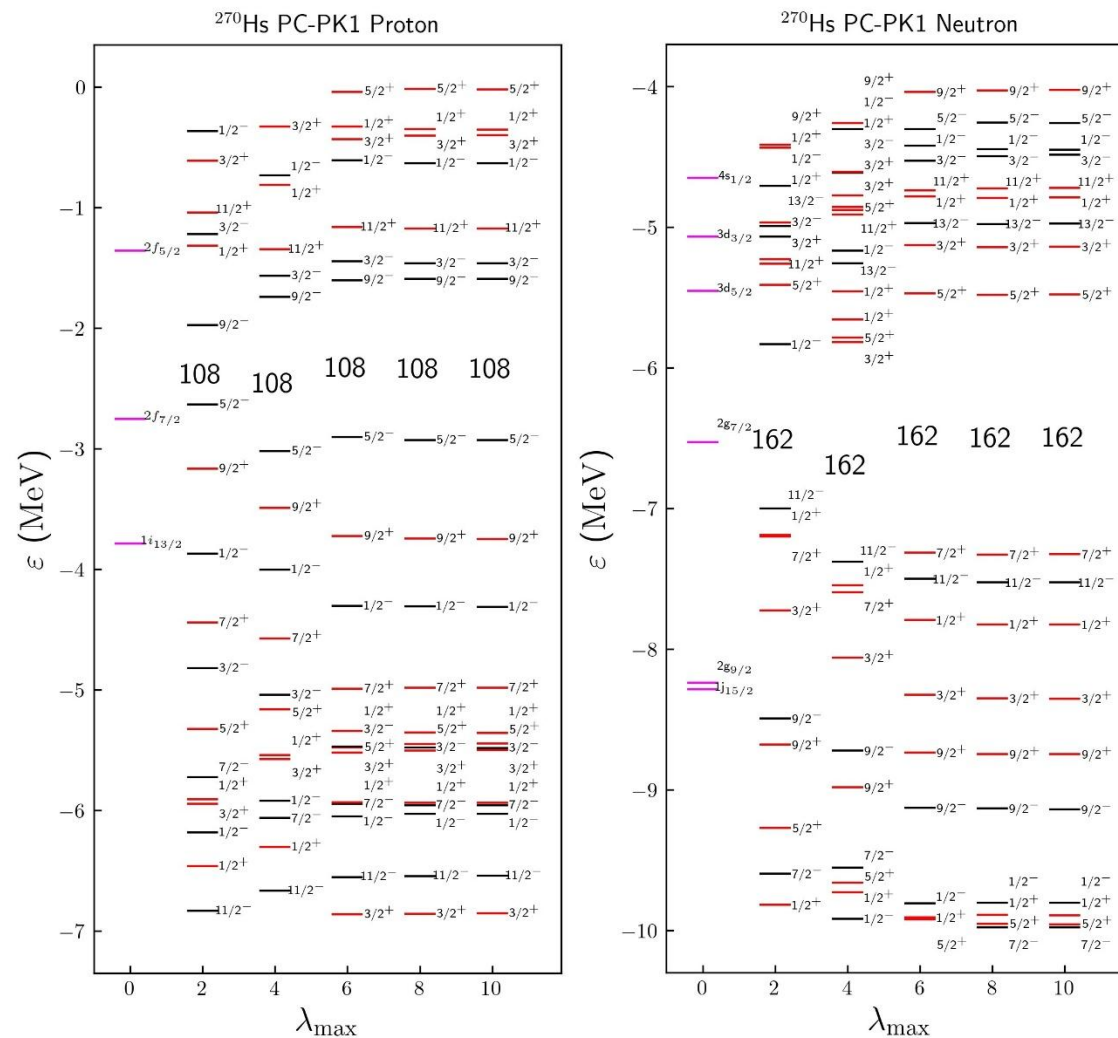
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

Effects of higher-order deformations



Changes in E_B & s.p. shell gap (in MeV)

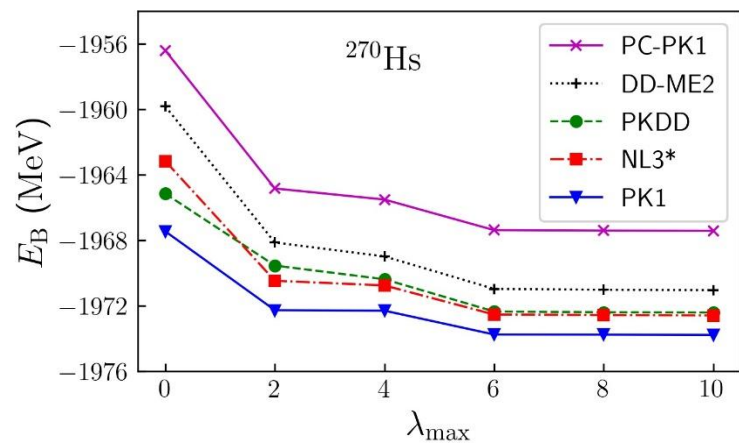
λ_{\max}	2	4	6	8	10
ΔE_B	8.43	0.68	1.87	0.03	0.01
Δ^p_{sh}	0.66	1.28	1.30	1.34	1.34
Δ^n_{sh}	1.17	1.56	1.84	1.85	1.85



Courtesy of Xiao-Qian Wang (王晓倩)

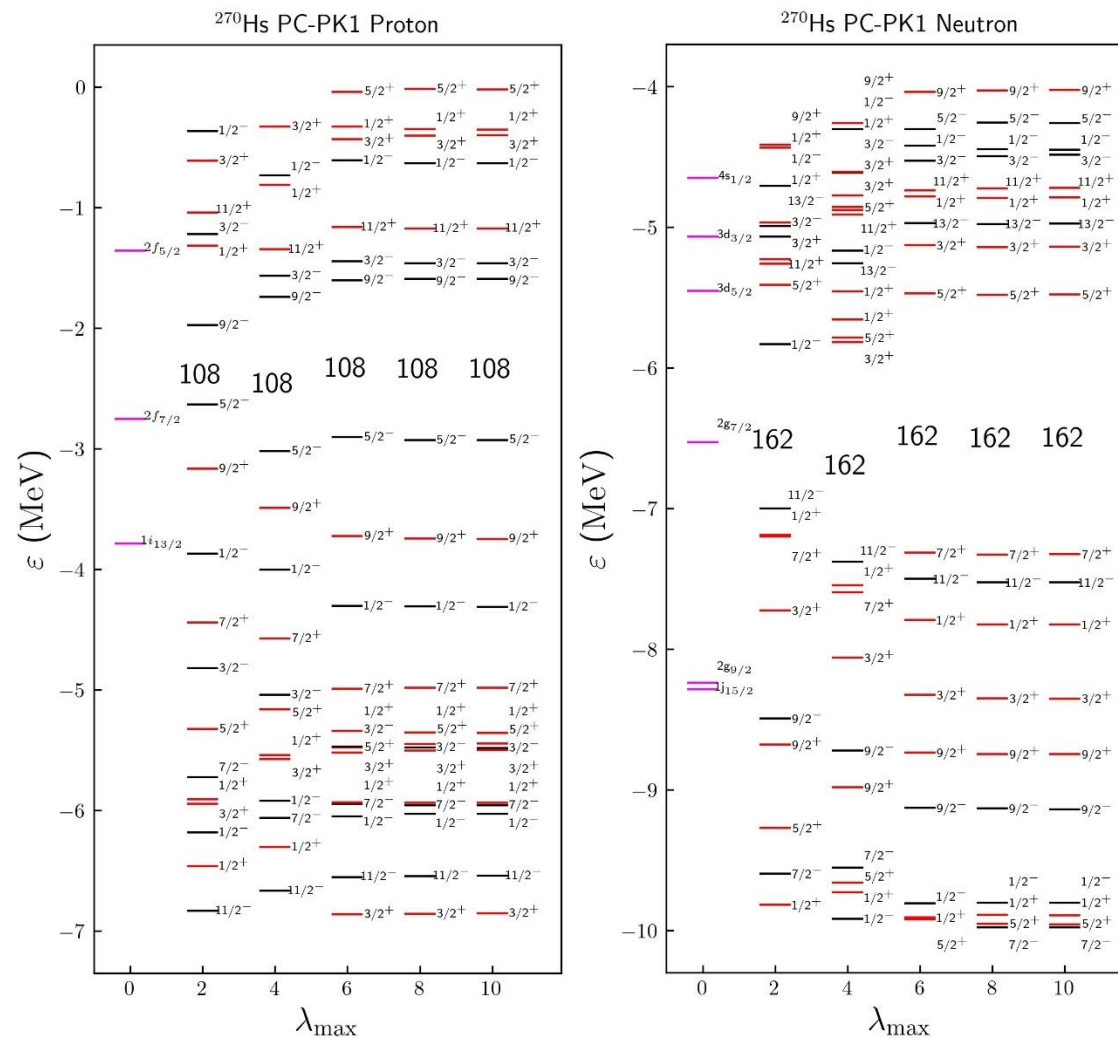
$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

Effects of higher-order deformations



Changes in E_B & s.p. shell gap (in MeV)

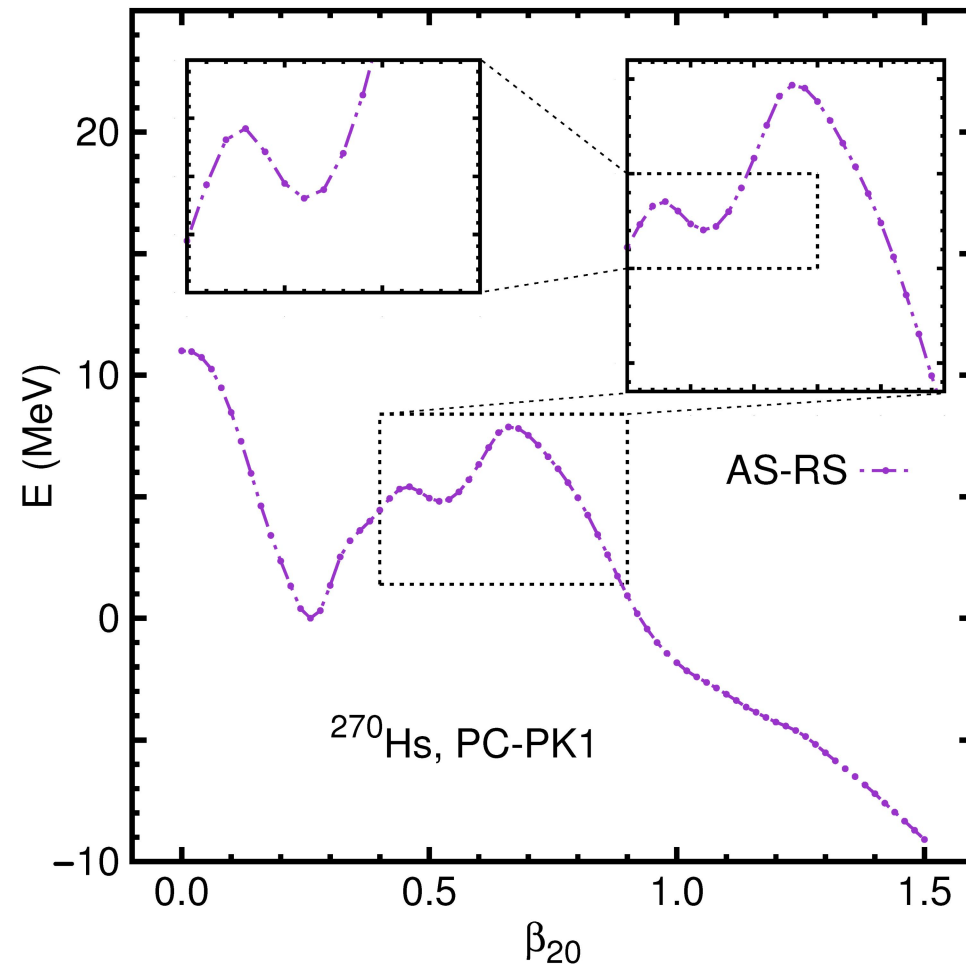
λ_{\max}	2	4	6	8	10
ΔE_B	8.43	0.68	1.87	0.03	0.01
Δ^p_{sh}	0.66	1.28	1.30	1.34	1.34
Δ^n_{sh}	1.17	1.56	1.84	1.85	1.85



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$$R(\theta, \varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \varphi) \right]$$

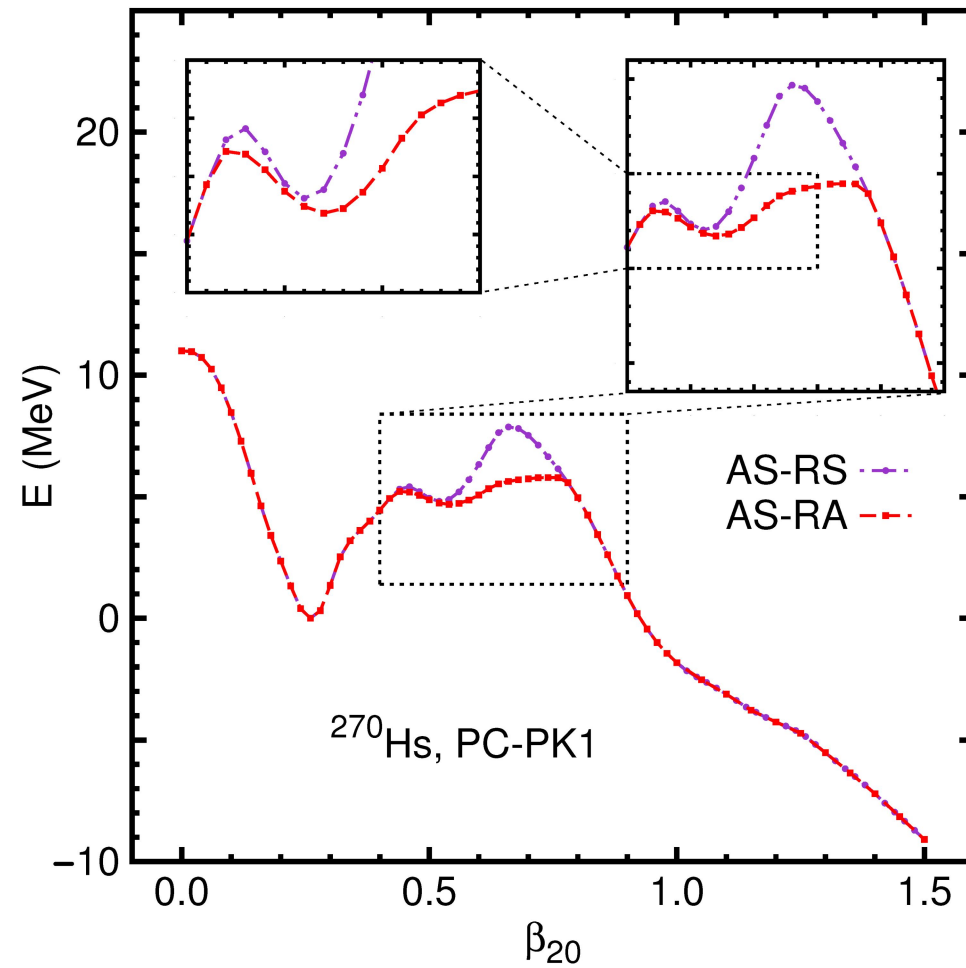
^{270}Hs : 1D PEC from MDC-RMF calc.



AS-RS: Axially-Symmetric &
Reflection Symmetric

Courtesy of Xu Meng (孟旭)

^{270}Hs : 1D PEC from MDC-RMF calc.

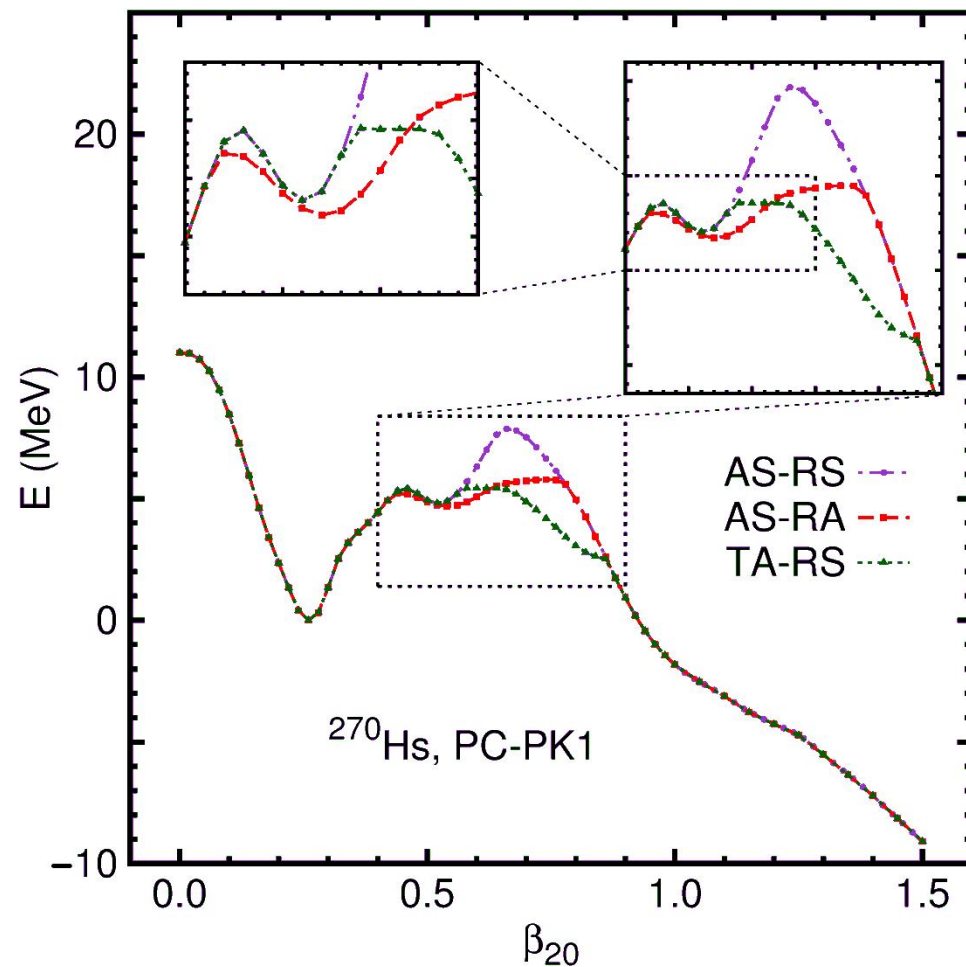


AS-RS: Axially-Symmetric &
Reflection Symmetric

AS-RA: Axially-Symmetric &
Reflection Asymmetric

Courtesy of Xu Meng (孟旭)

^{270}Hs : 1D PEC from MDC-RMF calc.



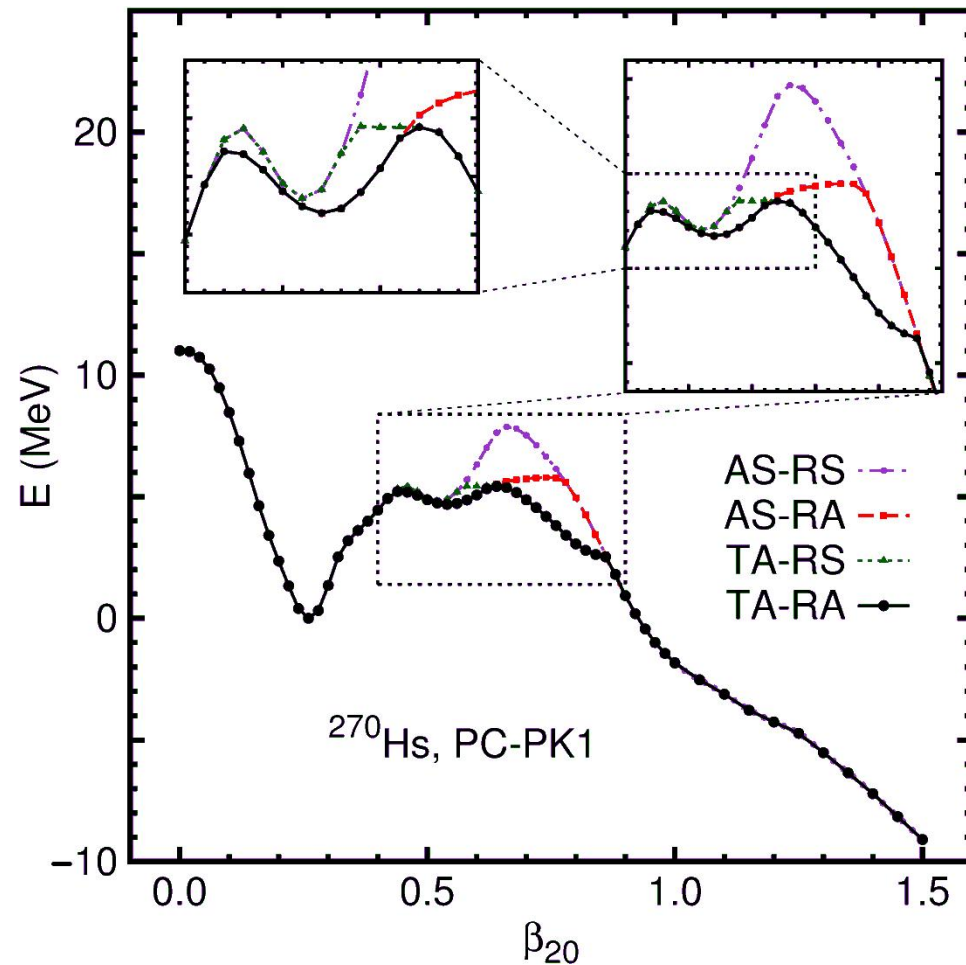
AS-RS: Axially-Symmetric &
Reflection Symmetric

AS-RA: Axially-Symmetric &
Reflection Asymmetric

TA-RS: TriAxial &
Reflection Symmetric

Courtesy of Xu Meng (孟旭)

^{270}Hs : 1D PEC from MDC-RMF calc.



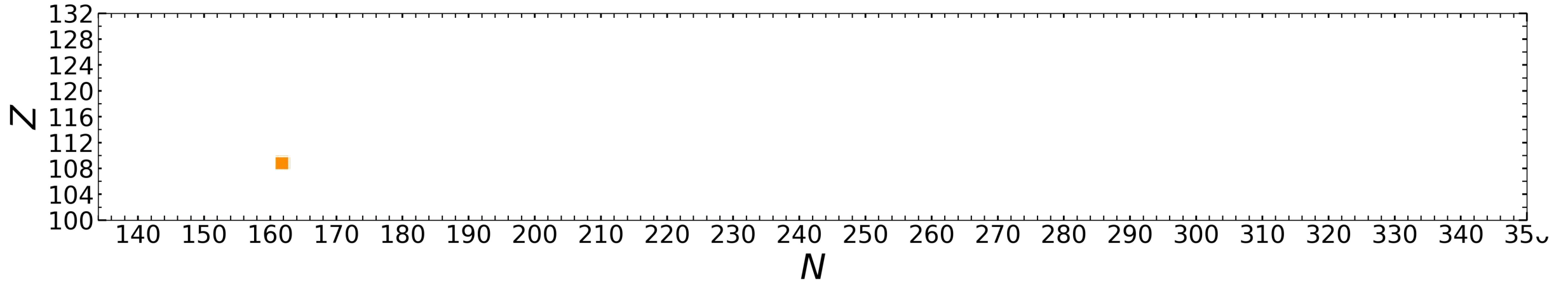
AS-RS: Axially-Symmetric &
Reflection Symmetric

AS-RA: Axially-Symmetric &
Reflection Asymmetric

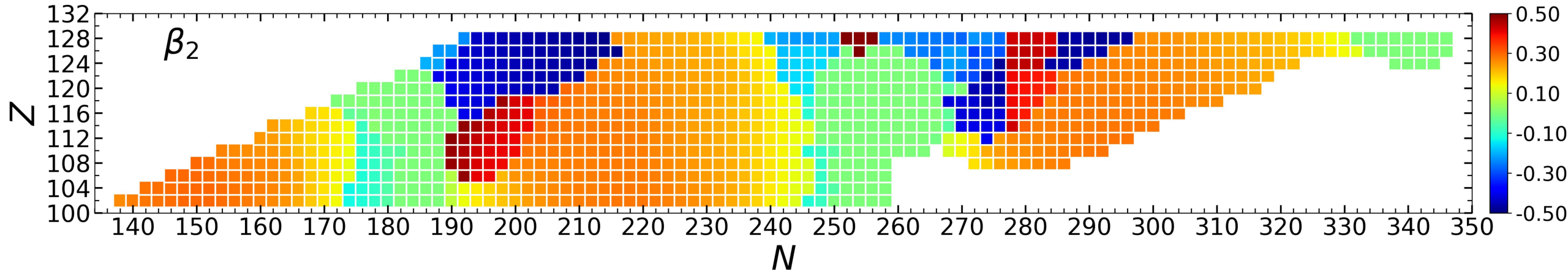
TA-RS: TriAxial &
Reflection Symmetric

TA-RA: TriAxial &
Reflection Asymmetric

A systematic study of even-even superheavy nuclei



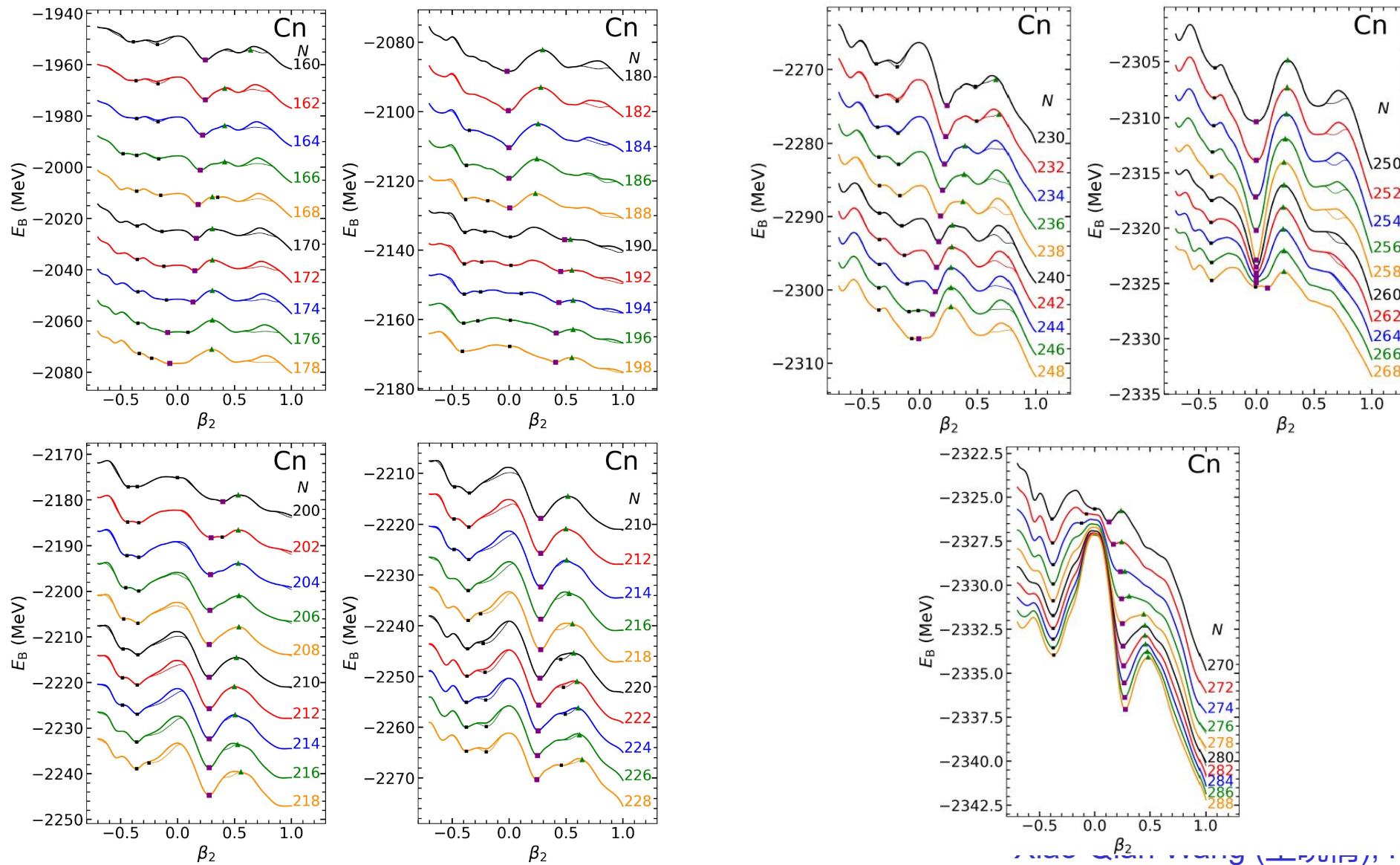
A systematic study of even-even superheavy nuclei



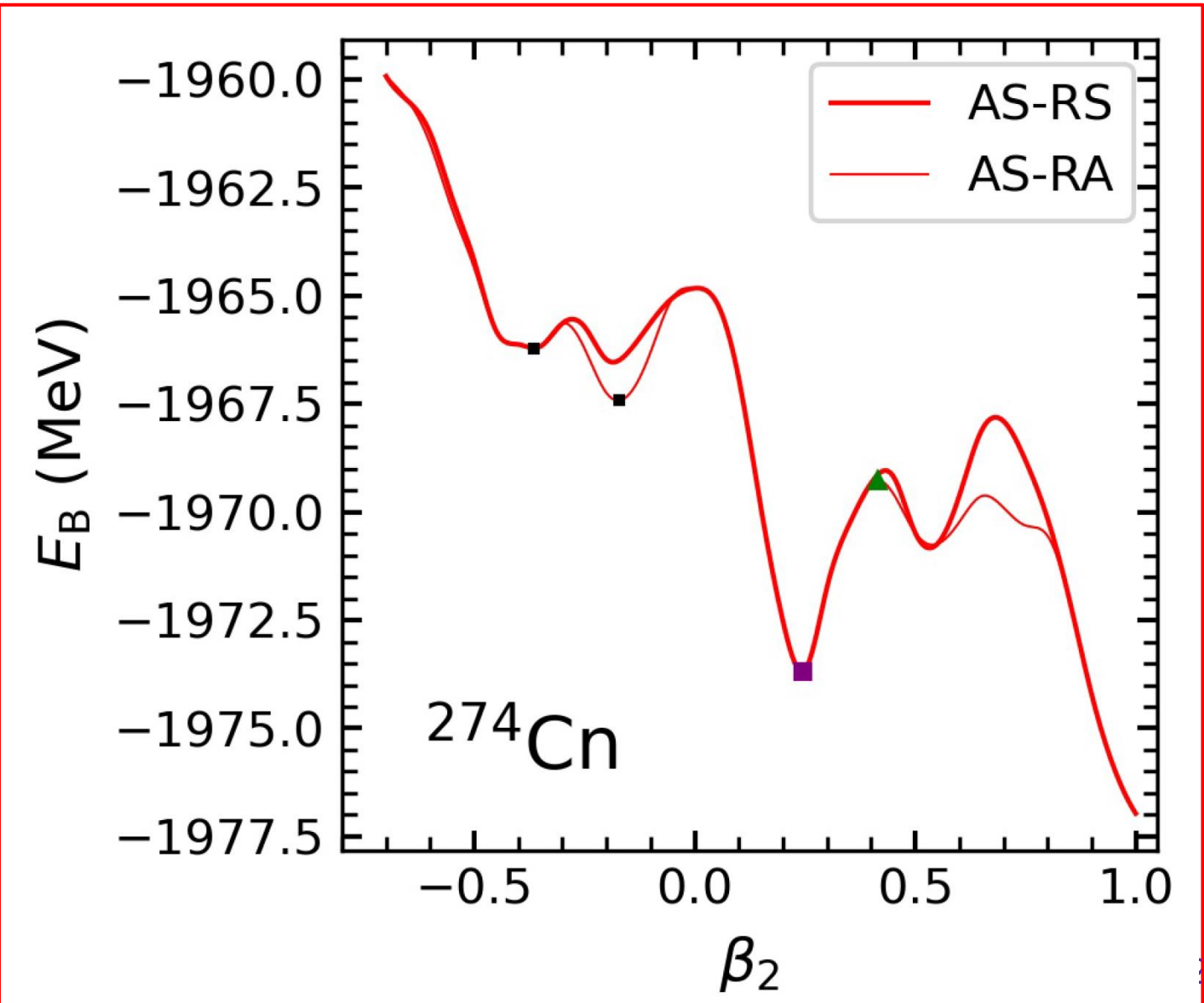
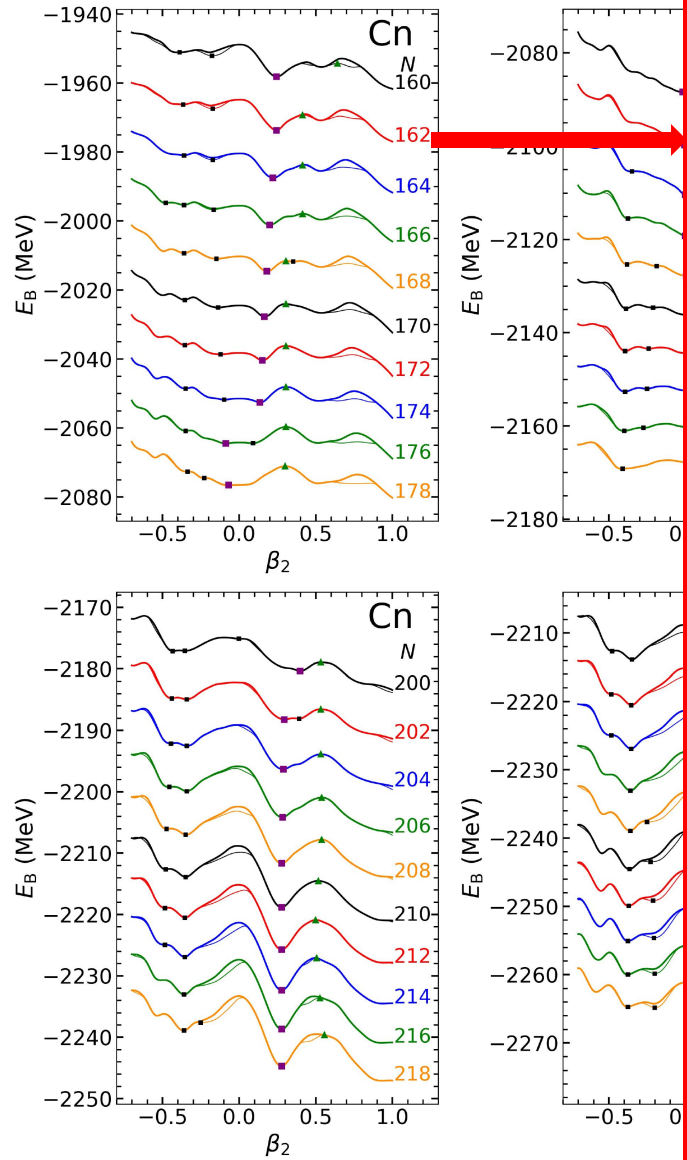
MDC-RMF calculations

- $102 \leq Z \leq 128$, proton drip line to neutron drip line
- Effective interaction: PC-PK1
- BCS w/ separable pairing force of finite range

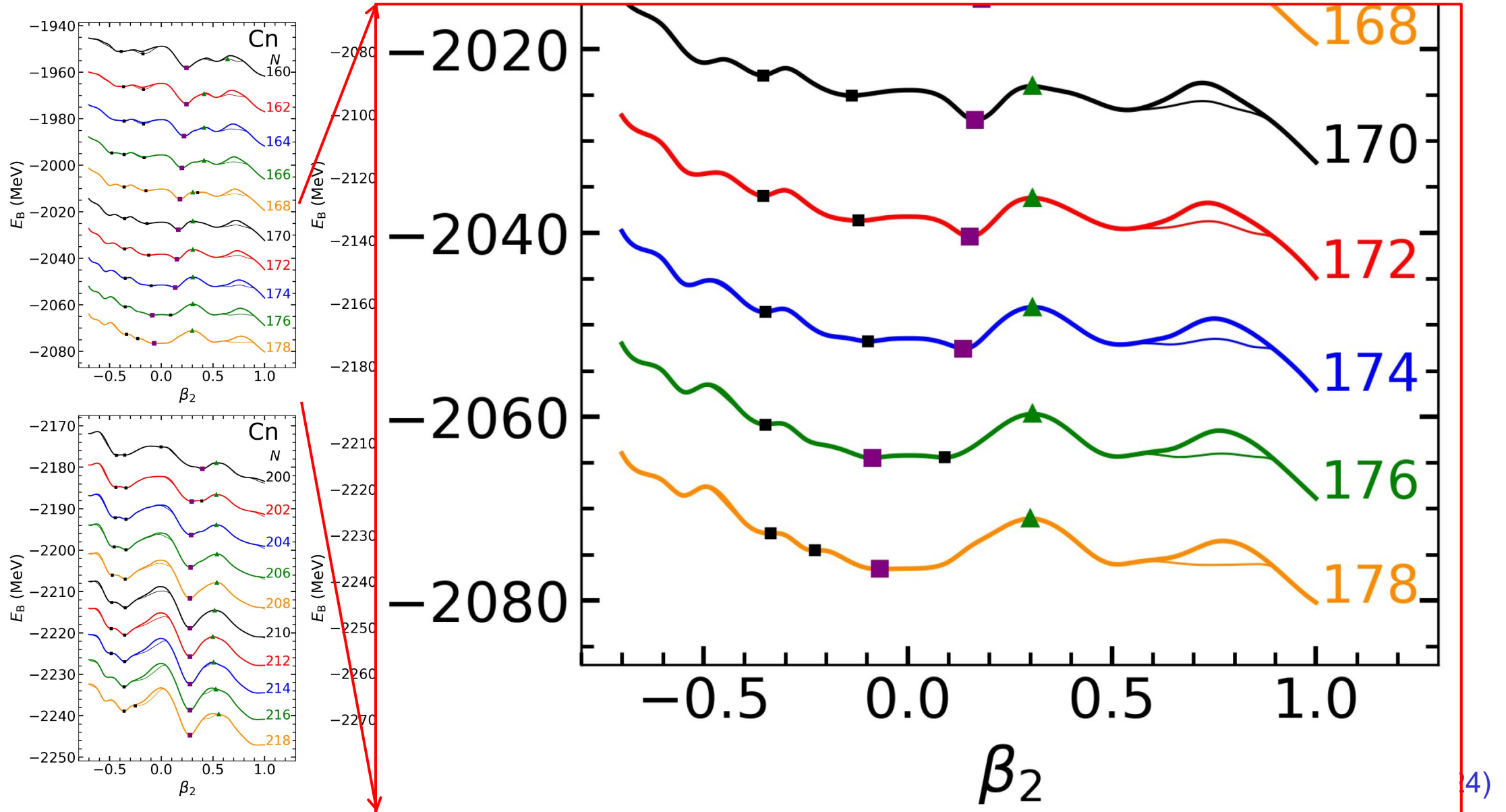
PECs of even- N Cn isotopes ($160 \leq N \leq 288$)



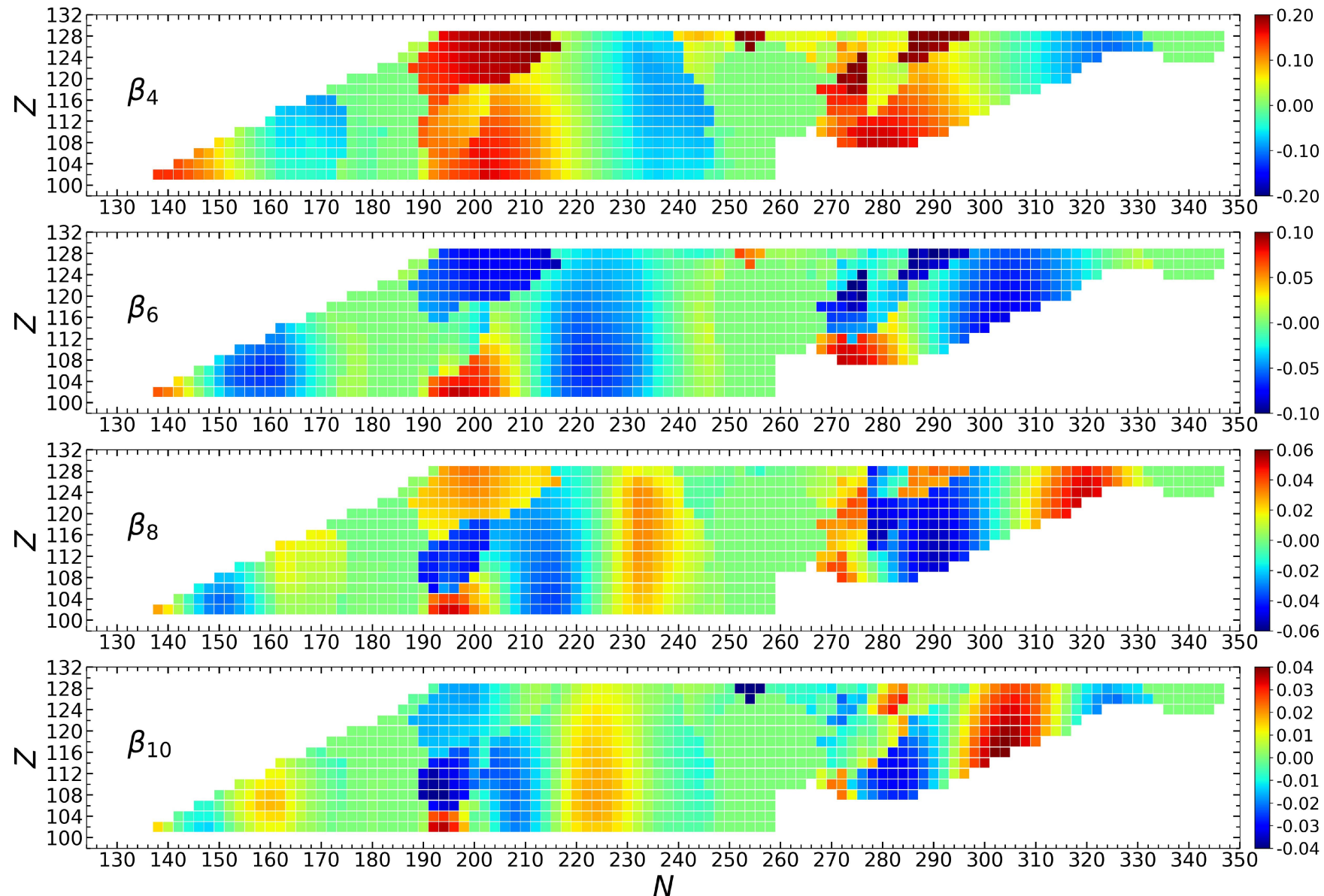
PECs of ^{274}Cn



PECs of $^{282,284,286,288,290}\text{Cn}$ isotopes



A systematic study of even-even superheavy nuclei



Summary & perspectives

- Fission barrier is crucial for the description of fission & various shapes may appear during fission
- MultiDimensionally-Constrained Covariant Density Functional Theories
- Potential energy surfaces & fission barriers: ^{240}Pu ; actinides & ^{270}Hs ; superheavies
 - Inner barrier: triaxial deformation
 - Outer barrier: octupole & triaxial deformations
 - A systematic study of even-even superheavy nuclei (w/o triaxiality)

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Thank you !