



Manifestation of clustering in fission of heavy nuclei

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- Description of the model
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 - *Mass, charge, TKE and <n> distributions*
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Model



<u>The interaction potential between the fragments</u>: $V(R, Z, A, J, \beta_1, \beta_2) = V_N + V_C$

$$V_c(R, Z_1, Z_2, \beta_1, \beta_2) = \frac{e^2 Z_1 Z_2}{R} + \left(\frac{9}{20\pi}\right)^{1/2} \frac{e^2 Z_1 Z_2}{R^3} \sum_{i=1}^2 R_i^2 \beta_i \left[1 + \frac{2}{7} \left(\frac{5}{\pi}\right)^{1/2} \beta_i\right] P_2(\cos\theta_i)$$

$$V_{N} = \int \rho_{1}(r_{1})\rho_{2}(R - r_{2})F(r_{1} - r_{2})dr_{1}dr_{2} \qquad F(r_{1} - r_{2}) = C_{0}[F_{in}\frac{\rho_{1}(r_{1})}{\rho_{00}} + F_{ex}(1 - \frac{\rho_{1}(r_{1})}{\rho_{00}})]\delta(r_{1} - r_{2})$$

$$\rho_{0}(r) = \rho_{1}(r) + \rho_{2}(R - r)$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex}\frac{(N - Z)(N_{2} - Z_{2})}{(N + Z)(N_{2} + Z_{2})}$$

$$C_{0} = 300MeVfm^{3}$$

$$f_{in} = 0.09, f_{ex} = -2.59$$

$$\rho_{00} = 0.17fm^{-3}, a = 0.51 - 0.56fm$$

Sh. A. Kalandarov, G.G. Adamian, N.V. Antonenko, W. Scheid, Phys. Rev. C 82, 044603 (2010)



$$U_{i}^{L.D.}(A_{i}, Z_{i}, \beta_{i}) = U_{i}^{Surface}(A_{i}, Z_{i}, \beta_{i}) + U_{i}^{C}(A_{i}, Z_{i}, \beta_{i}) + U_{i}^{Sym}(A_{i}, Z_{i})$$

• Liquid drop terms:

$$U_i^{sym}(A_i, Z_i) = 27.612 \frac{(N_i - Z_i)^2}{A_i}$$

$$U_i^C(A_i, Z_i, \beta_i) = \frac{3}{5} \frac{(Z_i e)^2}{R_{0,i}} \frac{\beta_i^{1/3}}{\sqrt{\beta_i^2 - 1}} ln(\beta_i + \sqrt{\beta_i^2 - 1})$$

Surface energy with variable surface tension:

$$U_{i}^{Surface}(A_{i}, Z_{i}, \beta_{i}) = \sigma_{i}S_{i}$$

$$\sigma_{i} = \sigma_{0,i}(1 + k_{i}(\beta_{i} - \beta_{i}^{g.s.})^{2})$$

$$\sigma_{0,i} = 0.9517(1 - 1.7826((N_{i} - Z_{i})^{2})/A_{i})^{2})$$

$$k_{i} = \frac{1}{1 + exp[-0.063(C_{vib}(Z_{i}, A_{i}) - 67)]}$$

$$C_{vib}(Z_{i}, A_{i}) = \frac{\hbar\omega_{vib}^{i}(3Z_{i}eR_{0,i}^{2}/(4\pi))^{2}}{2B(E2)_{vib}^{i}}$$

$$B(E2)_{vib} = E_{2^{+}}^{i}B(E2)_{rot}^{i}/(\hbar\omega_{vib}^{i})$$

Shell corrections are calculated as in : J. Maruhn and W. Greiner, Z. Phys. 251, 431 (1972).



<u>Model</u>

Excitation energy of the scission configuration can be calculated as a sum of the initial excitation energy of the fissioning nucleus and the difference of the potential energies of the fissioning nucleus and scission configuration:

$$E^*(A_i, Z_i, \beta_i, R_m) = E^*_{CN} + [U_{CN}(A, Z, \beta) - U(A_i, Z_i, \beta_i, R_m)].$$

$$T_{DNS}(E^*) = \sqrt{E^*/a} , a = A/12 \, MeV^{-1}$$

Temperature dependence of LD terms:

 $U_{i}^{sym}(A_{i}, Z_{i}, T) = U_{i}^{sym}(A_{i}, Z_{i}, T = 0)(1 + 6 * 10^{-4}E_{i}^{*}/A_{i}),$ $U_{i}^{C}(A_{i}, Z_{i}, \beta_{i}, T) = U_{i}^{C}(A_{i}, Z_{i}, \beta_{i}, T = 0)(1 - 0.12E_{i}^{*}/A_{i}),$ $U_{i}^{Surf}(A_{i}, Z_{i}, \beta_{i}, T) = U_{i}^{Surf}(A_{i}, Z_{i}, \beta_{i}, T = 0)(1 + 0.102E_{i}^{*}/A_{i}),$ $k_{i}(E_{i}^{*}) = k_{i} * exp[-E_{i}^{*}/E_{k}]$

<u>Shell damping:</u> $\delta U_i^{shell}(A_i, Z_i, \beta'_i, E_i^*) = \delta U_i^{shell}(A_i, Z_i, \beta'_i, E_i^* = 0) exp[-E_i^*/E_D]$

G. Sauer, H. Chandra, U. Mosel, Nucl. Phys. A 264 (1976) 221-243

<u>Model</u>

<u>Yields</u>:

Using
$$P_{Z,A}(E_{CN}^*, \beta_1, \beta_2) \sim exp\{-U(R_m, Z, A, \beta_1, \beta_2)/T\}$$

 $P_{Z,A,\beta_1,\beta_2}^{decay} \sim exp\{-B_{qf}(Z, A, \beta_1, \beta_2)/T\}$
 $w(A_i, Z_i, \beta_i, E^*) = N_0 exp[-\frac{U(A_i, Z_i, \beta_i, R_m) + B_{qf}(A_i, Z_i, \beta_i)}{T}]$

The different yields can be calculate by integrating over the deformations:

$$Y(A_i, Z_i) = N_0 \int \int w(A_i, Z_i, \beta_1, \beta_2, E^*) \,\mathrm{d}\beta_1 \mathrm{d}\beta_2,$$

$$\begin{split} Y(A_{i}) &= N_{0} \sum_{Z_{i}} \int \int w(A_{i}, Z_{i}, \beta_{1}, \beta_{2}, E^{*}) \, \mathrm{d}\beta_{1} \mathrm{d}\beta_{2}, \\ Y(Z_{i}) &= N_{0} \sum_{A_{i}} \int \int w(A_{i}, Z_{i}, \beta_{1}, \beta_{2}, E^{*}) \, \mathrm{d}\beta_{1} \mathrm{d}\beta_{2}, \\ TKE(A_{i}, Z_{i}) &= V_{c}(A_{i}, Z_{i}, \beta_{1}, \beta_{2}) + V_{n}(A_{i}, Z_{i}, \beta_{1}, \beta_{2}) \\ < TKE > (A_{i}) &= \frac{\sum_{Z_{i}} \int TKE(A_{i}, Z_{i}, \beta_{1}, \beta_{2}) w(A_{i}, Z_{i}, \beta_{1}, \beta_{2}, E^{*}) d\beta_{1} d\beta_{2}}{\sum_{Z_{i}} \int w(A_{i}, Z_{i}, \beta_{1}, \beta_{2}, E^{*}) d\beta_{1} d\beta_{2}} \end{split}$$

<u>Results</u>



- W. Lang, H.G. Clerc, W. Wohlfarth, H. Schrader, and K.-H. Schmidt, Nucl. Phys. A 345, 34(1980);
- G. Siegert, H. Wollnik, J. Grief, R. Decker, G. Fiedler, and B. Pfeiffer, Phys.Rev.C 14, 1864 (1976);
- G. Mariolopoulos, Ch. Hamelin, J. Blachot, J.P. Boucqet, B. Brissot, J. Crancon, H. Nifenecker, and Ch. Ristori, Nucl. Phys. A 361, 213 (1981).
- T.R. England and B.F. Rider, LANL report no. LA-UR-94-3106 (1994).
- P. Schillebeeckx, C. Wagemans, A.J. Deruytter, and R. Barthelemy, Nucl. Phys. A 545, 623 (1992).
- R. Schmidt and H. Henschel, Nucl. Phys. A 395, 15 (1983).

<u>Results</u>



- W. Lang, H.G. Clerc, W. Wohlfarth, H. Schrader, and K.-H. Schmidt, Nucl. Phys. A 345, 34(1980);
- G. Siegert, H. Wollnik, J. Grief, R. Decker, G. Fiedler, and B. Pfeiffer, Phys.Rev.C 14, 1864 (1976);
- G. Mariolopoulos, Ch. Hamelin, J. Blachot, J.P. Boucqet, B. Brissot, J. Crancon, H. Nifenecker, and Ch. Ristori, Nucl. Phys. A 361, 213 (1981).
- T.R. England and B.F. Rider, LANL report no. LA-UR-94-3106 (1994).
- P. Schillebeeckx, C. Wagemans, A.J. Deruytter, and R. Barthelemy, Nucl. Phys. A 545, 623 (1992).

<u>Results (and more modelling)</u>

$$\langle TKE \rangle (A_i) = \frac{\sum_{Z_i} \int d\beta_L d\beta_H TKE(A_i, Z_i, \beta_i) w(A_i, Z_i, \beta_i, E^*)}{\sum_{Z_i} \int d\beta_L d\beta_H w(A_i, Z_i, \beta_i, E^*)} .$$

$$TKE(\{A_i, Z_i, \beta_i\}) = V^N(\{A_i, Z_i, \beta_i\}) + V^C(\{A_i, Z_i, \beta_i\})$$



- R. Schmidt and H. Henschel, Nucl. Phys. A 395, 15 (1983).
- C. Wagemans, E. Allaert, A. Deruytter, R. Barthelemy, and P. Schillebeeckx, Phys. Rev. C 30, 218 (1984).
- T.R. England and B.F. Rider, LANL report no. LA-UR-94-3106 (1994).

Results (and more modelling)



- K. Nishio, et.al., Nucl. Phys. A 632, 540 (1998); K. Nishio, et.al., J. Nucl. Sci. Technol. 32, 404 (1995); C. Tsuchiya, et.al., J. Nucl. Sci. Technol. 37, 941 (2000); V.F. Apalin, et.al. Nucl. Phys. A 71, 553 (1965).
- R. Schmidt and H. Henschel, Nucl. Phys. A 395, 29 (1983).

Results (and more modelling)



• B.C. Diven, H.C. Martin, R.F. Taschek, and J. Terrell, Phys. Rev. 101, 1012 (1956).

• N. E. Holden and M. S.Zucker, Brookhaven National Laboratory Report No. BNL-NCS-35513 (1985).

• Z. Huanqiao, L. Zuhua, D. Shengyao, and L. Shaoming, Nucl. Sci. Eng. 86, 315 (1984).

<u>Results - thorium isotopes</u>



• K.-H. Schmidt et al., Nucl. Phys. A 665, 221 (2000); 693, 169 (2001).

• A. Chatillon et al., Phys. Rev. Lett. 124, 202502 (2020)





80 100 120 **A**i

<u>Results - <N/Z> ratios in primary fragments</u>



<u>Results - <N/Z> ratios in primary fragments</u>





<u>Focus: ^{252}Cf </u> 10^{7} 10^{6} Two fission modes? 10^{5} Ba-Mo Relative Yields (arbitrary units) Ba-Mo: 1st mode: $\langle TKE \rangle = 189 \pm 1$ MeV $<\nu>=3-4$ 10^{4} 2nd mode: $\langle TKE \rangle = 153 \pm 3 \text{ MeV}$ 10^{3} $<\nu>\simeq 8$ 10^{2} Ce-Zr 10^{1}

G.M. Ter-Akopian et.al., PRC55 1146 (1997): "(...) data clearly indicate that most of the neutrons in the high neutron emission events come out of the Ba fragments – not the Mo fragments."



<u>Theory</u>: $TKE(\beta_1 = \beta_2 = 2.2) = 173.4$ MeV (Ba-Mo) and 171.0 MeV (Ce-Zr), respectively.

Maximum excitation energy: $\epsilon_{Ba}^* = E_{Ba}^* + U_{Ba}^{def} = 28.1 \text{ MeV}$ and $\epsilon_{Ce}^* = E_{Ce}^* + U_{Ce}^{def} = 24.1 \text{ MeV}$, respectively. Theory: $\langle TKE \rangle = 192.4$ MeV (Ba-Mo) $\langle TKE \rangle = 189.1$ MeV (Ce-Zr) $\langle \nu \rangle \simeq 3.5$

FWHM = 3.1

Neutron Loss Number

B. M. Musangu et.al., PRC **101**, 034610 (2020) see also: G.M. Ter-Akopian et.al., PRL**77**, 32 (1996) and G.M. Ter-Akopian et.al., PRC**55** 1146 (1997)



To obtain *TKE* ~ 153 MeV one needs $\beta_{Ba,Mo} \ge 2.5$ (>3 PRC55 1146 (1997)) - unrealistic.

<u>Focus: ^{252}Cf </u>

Competing mechanisms? Ideea supported by: I. Tsekhanovich *et. al.*, PRC**67** 034610 (2003), Yu.V. Pyatkov *et.al.*, Physics of Atomic Nuclei **86**/4 (2023), ...



Schematic illustration of normal binary decay and ternary fission processes, as well as the ternary cluster mechanism leading to a hyperexcited heavy fragment.

<u>Focus: ^{252}Cf </u>

Competing mechanisms? Ideea supported by: I. Tsekhanovich *et. al.*, PRC67 034610 (2003), Table I (right)



Ba (left) and Ce (right) nuclei have sufficient ϵ^* to be energetically feasible to be represented as clusters.

Notable third fragments: α , ${}^{10}Be$, ${}^{14}C$, ${}^{20}O$, ${}^{24, 26}Ne$ (neutron rich nuclei).

TABLE I. Mean kinetic energy \overline{E} , width of energy distribution σ_E and absolute yield of ternary particles.
Values given in <i>italics</i> are those from an "enforced" fit, as explained in the text; they should be considered
as preliminary.

	\overline{E} (Mev)	σ_E (MeV)	Yield
⁸ Li	15.1±1.4	7.1±1.3	$(2.6\pm0.7)\times10^{-6}$
⁹ Li	12.5 ± 0.9	5.5 ± 1.0	$(3.8\pm1.0)\times10^{-6}$
¹⁰ Be	17.5±0.4	7.7±0.6	$(3.8\pm0.7)\times10^{-5}$
¹¹ Be	16.5 ± 1.3	7.4 ± 0.9	$(4.7\pm1.2)\times10^{-6}$
¹² Be	15.1 ± 1.1	7.1 ± 1.1	$(2.7\pm0.7)\times10^{-6}$
^{12}B	21.8 ± 0.8	8.2 ± 1.8	$(1.5\pm0.4)\times10^{-6}$
¹³ B	20.1 ± 1.1	8.1 ± 0.9	$(2.4\pm0.6)\times10^{-6}$
^{14}B	17.0 ± 1.2	7.3 ± 0.7	$(1.4\pm0.4)\times10^{-7}$
¹⁵ B	16.8±1.9	7.0 ± 1.0	$(9.1\pm4.1)\times10^{-8}$
¹⁴ C	27.0 ± 0.3	9.9±0.5	$(1.3\pm0.2)\times10^{-5}$
¹⁵ C	25.1 ± 0.5	8.9 ± 0.7	$(5.3\pm1.1)\times10^{-6}$
¹⁶ C	24.4 ± 1.1	9.6±1.2	$(4.8 \pm 1.1) \times 10^{-6}$
¹⁷ C	21.3 ± 1.7	8.3 ± 0.9	$(7.5\pm2.8)\times10^{-7}$
¹⁸ C	20.4 ± 2.8	8.5 ± 1.4	$(2.4\pm0.7)\times10^{-7}$
¹⁶ N	25.9 ± 2.2	9.8 ± 1.7	$(1.5\pm0.4)\times10^{-7}$
¹⁷ N	25.0 ± 1.6	9.4 ± 1.2	$(8.1\pm2.0)\times10^{-7}$
¹⁸ N	23.8 ± 1.5	9.9±1.2	$(4.5\pm1.1)\times10^{-7}$
²⁰ N	fixed	7.0±0.9	1.3×10^{-8}
²¹ N	fixed	fixed	3.4×10^{-9}
²⁰ O	31.4±1.7	10.6 ± 1.9	$(2.5\pm0.7)\times10^{-6}$
²¹ O	24.2 ± 1.2	10.7 ± 0.7	$(6.4\pm1.3)\times10^{-7}$
²² O	33.0 ± 7.4	14.3 ± 4.2	$(4.2\pm1.6)\times10^{-7}$
²⁴ O	fixed	9.5±3.2	5.8×10^{-8}
²⁰ F	25.4±3.3	fixed	9.7×10^{-9}
²¹ F	26.5 ± 2.1	9.8 ± 1.3	$(1.6\pm0.4)\times10^{-7}$
²² F	33.8 ± 10.5	12.2 ± 4.6	$(1.4\pm0.8)\times10^{-7}$
²⁴ F	26.3 ± 2.8	12.1 ± 2.0	$(8.3\pm4.0)\times10^{-8}$
²⁴ Ne	33.9 ± 2.9	14.2 ± 1.9	$(2.4\pm0.6)\times10^{-7}$
²⁷ Ne	35.9±5.9	fixed	$2.0 imes 10^{-8}$
²⁸ Ne	fixed	fixed	1.8×10^{-8}
²⁷ Na	38.4 ± 8.2	16.3 ± 4.5	$(8.2\pm3.2)\times10^{-8}$
²⁸ Na	fixed	fixed	1.0×10^{-7}
³⁰ Na	31.7±8.6	11.9 ± 6.1	$(2.2\pm2.2)\times10^{-8}$
³⁰ Mg	34.9 ± 3.7	13.0 ± 1.8	$(1.3\pm0.4)\times10^{-7}$
³² Mg	fixed	10.8±2.7	3.7×10^{-8}
³⁴ Mg	fixed	fixed	1.0×10^{-9}
³⁰ A1	fixed	fixed	9.0×10^{-9}
³² A1	fixed	fixed	1.1×10^{-8}
³³ Al	fixed	fixed	1.8×10^{-8}
³² Si	fixed	12.0±1.7	8.9×10 ⁻⁹
³³ Si	fixed	11.3±1.4	1.5×10^{-8}
³⁴ Si	fixed	11.3±1.3	2.2×10^{-8}
³⁷ Si	fixed	fixed	2.0×10^{-9}
³⁹ P	fixed	fixed	<5.6×10 ⁻⁹
³⁷ S	fixed	fixed	4.7×10^{-9}
⁴⁰ S	fixed	fixed	<3.3×10 ⁻⁹

Focus: ²⁵²Cf

Competing mechanisms? Ideea supported by: I. Tsekhanovich *et. al.*, PRC67 034610 (2003), Yu.V. Pyatkov *et.al.*, Physics of Atomic Nuclei 86/4 (2023), ...



Interaction potentials favor the formation of the cluster composing of the heaviest nucleus with the third particle.

G.M. Ter-Akopian et.al., PRC55 1146 (1997): "(...) data clearly indicate that most of the neutrons in the high neutron emission events come out of the Ba fragments – not the Mo fragments."

Focus: ²⁵²Cf

Final excitation energy of the heavy fragment is: $\varepsilon^{**} = \varepsilon^* + \Delta V - Q$, where $\varepsilon^* = E_i^* + U^{def}$;

 ΔV - the reduction of the interaction potential due to the third fragment;

Q is the fusion Q-value.



	$^{148}\text{Ce} + ^{104}\text{Z}$	$\mathbf{r} = \epsilon_0^2$	$E_{Ce}^* = E_{Ce}^* + U_{Ce}^*$	$\frac{def}{Ce} = 24.1 \text{ MeV}$
	$\Delta V \; ({ m MeV})$	$Q ({ m MeV})$	ε_{H}^{**} (MeV)	TKE (MeV)
$(^{144}\text{Ba}+lpha)+^{104}\text{Zr}$	8.3	1.1	31.3	162.0
$(^{138}\text{Xe}+^{10}\text{Be})+^{104}\text{Ze}$	r 13.9	2.9	35.1	140.5
$(^{134}\text{Te}+^{14}\text{C})+^{104}\text{Zr}$	30.8	-9.2	64.0	150.3
$(^{128}Sn+^{20}O)+^{104}Zr$	39.6	-9.2	72.7	153.6
$(^{122}Cd+^{26}Ne)+^{104}Zr$	47.4	-9.9	81.4	144.5
$(^{124}Cd+^{24}Ne)+^{104}Zr$	45.8	-11.0	80.9	144.2

Large variations in ΔV , especially for heavy third nuclei.

 $^{146}Ba + {}^{106}Mo$

 $\epsilon^*_{Ba} = E^*_{Ba} + U^{def}_{Ba} = 28.1 \text{ MeV}$

 ΔV (MeV) Q (MeV) ε_{H}^{**} (MeV) TKE (MeV)

$(^{142}Xe+\alpha)+^{106}Mo$	12.7	2.0	38.8	166.6
$(^{136}\text{Te}+^{10}\text{Be})+^{106}\text{Me}$	13.7	3.2	38.6	144.4
$(^{132}Sn + ^{14}C) + ^{106}Mo$	30.5	-8.5	67.1	155.9
$(^{126}Cd+^{20}O)+^{106}Mo$	39.0	-3.5	70.6	160.6
$(^{120}Pd+^{26}Ne)+^{106}Mo$	45.0	-4.7	77.8	157.1
$(^{122}Pd+^{24}Ne)+^{106}Mo$	44.8	-5.6	78.5	158.0

Focus: ²⁵²Cf

Final excitation energy of the heavy fragment is: $\varepsilon^{**} = \varepsilon^* + \Delta V - Q$, where $\varepsilon^* = E_i^* + U^{def}$;

 ΔV - the reduction of the interaction potential due to the third fragment;

Q is the fusion Q-value.



	Large	variations	in ΔV ,	especially	for heavy	third nuclei.
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Large negative *Q*-values enhance ε^{**} , especially for heavy third nuclei.

	$\Delta V \; ({ m MeV})$	$Q \;({ m MeV})$	ε_{H}^{\sim} (MeV)	IKE (Mev)
$(^{144}\mathrm{Ba}{+}\alpha){+}^{104}\mathrm{Zr}$	8.3	1.1	31.3	162.0
$(^{138}Xe+^{10}Be)+^{104}Zr$	13.9	2.9	35.1	140.5
$(^{134}\text{Te}+^{14}\text{C})+^{104}\text{Zr}$	30.8	-9.2	64.0	150.3
$(^{128}Sn + ^{20}O) + ^{104}Zr$	39.6	-9.2	72.7	153.6
$(^{122}{\rm Cd}{+}^{26}{\rm Ne}){+}^{104}{\rm Zr}$	47.4	-9.9	81.4	144.5
$(^{124}Cd+^{24}Ne)+^{104}Zr$	45.8	-11.0	80.9	144.2
14	$^{6}\text{Ba} + {}^{106}\text{Me}$	0 ε	$_{Ba}^{*} = E_{Ba}^{*} + U$	$a_{Ba}^{def} = 28.1 \text{ MeV}$
14	$^{6}\mathrm{Ba}$ + $^{106}\mathrm{Me}$ $\Delta V (\mathrm{MeV})$	$ ho \qquad \epsilon Q \ ({ m MeV}) \ \delta Q \ ({ m MeV}) \ \delta Q \ \delta $	$\begin{aligned} & \stackrel{*}{}_{Ba} = E^{*}_{Ba} + U \\ & \stackrel{*}{}_{H} \varepsilon^{**}_{H} \text{ (MeV)} \end{aligned}$	$T_{Ba}^{def} = 28.1 \text{ MeV}$ TKE (MeV)
$(^{142}Xe+\alpha)+^{106}Mo$	$^{6}{ m Ba} + {}^{106}{ m Mo}$ $\Delta V \ ({ m MeV})$ 12.7	$ m o$ ϵ $Q \ (MeV) \ a$ 2.0	$\begin{aligned} *_{Ba} &= E_{Ba}^* + U\\ \varepsilon_H^{**} \text{ (MeV)}\\ 38.8 \end{aligned}$	$\frac{d^{def}_{Ba} = 28.1 \text{ MeV}}{\text{TKE (MeV)}}$ 166.6
$(^{142}Xe + \alpha) + ^{106}Mo$ $(^{136}Te + ^{10}Be) + ^{106}Mo$	${}^{6}\mathrm{Ba}$ + ${}^{106}\mathrm{MeV}$ $\Delta V (\mathrm{MeV})$ 12.7 13.7	$\begin{array}{c} \mathrm{o} & \epsilon \\ Q & (\mathrm{MeV}) & \epsilon \\ 2.0 \\ 3.2 \end{array}$	$s_{Ba}^{*} = E_{Ba}^{*} + U$ $\varepsilon_{H}^{**} (MeV)$ 38.8 38.6	$T_{Ba}^{def} = 28.1 \text{ MeV}$ TKE (MeV) 166.6 144.4
$(^{142}Xe+\alpha)+^{106}Mo$ $(^{136}Te+^{10}Be)+^{106}Me$ $(^{132}Sn+^{14}C)+^{106}Mo$	6 Ba + 106 Mo $\Delta V (MeV)$ 12.7 13.7 30.5	$\begin{array}{c} 0 & \epsilon \\ Q \text{ (MeV) } \\ \hline 2.0 \\ 3.2 \\ \hline -8.5 \end{array}$	$s_{Ba}^{*} = E_{Ba}^{*} + U$ $\varepsilon_{H}^{**} (MeV)$ 38.8 38.6 67.1	$T_{Ba}^{def} = 28.1 \text{ MeV}$ TKE (MeV) 166.6 144.4 155.9
$(^{142}Xe+\alpha)+^{106}Mo$ $(^{136}Te+^{10}Be)+^{106}Me$ $(^{132}Sn+^{14}C)+^{106}Mo$ $(^{126}Cd+^{20}O)+^{106}Mo$	6 Ba + 106 Mo $\Delta V (MeV)$ 12.7 13.7 30.5 39.0	$\begin{array}{c} 0 & \epsilon \\ Q & (MeV) \\ \hline 2.0 \\ 3.2 \\ \hline -8.5 \\ -3.5 \\ \end{array}$	$s_{Ba}^{*} = E_{Ba}^{*} + U$ $\varepsilon_{H}^{**} (MeV)$ 38.8 38.6 67.1 70.6	$T_{Ba}^{def} = 28.1 \text{ MeV}$ TKE (MeV) 166.6 144.4 155.9 160.6
$(^{142}Xe + \alpha) + ^{106}Mo$ $(^{136}Te + ^{10}Be) + ^{106}Mo$ $(^{132}Sn + ^{14}C) + ^{106}Mo$ $(^{126}Cd + ^{20}O) + ^{106}Mo$ $(^{120}Pd + ^{26}Ne) + ^{196}Mo$	${}^{6}\text{Ba} + {}^{106}\text{Me}$ $\Delta V \text{ (MeV)}$ 12.7 13.7 30.5 39.0 45.0	$ \begin{array}{c} 0 & \epsilon \\ Q (MeV) \\ 2.0 \\ 3.2 \\ -8.5 \\ -3.5 \\ -4.7 \\ \end{array} $	$s_{Ba}^{*} = E_{Ba}^{*} + U$ $\varepsilon_{H}^{**} (MeV)$ 38.8 38.6 67.1 70.6 77.8	$T_{Ba}^{def} = 28.1 \text{ MeV}$ TKE (MeV) 166.6 144.4 155.9 160.6 157.1

¹⁴⁸Ce + ¹⁰⁴Zr $\epsilon_{Ce}^* = E_{Ce}^* + U_{Ce}^{def} = 24.1 \text{ MeV}$

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<u>Focus: ^{252}Cf </u>

Final excitation energy of the heavy fragment is: $\varepsilon^{**} = \varepsilon^* + \Delta V - Q$,

where $\varepsilon^* = E_i^* + U^{def};$

 ΔV - the reduction of the interaction potential due to the third fragment;

Q is the fusion Q-value.



<u>Focus</u>: ^{252}Cf

Competing mechanisms?

The probability of emitting exactly ν neutrons, taking into account binary and tripartition contribution, can be written as:

 $P(\nu) = \lambda_d P_{\nu}(A_i, Z_i, \beta_i, \varepsilon^*) + \lambda_t P_{\nu}(A_i, Z_i, \beta_i, \varepsilon^{**})$ $P_{\nu}(A_i, Z_i, \beta_i, \varepsilon) = \sum_{\nu=0}^{\nu} \int_0^{\varepsilon - U_H^{\text{def}} - U_L^{\text{def}}} dEP_C(E) P_{\nu_L}(U_L^{\text{def}} + E) P_{\nu - \nu_L}(\varepsilon - U_L^{\text{def}} - E)$

The probability of emitting the third cluster is the product of the probabilities for formation of ternary system and three body decay.

- $\rightarrow \lambda_t$ can be estimated using the experimental values.
- \rightarrow our estimates: < 10⁻² compared to binary decay.

TABLE I. Mean kinetic energy \overline{E} , width of energy distribution σ_E and absolute yield of ternary particles.
Values given in <i>italics</i> are those from an "enforced" fit, as explained in the text; they should be considered
as preliminary.

	\overline{E} (Mev)	σ_E (MeV)	Yield
⁸ Li	15.1 ± 1.4	7.1±1.3	$(2.6\pm0.7)\times10^{-6}$
⁹ Li	12.5 ± 0.9	5.5 ± 1.0	$(3.8\pm1.0)\times10^{-6}$
¹⁰ Be	17.5 ± 0.4	7.7±0.6	$(3.8\pm0.7)\times10^{-5}$
¹¹ Be	16.5 ± 1.3	7.4 ± 0.9	$(4.7\pm1.2)\times10^{-6}$
¹² Be	15.1 ± 1.1	7.1 ± 1.1	$(2.7\pm0.7)\times10^{-6}$
${}^{12}B$	21.8 ± 0.8	8.2 ± 1.8	$(1.5\pm0.4)\times10^{-6}$
^{13}B	20.1 ± 1.1	8.1 ± 0.9	$(2.4\pm0.6)\times10^{-6}$
$^{14}\mathbf{B}$	17.0 ± 1.2	7.3 ± 0.7	$(1.4\pm0.4)\times10^{-7}$
¹⁵ B	16.8 ± 1.9	7.0 ± 1.0	$(9.1\pm4.1)\times10^{-8}$
¹⁴ C	27.0 ± 0.3	9.9±0.5	$(1.3\pm0.2)\times10^{-5}$
¹⁵ C	25.1 ± 0.5	8.9±0.7	$(5.3\pm1.1)\times10^{-6}$
¹⁶ C	24.4 ± 1.1	9.6±1.2	$(4.8 \pm 1.1) \times 10^{-6}$
¹⁷ C	21.3 ± 1.7	8.3 ± 0.9	$(7.5\pm2.8)\times10^{-7}$
¹⁸ C	20.4 ± 2.8	8.5 ± 1.4	$(2.4\pm0.7)\times10^{-7}$
¹⁶ N	25.9 ± 2.2	9.8±1.7	$(1.5\pm0.4)\times10^{-7}$
¹⁷ N	25.0 ± 1.6	9.4 ± 1.2	$(8.1\pm2.0)\times10^{-7}$
¹⁸ N	23.8 ± 1.5	9.9 ± 1.2	$(4.5\pm1.1)\times10^{-7}$
²⁰ N	fixed	7.0±0.9	1.3×10^{-8}
²¹ N	fixed	fixed	3.4×10^{-9}
²⁰ O	31.4±1.7	10.6 ± 1.9	$(2.5\pm0.7)\times10^{-6}$
²¹ O	24.2 ± 1.2	10.7±0.7	$(6.4\pm1.3)\times10^{-7}$
²² O	33.0 ± 7.4	14.3 ± 4.2	$(4.2\pm1.6)\times10^{-7}$
²⁴ O	fixed	9.5±3.2	5.8×10^{-8}
²⁰ F	25.4±3.3	fixed	9.7×10^{-9}
²¹ F	26.5 ± 2.1	9.8 ± 1.3	$(1.6\pm0.4)\times10^{-7}$
²² F	33.8 ± 10.5	12.2 ± 4.6	$(1.4\pm0.8)\times10^{-7}$
²⁴ F	26.3 ± 2.8	12.1 ± 2.0	$(8.3\pm4.0)\times10^{-8}$
²⁴ Ne	33.9 ± 2.9	14.2 ± 1.9	$(2.4\pm0.6)\times10^{-7}$
²⁷ Ne	35.9±5.9	fixed	$2.0 imes 10^{-8}$
²⁸ Ne	fixed	fixed	1.8×10^{-8}
²⁷ Na	38.4±8.2	16.3 ± 4.5	$(8.2\pm3.2)\times10^{-8}$
²⁸ Na	fixed	fixed	1.0×10^{-7}
³⁰ Na	31.7 ± 8.6	11.9 ± 6.1	$(2.2\pm2.2)\times10^{-8}$
³⁰ Mg	34.9 ± 3.7	13.0 ± 1.8	$(1.3\pm0.4)\times10^{-7}$
³² Mg	fixed	10.8±2.7	3.7×10^{-8}
³⁴ Mg	fixed	fixed	1.0×10^{-9}
³⁰ Al	fixed	fixed	9.0×10^{-9}
³² Al	fixed	fixed	1.1×10^{-8}
³³ Al	fixed	fixed	1.8×10^{-8}
³² Si	fixed	12.0±1.7	8.9×10 ⁻⁹
³³ Si	fixed	11.3±1.4	1.5×10^{-8}
³⁴ Si	fixed	11.3±1.3	2.2×10^{-8}
³⁷ Si	fixed	fixed	2.0×10^{-9}
³⁹ P	fixed	fixed	$< 5.6 \times 10^{-9}$
³⁷ S	fixed	fixed	4.7×10^{-9}
⁴⁰ S	fixed	fixed	<3.3×10 ⁻⁹

Focus: ²⁵²Cf

Competing mechanisms? - Neutron multiplicity



Focus: ²⁵²*Cf*

Competing mechanisms? - Neutron multiplicity



144.5

144.2

1	148 Ce + 104 Zr		$\epsilon_{Ce}^* = E_{Ce}^* + U_{Ce}^{def} = 24.1 \text{ MeV}$		
	ΔV (MeV) (Q (MeV)) ε_{H}^{**} (MeV)	TKE (MeV)	
$(^{144}\mathrm{Ba}{+}\alpha){+}^{104}\mathrm{Zr}$	8.3	1.1	31.3	162.0	
$(^{138}\text{Xe}+^{10}\text{Be})+^{104}\text{Zr}$	13.9	2.9	35.1	140.5	
$(^{134}\text{Te}+^{14}\text{C})+^{104}\text{Zr}$	30.8	-9.2	64.0	150.3	
$(^{128}Sn + ^{20}O) + ^{104}Zr$	39.6	-9.2	72.7	153.6	

-9.9

-11.0

81.4

80.9

47.4

45.8

 $(^{122}Cd+^{26}Ne)+^{104}Zr$

 $(^{124}Cd+^{24}Ne)+^{104}Zr$

 $^{146}\text{Ba} + ^{106}\text{Mo}$

 $\epsilon^*_{Ba} = E^*_{Ba} + U^{def}_{Ba} = 28.1 \text{ MeV}$

ΔV (MeV) Q (MeV) ε_{H}^{**} (MeV) TKE (MeV)

$(^{142}Xe + \alpha) + ^{106}Mo$	12.7	2.0	38.8	166.6
$(^{136}\text{Te}+^{10}\text{Be})+^{106}\text{Mo}$	13.7	3.2	38.6	144.4
$(^{132}Sn + ^{14}C) + ^{106}Mo$	30.5	-8.5	67.1	155.9
$(^{126}Cd+^{20}O)+^{106}Mo$	39.0	-3.5	70.6	160.6
$(^{120}\text{Pd}+^{26}\text{Ne})+^{106}\text{Mo}$	45.0	-4.7	77.8	157.1
$(^{122}Pd+^{24}Ne)+^{106}Mo$	44.8	-5.6	78.5	158.0

<u>Focus: ^{252}Cf </u>

Competing mechanisms? - Neutron multiplicity



<u>Focus</u>: ^{252}Cf

Competing mechanisms? - Neutron multiplicity



Focus: ²⁵²*Cf*

Competing mechanisms? - Neutron multiplicity



1	148 Ce + 104 Zr		$\epsilon_{Ce}^* = E_{Ce}^* + U_{Ce}^{def} = 24.1 \text{ MeV}$		
	$\Delta V ~({ m MeV})$	$Q \; ({ m MeV})$	ε_{H}^{**} (MeV)	TKE (MeV)	
$(^{144}\mathrm{Ba}{+}lpha){+}^{104}\mathrm{Zr}$	8.3	1.1	31.3	162.0	
$(^{138}Xe + ^{10}Be) + ^{104}Zr$	13.9	2.9	35.1	140.5	
$(^{134}\text{Te}+^{14}\text{C})+^{104}\text{Zr}$	30.8	-9.2	64.0	150.3	
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144.2

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 $\epsilon_{Ba}^* = E_{Ba}^* + U_{Ba}^{def} = 28.1 \text{ MeV}$

ΔV (MeV) Q (MeV) ε_{H}^{**} (MeV) TKE (MeV)

$(^{142}Xe + \alpha) + ^{106}Mo$	12.7	2.0	38.8	166.6
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Focus: ²⁵²Cf

Competing mechanisms? - Neutron multiplicity



 $(^{142}Xe + \alpha) + ^{106}Mo$

 $(^{136}\text{Te}+^{10}\text{Be})+^{106}\text{Mo}$

 $(^{132}Sn + {}^{14}C) + {}^{106}Mo$

 $(^{126}Cd+^{20}O)+^{106}Mo$

(¹²⁰Pd+²⁶Ne)+¹⁰⁶Mo

(¹²²Pd+²⁴Ne)+¹⁰⁶Mo

12.7

13.7

30.5

39.0

45.0

44.8

2.0

3.2

-8.5

-3.5

-4.7

-5.6

38.8

38.6

67.1

70.6

77.8

78.5

166.6

144.4

155.9

160.6

157.1

158.0

	(MeV)	Q	(MeV)	ε_{H}^{**}	(MeV)	TKE	(MeV)	
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Focus: ²⁵²Cf

Competing mechanisms? - Neutron multiplicity

¹⁴⁸Ce + ¹⁰⁴Zr $\epsilon_{Ce}^* = E_{Ce}^* + U_{Ce}^{def} = 24.1 \text{ MeV}$

 ΔV (MeV) Q (MeV) ε_{H}^{**} (MeV) TKE (MeV)



Other observables in the 2nd mode for Ba-Mo:

Total kinetic energy:

Neutron multiplicity:

Excitation energy: G.M. Ter-Akopian et.al., PRL77, 32 (1996)

 $< TKE >_{exp} = 153 \pm 3 \text{ MeV}$ $< TKE >_{th} = 155,9 \text{ MeV}$

$$\left(\frac{E^*_{Ba,mode1}}{E^*_{Ba,mode2}}\right)_{th} = 2.4$$

 $\left(E_{Ba,mode2}^{*} \right)_{exp}$

$(^{144}\mathrm{Ba}+lpha)+^{104}\mathrm{Zr}$	8.3	1.1	31.3	162.0
$(^{138}\text{Xe}+^{10}\text{Be})+^{104}\text{Zr}$	13.9	2.9	35.1	140.5
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 $^{146}\text{Ba} + {}^{106}\text{Mo}$

 $\epsilon_{Ba}^* = E_{Ba}^* + U_{Ba}^{aej} = 28.1 \text{ MeV}$

ΔV (MeV) Q (MeV) ε_{H}^{**} (MeV) TKE (MeV)

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<u>Focus: ^{252}Cf </u>

Other fragmentations - is the second mode present?

Nd+Sr fragmentation:

 \rightarrow Binary decay contribution much narrower than experimental data;

 \rightarrow PES minimum is much narrower than previous cases.

 \rightarrow example $P(\nu = 7)$ is at least two orders of magnitude less than previous cases.

- \rightarrow Most probable cluster is $Nd \rightarrow Xe + C$.
- $\rightarrow 2^{nd}$ mode overlaps significantly with 1st mode.

 $\rightarrow O, Ne$ clusters offer just a "tail" due to low probabilities.

Configuration	ΔV	Q	ϵ^{**}
$(^{150}Ce + \alpha) + ^{98}Sr$	9,7	4,12	29,4
$(^{144}Ba + {}^{10}Be) + {}^{98}Sr$	14,7	6,53	31,9
$(^{140}Xe + {}^{14}C) + {}^{98}Sr$	28,75	-4,3	50,09
$(^{134}Te + ^{20}O) + ^{98}Sr$	41,8	-13,1	78,6
$(^{128}Sn + ^{26}Ne) + ^{98}Sr$	56,04	-17,2	97





Conclusions

- The mass, charge, TKE and neutron distributions can be calculated within the same model.
- Clusterisation of the heavy fragment can lead to a highly excited fission fragment.
- Competing mechanisms are possible, and they provide key signatures in $\langle n \rangle$ and *TKE* values.
- Evidence for cluster effects for can be "hidden" within existing experimental data.
- Question : Is this mechanism more common than previously thought ?

Thank you!

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