Level density parameters and fission probability along fission path

1. Introduction
2. Survival probability
3. Potential energy surface, fission path, fission barrier/threshold
4. Level density
5. Energy dependence
6. Summary

A. Rahmatinejad, T. M. Shneidman, G. G. Adamian, N. V. Antonenko, P. Jachimowicz, M. Kowal

- Most of superheavy nuclei have been produced in the $1n$ (cold)- $4n$ (hot) evaporation channels, with excitation energies of the compound nucleus in the $E_{CN} = 10\text{–}40$ MeV.

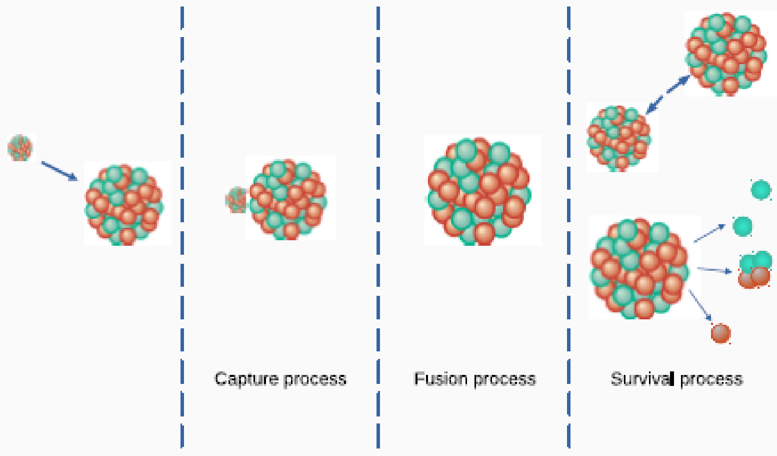
- One of the possibilities for expanding the table of elements is producing new isotopes of superheavy nuclei at larger excitation energies ($E_{CN} \geq 50$) MeV.

- The survival probability.

- Fission barrier. Fission paths. Fission threshold.

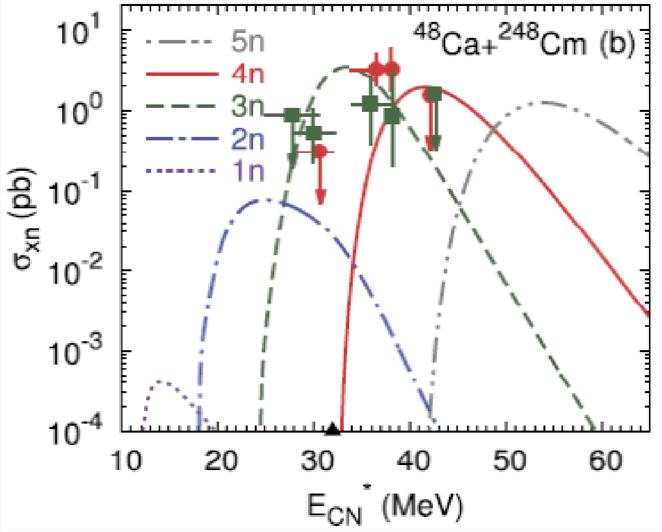
Survival probability: Competition between particle emission and fission

Juhee Hong, G.G. Adamian, N.V. Antonenko, P.Jachimowicz, M.Kowal, Phys. Lett. B 809, 135760 (2020).



$$\underbrace{\sigma}_{\sim 10^{-12}} \propto \underbrace{\sigma_{capture}}_{\sim 10^{-1}} \underbrace{P_{fusion}}_{\sim 10^{-2}} \underbrace{W_{survival}}_{\sim 10^{-9}}$$

$^{48}Ca + ^{248}Cm \rightarrow ^{296-xn}Lv + xn$



$$W_s(U) = P(s, U) \prod_{i_s=1}^{x_s} \frac{\Gamma_{i_s}(U_{i_s})}{\Gamma_t(U_{i_s})}$$

Challenges

1 Potential energy surface
- Fission paths and saddle points
- Ground-state and saddle point masses

2 Nuclear level density model
- Pairing and shell-correction
- Energy dependent shell and pairing effects based on microscopic calculations in the phenomenological analysis of various channels.

3 Damping of potential energy surface
- Consistent calculation of damping of potential energy and energy dependent thermodynamic quantities
- Variation of fission path and saddle points with excitation energy

METHOD

PES: take into account all the collective variables that we know play an important role in the process

⇩

NLD: determine the density of states at each point of a multidimensional network in a fundamental way so that the available phase space determines the transition

⇩

ME; take into account the stochastical nature (Master Equation) of the process and give the system as much freedom as possible to choose the path

⇩

RATES

^{232}Th

Microscopic-macroscopic method

$$E_{tot}(\beta_{\lambda\mu}) = E_{macro}(\beta_{\lambda\mu}) + E_{micro}(\beta_{\lambda\mu})$$

$$E_{macro}(\beta_{\lambda\mu}) = \text{Yukawa} + \text{exponential}$$

$$E_{micro}(\beta_{\lambda\mu}) = \text{Woods - Saxon} + \text{pairing BCS}$$

$$R(\theta,\phi)=cR_0\{1 + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{+\lambda} \beta_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)\}$$

Z. Muntian, J. Patyk, and A. Sobiczewski, Acta Phys. Pol. B **32**, 691 (2001).
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S. Ćwiok, J. Dudek, W. Nazarewicz, J. Skalski, and T. Werner, Comput. Phys. Commun. **46**, 379 (1987).
P. Jachimowicz, M. Kowal, and J. Skalski, At. Data Nucl. Data Tables **138**, 101393 (2021).
P. Jachimowicz, M. Kowal, and J. Skalski, Phys. Rev. C **101**, 014311 (2020).
P. Jachimowicz, M. Kowal, and J. Skalski, Phys. Rev. C **95**, 014303 (2017).

Microscopic–macroscopic method

$$E_{tot}(Z,N,\beta) = E_{mac}(Z,N,\beta) + E_{mic}(Z,N,\beta)$$

$$R(\vartheta,\varphi) = cR_0\{1+\beta_{20}Y_{20}+\frac{\beta_{22}}{\sqrt{2}}[Y_{22}+Y_{2-2}]+\beta_{40}Y_{40}+\beta_{60}Y_{60}+\beta_{80}Y_{80}\}$$

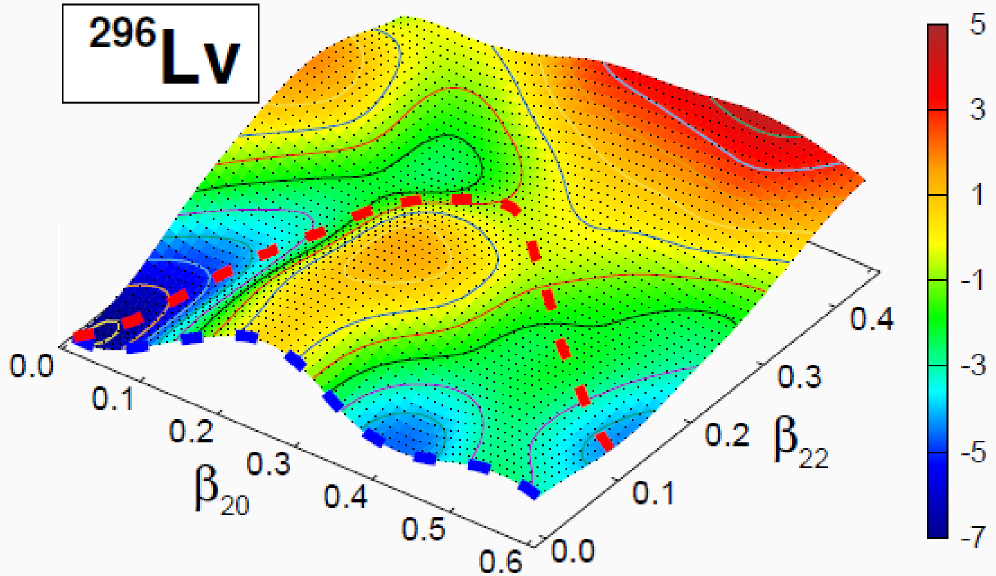
$\beta_{20} = 0.00\ (0.05)\ 0.60$

$\beta_{22} = 0.00\ (0.05)\ 0.45$

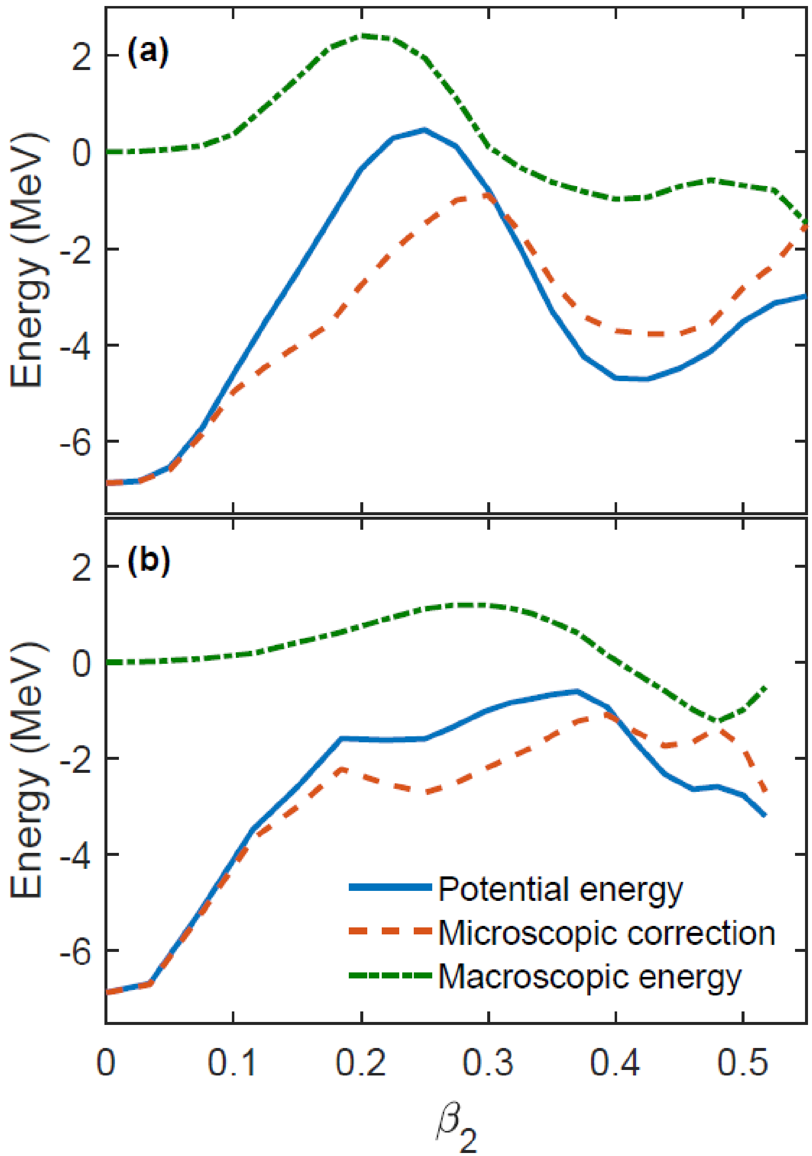
$\beta_{40} = -0.20\ (0.05)\ 0.20$

$\beta_{60} = -0.10\ (0.05)\ 0.10$

$\beta_{80} = -0.10\ (0.05)\ 0.10$



M. Kowal, P. Jachimowicz, A. Sobiczewski, Phys. Rev. C **82**, 014303 (2010); P. Jachimowicz, M. Kowal, J. Skalski, Phys. Rev. C **89**, 024304 (2014); Phys. Rev. C 95, 034329 (2017).



The variation of potential energy, liquid-drop energy, and microscopic correction at zero excitation energy is depicted as a function of the deformation parameter β_2 for axial (a) and triaxial (b) paths of fission of ^{296}Lv.

EPJA **60**, 214 (2024)

Thermodynamic quantities

$$E_{N(Z)}(T) = 2\sum_k \varepsilon_k n_{\Delta,k}^{N(Z)} - \frac{\Delta_{N(Z)}^2}{G_{N(Z)}}$$

$$U_{N(Z)}(T) = E_{N(Z)}(T) - E_{N(Z)}(0)$$

$$S_{N(Z)}(T) = \sum_k \left\{ \ln\left[1 + \exp\left(-\frac{E_k^{N(Z)}}{T}\right)\right] + \frac{E_k^{N(Z)}}{T\left[1 + \exp\left(\frac{E_k^{N(Z)}}{T}\right)\right]} \right\}$$

$$\rho = \frac{\exp(S)}{(2\pi)^{\frac{3}{2}}\sqrt{D}}$$

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{\frac{1}{4}}(U-\Delta)^{\frac{5}{4}}} \exp(2\sqrt{a(U-\Delta)})$$

P. Decowski, W. Grochulski, A. Marcinkowski, K. Siwek,and Z. Wilhelmi, Nucl. Phys. A 110, 129 (1968), G.D. Adeev and P.A. Cherdantsev, Yad. Fiz. 21, 491(1975)

BCS equations

Neutrons:

$$N = 2\sum_k n^N_{\Delta,k}(T)$$

$$\frac{2}{G_N} = \sum_k \frac{1}{E_k^N} \tanh \frac{E_k^N}{2T}$$

$$n^{N(Z)}_{\Delta,k}(T) = \frac{1}{2}\left(1 - \frac{\varepsilon_k^{N(Z)} - \lambda_N}{E_k^{N(Z)}} \tanh \frac{E_k^{N(Z)}}{2T}\right)$$

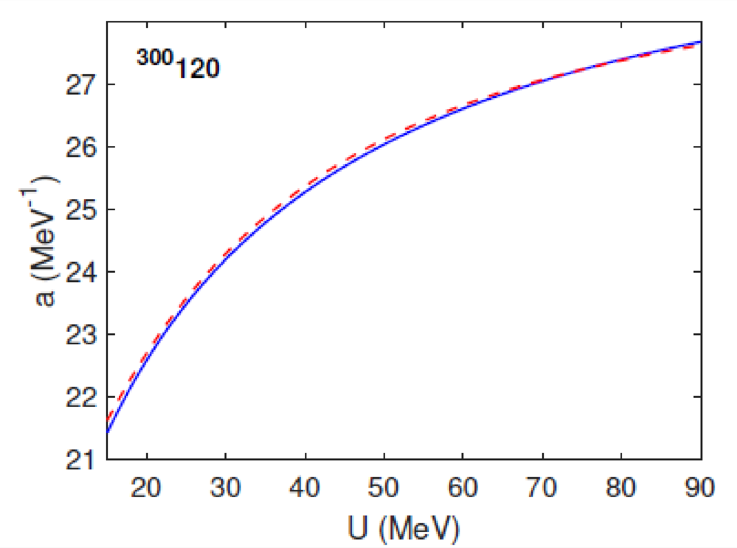
Protons:

$$Z = 2\sum_k n^Z_{\Delta,k}(T)$$

$$E_k^{N(Z)} = \sqrt{(\varepsilon_k^{N(Z)} - \lambda_{N(Z)})^2 + \Delta^2_{N(Z)}}$$

$$\frac{2}{G_Z} = \sum_k \frac{1}{E_k^Z} \tanh \frac{E_k^Z}{2T}$$

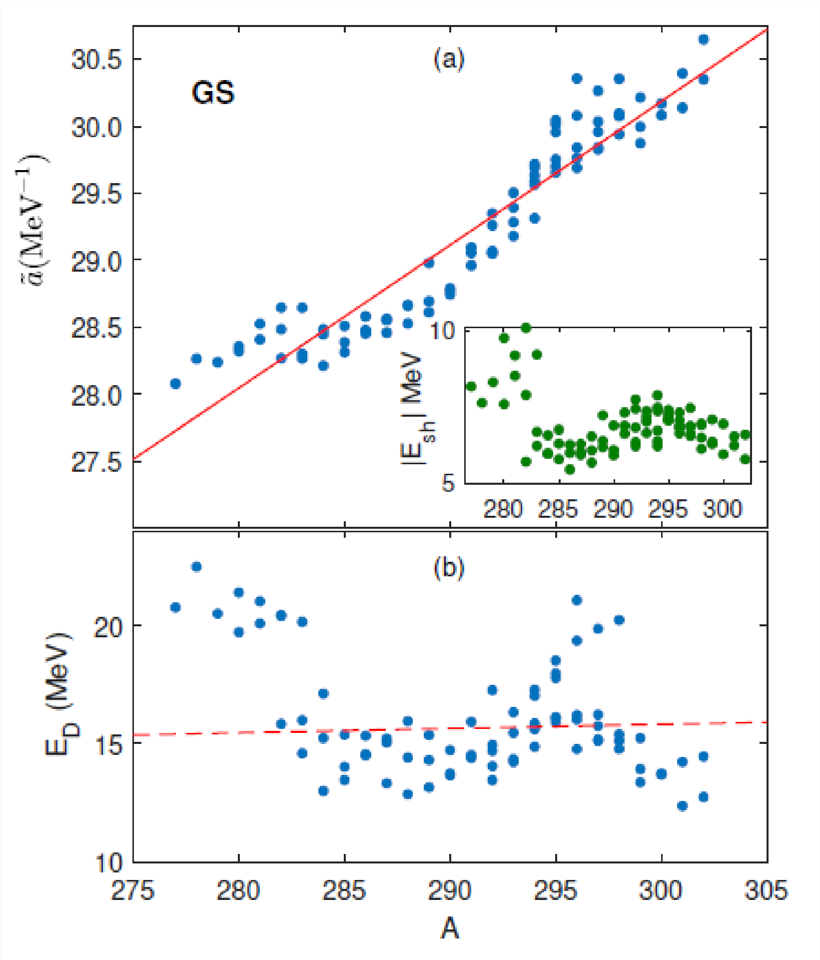
GS Level-density parameter



$$a(A,U) = \tilde{a}(A)\left[1 + \frac{1-\exp(-\frac{U}{E_D})}{U}\delta E_{sh}\right]$$

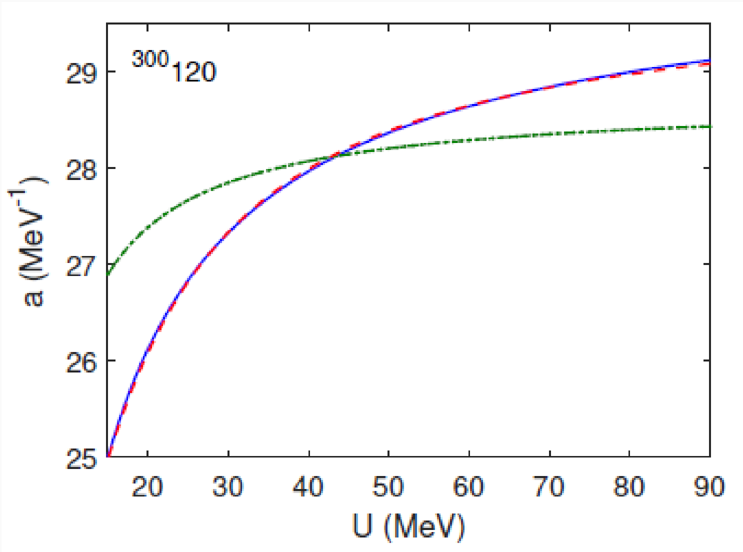
$$\tilde{a} = a_1 A + a_2 A^2$$

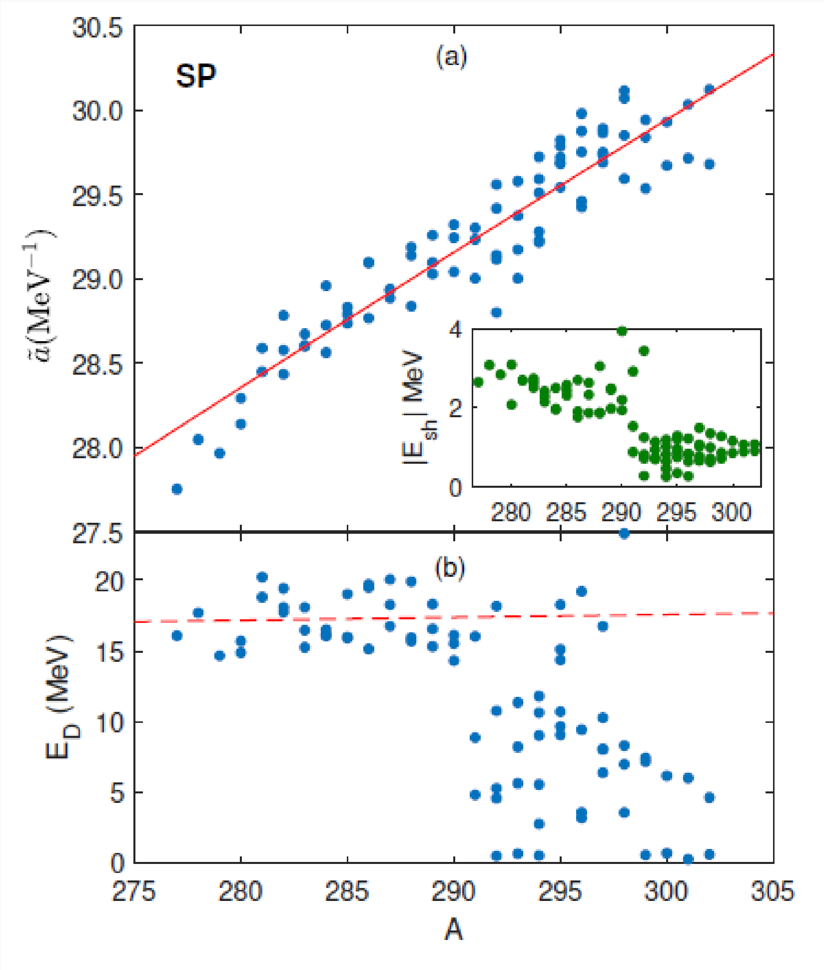
$a_1 = 0.09$ MeV^{-1},
$a_2 = 2.89 \times 10^{-4}$ MeV^{-1},
$E_D \approx 15$ MeV.



A. Rahmatinejad, et. al, Phys. Rev. C **105** 044328 (2022)/Phys. Rev. C **103**, 034309, (2021).

SP Level-density parameter





$$|\delta E_{sh}| \leq 1.7\text{MeV}$$
$$\delta E_{sh} \rightarrow (\delta E_{sh} - \Delta)$$

$$\tilde{a} = a_1 A + a_2 A^2$$

$a_1 = 0.1217 \text{ MeV}^{-1}$,
$a_2 = -7.3 \times 10^{-5} \text{ MeV}^{-1}$,
$E_D \approx 17$ MeV.

A. Rahmatinejad, et. al, Phys. Rev. C **105** 044328 (2022)/Phys. Rev. C **103**, 034309, (2021).

Survival probability calculation

$$W_s(U) = P(s,U)\prod_{i_s=1}^{x_s}\frac{\Gamma_{i_s}(U_{i_s})}{\Gamma_t(U_{i_s})}$$

Jackson analytic formula

$$P(xn,U) = I(\Delta_x, 2x-3) - I(\Delta_{x+1}, 2x-1),$$
$$I(\Delta_x, 2x-3) = 1 - \exp[-\Delta_x]\left[\sum_{i=0}^{2x-3}\frac{(\Delta_x)^i}{i!}\right],$$

$$\Delta_x = (U - B_{xn})/T,\ B_{xn} = \sum_{i=1}^{x} B_i,$$

T is the effective temperature, and B_i is the neutron binding energy

xn $T = T_{CN}/\sqrt{2}$

$$E_i = U - \sum_{j=1}^{i-1}(B_j + \varepsilon_j)$$

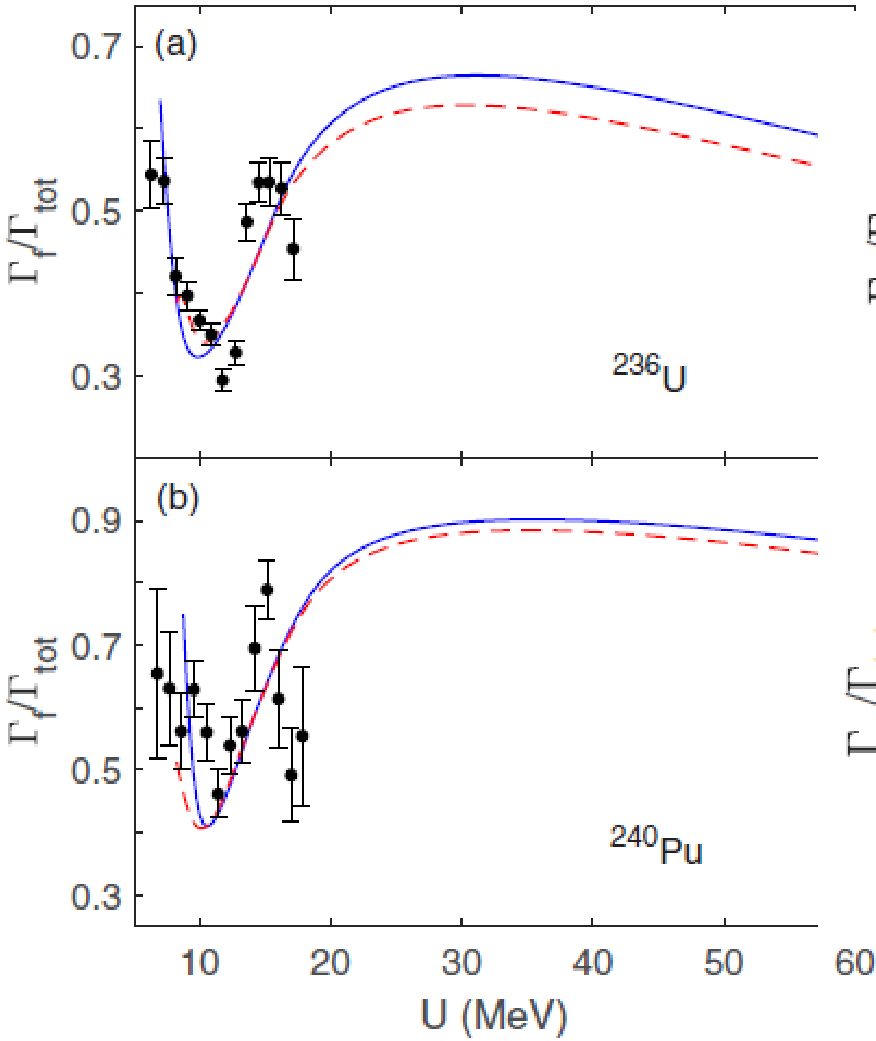
pxn, *αxn* $T = T_{CN}/\sqrt{2.25}$

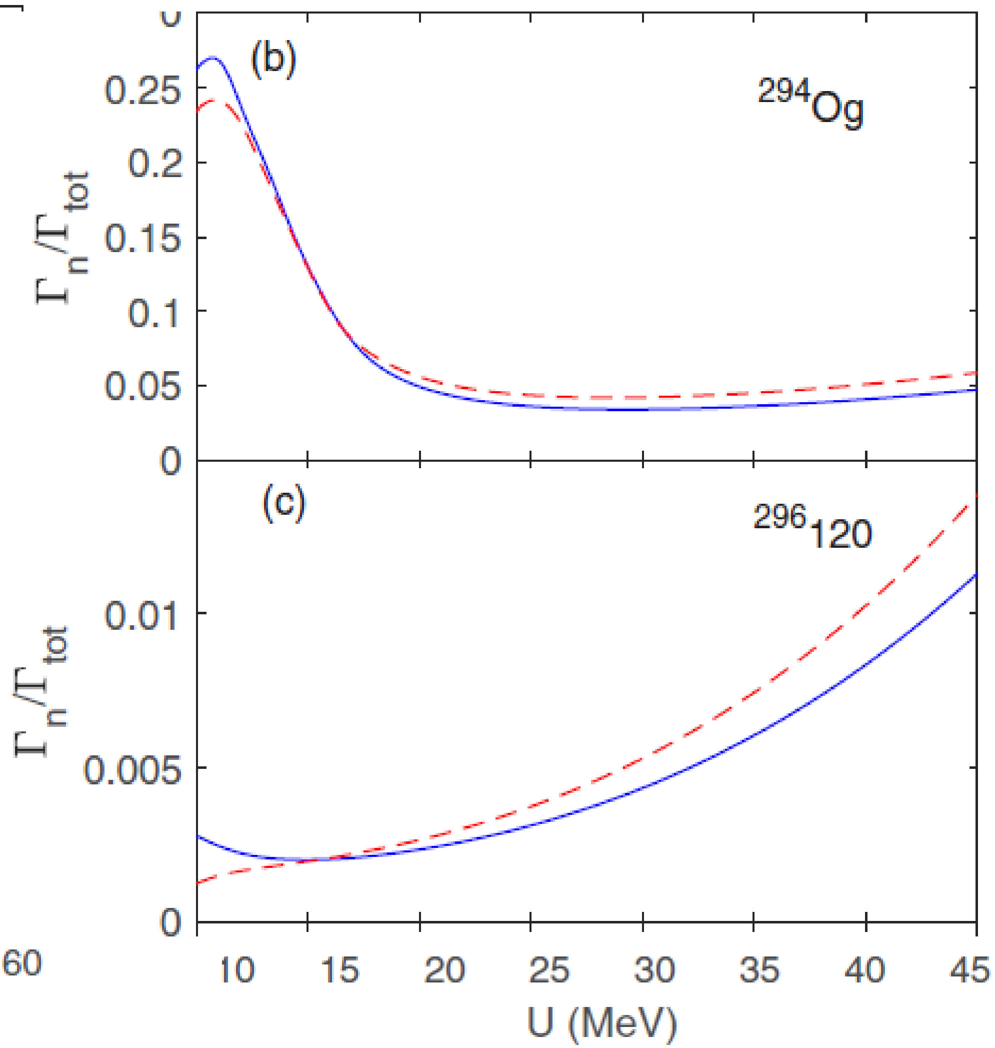
$$\bar{\varepsilon}_i(xn,U) = 2T_i\left(1 - e^{-\frac{U-B_{xn}}{x}}\right)$$

the average kinetic energy in the *xn*-evaporation channel

$$\frac{\Gamma_n}{\Gamma_f} = \frac{4A^{2/3}a_f(U - B_n - \Delta_n)}{K_0 a_n[2a_f^{1/2}(U - B_f - \Delta_f)^{1/2} - 1]} \times \exp\left[2a_n^{1/2}(U - B_n - \Delta_n)^{1/2} - 2a_f^{1/2}(U - B_f - \Delta_f)^{1/2}\right],$$

$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3}}{K_0} \frac{\int_0^{U-B_n} \varepsilon \rho_{GS}(U - B_n - \varepsilon) d\varepsilon}{\int_0^{U-B_f} \rho_{SP}(U - B_f - \varepsilon) d\varepsilon},$$





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Possibilities of direct production of superheavy nuclei with Z=112–118 in different evaporation channels

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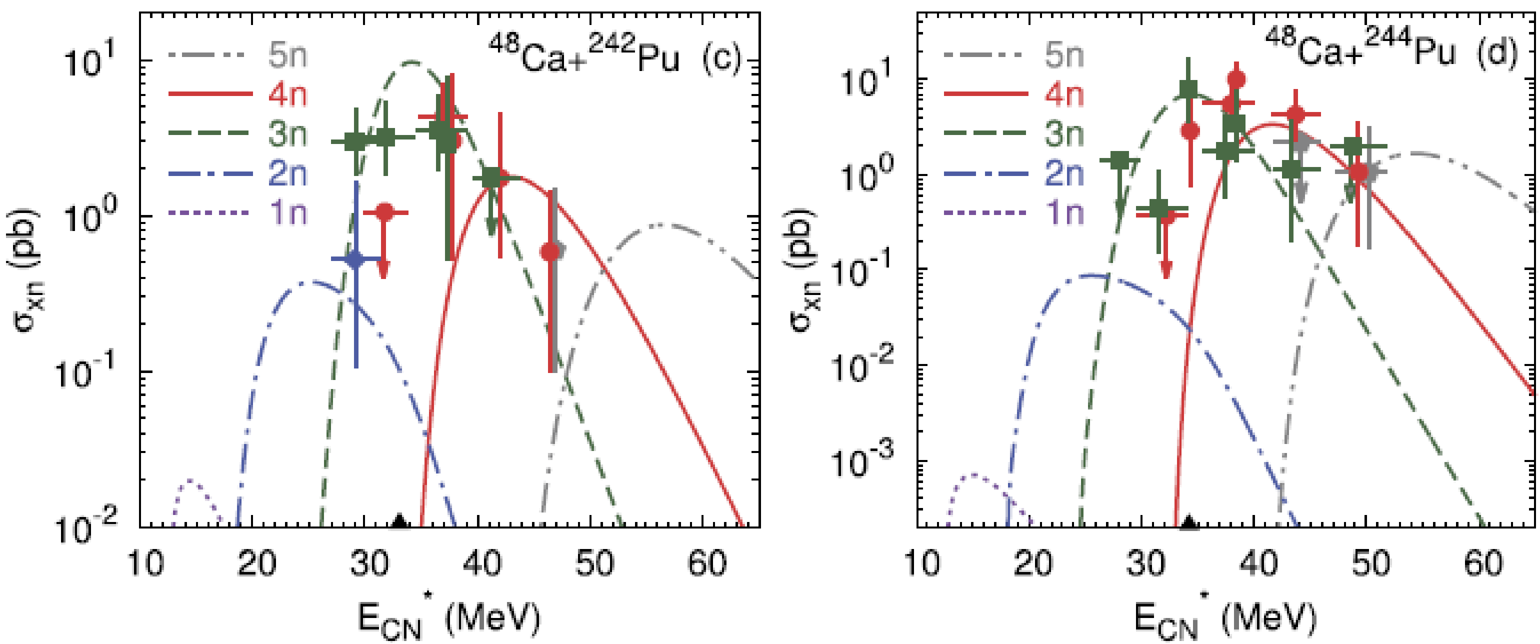


Shell correction

$$\delta E \longrightarrow \delta F = E(T) - \tilde{E}(T) - T\left[S(T) - \tilde{S}(T)\right]$$

$$E(T) = 2\int_{-\infty}^{\infty} n(T)\varepsilon g(\varepsilon)d\varepsilon$$

$$S(T) = -\int_{-\infty}^{+\infty} [(1 - n(T))\log(1 - n(T)) + n(T)\log n(T)]g(\varepsilon)d\varepsilon$$

Discrete levels

$$g(\varepsilon) = \sum_k \delta(\varepsilon - \varepsilon_k)$$

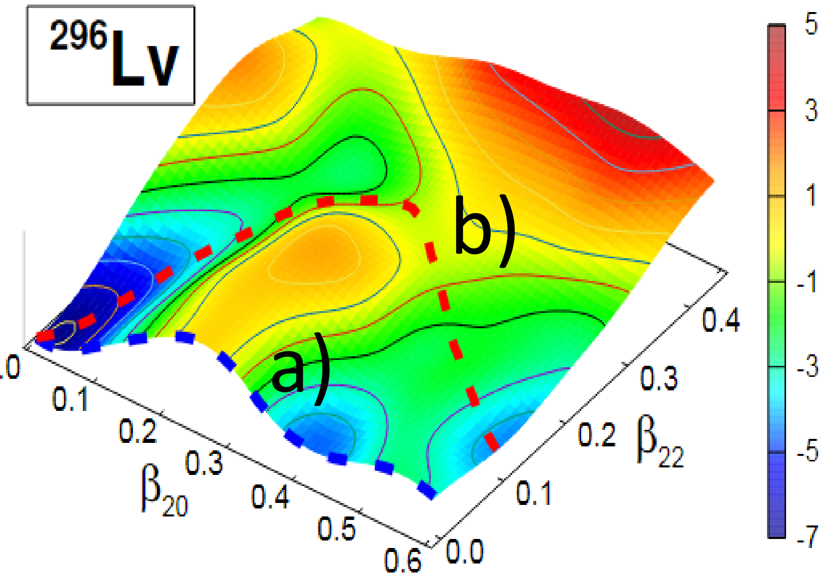
$$n_k(T) = 1/(1 + e^{(\varepsilon_k - \lambda)/T})$$

Smooth spectrum

$$\tilde{g}(\varepsilon) = \frac{1}{\gamma\sqrt{\pi}} \sum_{k=1} e^{-x^2} \sum_{n=0,2,\ldots}^{6} c_n H_n(x),$$

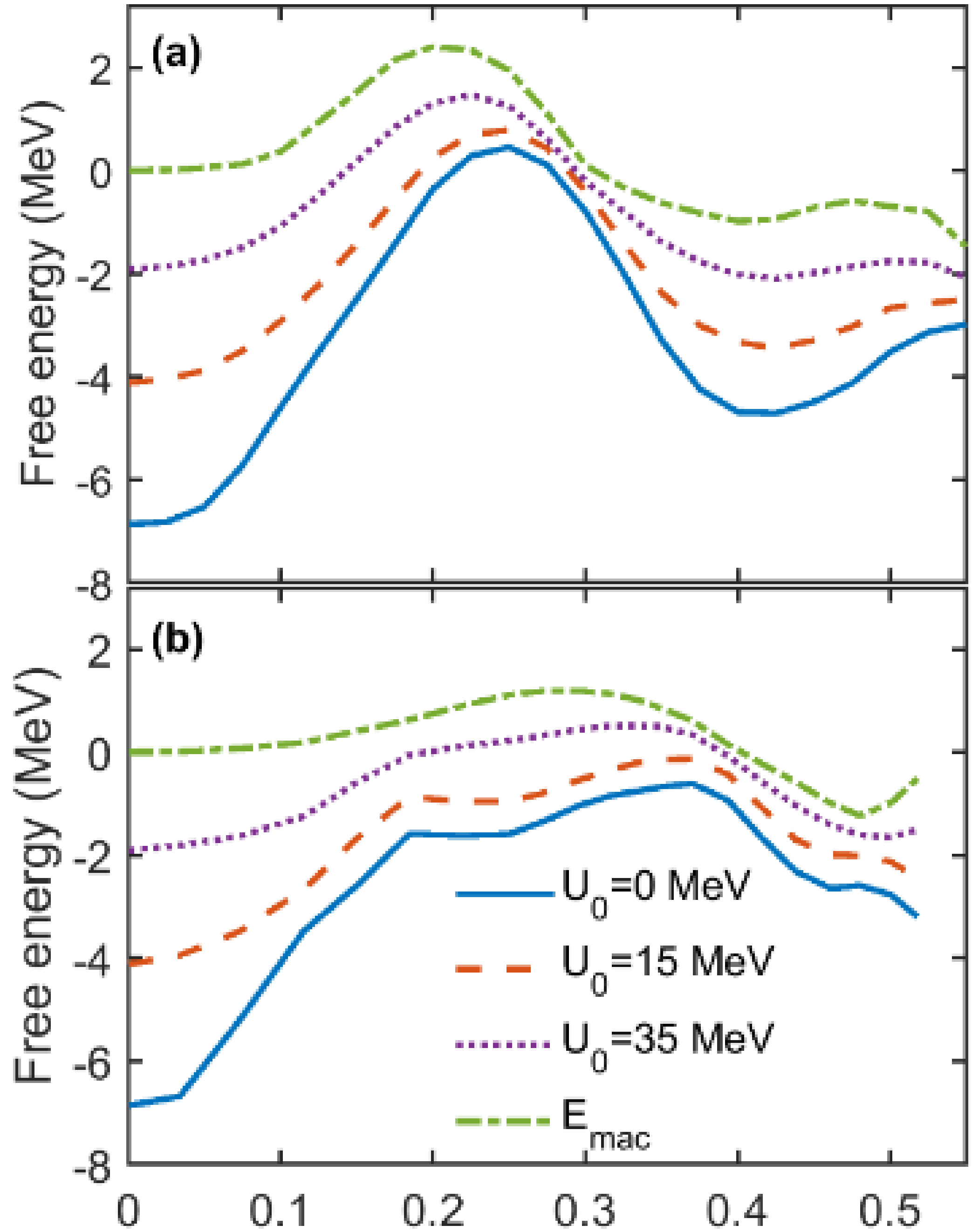
$$n_\varepsilon(T) = 1/(1 + e^{(\varepsilon - \tilde{\lambda})/T})$$

$$N(Z) = 2\sum_k n_k(T) = 2\int_{-\infty}^{+\infty} n_\varepsilon(T)\tilde{g}(\varepsilon)d\varepsilon$$

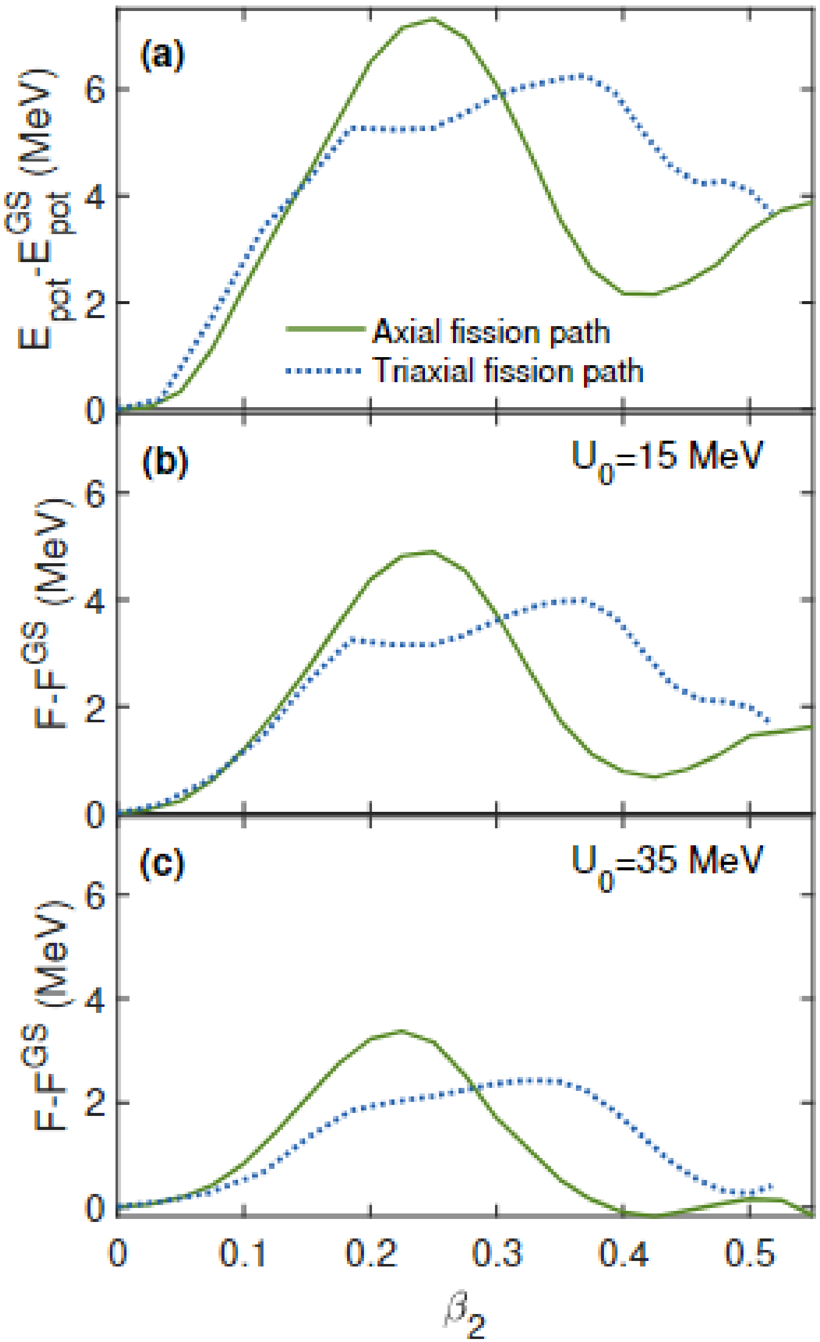


a) Axial

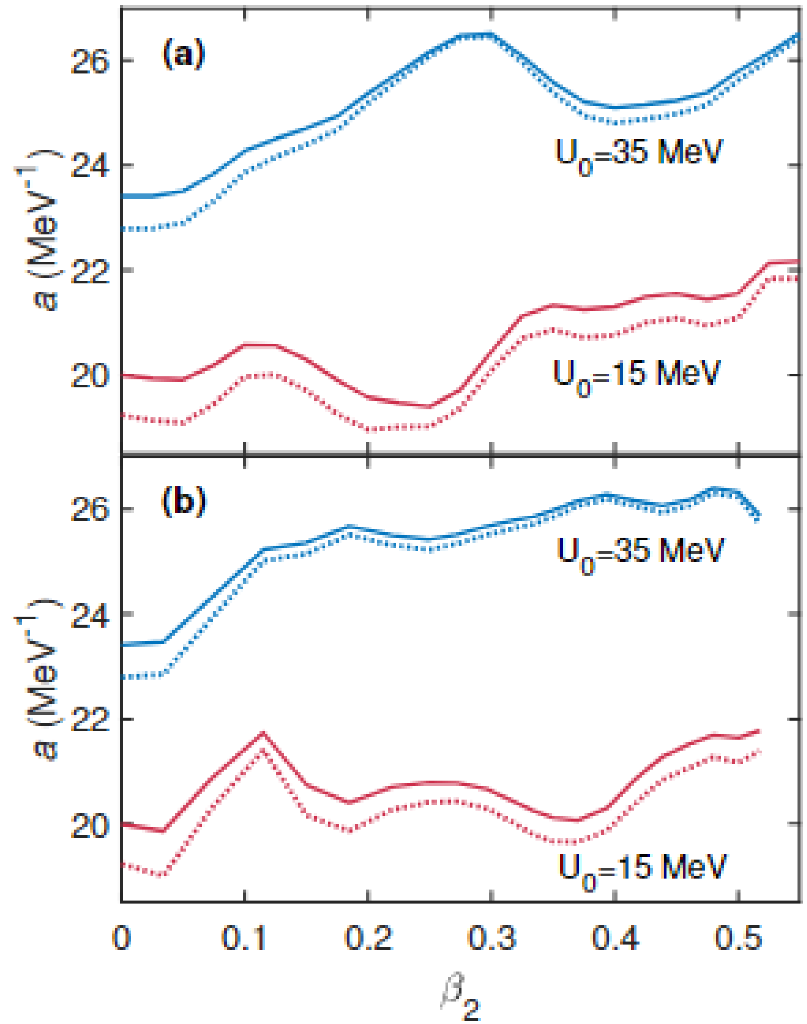
b) Traxial



a (U, def)=?

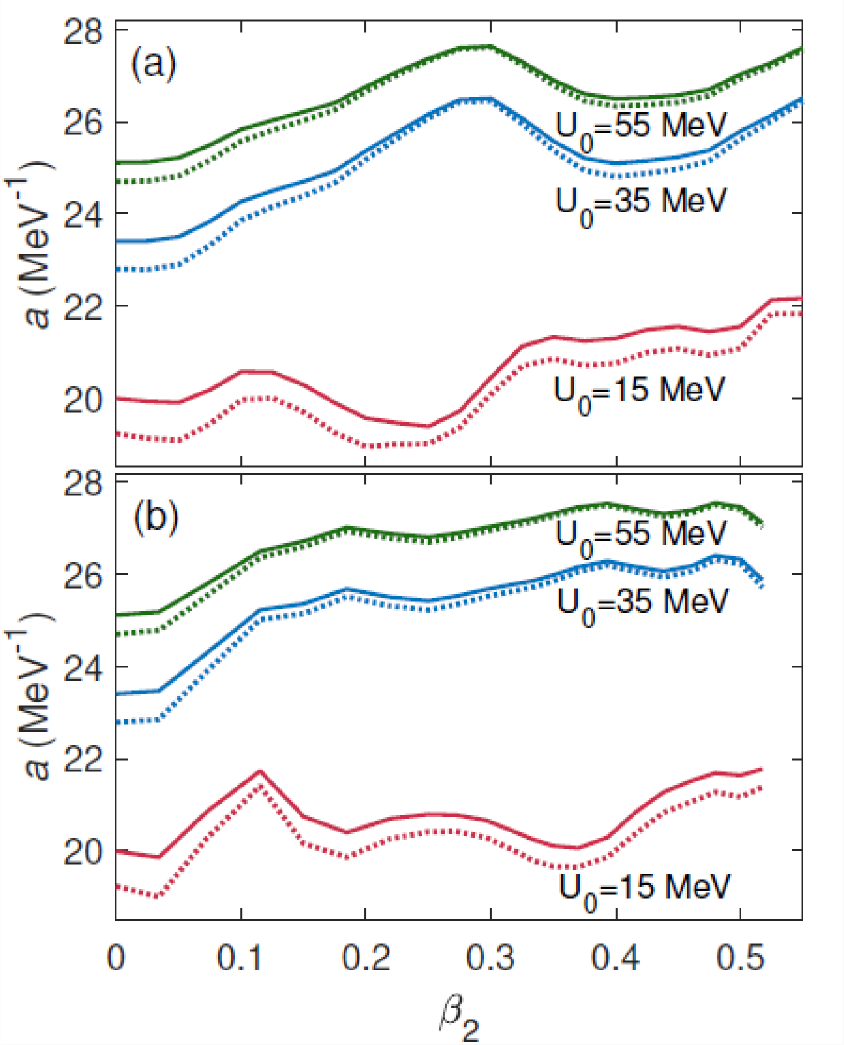


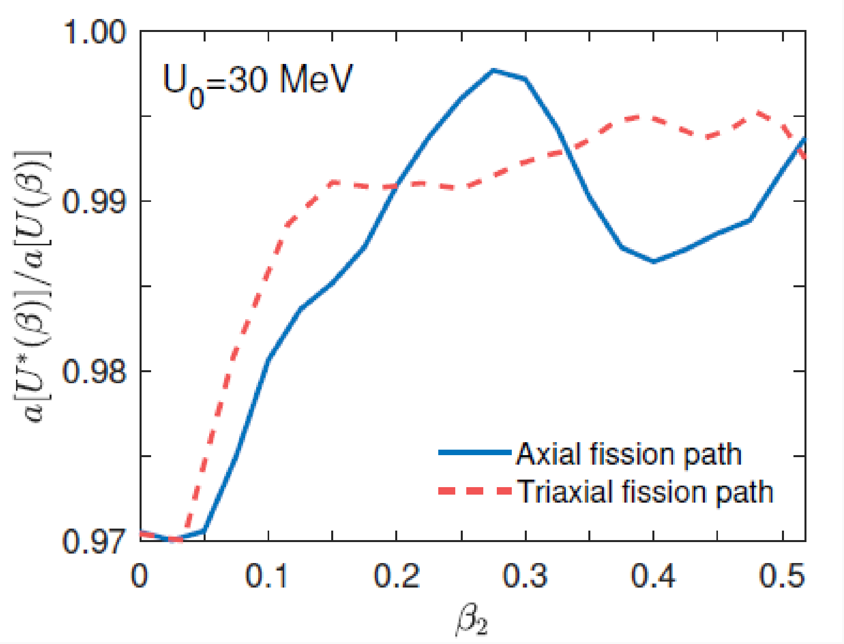
(a)
(b)
(c)
$E_{pot}-E_{pot}^{GS}$ (MeV)
F-F^{GS} (MeV)
F-F^{GS} (MeV)
Axial fission path
Triaxial fission path
U_0=15 MeV
U_0=35 MeV
0
2
4
6
0
0.1
0.2
0.3
0.4
0.5
β_2



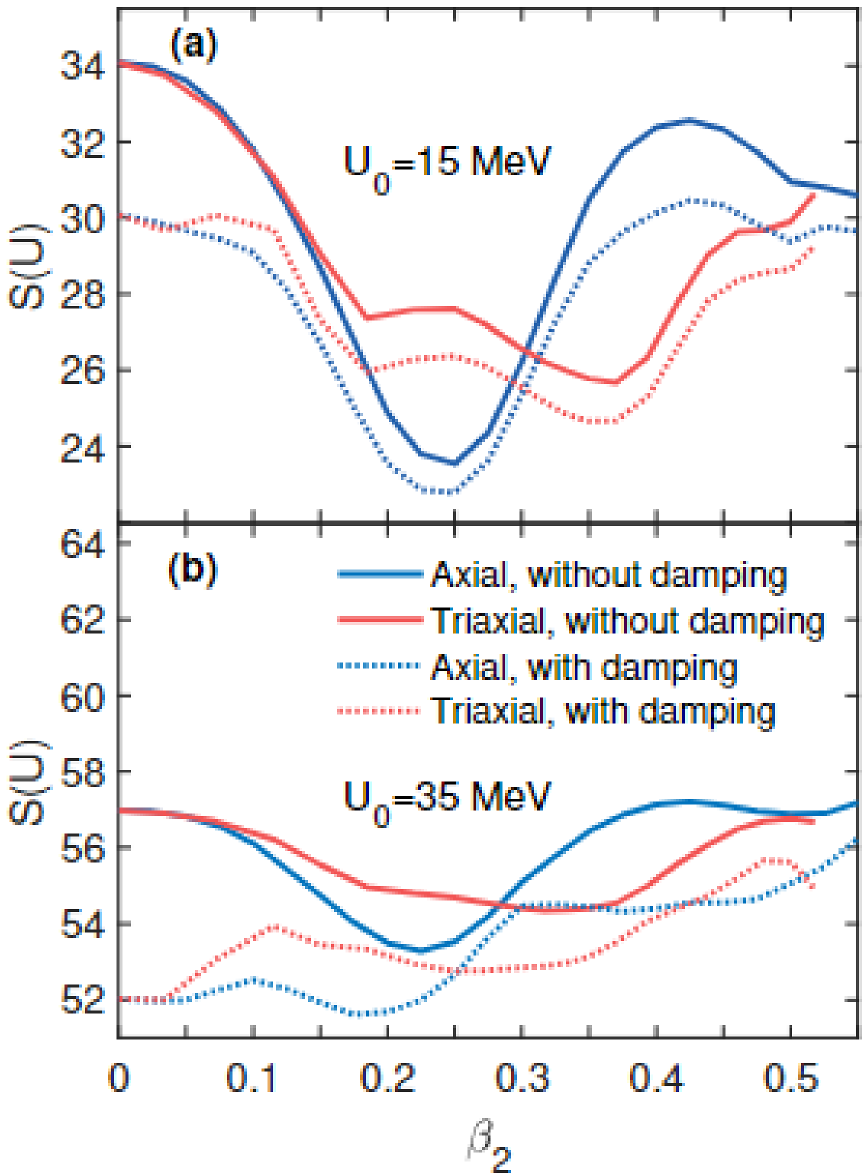
(a)
(b)
a (MeV^{-1})
a (MeV^{-1})
U_0=35 MeV
U_0=15 MeV
U_0=35 MeV
U_0=15 MeV
20
22
24
26
0
0.1
0.2
0.3
0.4
0.5
β_2

Level density parameter along the fission pathways





Entropy



1. Damping Effect - Damped microscopic corrections lower entropy along both axial and triaxial paths, especially at higher energies.

2. Low Energy - At low energies, the GS has higher entropy due to excitation energy, but entropy decreases with increasing deformation towards the SP.

3. High Energy - At higher energies, the entropy at the GS becomes lower than at the SP, with damping further reducing GS entropy due to reduced excitation energy.

4. Axial vs. Triaxial - The entropy difference between axial and triaxial paths decreases with energy

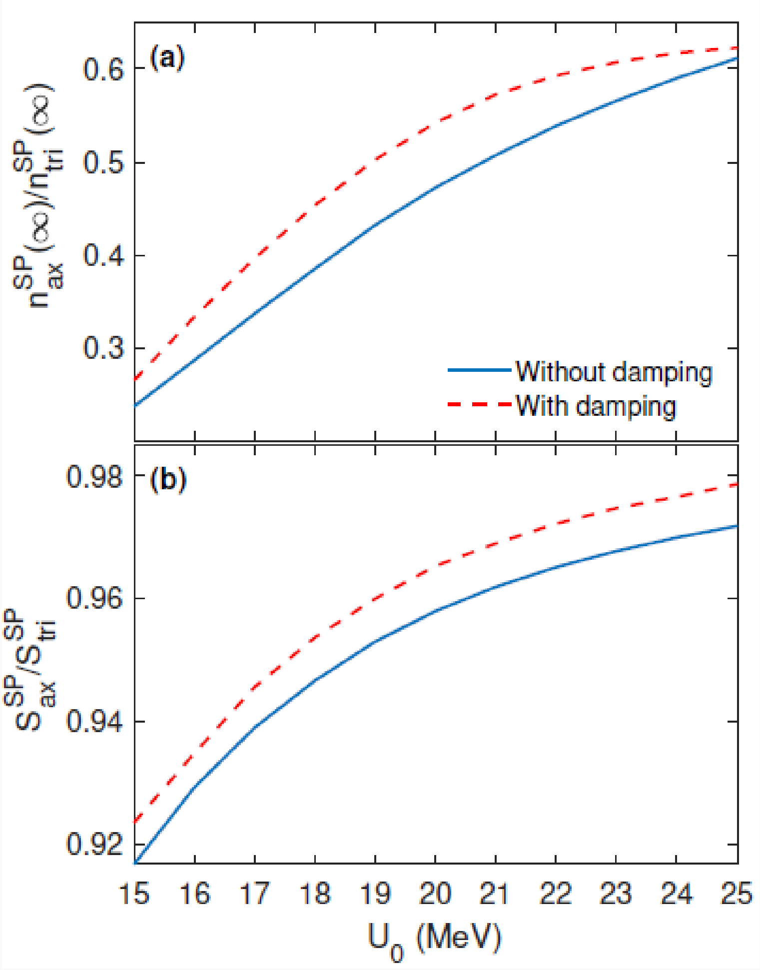
Fission probability

The probability of fission through axial and triaxial pathways get closer to each other with increasing excitation energy.

$$\frac{dn(\beta_i)}{dt} = [\Lambda_{i,i\pm1} n(\beta_{i\pm1}) - \Lambda_{i\pm1,i} n(\beta_i)]$$

$$\Lambda_{if} = \lambda_{if} \rho(\beta_f);$$

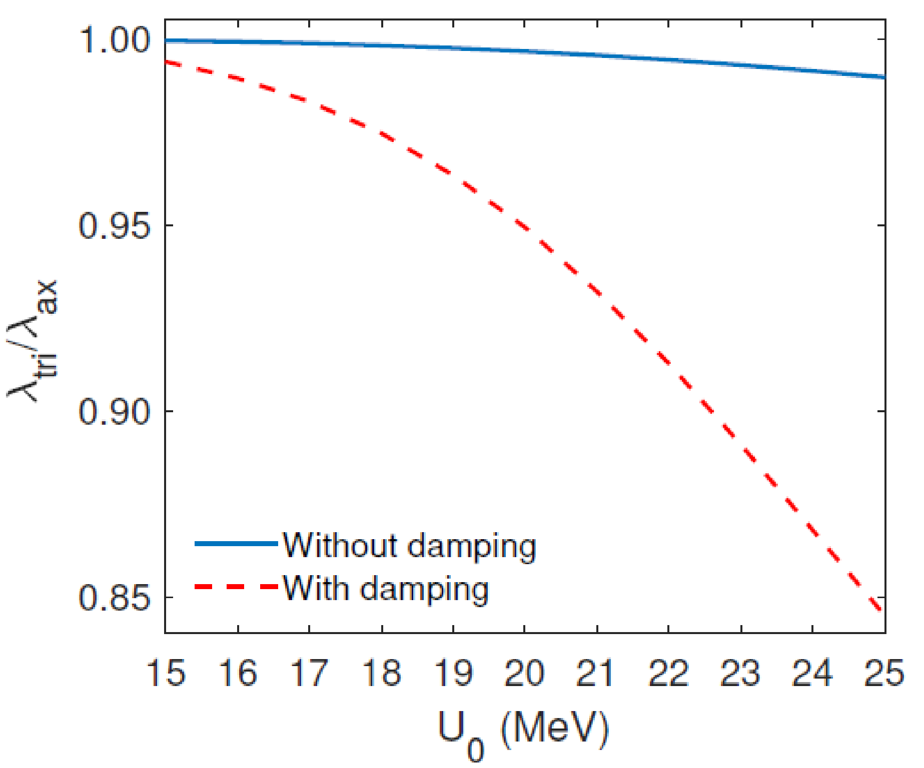
$$\lambda_{if} = \lambda_{fi}, \lambda_{if} = \frac{\lambda_0}{\sqrt{\rho(\beta_i)\rho(\beta_f)}}.$$



The ratio of decay constants corresponding to the triaxial and axial pathways

$$n^{SP}(t) = n^{SP}(\infty)(1 - e^{-\lambda t})$$

With account of the damping effect the axial path reaches its asymptotic probability faster than triaxial path and this effect increases with excitation energy.



Summary

- The analytical Jackson formula, with effective temperatures of $T = T_{CN}/\sqrt{2}$ and $T = T_{CN}/\sqrt{2.25}$, accurately reproduces the probabilities of *xn*- and *pxn/αxn*-evaporation channels in moderately excited SHN, as calculated using the microscopic model.

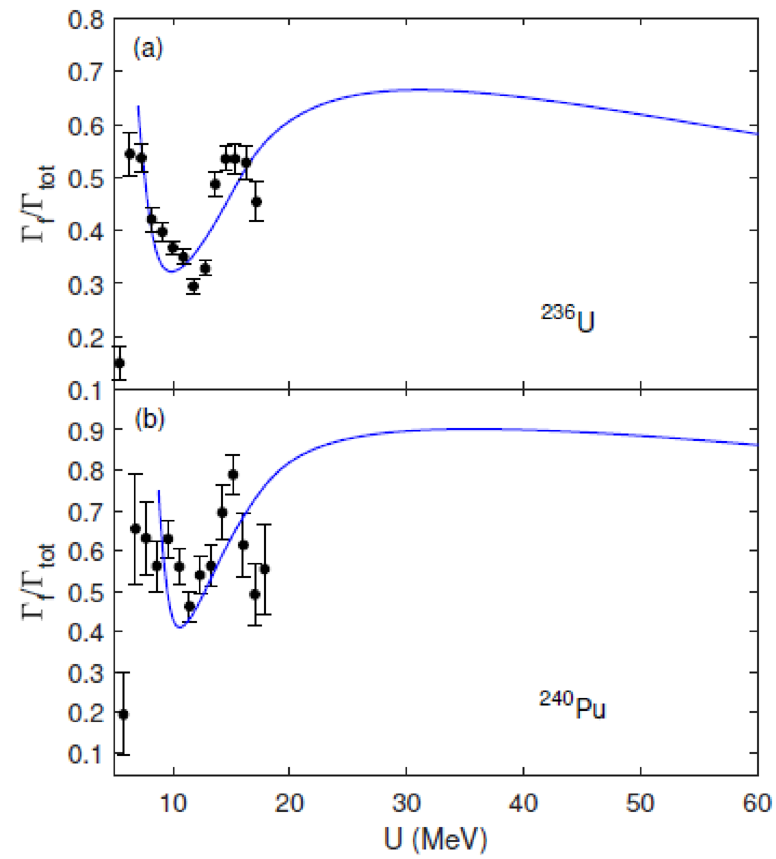
- Suppressing shell effects can alter the fission scenario depending on the available excitation energy. The entropy incorporates both energy and structural effects.

- To study the fission barrier height / fission threshold, the **cold fusion** reactions are required.

Fission and neutron emission probabilities

$$\frac{\Gamma_n}{\Gamma_f} = \frac{gA^{2/3}}{K_0} \frac{\int_0^{U-B_n} \varepsilon \rho_{GS}\left(U - B_n - \varepsilon\right) d\varepsilon}{\int_0^{U-B_f} \rho_{SP}\left(U - B_f - k\right) dk}$$

$$\frac{\Gamma_f}{\Gamma_{tot}} = \frac{1}{1 + \Gamma_n/\Gamma_f}$$



A. Rahmatinejad, et al., Phys. Rev. C **103**, 034309, (2021).

Experimental data: E. Cheifetz, et al., Phys. Rev. C**24**, 519 (1981).