Charmonia production within k_T-factorization approach QCD at LHC energies

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Introduction: Nonrelativistic QCD

- **Nonrelativistic QCD** (NRQCD) *[Phys.Rev.D 53,150 (1996)],[Phys.Rev.D 51,1125 (1995)]*:

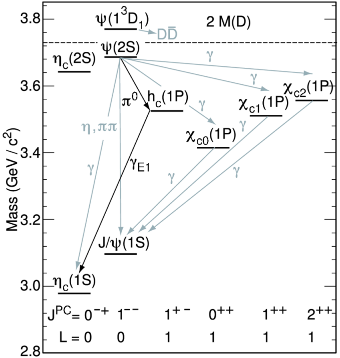
$$\sigma(pp \to J/\psi + X) = \sum_n \sigma(pp \to Q\bar{Q}[^{2S+1}L_J^{(a)}] + X)\langle \mathcal{O}^{J/\psi}[n]\rangle$$

- $\sigma(pp \to Q\bar{Q}[^{2S+1}L_J^{(a)}] + X)$ cross-section of heavy quark pair production $Q\bar{Q}$ in Fock state $n = ^{2S+1} L_J^{(a)}$ with definite spin S, orbital L и total J angular momenta and color representation a (color singlet (CS) [1] and color octet (CO) [8]) - can be calculated in the framework of perturbative QCD.

- **LDME** (nonperturbative matrix element) $\langle \mathcal{O}^{J/\psi}[n]\rangle$ describe the transition of intermediate state $Q\bar{Q}[n]$ into physical quarkonium via emission of soft gluons.

- Decomposition of wave functions of S- and P-wave charmonia over the small parameter v of quark relative motion and Fock states:

$$|\psi\rangle \quad = O(v^0)|c\bar{c}[^3S_1^{(1)}]\rangle + O(v^1)|c\bar{c}[^3P_J^{(8)}]g\rangle + O(v^2)|c\bar{c}[^3S_1^{(8)}]gg\rangle + ...$$
$$|\chi_{cJ}\rangle = O(v^0)|c\bar{c}[^3P_J^{(1)}]\rangle + O(v^1)|c\bar{c}[^3S_1^{(8)}]g\rangle + O(v^2)|c\bar{c}[^1P_1^{(8)}]gg\rangle + ...$$



Charmonia production: recent status and challenges

- Nonrelativistic QCD in collinear factorization approach at the NLO:
  $\hookrightarrow$ good description for the charmonia $(J/\psi, \psi', \chi_c)$ and bottomonia (Υ, χ_b) transverse momentum distributions *[Phys.Lett.B 673, 197 (2009); Phys.Rev.D 84, 051501 (2011); Phys.Rev.D 83, 111503 (2011); Phys.Rev.Lett. 106, 172002 (2012);Phys.Rev.D 90, 074021 (2014); Phys.Rev.D 83, 114021 (2011)]*
  $\hookrightarrow$ tree-level NNLO calculations to the color singlet contributions *[Phys.Rev.Lett. 101, 152001 (2008); Phys.Lett.B 695, 149 (2011)]*
  $\hookrightarrow$ problem with polarization *[Phys.Rev.Lett.108, 172002 (2012)]*.
  $\hookrightarrow$ global fit for $\psi(2S)$ (NLO calculations) *[Phys. Rev. D 107, 034003 (2023)]*: a satisfactory fit can be achieved only at $p_T > 7$ GeV, polarization problem is not solved

- A possible solution of polarization problem *[Phys.Rev.D 93, 054037 (2016)]*.

- Calculation in k_T-factorization approach at the LO:
  $\hookrightarrow$ good agreement with LHC data, including the polarization observables for $J/\psi, \psi', \chi_c$ and $\Upsilon(nS), \chi_b$ *[Phys.Rev.D 100, 114021 (2019); Eur.Phys.J. C 79, 621 (2019); 80, 1022 (2020); 79, 830 (2019);80, 486 (2020); 81, 1085 (2021)]*
  $\hookrightarrow$ Results for P-wave quarkonia χ_c, χ_b are in contradictions with the Heavy Quark Spin Symmetry (HQSS) relations
  $\hookrightarrow$ Possibly the higher-order correction could restore the HQSS.

k_T-factorization approach

- In the region of high energies (small x) parton model assumptions about collinear factorization of parton distributions and partonic cross sections are violated. Along with contributuions $\sim \alpha_s^n \ln^n \mu^2/\Lambda^2_{\text{QCD}}$ arises contributions $\sim \alpha_s^n \ln^n 1/x$, which can be summarized up to all orders of perturbative theory with evolution equations BFKL (Balitsky-Fadin-Kuraev-Lipatov) or CCFM (Catani-Ciafaloni-Fiorani-Marchesini)

- In **k_T-factorization approach** *[Phys.Rep. 100,1 (1983)],[Sov.J.Nucl.Phys, 53,657 (1991)],[Nucl.Phys.B 366,135 (1991)]*:

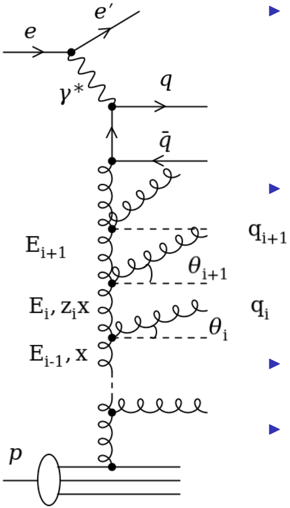
$$d\sigma = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} d\mathbf{k}^2_{1T} d\mathbf{k}^2_{2T} \frac{d\phi_1}{2\pi}\frac{d\phi_1}{2\pi} f_1(x_1, \mathbf{k}^2_{1T}, \mu_F^2) f_2(x_2, \mathbf{k}^2_{2T}, \mu_F^2) d\hat{\sigma}^*(k_1, k_2, \mu_R^2),$$

  where $f(x, \mathbf{k}^2_T, \mu^2_F)$ - TMD (transverse momentum dependent) parton distribution function in proton which obey the BFKL or CCFM evolution equation.

- LO in k_T-factorization approach can include the large piece of high-order corrections from collinear QCD approach (partially NLO + NNLO + ...) taking them into account in the form of TMD gluon distributions $f(x, \mathbf{k}^2_T, \mu^2_F)$ in the proton.

- Gauge invariance of off-shell amplitudes are achieved by using the effective vertex *[Nucl. Phys. B 452,369 (1995)]*

CCFM evolution equation and multiple gluon radiation



- **CCFM evolution equation** *[Nucl. Phys. B 296, 49 (1988)],[PLB 234,339 (1990)]* at leading logarithmic approximation:

$$f_g(x,\mathbf{k}_T^2,q^2) = f_g(x,\mathbf{k}_T^2,q_0^2)\,\Delta_s(q,q_0) + \int dz \int \frac{dq'^2}{q'^2}\Theta(q-zq')\times$$
$$\times\,\Delta_s(q,zq')\tilde{P}_{gg}(z,q',\mathbf{k}_T)f_g(x/z,\mathbf{k'}_T^2,q'^2),$$

- Emitted gluons obey the angular ordering conditions (due to color coherence effect):

$$q > z_n q_n > z_{n-1}q_{n-1} > ... > q_1 > q_0,$$

$\hookrightarrow$ at $z \to 0$: no constraints on q_i - BFKL conditions
$\hookrightarrow$ at $z \to 1$: ordering on q_i - DGLAP conditions

- **Multiple gluon radiation** can serve as a source of J/ψ produced via fragmentation mechanism (emitted gluons have $\mathbf{p}_T \neq 0$).
- CCFM evolution equation and k_T-factorization approach QCD give us unique opportunity to calculate such contributions with inclusion of high order corrections.

Challenges in NRQCD within k_T-factorization approach

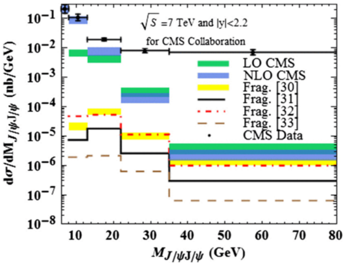
- Pair charmonia production $\psi + \psi$
- Associative charmonia production $J/\psi + Z/W^{\pm}$
- Violation of HQSS for χ_{cJ} and η_c

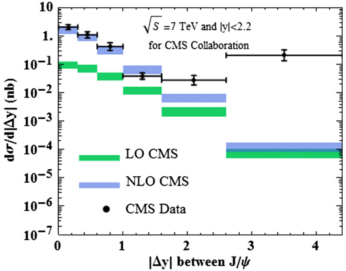
Chapter 1: J/ψ pair production

- **Significant discrepancy** between NRQCD predictions and experimental data *ATLAS* *[Eur. Phys. J. C 77, 76 (2017)]* and *CMS* *[JHEP 09, 094 (2014)]*, especially at the region of high $m(J/\psi, J/\psi)$ and $\Delta y(J/\psi, J/\psi)$.

- **Progress within NRQCD**:
  $\hookrightarrow$ full LO CS and CO contributions [Phys. Rev. Lett. 115.022002 (2015)];
  $\hookrightarrow$ NLO CS corrections *[Phys. Rev. D 94.074033 (2016)]*;
  $\hookrightarrow$ partial NLO* corrections for both CS and CO *[Phys. Rev. Lett. 111.122001 (2013)]*.

- Such processes are very important for additional verification of NRQCD formalism





DPS and heavy quarkonia pair production

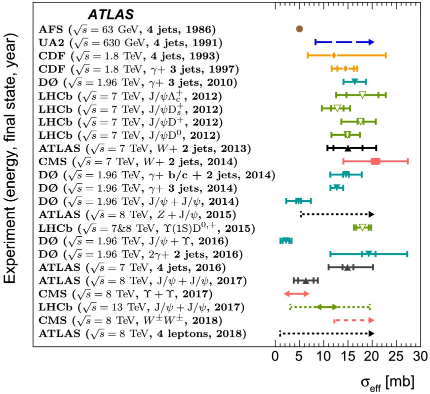
The **double parton scattering** (DPS) is of particular interest for heavy quarkonia pair production.

At high energies DPS cross section can be presented in the factorized form (omitting correlation and interference):

$$\sigma_{AB}^{DPS} = \frac{1}{1+\delta_{AB}} \frac{\sigma(A)\ \sigma(B)}{\sigma_{\text{eff}}},$$

where σ_{eff} - **effective cross section**, which determine the effective transverse overlap of parton interactions.

- Typical value of σ_{eff} 15 mb (12 - 20 mb) from experimental data.
- Processes with heavy quarkonia, such as $J/\psi + J/\psi$, $J/\psi + \Upsilon$, $\Upsilon + \Upsilon$, give a significantly lower estimations: $\sigma_{\text{eff}} \sim$ 3-6 mb.



Heavy quarkonia production mechanisms

- Creation of two $c\bar{c}$ pair at the definite quantum states $n, m = [^{2S+1}L_J^{(1,8)}]$ with subsequent nonperturbative transition into physical quarkonia $\mathcal{H}$ и $\mathcal{H}'$:

$$p + p \to c\bar{c}[n] + c\bar{c}[m] \to \mathcal{H} + \mathcal{H}'$$

- Charmonia pair production via **CS mechanism** within k_T-factorization approach at LO QCD $\mathcal{O}(\alpha_s^4)$:

$$g^* + g^* \to c\bar{c}[^3S_1^{(1)}] + c\bar{c}[^3S_1^{(1)}] \qquad (\sim 1/p_T^8)$$

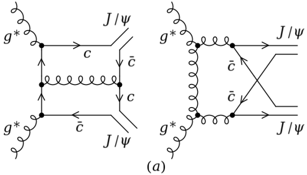
- Role of **CS-CO and CO-CO mechanisms** increase with p_T growth

$$g^* + g^* \to c\bar{c}[^3S_1^{(1)}] + c\bar{c}[^3P_J^{(8)}] + g \qquad (\sim v^2/p_T^8)$$

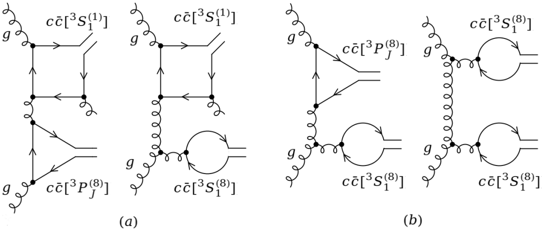
$$g^* + g^* \to c\bar{c}[^3S_1^{(1)}] + c\bar{c}[^3S_1^{(8)}] + g \qquad (\sim v^4/p_T^6)$$

$$g^* + g^* \to c\bar{c}[^3P_J^{(8)}] + c\bar{c}[^3S_1^{(8)}] \qquad (\sim v^6/p_T^6)$$

$$g^* + g^* \to c\bar{c}[^3S_1^{(8)}] + c\bar{c}[^3S_1^{(8)}] \qquad (\sim v^8/p_T^4)$$



Color singlet (CS) mechanism



Singlet-octet (CS-CO) and octet-octet (CO-CO) mechanisms

Fragmentation mechanism

- **Factorization approach** propose separation of parton production in hard interaction and their subsequent transition to hadrons.
- Cross section of **fragmentation production** (a → A) :

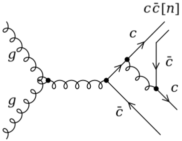
$$\sigma(pp \to A + X) = \sum_a \int \hat{\sigma}(pp \to a + X) D_a^A(z, \mu^2) \delta\left(z - \frac{p_A^+}{p_a^+}\right) dz,$$

where $\mathcal{D}_a^A(z, \mu^2)$ - fragmentation function.

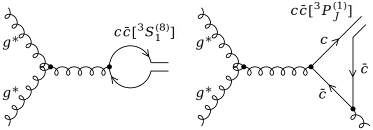
- **NRQCD formalism** for fragmentation function at the initial scale $\mu_0^2 \sim m_\psi^2$:

$$D_{g/q}^{\mathcal{H}}(z, \mu_0^2) = \sum_{|Q\bar{Q}(n)|} d_{g/q \to |Q\bar{Q}(n)|}(z, \mu_0^2) \frac{\langle \mathcal{O}^{\mathcal{H}}_{|Q\bar{Q}(n)|} \rangle}{m_Q^{2L+3}}$$

- DGLAP evolution $\Longrightarrow \mathcal{D}_c^{\mathcal{H}}(z, \mu^2)$ and $\mathcal{D}_g^{\mathcal{H}}(z, \mu^2)$ at any μ^2:



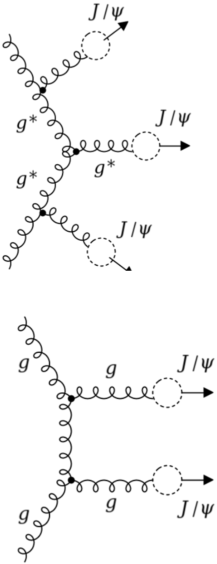
Charm quark fragmentation



Gluon fragmentation

Role of noncollinear evolution

- k_T-factorization approach allow to **effectively include the high order corrections of collinear QCD** bounded with real gluon emissions at the initial state (multiple gluon radiation).
- Emitted gluons have $\mathbf{k_T} \neq 0 \Rightarrow$ source of quarkonia production (c $\mathbf{p}_T^{J/\psi} \neq 0$) via fragmentation mechanism.
- **Fragmentation contributions from multiple gluon radiation**:
  $\hookrightarrow$ play the dominant role at the region of high $m(J/\psi, J/\psi)$ and $\Delta y(J/\psi, J/\psi)$ at the central rapidities (ATLAS, CMS)
  $\hookrightarrow$ negligibly small at forward rapidities (LHCb)



DPS σ_{eff} extraction from LHCb data

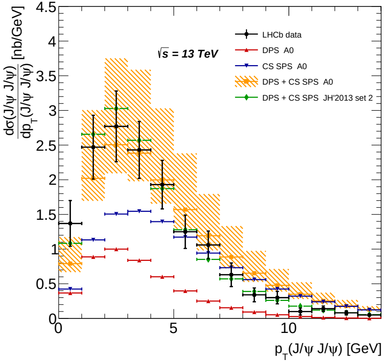
σ_{eff} extraction from available LHCb data: J/ψ pair production at $\sqrt{s} = 7$ TeV *[Phys. Lett. B707, 52 (2012)]* and $\sqrt{s} =$ 13 TeV *[JHEP 06, 047 (2017)]*

- Only CS + DPS contributions are considered within k_T-factorization approach with TMD gluon distributions A0 и JH'2013 set 2 *[Nucl.Phys.B. 883,1 (2014)]*

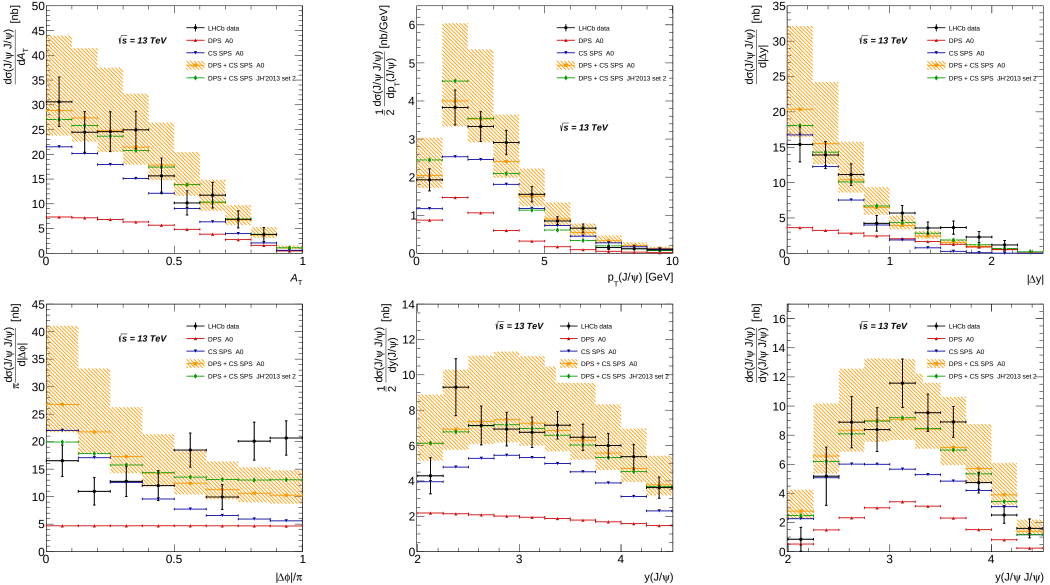
- Factorized DPS formula:

$$\sigma_{DPS} = \frac{1}{2}\frac{\sigma^2(pp \to J/\psi + X)}{\sigma_{eff}}$$

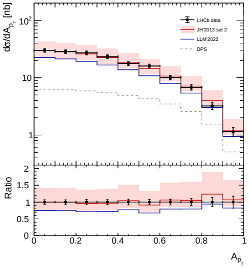
- Direct J/ψ and feed-down χ_c и ψ' contributions
- Results are published *[Eur.Phys.J.C 80, 1046 (2020)]*:
  - $\hookrightarrow$ $\sigma_{\text{eff}} = 17.5 \pm 4.1$ mb for A0
  - $\hookrightarrow$ $\sigma_{\text{eff}} = 13.8 \pm 0.9$ mb for JH'2013 set 2
- Results are compatible with many other estimations based on essentially different final states.

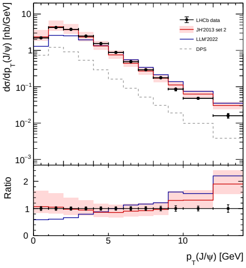


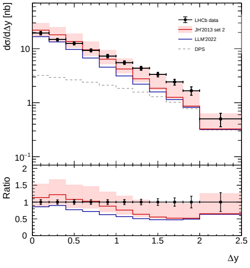
Comparison with LHCb data *(see [Eur.Phys.J.C 80, 1046 (2020)])*

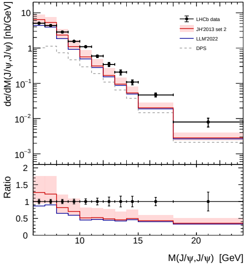


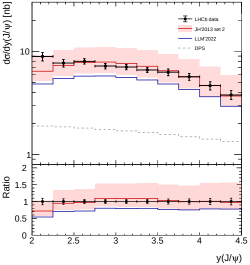
New LHCb data *[JHEP 03 (2024) 088]*: $J/\psi, J/\psi$ production *(see [Phys.Rev.D 110, 054001 (2024)])*

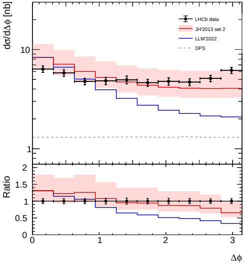




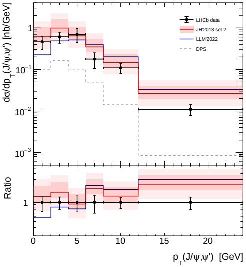


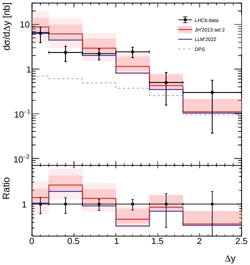


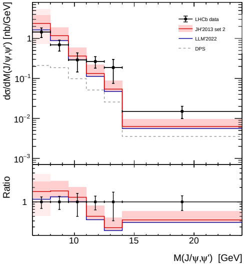


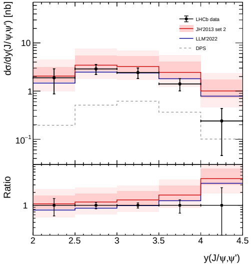


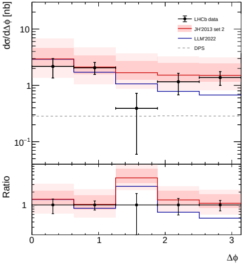
New LHCb data *[JHEP 05 (2024) 259]*: $J/\psi, \psi(2S)$ production *(see [Phys.Rev.D 110, 054001 (2024)])*





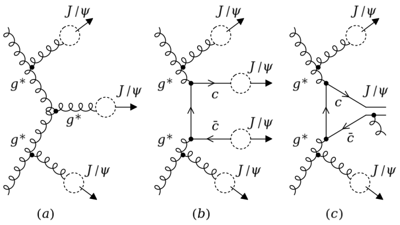




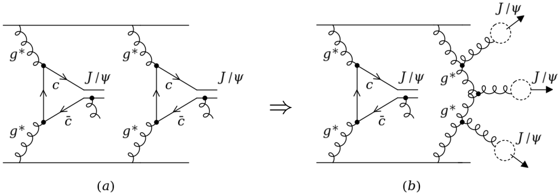


J/ψ pair production at central rapidities

- Fragmentation mechanism + multiple gluon radiation allow us to include (within k_T-factorization):
  - ↪ main CS-CO и CO-CO contributions
  - ↪ high order collinear QCD corrections
  - ↪ additional fragmentation contributions
- Modified scheme of DPS calculation include fragmentation contribution
  - ↪ direct production $g^* g^* \to J/\psi$ via intermediate state ${}^3S_1^{[8]}$ can be replaced by $g^* g^* \to g^*$ in each of single parton interaction with subsequent fragmentation to J/ψ with multiple gluon radiation
  - ↪ additional subprocess $g^* g^* \to c\bar{c}$



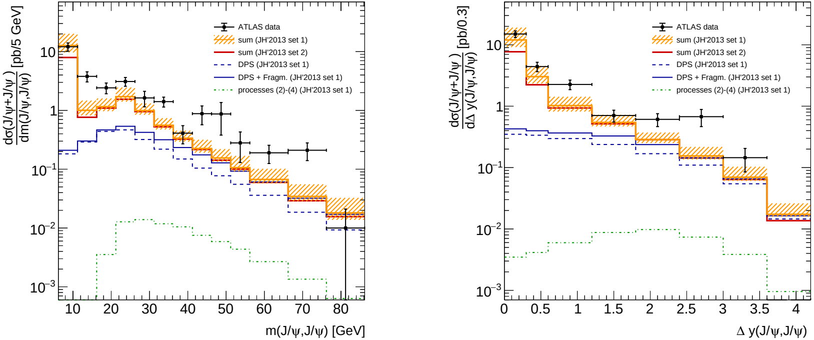
J/ψ production with fragmentation



Modified scheme of DPS

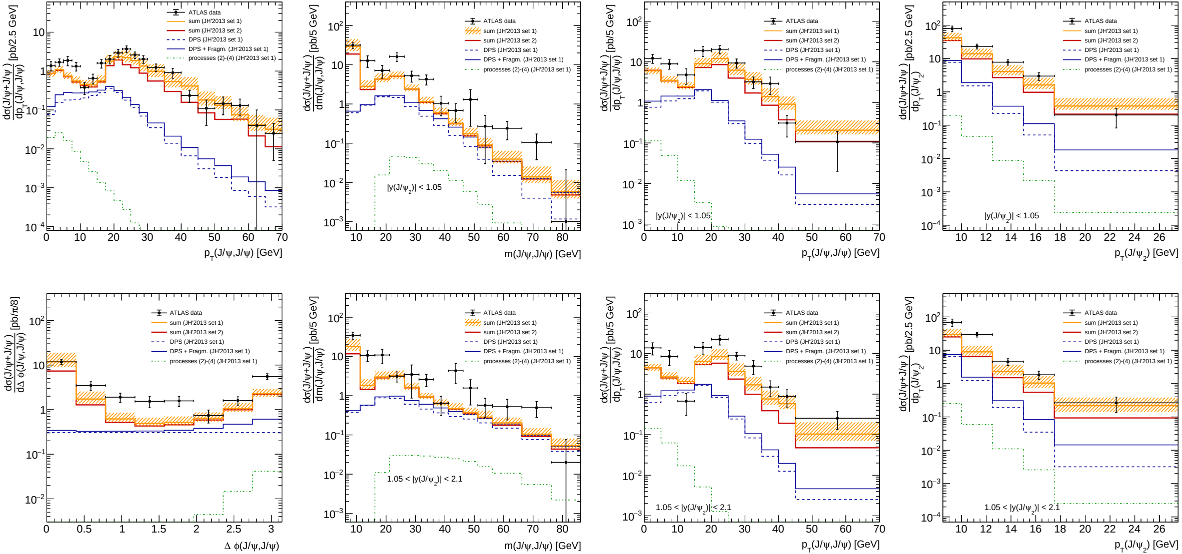
Comparison with ATLAS data

ATLAS data [Eur. Phys. J. C 77, 76 (2017)]



- DPS effective cross section $\sigma_{\text{eff}} = 13.8$ мбн.
- Inclusion of fragmentation contributions from multiple gluon radiation allow to strongly improve the agreement between theoretical predictions within NRQCD and ATLAS data at $\sqrt{s} = 8$ TeV at central rapidities especially at the region of high $m(J/\psi, J/\psi)$ and $\Delta y(J/\psi, J/\psi)$. *(see [Phys.Rev.D 106, 034020 (2022)])*
- The sensitivity of calculations to the choice of TMD distributions within theoretical uncertainties

Comparison with ATLAS data

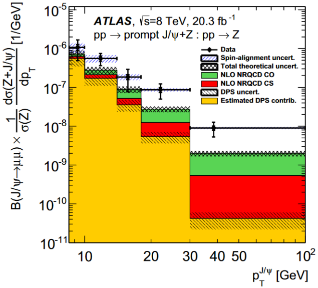


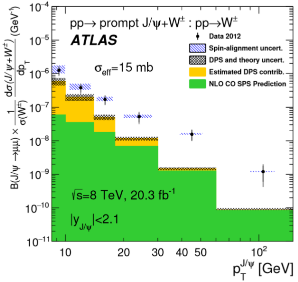
Chapter 2: associative $J/\psi + Z/W^\pm$ production

- Theoretical predictions of NRQCD **significantly underestimate** latest experimental data *ATLAS [Eur. Phys. J. C 75, 229 (2015)]* and *ATLAS [JHEP 2020,95 (2020)]*.

- **Progress within NRQCD**:
  $\hookrightarrow$ complete NLO CS and CO contributions *[Phys.Rev.D 66,114002 (2002); Phys.Rev.D 83, 014001 (2011); JHEP 02,71 (2011)]*;
  $\hookrightarrow$ NLO corrections play a significant role.

- Complex test of NRQCD and electroweak theory





Associative J/ψ and gauge $Z/W^{\pm}$ bosons production

- Possibility to investigate role of fragmentation mechanisms of charmonia production and multiple gluon radiation at new scales $\mu \sim m(Z, W^{\pm})$

- Main source of fragmentation contribution to the process of $J/\psi + Z/W^{\pm}$ - **excitation of c quark for Z boson** (or s for $W^{\pm}$) with subsequent fragmentation:

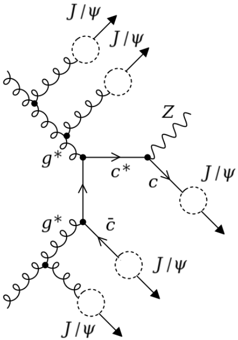
$$g + c \to Z + c, \quad g + s \to W^{-} + c, \quad c \to J/\psi + c$$

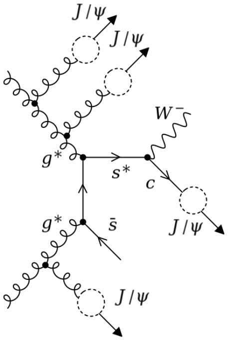
- Such fragmentation contributions are not included in NLO calculations *[Phys.Rev.D 66,114002 (2002); Phys.Rev.D 83, 014001 (2011); JHEP 02,71 (2011)]*, (new subprocesses contain additional quarks at the final state).

- Within k_T-factorization approach these processes turn to:

$$g^{*} + g^{*} \to Z/W^{\pm} + c + \bar{c}$$

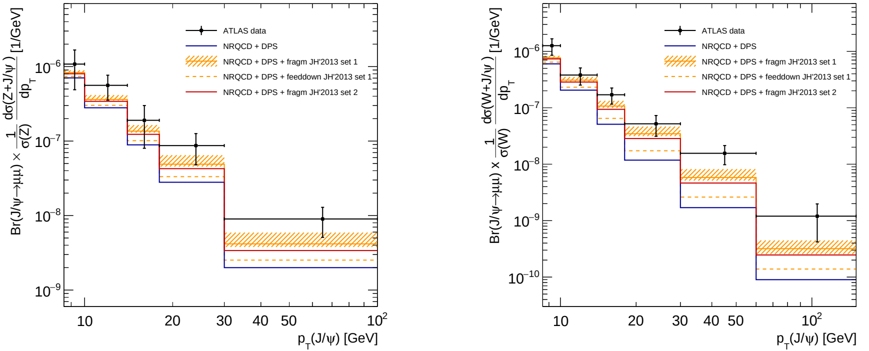
- Additional source of J/ψ: **multiple gluon radiation**





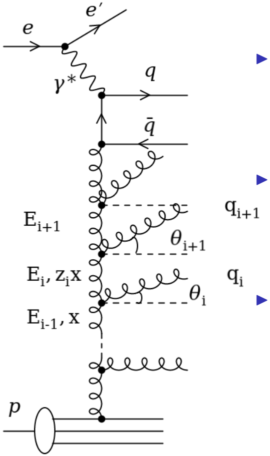
Comparison with ATLAS data

Data ATLAS [Eur. Phys. J. C 75, 229 (2015)] and ATLAS [JHEP 2020,95 (2020)]



- Fragmentation contributions come from charm quarks and gluons significantly improve the NRQCD predictions especially at the region of high J/ψ transverse momenta (at $p_T^{J/\psi} \geq$ 20-30 GeV contributions $\sim$ NLO NRQCD predictions + DPS ($\sigma_{\text{eff}} = 15$ mb)) *(see [Phys.Rev.D 104, 034018 (2021)])*.
- Feed-down contributions ψ', χ_{cJ} also important ($\sim$ 30% to the direct J/ψ production).
- The sensitivity of calculations to the choice of TMD distributions within theoretical uncertainties

Chapter 3: beyond the LO calculations in k_T-factorization approach



- Cross section beyond the leading order:

$$\sigma = \sigma_{\text{LO}} + \alpha_s(\sigma_{\text{NLO real}} + \sigma_{\text{NLO virtual}}) + \dots .$$

- NLO calculations in collinear QCD approach contain some challenges: the loop calculations, cancellation of divergences and some sort of processes with complex topology.

- LO in k_T-factorization approach can include the large piece of high-order corrections from collinear QCD approach (partially NLO + NNLO + ...) taking them into account in the form of TMD gluon distributions $f(x, \mathbf{k}_T^2, \mu_F^2)$ in the proton.

- NLO in k_T-factorization approach:
  $\hookrightarrow$ more complicated calculation of off-shell amplitudes (extended set of Feynman diagrams)
  $\hookrightarrow$ Double counting problem for LO and NLO: some contributions (parton emissions) can be counted twice - from tree-level amplitude and initial gluon radiation (from evolution of TMD gluon distributions)

χ_{cJ} production at NLO

- Progress in collinear QCD:
  $\hookrightarrow$ LO calculations *[Phys.Rev.D 90, 0974021 (2014)]*: fits point to the large $|\mathcal{R}'^{\,(1)}(0)|^2 \gg$ typical value from potential models $|\mathcal{R}'^{\,(1)}(0)|^2 = 0.075$ GeV^5
  $\hookrightarrow$ NLO calculations *[Phys.Rev.D 93, 054033 (2016)]*: good agreement with p_T spectra, $|\mathcal{R}'^{\,(1)}(0)|^2 = 0.075$ GeV^5
- Progress in k_T-factorization approach (links) LO:
  $\hookrightarrow$ good agreement with data (spectra + polarization) for χ_c and χ_b
  $\hookrightarrow$ HQSS violation

- Heavy Quark Spin Symmetry (HQSS) in NRQCD:

$$\langle \mathcal{O}^{\chi_{cJ}}[^3P_J^{(1)}] \rangle = (2J+1)\langle \mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}] \rangle$$

$$\langle \mathcal{O}^{\chi_{cJ}}[^3S_1^{(8)}] \rangle = (2J+1)\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}] \rangle$$

- Can NLO contributions restore the HQSS?
- LDME for P-wave quarkonia $\langle \mathcal{O}^{\chi_{cJ}}[^3P_J^{(1)}] \rangle = 6N_c(2J+1)\frac{|\mathcal{R}'^{\,(1)}(0)|^2}{4\pi}$, where $|\mathcal{R}'^{\,(1)}(0)|^2 = 0.075$ GeV^5 from potential model.
- Merging scheme of LO and NLO should exclude the double counting: we propose two possible solutions scenario A and scenario B

$LO + NLO^{\dagger}$ merging scheme: scenario A

- CS contributions: $\left| \begin{array}{ll} \text{LO} \;\; 2 \to 1: & g^*(k_1) + g^*(k_2) \to c\bar{c}[^3P_J^{(1)}](p) \\ \text{NLO} \;\; 2 \to 2: & g^*(k_1) + g^*(k_2) \to c\bar{c}[^3P_J^{(1)}](p) + g(p_g) \end{array} \right.$
- CO contributions: $\left| \begin{array}{ll} \text{LO} \;\; 2 \to 1: & g^*(k_1) + g^*(k_2) \to c\bar{c}[^3S_1^{(8)}](p), \\ \text{NLO} \;\; 2 \to 2: & g^*(k_1) + g^*(k_2) \to c\bar{c}[^3S_1^{(8)}](p) + g(p_g) \end{array} \right.$
- k_T - transverse momenta of initial gluon can serve as characteristic momentum of emitted gluons (as a result of evolution) with a good accuracy:

$$\langle k_T \rangle > \langle p_T^{\text{gluon emissions}} \rangle$$

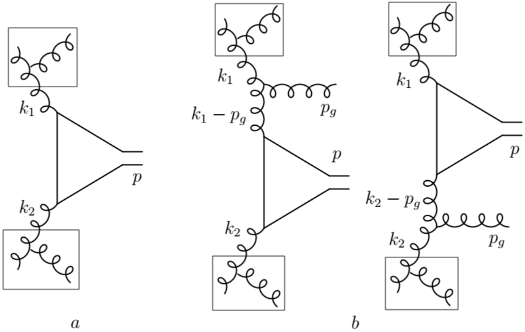
- Scenario A: merging scheme of LO + tree-level NLO (NLO†):
  - ⋆ 2 → 1: without cuts
  - ⋆ 2 → 2: $p_{gT} > \max(k_{1T}, k_{2T})$

↪ separation of LO and NLO contributions

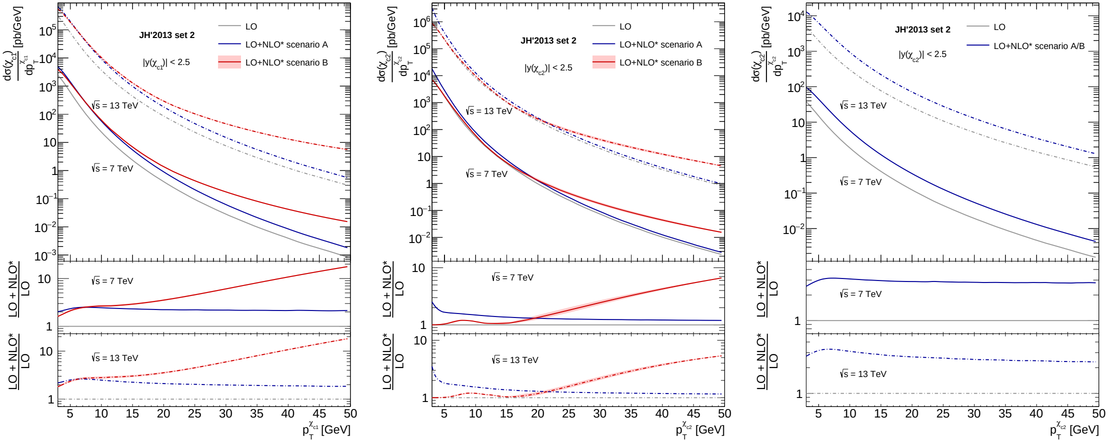
↪ separation of gluon $g(p_g)$ from amplitude $2 \to 2$ and emissions from TMD parton distributions $f(x, \mathbf{k}_T^2, \mu_F^2)$.

$LO + NLO^{\dagger}$ merging scheme: scenario B

- Only certain sets of $2 \to 2$ diagrams can contribute to the double counting → more target restrictions
- Choice of polarization tensor in the form $\sum \epsilon^{\mu}(k)\epsilon^{*\nu}(k) = \frac{k_T^{\mu} k_T^{\nu}}{\mathbf{k}_T^2}$ allow us to exclude contributions from some of nonfactorizable diagrams
- In case of χ_{cJ} production via ${}^3P_J^{[1]}$ mechanism only 2 diagrams create the double counting.
- Parameter of merging k_T^{cut} can be obtained from $\frac{d\sigma}{dk_T}$ distributions:
  - ↪ 2 → 1: k_T - transverse momentum of initial gluon
  - ↪ 2 → 2: $t_{\text{min. gluon}} = \min\{(k_1 - p_g)^2, (k_2 - p_g)^2\}$ - minimal gluon propagator
- Scenario B merging scheme
  - $2 \to 1 : \begin{cases} P = 1/2 : k_{1T} < k_T^{\text{cut}},\ k_{2T} \text{ without cuts} \\ P = 1/2 : k_{1T} \text{ without cuts, } k_{2T} < k_T^{\text{cut}} \end{cases}$
  - $2 \to 2 : \sqrt{t} > k_T^{\text{cut}}$

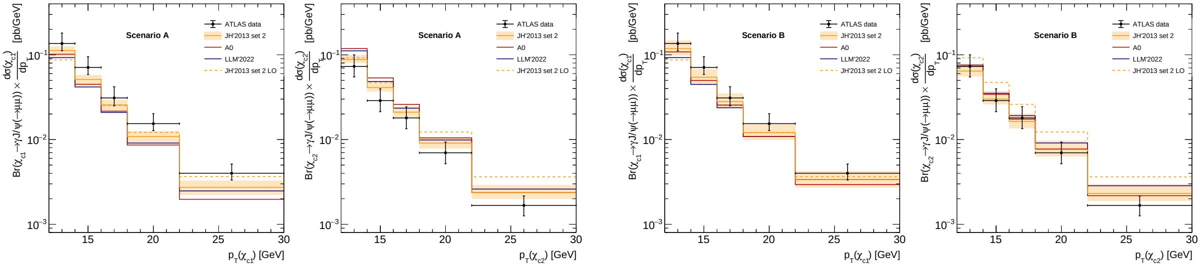


Comparison of $LO + NLO^{\dagger}$ with LO in ATLAS/CMS kinematics



Determination of $\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{[8]}]\rangle$ *(see [Eur.Phys.J.C 84, 348 (2024)])*

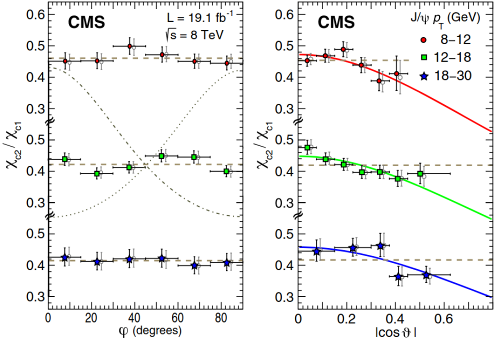
Fit: ATLAS data $\sqrt{s} = 7$ TeV *[JHEP 07, 154 (2014)]* and CDF data $\sqrt{s} = 1.8$ TeV *[Phys. Rev. Lett. 79, 578 (1997)]*



The fitted values of LDME $\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{[8]}]\rangle/\mathrm{GeV}^3$

	Scenario A	$\chi^2/\mathrm{n.d.f.}$	Scenario B	$\chi^2/\mathrm{n.d.f.}$
JH'2013 set 2	$(3.07 \pm 0.89) \times 10^{-4}$	0.78	$(1.74 \pm 0.62) \times 10^{-4}$	0.39
A0	$(1.91 \pm 1.91) \times 10^{-4}$	1.8	$(1.32 \pm 0.49) \times 10^{-4}$	0.65
LLM'2022	$(4.84 \pm 0.87) \times 10^{-4}$	1.09	$(3.86 \pm 0.76) \times 10^{-4}$	1.18

CMS χ_{c1} and χ_{c2} polarization $\sqrt{s} = 8$ TeV *[Phys.Rev.Lett.,124,162002 (2020)]*



- Parametrization of muon angular distributions:

$$\frac{d\sigma(J/\psi\to\mu\mu)}{d\cos\theta^*\ d\phi^*} \sim$$

$$\frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta\cos^2\theta^*+\lambda_\phi\sin^2\theta^*\cos 2\phi^*+\lambda_{\theta\phi}\sin 2\theta^*\cos\phi^*\right)$$

- Correlation between the $\lambda_\theta^{\chi c1}$ and $\lambda_\theta^{\chi c2}$ parameters from CMS analysis:

$$\lambda_\theta^{\chi c2} = (-0.94 + 0.90\lambda_\theta^{\chi c1}) \pm (0.51 + 0.05\lambda_\theta^{\chi c1}),$$

$$8 < p_T^{J/\psi} < 12 \text{ GeV},$$

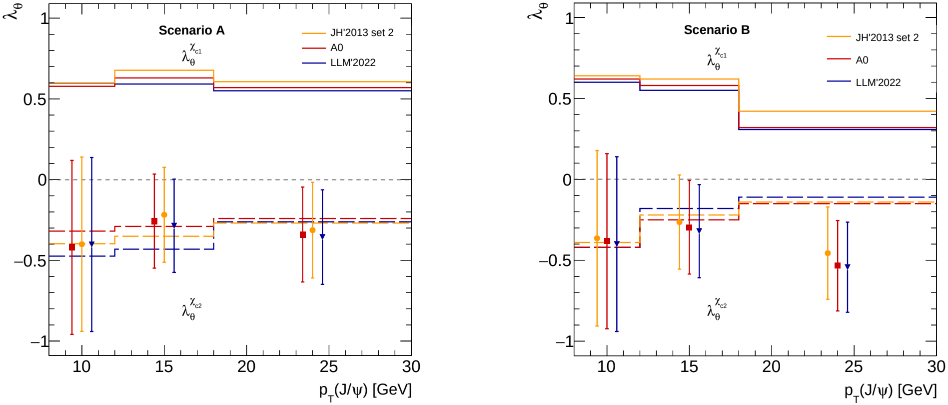
$$\lambda_\theta^{\chi c2} = (-0.76 + 0.80\lambda_\theta^{\chi c1}) \pm (0.26 + 0.05\lambda_\theta^{\chi c1}),$$

$$12 < p_T^{J/\psi} < 18 \text{ GeV},$$

$$\lambda_\theta^{\chi c2} = (-0.78 + 0.77\lambda_\theta^{\chi c1}) \pm (0.26 + 0.06\lambda_\theta^{\chi c1}),$$

$$18 < p_T^{J/\psi} < 30 \text{ GeV}.$$

Polarization predictions *(see [Eur.Phys.J.C 84, 348 (2024)])*



- $\lambda_\theta^{\chi_{c2}}$ points (expected values) are obtained from the CMS relations.
- The $2 \to 2$ contribution provide lower polarization of the ${}^3P_J^{[1]}$ mesons as compared to the $2 \to 1$ contribution (NLO* ${}^3P_J^{[1]}$ more important in scenario B).

Summary

- Fragmentation contributions from multiple gluon radiation arising during the noncollinear evolution of gluon distributions play a significant role in describing LHC data for the J/ψ pair production in central rapidity region (ATLAS, CMS experiments). In forward rapidity region at small transverse momenta such contributions can be safely omitted (LHCb experiments).

- Value of DPS effective cross section $\sigma_{\text{eff}} \simeq 15$ mb was obtained in forward rapidity region from the fit of LHCb measurements of J/ψ pair production. Additional fragmentation contributions in central rapidity region can point out to the same estimations of σ_{eff}.

- Taking into account new mechanisms associated with the fragmentation of quarks and gluons into charmonia allows us to significantly improve the agreement between NRQCD predictions and LHC data for associated J/ψ and gauge $Z/W^{\pm}$ boson production.

- NLO† corrections for χ_{cJ} production within k_T-factorization approach allow us to restore the HQSS relations and keep the descriptions of polarization observables.

Backup

TMD распределения глюонов в протоне

- TMD распределения глюонов в протоне: A0, JH'2013 set 1 и set 2.

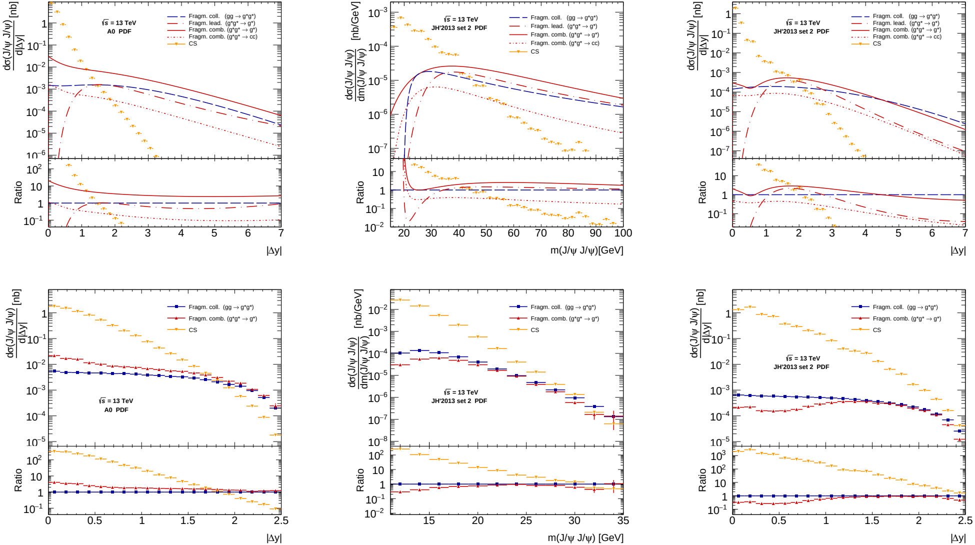
- Общая параметризация TMD распределений:

$$f_g(x, \mathbf{k}_T^2, q_0^2) = N x^{p_0} (1-x)^{p_1} \exp(-\mathbf{k}_T^2 / k_0^2)$$

- A0: при численном решении уравнения эволюции CCFM учитывались только сингулярные члены в функции расщепления ($z \to 1$ и $z \to 0$). Использовалось однопетлевое приближение $\alpha_s(\mu^2)$ с параметрами $n_f = 4$, $\Lambda_{\mathsf{QCD}} = 250$ МэВ. Параметры $p_1 = 4$, $p_0 = 0$, $k_0 = k_T^{cut} = 1.3$ ГэВ были получены из условия наилучшего описания данных коллабораций H1 и ZEUS для структурной функции протона $F_2(x, Q^2)$ в области $x < 5 \cdot 10^{-3}$ и $Q^2 > 4.5$ ГэВ.

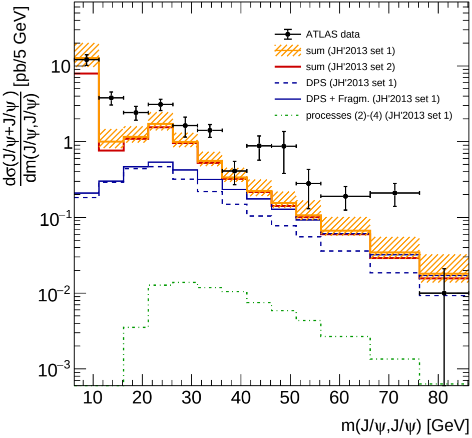
- JH'2013 set 1 и set 2: при численном решении уравнения эволюции CCFM учитывались также не сингулярные слагаемые. Использовалось двухпетлевое приближение $\alpha_s(\mu^2)$ с параметрами $n_f = 4$, $\Lambda_{\mathsf{QCD}} = 200$ МэВ.
  $\hookrightarrow$ JH'2013 set 1: параметры $p_1 = 6.5734, p_0 = -0.18074, k_0 = k_T^{cut} = 2.2$ ГэВ из условия наилучшего описания данных H1 и ZEUS для $F_2(x, Q^2)$ в области $x < 5 \cdot 10^{-3}$ и $Q^2 > 4.5$ ГэВ.
  $\hookrightarrow$ JH'2013 set 2: параметры $p_1 = 11.431, p_0 = -0.14739, k_0 = k_T^{cut} = 2.2$ ГэВ. Были дополнительно использованы данные для $F_2^c(x, Q^2)$ при $Q^2 > 2.5$ ГэВ.

Роль неколлинеарной эволюции

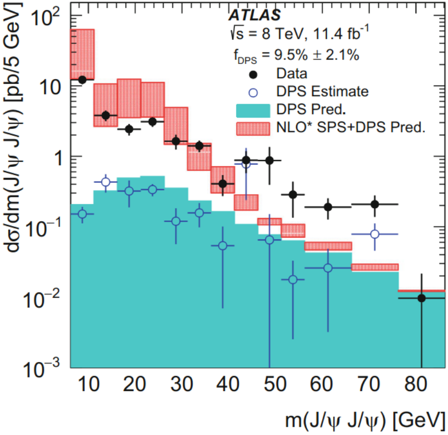


Сравнение с данными ATLAS

Данные ATLAS [Eur. Phys. J. C 77, 76 (2017)]



$\sigma_{\text{eff}} = 13.8$ мбн



$\sigma_{\text{eff}} = 6.3$ мбн

Парное рождение J/ψ в центральной области быстрот

Рассматриваемые подпроцессы:

- Вклады от J/ψ, ψ' и χ_{cJ}
- CS вклады: $$\left| \begin{array}{l} g^* + g^* \to c\bar{c}[^3S_1^{(1)}] + c\bar{c}[^3S_1^{(1)}], \quad g^* + g^* \to c\bar{c}[^3P_J^{(1)}] + c\bar{c}[^3P_J^{(1)}], \\ g^* + g^* \to c\bar{c}[^3P_J^{(1)}] + c\bar{c}[^3S_1^{(1)}] + g, \quad g^* + g^* \to c\bar{c}[^3S_1^{(1)}] + c\bar{c}[^3S_1^{(1)}] + g + g \end{array} \right.$$

  амплитуды вычислены С.П. Барановым *[Phys. Rev. D 84, 054012 (2011)]*
- CS-CO, CO-CO вклады (с учетом фрагментации): $$\left| \begin{array}{l} g^* + g^* \to g^*, \quad g^* + g^* \to c + \bar{c}, \quad g^* + g^* \to q + \bar{q}, \\ g^* + g^* \to c\bar{c}\left[\,^3P_J^{(1,8)}\right], \quad g^* + g^* \to c\bar{c}\left[\,^3S_1^{(1)}\right] + g \end{array} \right.$$
- Модифицированная схема ДПР на примере рождения J/ψ мезонов $$[^3S_1^{(1,8)},\, ^3P_J^{(8)}] \times [^3S_1^{(1,8)},\, ^3P_J^{(8)}] \quad \Longrightarrow \quad [g,\, c,\, ^3S_1^{(1)},\, ^3P_J^{(8)}] \times [g,\, c,\, ^3S_1^{(1)},\, ^3P_J^{(8)}]$$
- В функциях фрагментации $\mathcal{D}^{\mathcal{H}}_{c/g}(z, \mu^2)$ были учтены следующие вклады:

$$J/\psi, \psi' \quad \Longleftrightarrow \quad \begin{cases} g \to c\bar{c}[^3S_1^{(8)}] \\ g \to c\bar{c}[^3P_J^{(8)}] + g \\ c \to c\bar{c}[^3S_1^{(1)}] + c \end{cases} \qquad \chi_{cJ} \quad \Longleftrightarrow \quad \begin{cases} g \to c\bar{c}[^3P_J^{(1)}] + g \\ g \to c\bar{c}[^3S_1^{(8)}] \\ c \to c\bar{c}[^3P_J^{(1)}] + c \end{cases}$$

- JH'2013 set 1 и set 2 TMD плотности глюонов; Монте-Карло генератор событий CASCADE

Фрагментационные вклады в процесс $J/\psi + Z/W^{\pm}$

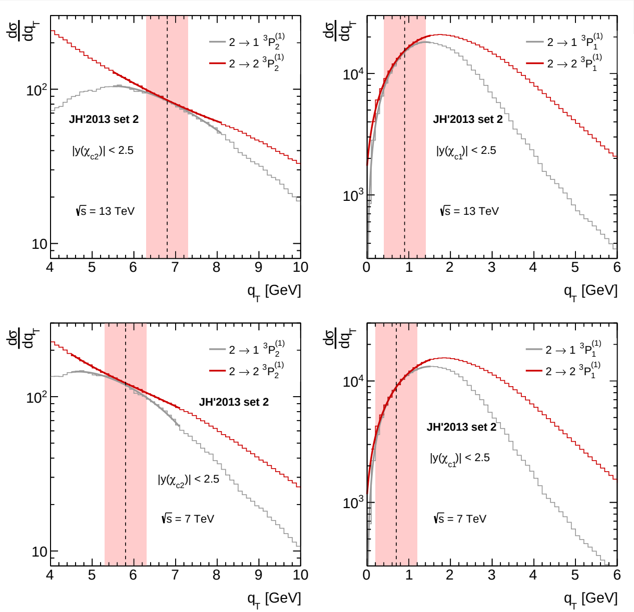
Рассматриваемые подпроцессы:

- Вклады от J/ψ, ψ' и χ_{cJ}
- В подходе k_T-факторизации: $\left| \begin{array}{l} g^* + g^* \to Z + c + \bar{c}, \quad g^* + g^* \to Z + q + \bar{q}, \\ g^* + g^* \to W^- + c + \bar{s}, \quad g^* + g^* \to W^- + q + \bar{q}' \end{array} \right.$
- Дополнительные коллинеарные подпроцессы: $\left| \begin{array}{l} q_v + \bar{q} \to Z/W^{\pm}, \quad q_v + g \to Z/W^{\pm} + q, \\ q + c \to Z + q + c, \quad q + q' \to W^{\pm} + q + c \end{array} \right.$
- В функциях фрагментации $\mathcal{D}^{\mathcal{H}}_{c/g}(z, \mu^2)$ были учтены следующие вклады:

$$J/\psi, \psi' \iff \begin{cases} g \to c\bar{c}[^3S_1^{(8)}] \\ c \to c\bar{c}[^3S_1^{(1)}] + c \end{cases} \qquad \chi_{cJ} \iff \begin{cases} g \to c\bar{c}[^3S_1^{(8)}] \\ c \to c\bar{c}[^3P_J^{(1)}] + c \end{cases}$$

- JH'2013 set 1 и set 2 TMD плотности глюонов для k_T-факторизационного подхода; Монте-Карло генератор CASCADE
- PDF MMHT2014LO для коллинеарных расчетов; Монте-Карло генератор PYTHIA

k_T^{cut} determination



- JH'2013 set 2 : $k_T^{cut}(^3P_1^{[1]}) = 5.8$ and 6.8 GeV, $k_T^{cut}(^3P_2^{[1]}) = 0.7$ and 0.9 GeV at $\sqrt{s} = 7$ and 13 TeV.