SPD Physics and MC Meeting
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Reconstruction of secondary vertex and short-lived particles

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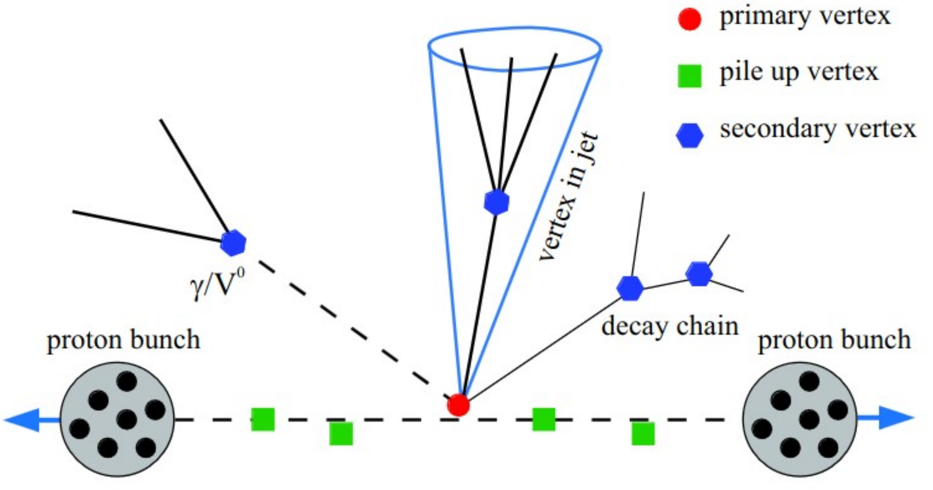
Introduction



1. Secondary vertex and short-lived particle reconstruction is very important task of many experiments.

2. In 2020 it was urgent request from SPD collaboration to create some software for short-lived particle reconstruction.

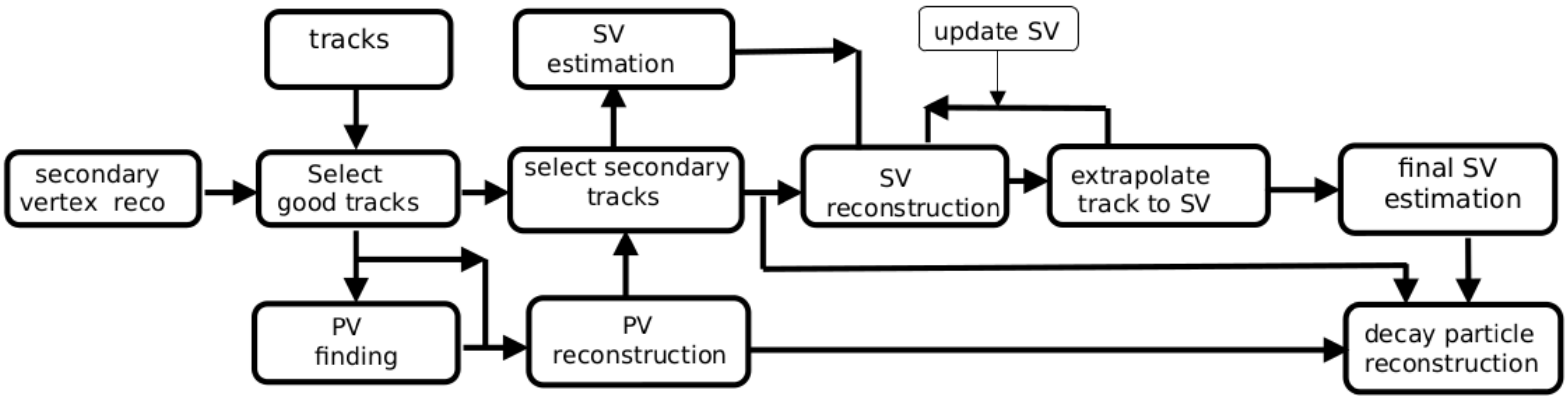
3. After some study KFParticle package was introduced in SPDroot software for these purposes.

4. Main advantage of KFParticle is that this package can be use as external package without big changes in SPD software.

5. Question - are there others packages for short-lived particle reconstruction ?

Short-lived particle reconstruction

General schema for secondary vertex and short-lived particle reconstruction can be presented as the next:



1. Selection good reconstructed (fitted) tracks.

2. Reconstruction of primary vertex or used already reconstructed.

3. Selection of secondary tracks (use some selection cuts at this step).

4. Reconstruction of secondary vertex.

5. Finally reconstruction of short-lived particle and it's parameters.

6. Important part of short-lived particle reconstruction is a kinematic fitting procedure which is usually used in all high energy physics experiments today.

Kinematic fitting

1. Kinematic fitting methods have been known in particle physics since the 1960s and were developed in the analysis of experimental data on bubble chambers.

2. Kinematic fitting can be defined as a mathematical procedure that uses physical laws, such as the law of four-momentum conservation, invariant mass, performed during interactions or decays of particles, in order to improve measurement results.

3. For example, the fact that three daughter particles in the decay of the D+ meson ($D^+ \to K^- \pi^+ \pi^+$) must come from a common space point can be used to improve the parameters of the daughter particles, thus improving the mass and momentum resolution of the D+ itself.

4. Using such procedure leads to an improvement in the ratio of the signal to the background and, thereby, increases the probability of measuring rare physical processes.

5. Kinematic fitting procedure uses the next available information from measurements: such as momentum, angles and energy of particle, and combines this information with physics constraints equations, such as four-momentum conservation in a production or displaced decay vertex, or the mass of an short-lived particle which can be determined through its decay products.

6. It is also assumed that the measured values of the particle parameters are random variables with a normal Gaussian distribution. The task of the kinematic fitting is to adjust the set of parameters in such a way that the parameters have the best estimation of true particle parameters, stay of random Gaussian distributed values and at the same time the laws of four-momentum conservation is valid.

7. The kinematic fitting procedure for analyzing experimental data has many different modifications and is currently widely used in almost all modern experiments in high-energy physics.

Kinematic fitting(2)

The kinematic fitting technique is based on the well-known Lagrange multiplier method. It is assumed that the constraint equations can be linearized and summarized in two matrices, D and d. Let α represent the parameters for a set of n tracks. Usually the next particle parameters $\alpha = (p_X, p_Y, p_Z, E, x, y, z)$ for constraint equations are used. Where (x,y,z) - space point on some particle trajectory, (p_X, p_Y, p_Z) – particle momentum at this space point and E – particle energy. It has the form of a column vector

$$\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$$

Initially the track parameters have the unconstrained values α_0 , obtained from the reconstruction. The r functions describing the constraints can be written generally as $H(\alpha) \equiv 0$, where $H = H_1\ H_2 \cdots H_r$ are different constraint. Expanding H(α) around a some point α_A yields the linearized equations

$$\mathbf{H}(\alpha) = 0 \approx H(\alpha_0) + \frac{\partial H(\alpha_A)}{\partial \alpha}(\alpha - \alpha_0) = \mathbf{d} + \mathbf{D}\delta\alpha,$$

where $\delta\alpha = \alpha - \alpha_A$. Thus we see that

$$\mathbf{d} = \begin{pmatrix} H_1(\alpha_A) \\ H_2(\alpha_A) \\ . \\ . \\ H_r(\alpha_A) \end{pmatrix} \qquad \mathbf{D} = \begin{pmatrix} \frac{\partial H_1}{\partial \alpha_1} & \frac{\partial H_1}{\partial \alpha_2} & \cdots & \frac{\partial H_1}{\partial \alpha_n} \\ \frac{\partial H_2}{\partial \alpha_1} & \frac{\partial H_2}{\partial \alpha_2} & \cdots & \frac{\partial H_2}{\partial \alpha_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial H_r}{\partial \alpha_1} & \frac{\partial H_r}{\partial \alpha_2} & \cdots & \frac{\partial H_r}{\partial \alpha_n} \end{pmatrix}.$$

Kinematic fitting (3)

In the fitting the constraints are incorporated using the method of Lagrange multipliers in which the χ2 is written as a sum of two term

$$\chi^2 = (\alpha - \alpha_0) V_{\alpha_0}^{-1} (\alpha - \alpha_0) + 2\lambda^T (\mathbf{d} + \mathbf{D}\delta\alpha).$$

where λ is a vector of r unknown the Lagrange multipliers. Minimizing the χ2 with respect to α and λ yields two vector equations which can be solved for parameters α and their covariance matrix

$$\begin{aligned} V_{\alpha_0}^{-1}(\alpha - \alpha_0) + \mathbf{D}^T\lambda &= 0 \\ \mathbf{d} + \mathbf{D}\delta\alpha &= 0. \end{aligned}$$

The solution can be written

$$\begin{gathered} \alpha = \alpha_0 - V_{\alpha_0}\mathbf{D}^T\lambda \\ \lambda = V_D(\mathbf{D}\delta\alpha_0 + \mathbf{d}) \\ V_D = \left(\mathbf{D}V_{\alpha_0}\mathbf{D}^T\right)^{-1} \\ V_\alpha = V_{\alpha_0} - V_{\alpha_0}\mathbf{D}^T V_D \mathbf{D} V_{\alpha_0} \\ \chi^2 = \lambda^T V_D^{-1}\lambda = \lambda^T(\mathbf{D}\delta\alpha_0 + \mathbf{d}) \end{gathered}$$

where $\delta\alpha_0 = \alpha_0 - \alpha_A$. The changes to α caused by the constraints are propagated by matrix 2 multiplication. The χ2 is a sum of r distinct terms, one per constraint. The contribution of each constraints is correlated with all others through matrix V_D.

Constraint equations example

1. In the analysis of experimental events in high energy physics, various constraint equations are used, which are determined both by the type of events themselves and by the conditions of the experiment.

2. For example, for physical event when the parameters of the primary particle and all secondary particles are measured. In this case, the constraint equations are based on the law of four-momentum conservation. It means that the sum of the energy and momentum of all particles in the final state is equal to the energy and momentum of the primary particle (or the initial state of the system) (**4C constraint**).

$$f = \begin{cases} \sum_{i=1}^{N} p_i \sin\theta_i \cos\phi_i - p_{\mathrm{ini},x} = 0 \quad (p_x), \\ \sum_{i=1}^{N} p_i \sin\theta_i \sin\phi_i - p_{\mathrm{ini},y} = 0 \quad (p_y), \\ \sum_{i=1}^{N} p_i \cos\theta_i - p_{\mathrm{ini},z} = 0 \quad (p_z), \\ \sum_{i=1}^{N} \sqrt{p_i^2 + m_i^2} - E_{\mathrm{ini}} = 0 \quad (E). \end{cases}$$

3. In the next example, the parameters of one of the particles which is involved in the interaction are unknown, when a neutral short-lived particle decays at some distance from the primary vertex (**3C constraint**).

   In this case, the constraint equations use the law of four-momentum conservation in the secondary vertex, in which the short-lived particle decays into several secondary particles, which parameters are measured.

   In addition to the law of four-momentum conservation, information that short-lived particle is formed in primary vertex is added. Then the primary and secondary vertices can be connected by a straight line $v - p * t = v^{PV}$. The additional parameter t indicates the trajectory length of a short-lived particle, normalized by its momentum.

Constraint equations example (2)

4. Then the additional three constraint equations can be represented as follows:

$$
\begin{aligned}
0 &= \Delta x - p^x \cdot t\,, \\
0 &= \Delta y - p^y \cdot t\,, \\
0 &= \Delta z - p^z \cdot t\,,
\end{aligned}
$$

where $\Delta x = (x^V - x^{PV})$, $\Delta y = (y^V - y^{PV})$, $\Delta z = (z^V - z^{PV})$. Since in this case the momentum of the short-lived particle p is unknown, the initial approximation of the momentum of a short-lived particle is estimated from the law of four-momentum conservation, taking into account the measured parameters of the decay products:

$$p_M = \sqrt{\left(\sum_i \sqrt{p_i^2 + m_i^2}\right)^2 - m_M^2}\,.$$

where the M index represents the mass of a short-lived particle, and the i index refers to reconstructed decay products. In this expression, the mass of a short-lived particle is also considered as a mass hypothesis. Since the momentum of the short-lived p_M particle, as well as the accuracy of its determination, are constraint values, these values are determined from the kinematic fitting procedure. The constraint equations are written taking into account the law of four-momentum conservation at the point of decay of a short-lived particle:

$$
f = \begin{cases}
\sum_{i=1}^{N} p_i \sin\theta_i \cos\phi_i - p_M \sin\theta_M \cos\phi_M = 0 \quad (p_x), \\
\sum_{i=1}^{N} p_i \sin\theta_i \sin\phi_i - p_M \sin\theta_M \sin\varphi_M = 0 \quad (p_y), \\
\sum_{i=1}^{N} p_i \cos\theta_i - p_M \cos\theta_M = 0 \quad (p_z), \\
\sum_{i=1}^{N} \sqrt{p_i^2 + m_i^2} - \sqrt{p_M^2 + m_M^2} = 0 \quad (E).
\end{cases}
$$

Kinematic fitting packages

There were analyzed several algorithms and packages for secondary vertex and short-lived particle reconstruction which are available in internet. The next experiment were considered:

CLEO (Fermilab), CMS (LHC), ATLAS (LHC), ALICE (LHC), CBM (FEAR), STAR (BNL), sPHENIX (BNL), BELLE (KEK), ILD (ILC), BM@N (NICA), MPD (NICA), CMD-3 (VEPP), H1 (HERA) and some others.

All the considered packages for reconstruction of primary and secondary vertices and determination parameters of a short-lived particle can be divided into two large groups:

- software packages for reconstruction and kinematic fitting, which are built into the appropriate software and optimized for the conditions of a particular experiment;

- packages which are not depended on the geometry and structure of a particular experiment are freely distributed and can be used in any experiment, including the SPD experiment.

Incorporated algorithms

CLEO - KWFIT
CMS - Kinematic Tree (KinFitter)
ATLAS – VkalVrt, VertexKinematicFitter
ALICE – AliKFparticle, KFParticle
CBM – CbmKFParticle
STAR – KFParticle
Belle – Kfit, TreeFitter, RAVE, OrcaKinFit
ILD – RAVE, MarlinKinfit (OPALFitter, OrcaKinFit)
HADES - KinFit
SPHENIX – KFParticle

BM@N - CbmKFParticle
MPD - KFParticle

Geometry independent

KWFIT
RAVE
KinFit
KinFitter
OrcaKinFit
CMD-3 (VEPP)
KLFitter
KFParticle

Kinematic fitting packages (2)

KWFIT - one of the first kinematic fit software packages written in a format independent of the geometry of the experiment. It has been used in the CLEO experiment software since 1990 and includes all possible constraint equations._The package is written on FORTRAN, but currently the text is not publicly available.

RAVE - a set of tools for reconstruction of primary and secondary interaction vertices, independent on the detector geometry, has been developed since 2003 and includes modern algorithms for vertex search and reconstruction and some kinematic fitting procedure. Initially, the algorithms were developed as part of the CMS experiment software, but later all algorithms were incorporated in the separate RAVE package. This package was used at the initial stage of software development in such experiments as ILD and Belle. Technical support for the RAVE package has been stopped since 2011, but this package is available at the following link https://rave .hepforge.org/.

KinFitter - it is a software package that implements all the kinematic fitting algorithms used in the CMS experiment, but in a way independent of the geometry of the experiment. The package does not depend on other special packages, with the exception of the ROOT software package. Unfortunately, this package has also not been supported since 2011. The KinFitter package is located at https://github.com/goepfert/KinFitter.git .

KinFit - also based on the Lagrange multiplier method, a kinematic fit package of interaction vertices was developed to fulfill the hyperon reconstruction program in the HADES experiment. This package is optimized for experiments with extended targets in which the initial position of the interaction vertex is unknown. The main disadvantage of this package is the assumption of absent magnetic field at interaction region and the particles trajectory is considered as straight line in this region. The KinFit package can be found at the link https://github.com/KinFit/KinFit.git

OrcaKinFit - a package is based on the MarlinKinFit kinematic fitting package used in the ILC project, which in turn is based on the algorithms kinematic fitting of the OPAL experiment. The OrcaKinFit package is free software independent of the geometry of the ILD detector or other specific experiment, however, this package has not been supported since 2014. This package is available in the following directory https://github.com/tferber/OrcaKinfit/tree/master .

Kinematic fitting packages (3)

KLFitter - a kinematic fitting package using a likelihood function-based reconstruction algorithm for arbitrary event topologies was developed for use in experiments at the Large Hadron Collider. The package has a modular structure that describes the physical processes and detector models independently. The implemented algorithms are common and can be transferred from one experiment to another. This package is available in the directory https://github.com/KLFitter/KLFitter

CMD-3 - a special kinematic fitting software package was developed for the CMD-3 experiment, located on the electron-positron collider VEPP-2000 with a maximum energy at the center of mass of 2 GeV and is currently actively used in the study of a number of physical processes. Despite the fact that this package was developed for the CMD-3 experiment, the authors believe that after some adaptation, the package can be used in similar experiments. To do this, it is necessary to make a number of experiment-specific changes related to the format of the input data and, possibly, to the description of the particle parameters. This package consists of two parts and is available in the following https://github.com/sergeigribanov/KFCmd and https://github.com/sergeigribanov/KFBase directories.

KFParticle – initially algorithms of short-lived particle reconstruction and kinematic fitting were developed for CBM experiment (CbmKFparticle), then it was developed inside ALICE software (AliKFparticle) and finally the detector independent package KFParticle was established. This package can reconstruct a variety of physical processes and particles, including strange particles, strange resonances, hyperons, light vector mesons, charmonium and particles with open charm.

KFParticle

The KFParticle package is based on the Kalman filter mathematics and is characterized by the following basic properties:

- the same set of parameters (x, y, z, p^X, p^Y, p^Z, E, l/p) is used to describe the primary (short-lived) particle and secondary particles (decay products),

- secondary particles are added to the primary particle or subtracted independently using the mathematics of the Kalman filter,

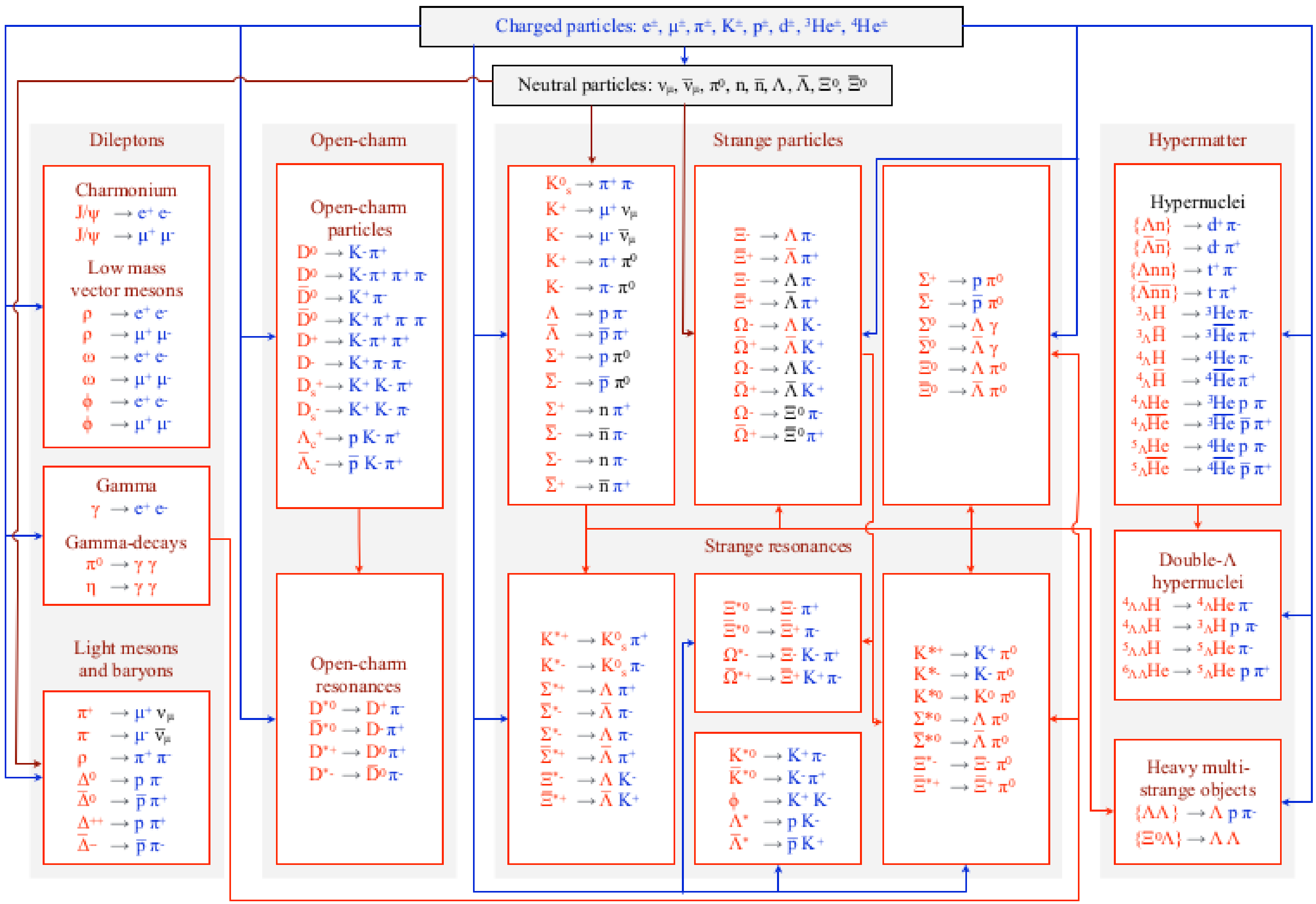
- the package allows to restore the entire decay chain for the case of sequential decay of several short-lived particles,

- the package does not depend on the geometry of detector and easy for the installation as external library.

The kinematic fitting procedure is also used in the KFParticle package, which allows during reconstruction of decay products to improve the parameters of secondary and primary neutral particles by applying certain constraint equations. These constraint equations in the KFParticle package are processed by the Kalman filter as ordinary additional dimensions.

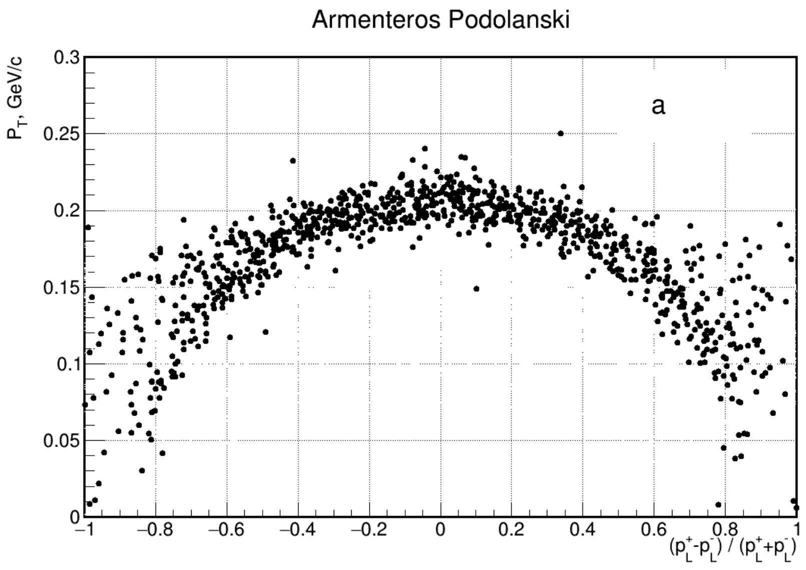
Currently, the KFParticle package includes more than 200 decays of short-lived particles, is actively developed and maintained, and is widely used in many experiments in high-energy physics by ALICE, CBM, STAR, sPHENIX and some others. See, for example, last KFParticle meeting https://indico.gsi.de/event/18341/ at 13 October 2023. All this makes the KFParticle package a universal platform for the reconstruction of short-lived particles and the analysis of physical events.

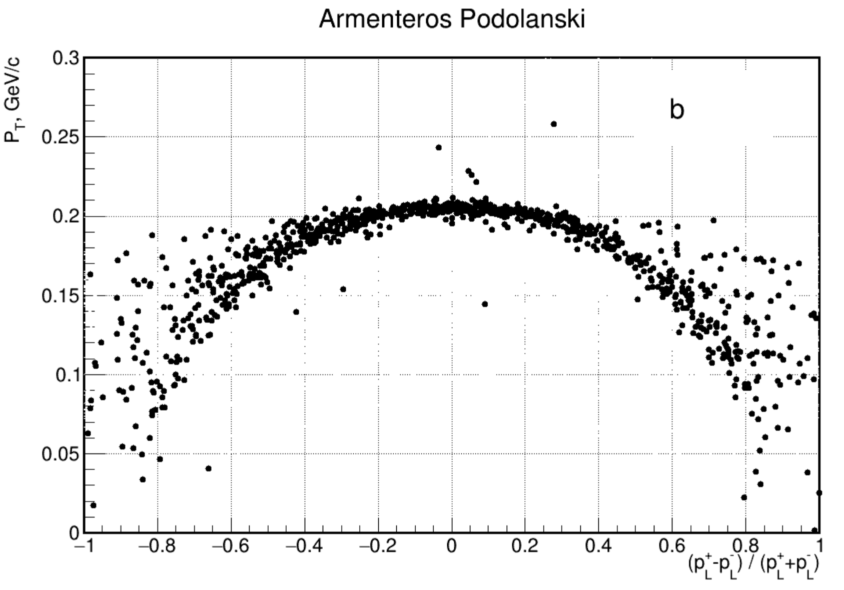
KFParticle short-lived particles decay modes



All these short-lived particles can be reconstructed in one job by **KFParticleFinder** method and with applying the corresponding cuts.

Example of SV constraint in KFParticle





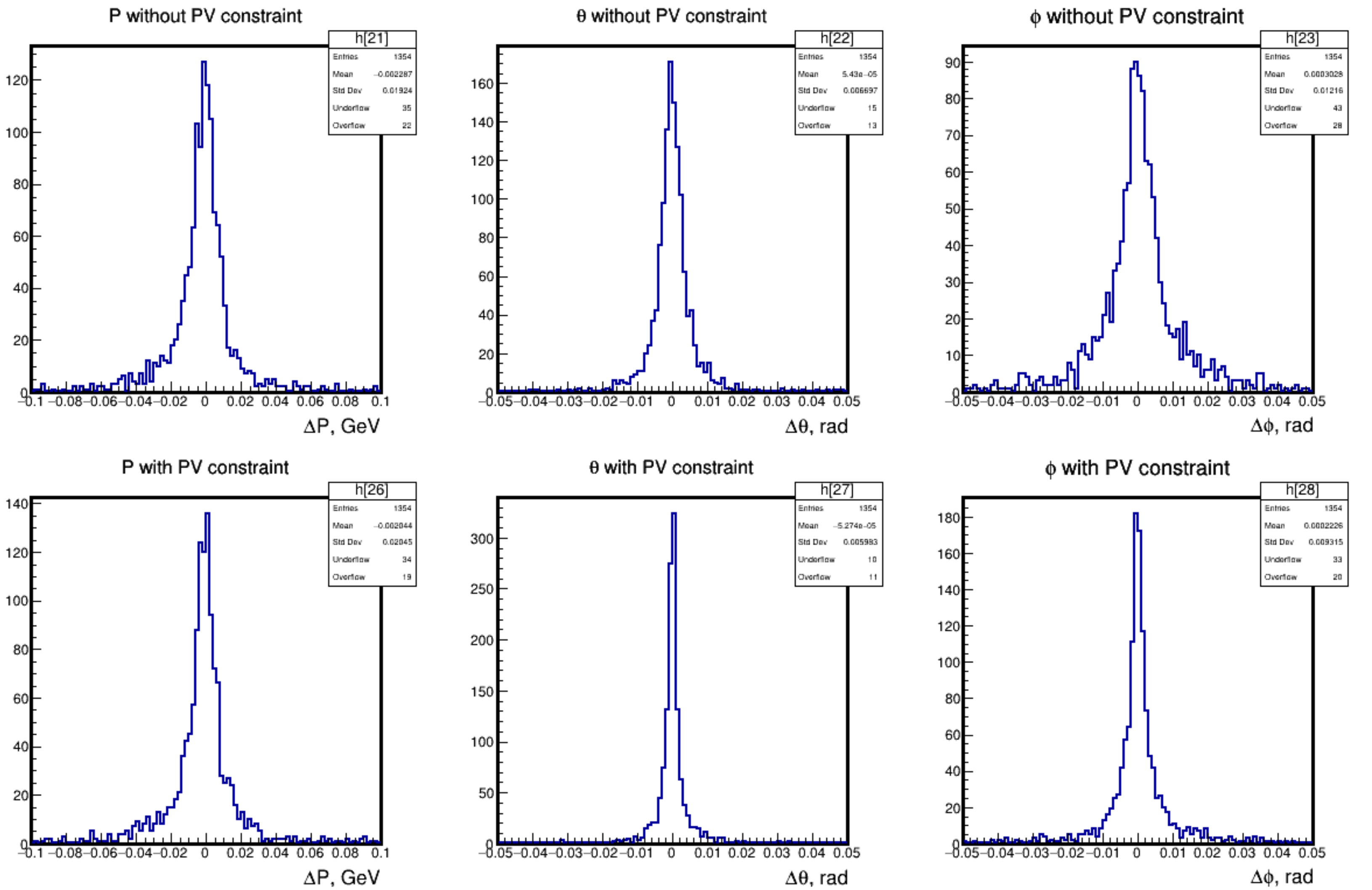
Example of the improvement parameters of the daughter particles for the decay of $K^0_S \rightarrow \pi+ \pi-$ using the constraint equation that both daughters pions are formed in the same reconstructed secondary vertex for Minimum Bias events:

a) for the initially reconstructed parameters of secondary pions,

b) for pions which parameters were tuned using the KFParticle package and the condition that the secondary tracks were formed at the same reconstructed vertex.

These improvement can be done in KFParticle with the next method - **SetProductionVertex(vtx)**

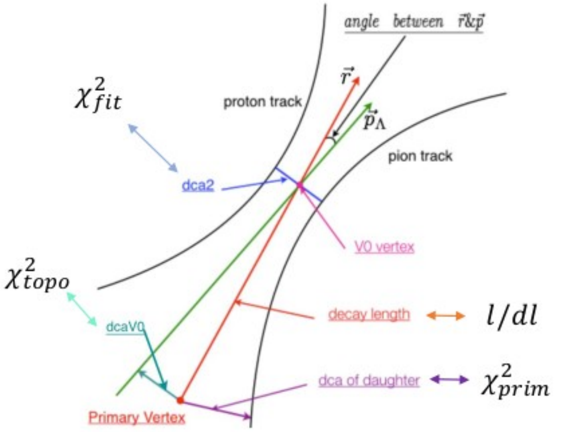
Short-lived particle parameters with PV constraint



This example shows the effect of constraint equation on the reconstructed parameters of short-lived particle, in this case the K^0_S meson for Minimum Bias events. The constraint condition is that the decayed particle was formed at the primary vertex. It can be seen that the application of this constraint equation significantly improves the parameters of a short-lived particle, which is especially for the reconstructed θ and φ - polar and azimuthal angles.

Construct(vDaughters,nDaughters,0,-1) – without PV constraint;
Construct(vDaughters,nDaughters,&pVtx,-1) - with PV constraint;

KFparticle selection cuts for secondary tracks



Default KFPF cuts

- $\chi^2_{fit} < 10$
- $l/dl > 5$
- $l > 5$ cm
- $\chi^2_{prim} > 18.6$
- $\chi^2_{topo} < 5$
- $d_{max} < 1$ cm

1. The variable $\chi2_{prim}$ is used at the first stage of a short-lived particle reconstruction to select secondary and primary vertex tracks.

2. The next variable $\chi2_{fit}$ refers to the geometric fitting procedure of the secondary vertex. It expresses the quality of a given kinematic fitting: small value of $\chi2_{fit}$ corresponds to good quality of the fitting, whereas large value means poor quality of the fitting. Therefore, the value of $\chi2_{fit}$ can be interpreted as whether the trajectories of the daughter tracks intersect within their measurement precision.

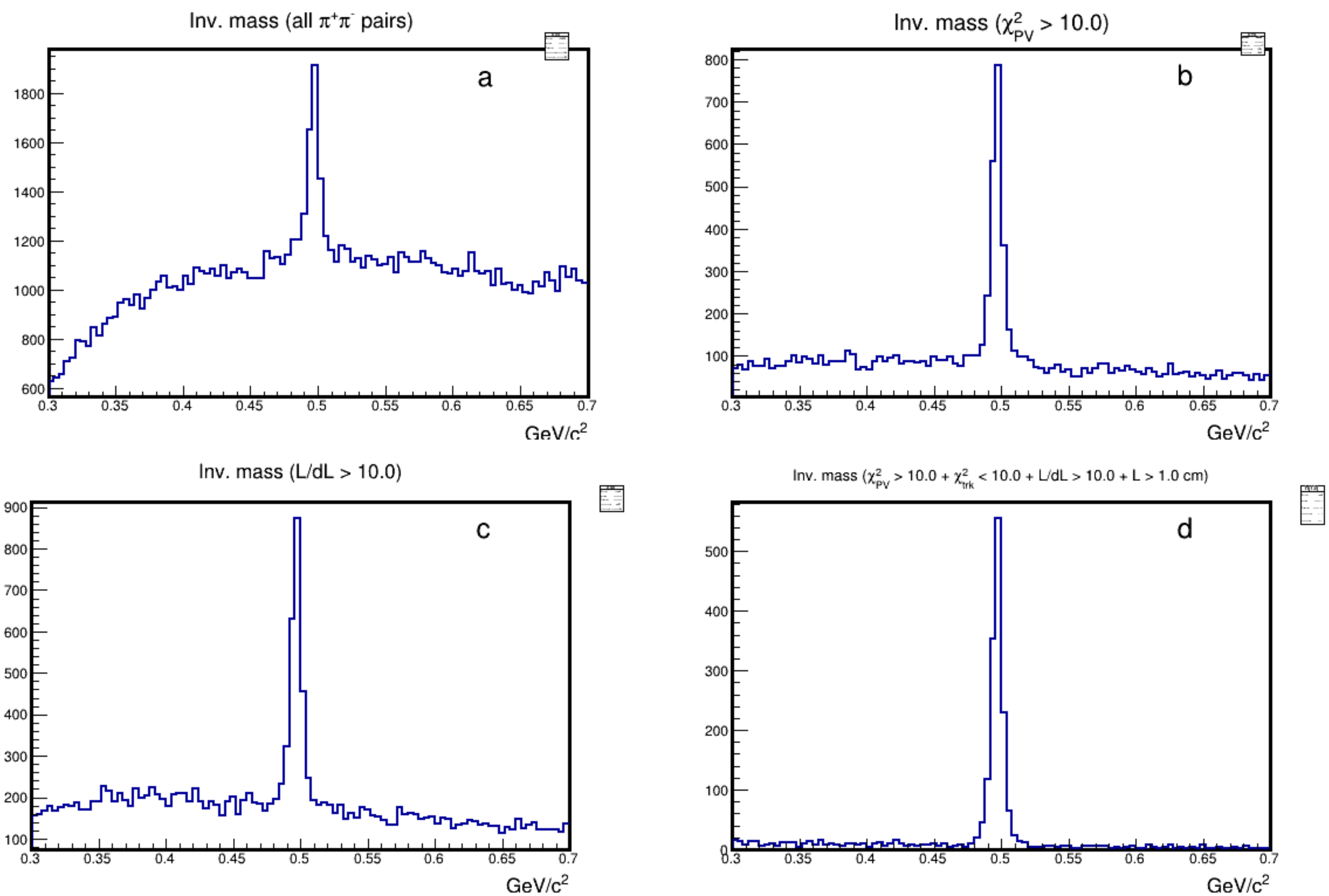
3. The variable $\chi2_{topo}$ is often used, which reflects the probability that the reconstructed short-lived particle actually formed at the primary vertex, taking into account measurement errors of vertex and tracks.

4. The next two variables that are also used for reconstruction of short-lived particles, and which can also significantly suppress the combinatorial contribution from primary tracks, are:

   - L is the decay length or the distance between the primary vertex and the decay point of the reconstructed short-lived particle,

   - the ratio of this distance L to the error of its determination (L/dL). This variable L/dL describes how far from the primary vertex the short-lived particle decayed, taking into account reconstruction errors.

Effect of cuts on the K^0_S selection



Example of effect of the next cuts ($\chi2_{prim}$, L and L/dL) on the selection of K^0_S meson ($K^0_S \rightarrow \pi+ \pi-$) for Minimum Bias events is presented. The lifetime of K^0_S meson is a significant amount (τ = 2.7 cm), and this allows to reliable separate the K^0_S mesons from the primary interaction vertex. The distribution of the invariant mass of two oppositely charged pions is shown on the figure:

a) all pions combinations,
b) applying PV cuts ($\chi2_{prim}$ > 10.0),
c) applying L/dL cut (L/dL > 10.0),
d) three cuts together: $\chi2_{prim}$ > 10.0 + L/dL > 10.0 + L > 1.0 cm.

Summary

1. KFParticle package is a universal platform for the reconstruction of short-lived particles and secondary vertex written in the detector independent way.

2. KFParticle also can be use for primary vertex reconstruction.

3. Maybe for some specific decays it needs to develop separate algorithms (?).