

Актуальные проблемы физики тяжелых ионов и адронов

Школа-интенсив по физике кварк-
глюонной (адронной) материи

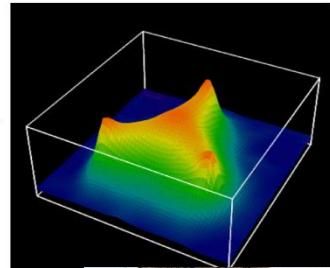
УНЦ ОИЯИ
20.11.2023

Теряев О.В.

ОИЯИ

teryaev@jinr.ru

Проблемы КХД как фундаментальной теории сильных взаимодействий (лекции В. Брагуты, А. Гуськова, Д. Казакова, В. Рябова)



КОНФАЙНМЕНТ!!

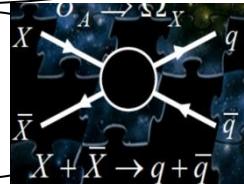
Адроны

Структура

Спектроскопия

HQ, Экзотика

Лептоны,
НФ



Тяжелые ионы

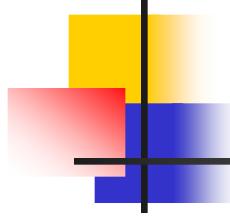
Фазы,

крит.точка

Потоки

Фемтоскопия

Транспорт



Некоторые общие для адронов и тяжелых ионов проблемы

Спиновая структура адронов

Гравитация

Аномалии

Проявления плотной среды через
спиновую поляризацию соударениях
тяжелых ионов

“Эффективная” (в соответствии с **принципом эквивалентности**) гравитация в столкновениях тяжелых ионов

УФН 193 (2023) 2, 113-154

• e-Print: 2204.00427 [hep-th]

Эффекты общей теории относительности в прецизионных спиновых экспериментах по проверке фундаментальных симметрий

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Действительно, локальную угловую скорость Ω , можно оценить, полагая, что скорость меняется на величину порядка скорости света c на масштабах, соответствующих размеру ядра R_A . Ее отношение к угловой скорости вращения Земли (6.8) удобно представить в виде

$$\eta_{\text{rot}} = \frac{\Omega}{\omega_{\oplus}} = \frac{c}{R_A} \cdot \frac{T_{\oplus}}{2\pi} = \frac{1}{2\pi} \cdot \frac{cT_{\oplus}}{R_A} \approx 10^{27} \quad (10.1)$$

отношения световых суток (расстояния, проходимого светом за время оборота Земли вокруг своей оси T_{\oplus} , и примерно в 150 раз превышающего ее расстояние до Солнца) и размера ядра.

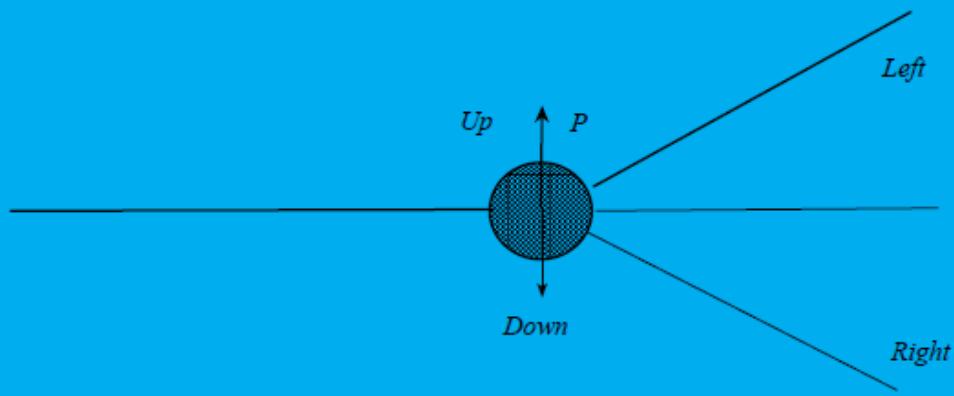
Оценку для ускорения можно связать с оценкой для угловой скорости, помножив и разделив очевидное выражение для нее на $T_{\oplus}/2\pi$:

$$\eta_{\text{acc}} = \frac{c}{R_A} \cdot \frac{c}{g_{\oplus}} = \eta_{\text{rot}} \frac{2\pi c}{T_{\oplus} g_{\oplus}} \approx 10^{30}. \quad (10.2)$$

Дополнительный фактор ~ 2000 пропорционален отношению скорости света к скорости, приобретаемой в течение суток при движении с ускорением g_{\oplus} .

Single Spin Asymmetries (vector polarization)

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \rightarrow \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$ \vec{n} is the normal to the scattering plane.

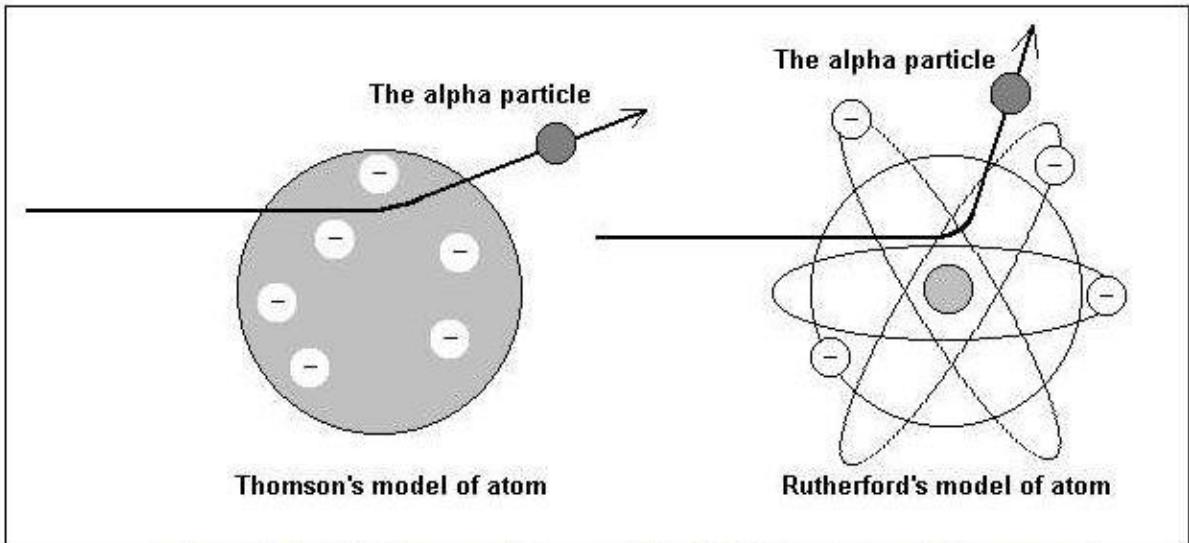
Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

Polarization data has often been the graveyard of fashionable theories. If theorists had their way, they might just ban such measurements altogether out of self-protection.

J.D. Bjorken
St. Croix, 1987

Вероятность рассеяния на определенном прицельном расстоянии пропорциональна сечению

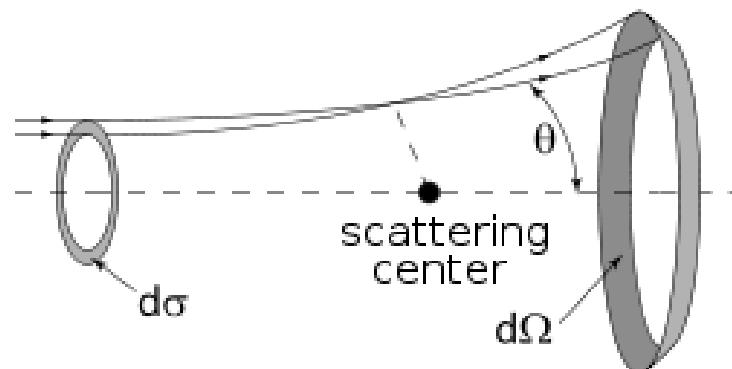


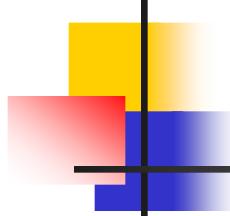
The models of the Thomson's atom and Rutherford's atom; and the expected aberrations of alpha particle in both cases.

$$\sigma(\Omega)d\Omega = \frac{\text{number of particles scattered into solid angle } d\Omega \text{ per unit time}}{\text{incident intensity}}$$

$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 m v_0^2} \cot \frac{\Theta}{2}. \quad 2\pi I b |db| = I \sigma d\Omega \quad \sigma = \frac{b}{\sin \Theta} \left| \frac{db}{d\Theta} \right|$$

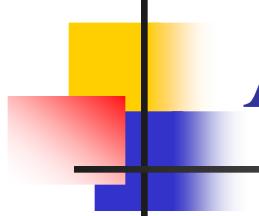
$$\sigma = \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 m v_0^2} \right)^2 \csc^4 \left(\frac{\Theta}{2} \right).$$





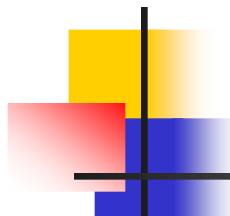
SSA

- Parity conservation – normal to (elastic or inclusive) scattering plane (**HIC – angular momentum**)
- Interference – LS coupling (**HIC- hydrodynamical axial anomaly**)
- T conservation – absorptive phases (**HIC : dissipation**)



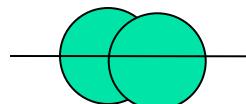
Λ -polarisation

- Self-analyzing (spin-momentum couplings) in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: '87): Randomization – smearing – of the scattering plane



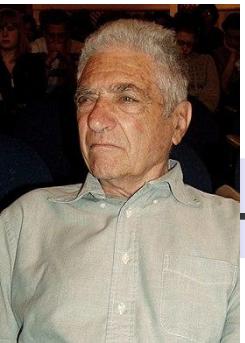
Global polarization

- Global polarization normal to REACTION plane



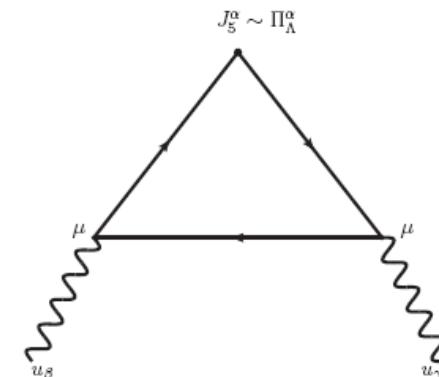
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?

Anomalous mechanism



Аномалии – нарушение законов сохранения квантовыми эффектами (впервые – Дж. Стейнбергер (1921-2020) в 1949 г. (в 1988 г. как экспериментатор стал лауреатом Нобелевской премии за открытие 2 типов нейтрино)

- 4-Velocity is also a **GAUGE FIELD** (V.I. Zakharov et al): $\mu \varrho = \mu J_0 V^0 \rightarrow \mu J \gamma V^\gamma$
 $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$
- **Triangle anomaly** leads to polarization of quarks and hyperons
(Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin
(Efremov, OT'88)
- **4-velocity instead of gluon field!**



“Anomalous” mechanism

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

- Prediction of decrease with energy (due to chemical potential)
- Prediction of $P \sim 1\%$

BAZNAT, GUDIMA, SORIN, AND TERYAEV

PHYSICAL REVIEW C 88, 061901(R) (2013)

$$\langle P_\Lambda \rangle \sim \frac{\langle \mu^2 \rangle N_c H}{2\pi^2 \langle N_\Lambda \rangle}$$

For numerical estimate at NICA energies, we take (see Fig. 3) $H = 30 \text{ fm}^2 (c = 1)$ and, as typical values, $\langle \mu^2 \rangle = 900 \text{ MeV}^2$, $\langle N_\Lambda \rangle = 15$ to get $\langle P_\Lambda \rangle \sim 0.8\%$. This value is

- Postdiction of larger polarization of antilambdas

ALEXANDER SORIN AND OLEG TERYAEV

PHYSICAL REVIEW C 95, 011902(R) (2017)

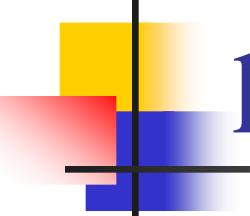
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)} \right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \quad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

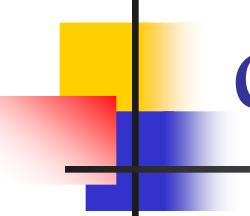
This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

The proportionality of the polarization to the square of the chemical potential related to C -even parity of axial current leads to the same sign of polarization of Λ and $\bar{\Lambda}$ hyperons. The smaller number of the latter should result in a larger fraction of the axial charge, corresponding to each antihyperon and to a larger absolute value of polarization. Detailed numerical sim-



Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity $\sim c/\text{Compton wavelength}$)
- ~ 27 orders of magnitude faster than Earth's rotation
- Differential rotation – vorticity
- P-odd :May lead to various P-odd effects
- Calculation in kinetic quark - gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes):
Baznat,Gudima,Sorin,OT, PRC'13,16



Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L = \sum r \times p$
- Differential pseudovector characteristics – vorticity
- $\omega = \text{rot } v$
- Pseudoscalar – helicity
- $H \sim \langle (v \cdot \text{rot } v) \rangle$
- Maximal helicity – Beltrami chaotic flows
 $v \parallel \text{curl } v$

Simulation in QGSM

(First calculation of **vorticity** in kinetic model;
Baznat, Gudima, Sorin, OT, PRC'13)

50 × 50 × 100 cells $dx = dy = 0.6 \text{ fm}$, $dz = 0.6/\gamma \text{ fm}$

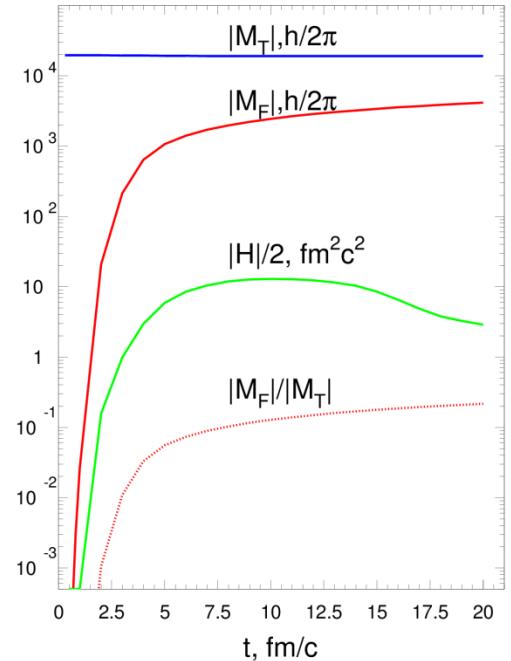
- Velocity

$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

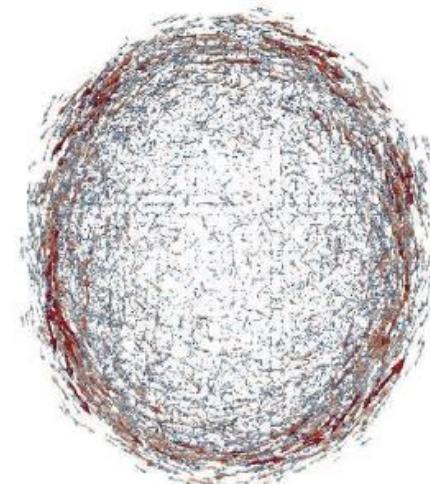
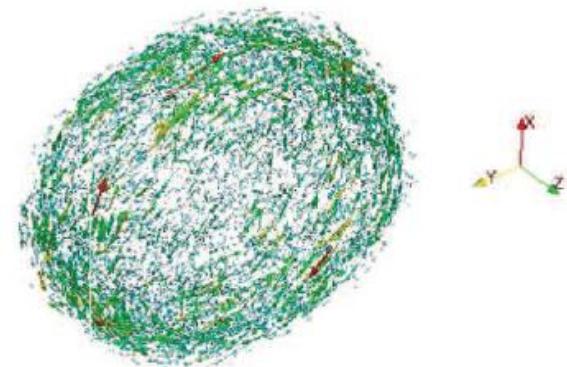
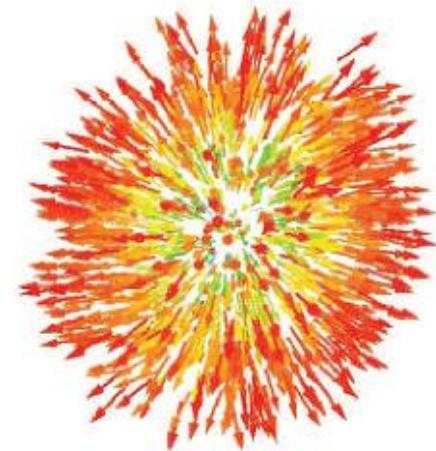
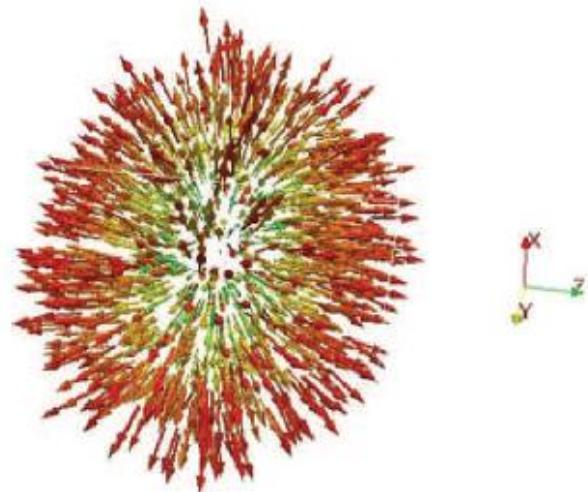
- Vorticity – from discrete partial derivatives

Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- ($\sim 10\%$ of total)
- Conservation of OAM
a good accuracy!

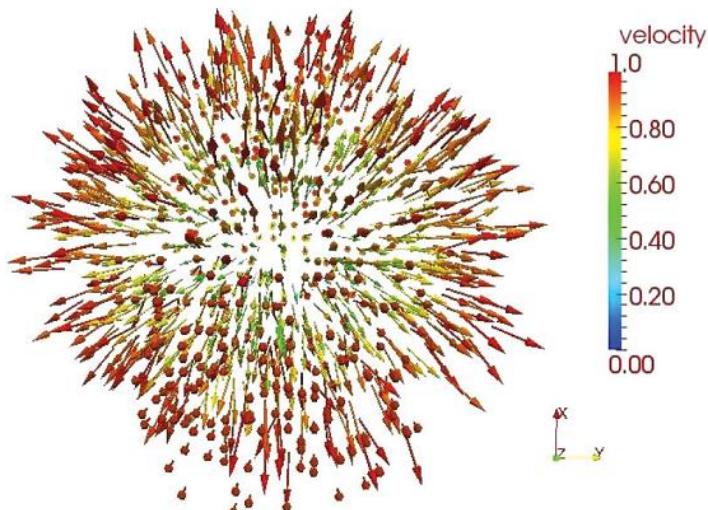
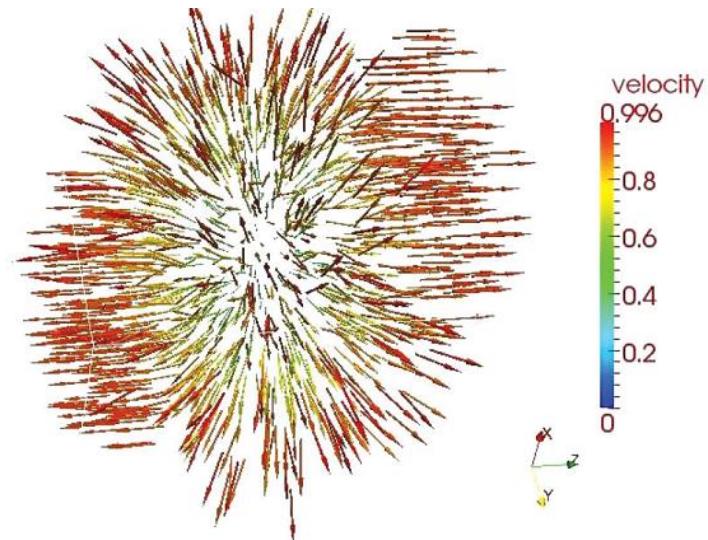


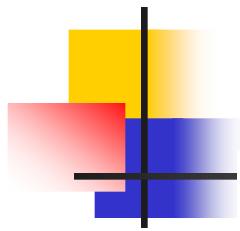
Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



Distribution of velocity (“Little Bang”)

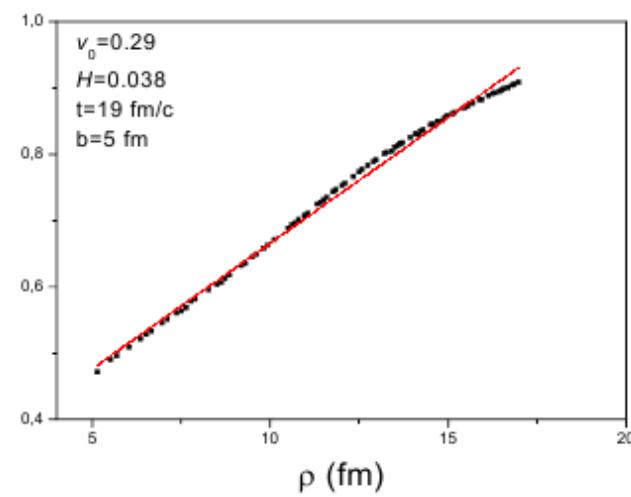
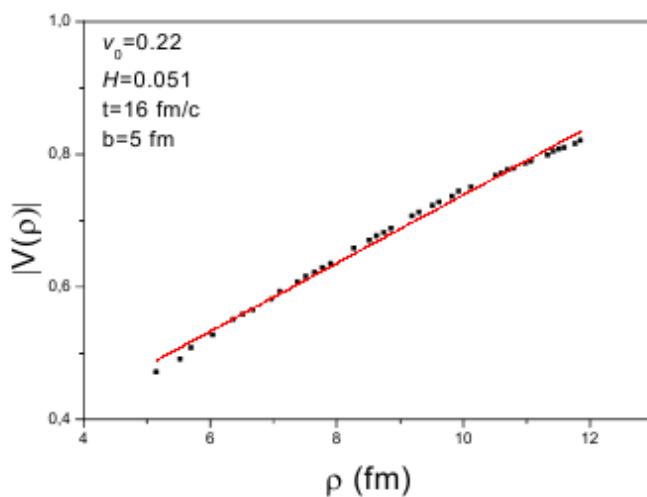
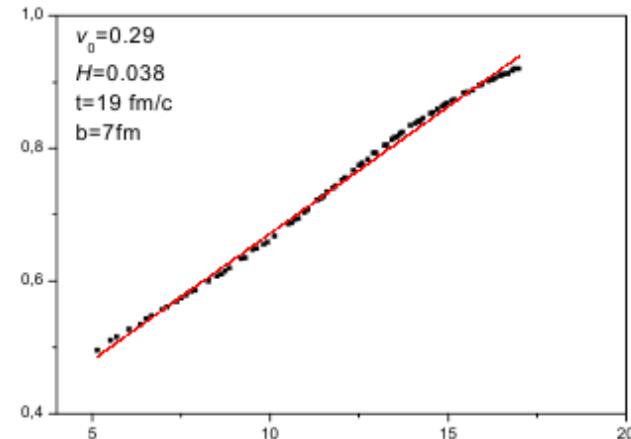
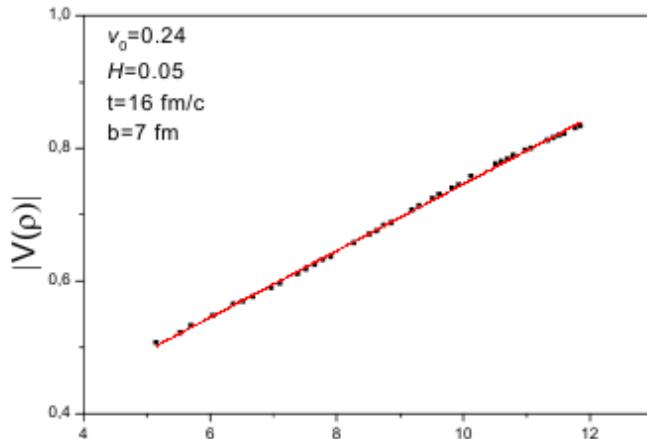
- 3D/2D projection
- z-beams direction
- x-impact parameter





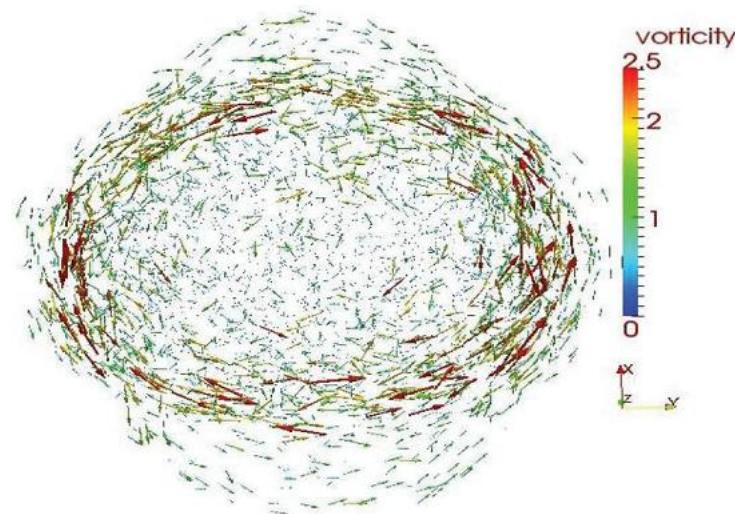
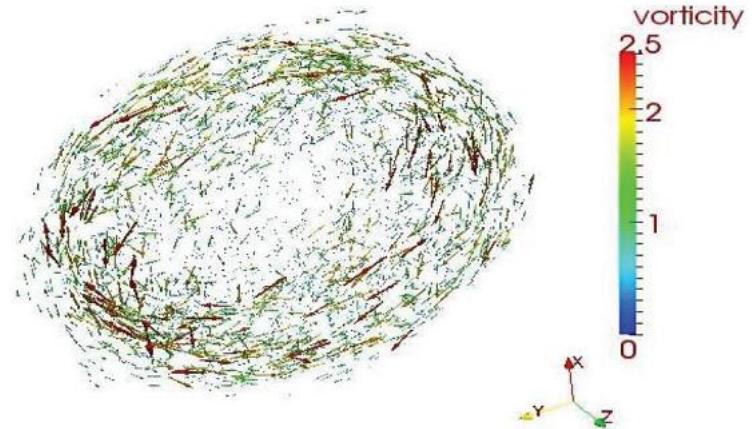
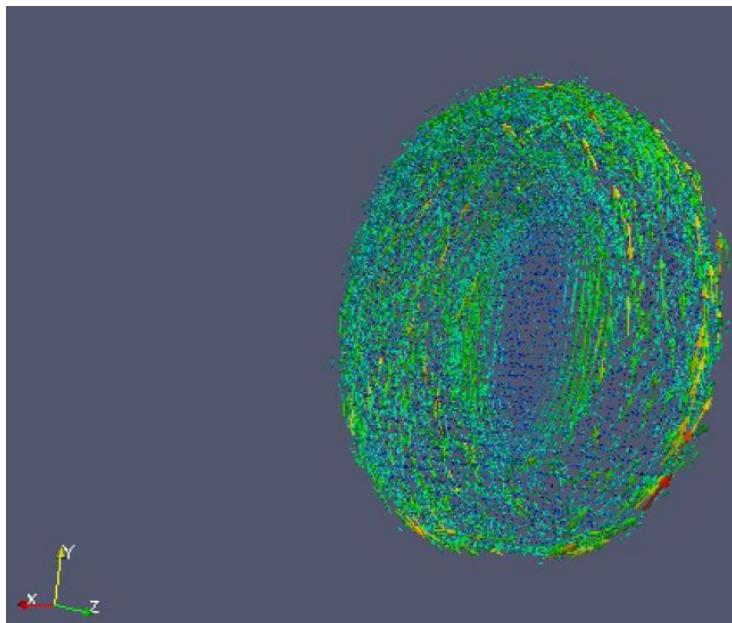
“Little Hubble” in PHSD

$$v=v_0+H\rho$$

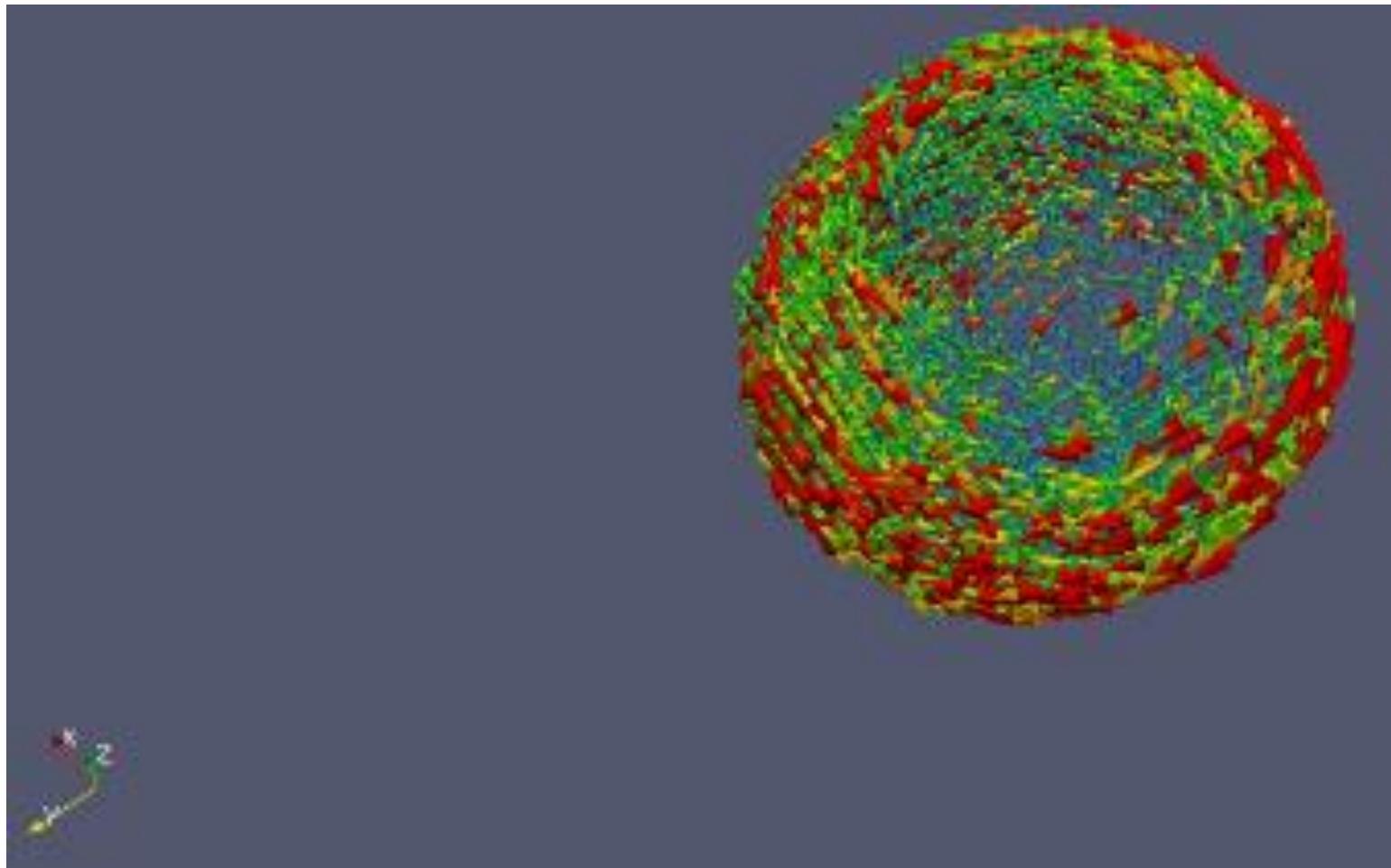


Distribution of vorticity (“Femto-cyclones”, “Little galaxies”)

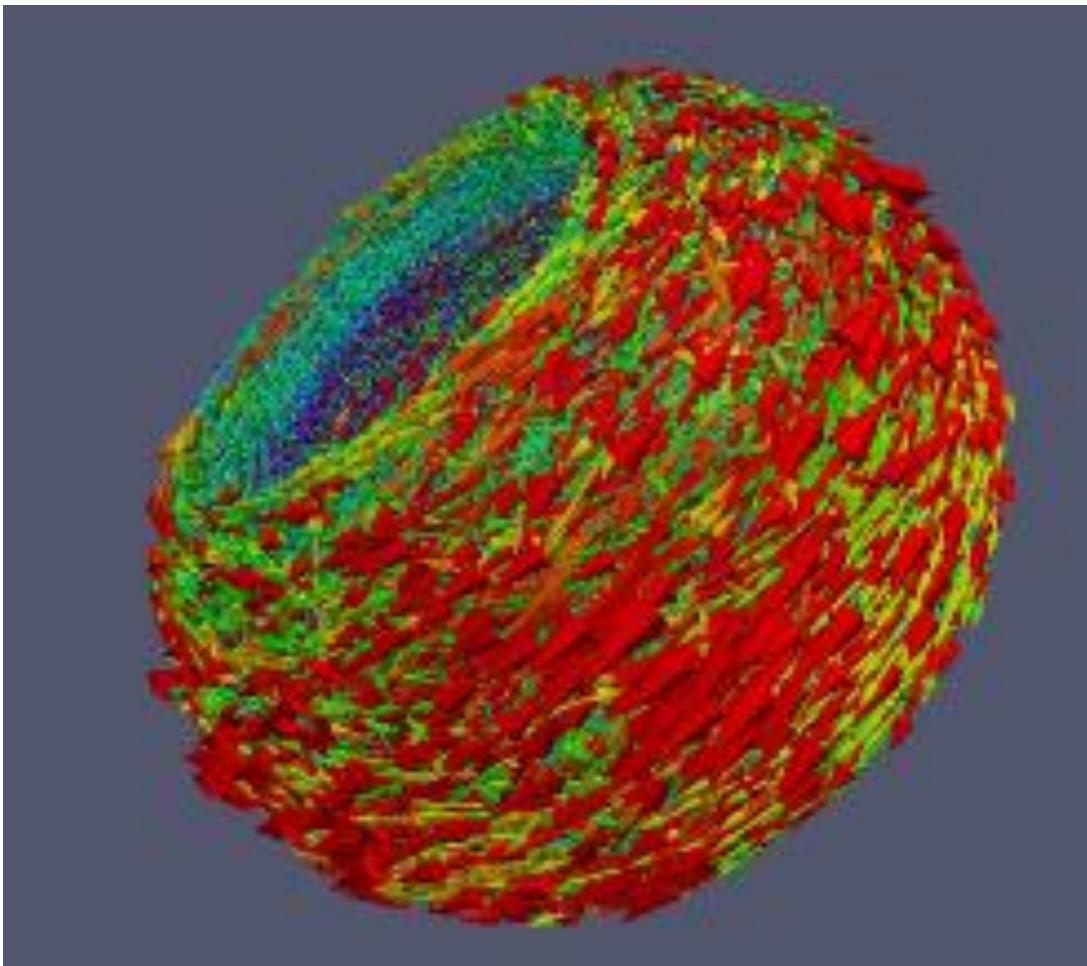
- Layer (on core - corona borderline) patterns



Vortex sheet (Femto-cyclone) with fixed direction of L

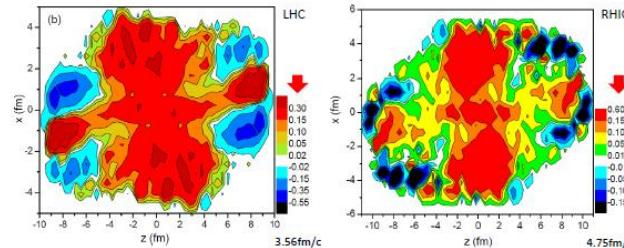


Vortex sheet (Average over L directions)

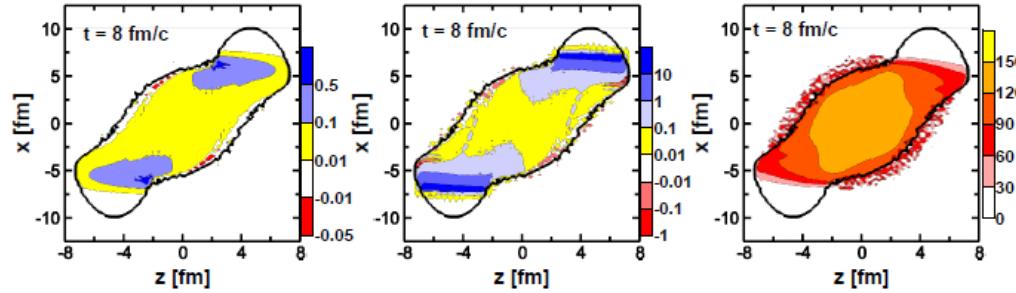


Vortex sheets

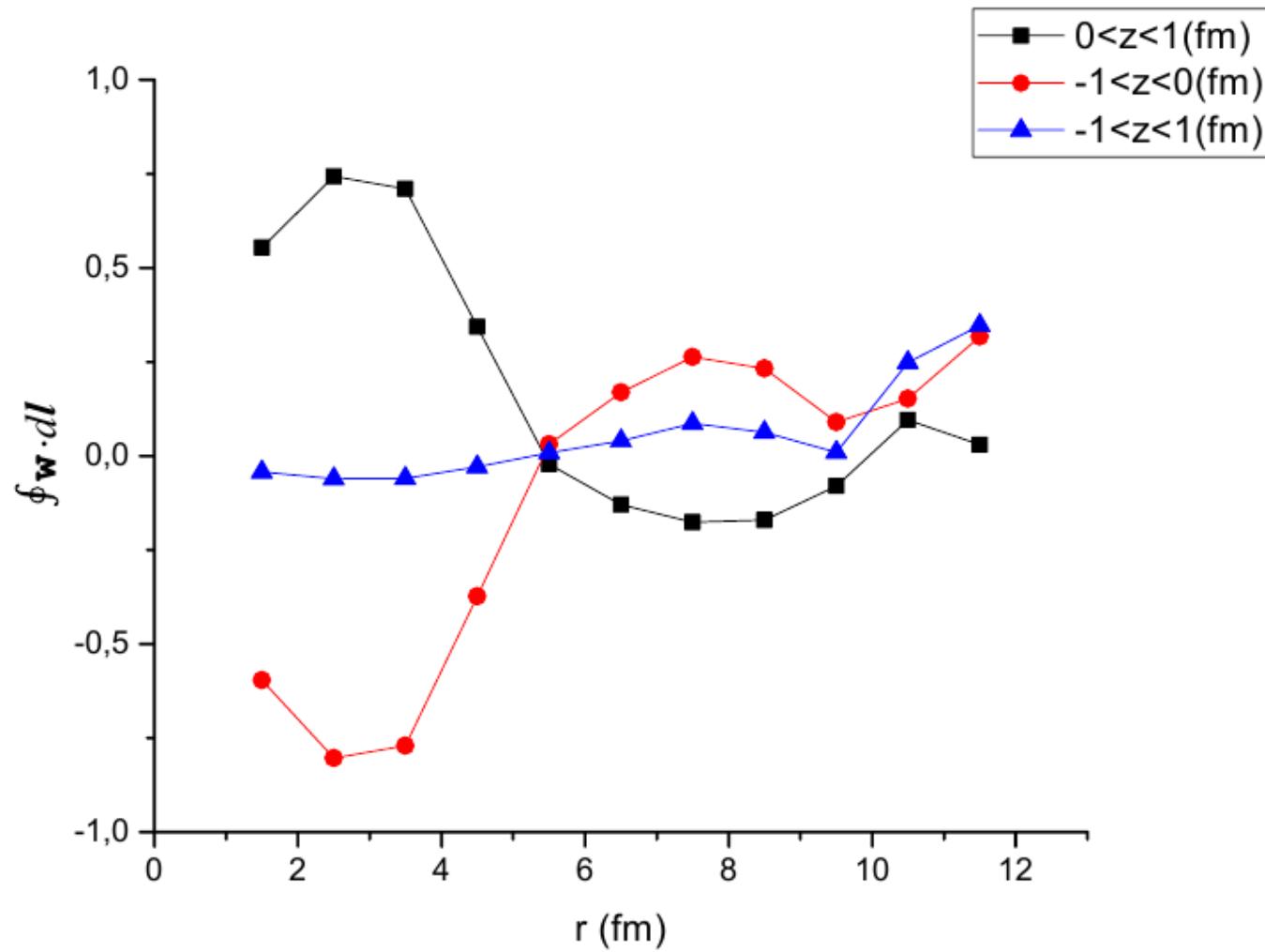
- Naturally appears in kinetic models
- Absent in viscous HD (L. Csernai et al)



- Appears in 3 fluid dynamics model (Yu. Ivanov, A. Soldatov, PRC'17)



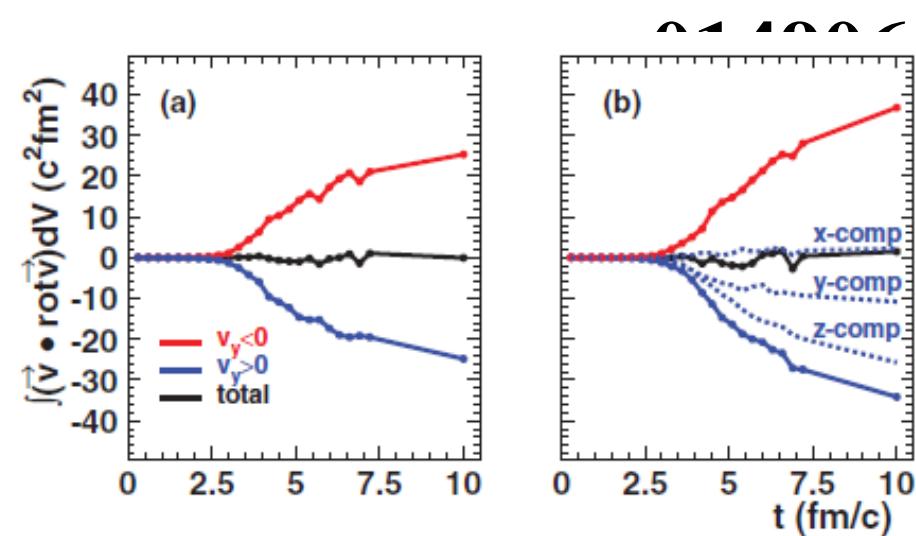
Mirror vortex rings in PHSD



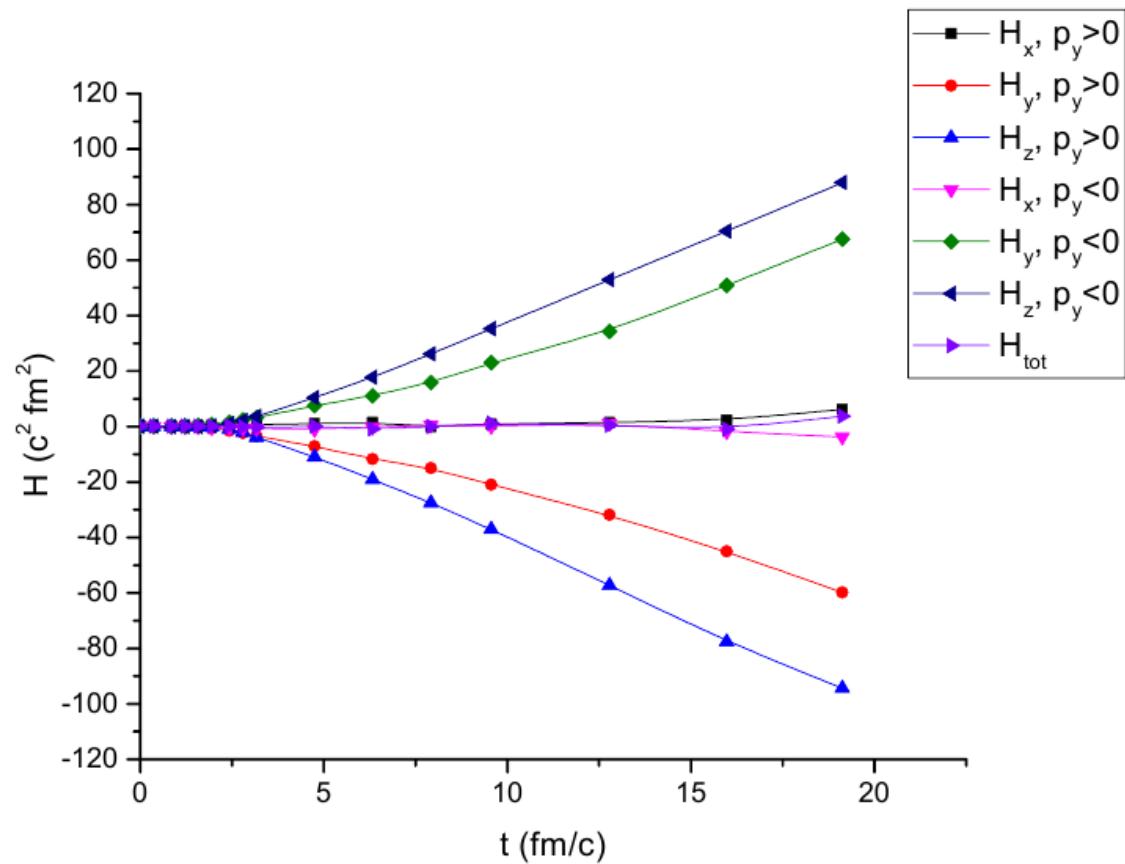
Helicity separation in QGSM

PRC88 (2013) 061901

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (OT, Usubov, PRC92 (2015))

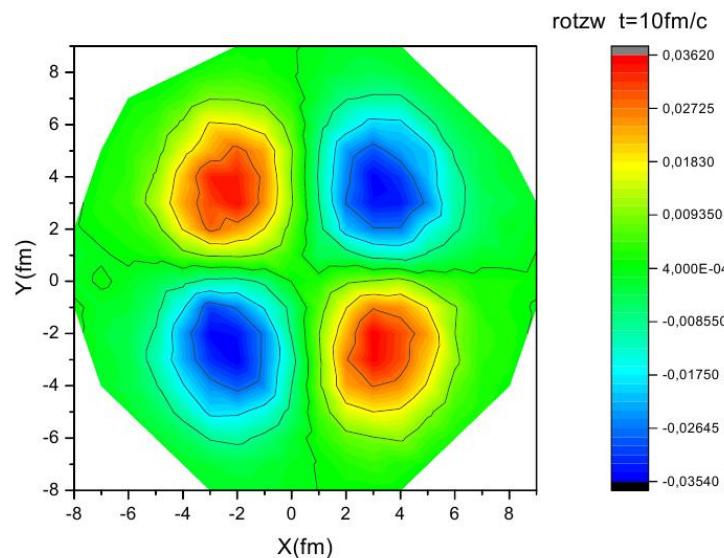


Helicity@PHSD

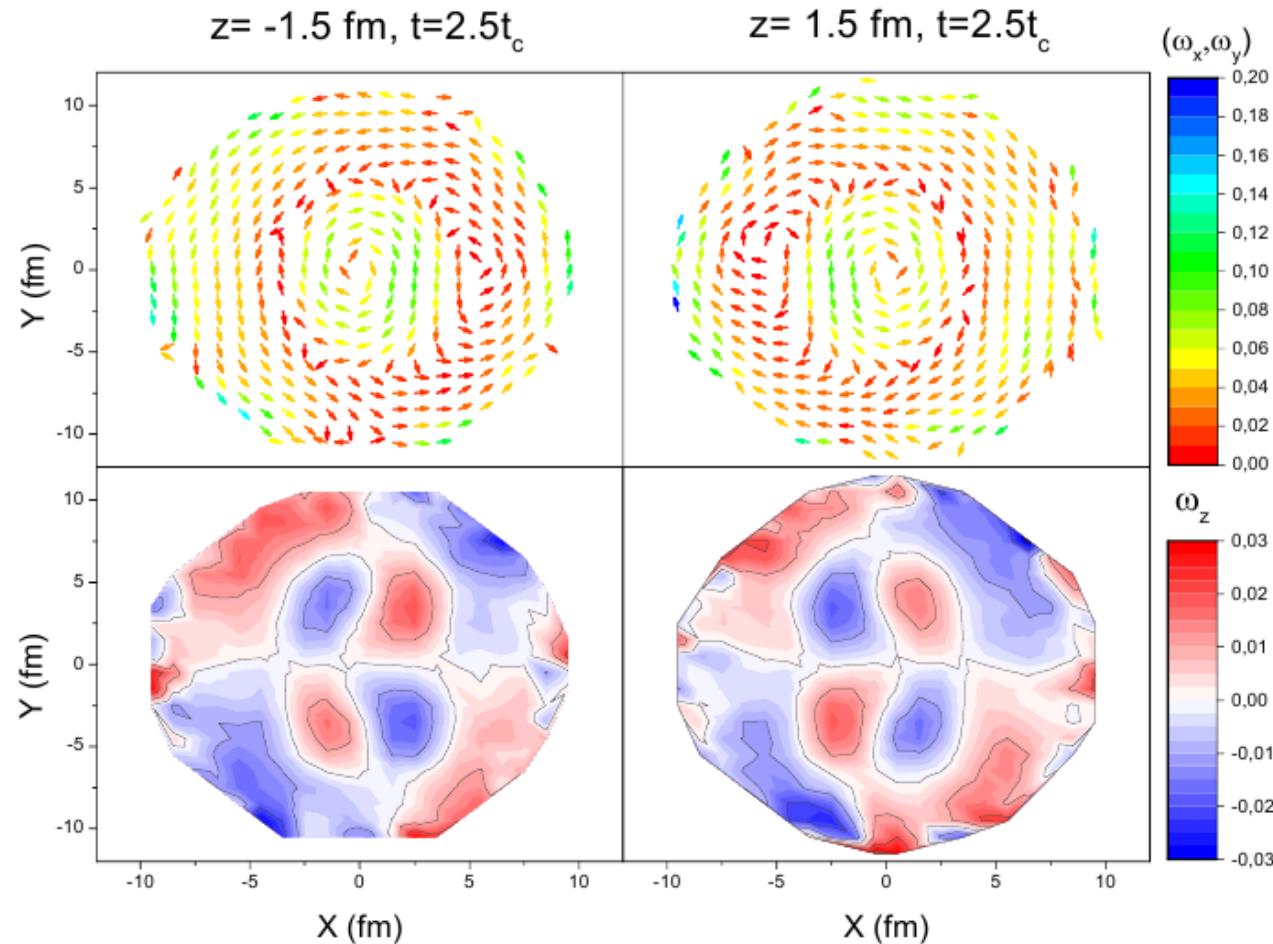


Structure of vorticity (Baznat, Gudima, Sorin, OT'17)

- y-component: constant vorticity, velocity changes sign
- **z-component: quadrupole structure of vorticity**

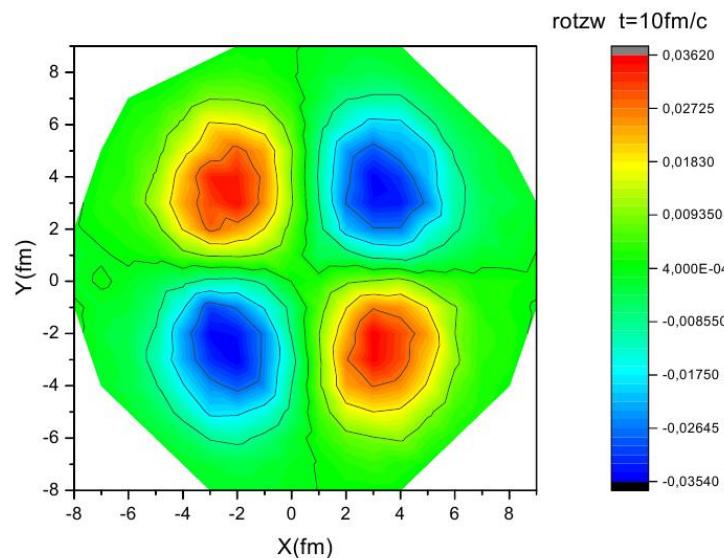


PHSD: 2nd Quadrupole Structure

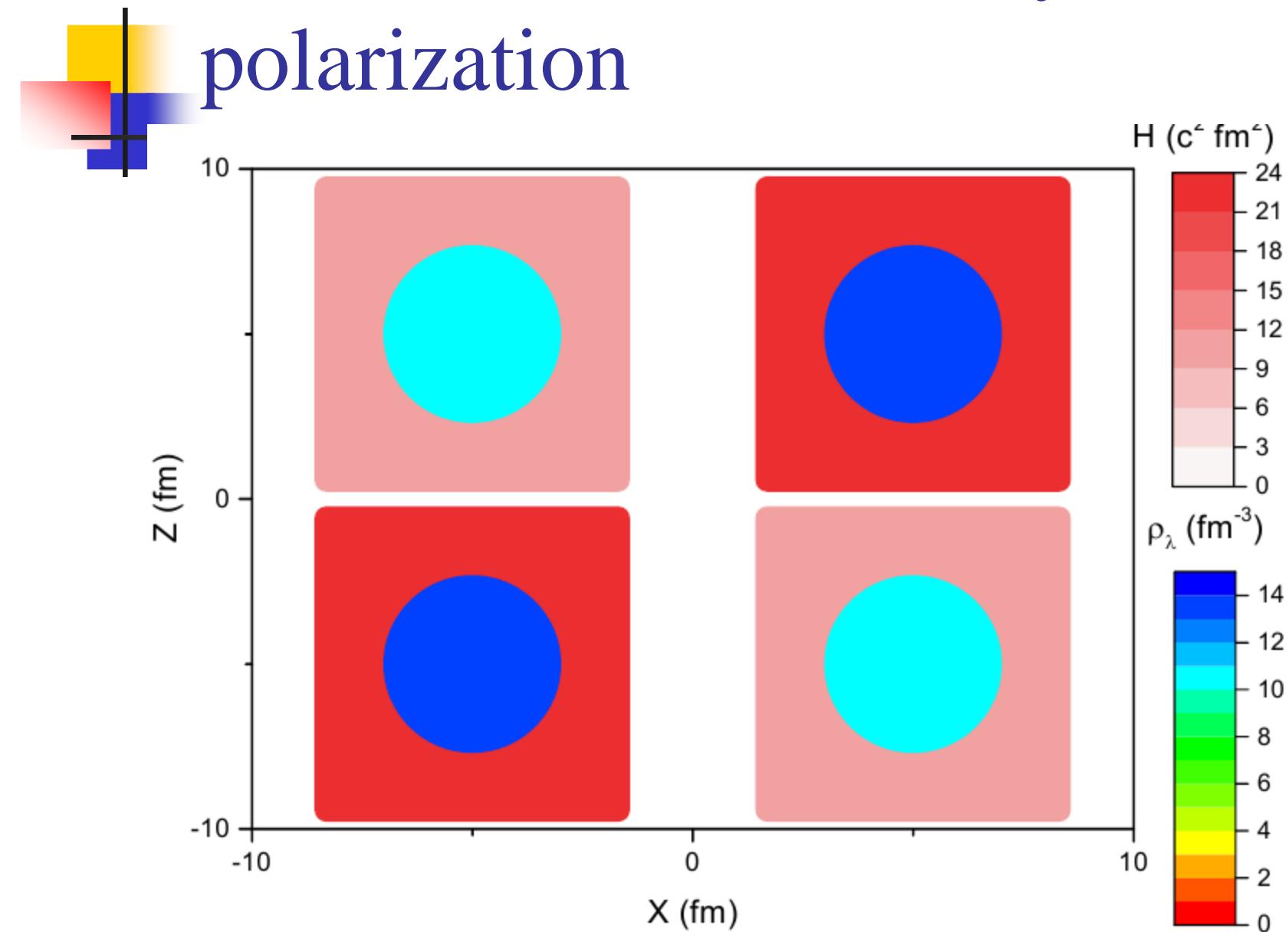


Structure of vorticity (Baznat, Gudima, Sorin, OT'17)

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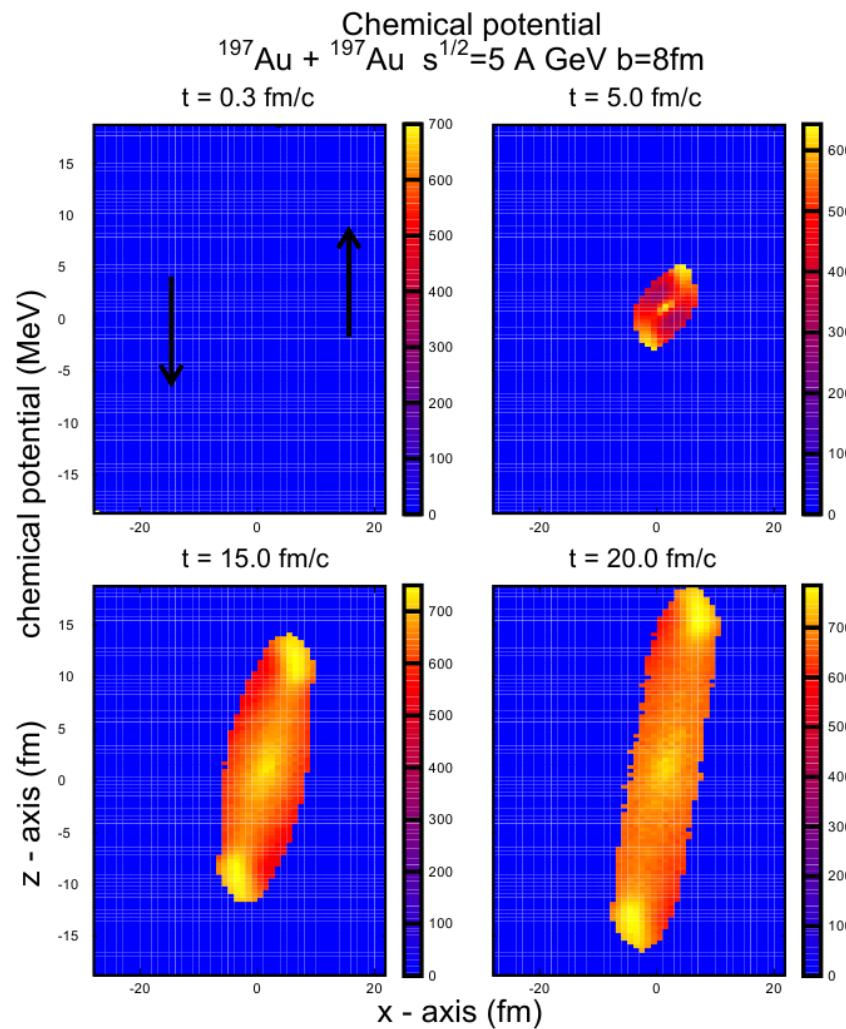


XZ- structure of helicity and polarization

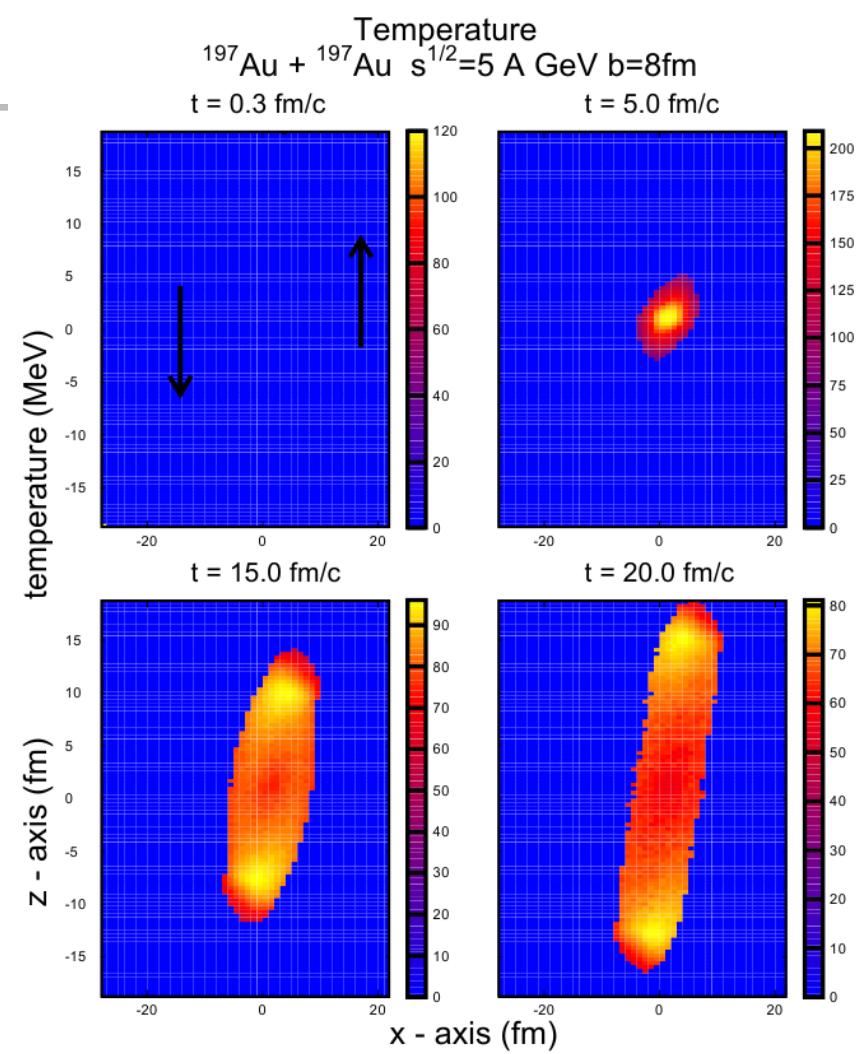


Chemical potential : Kinetics \rightarrow TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: $y=0$

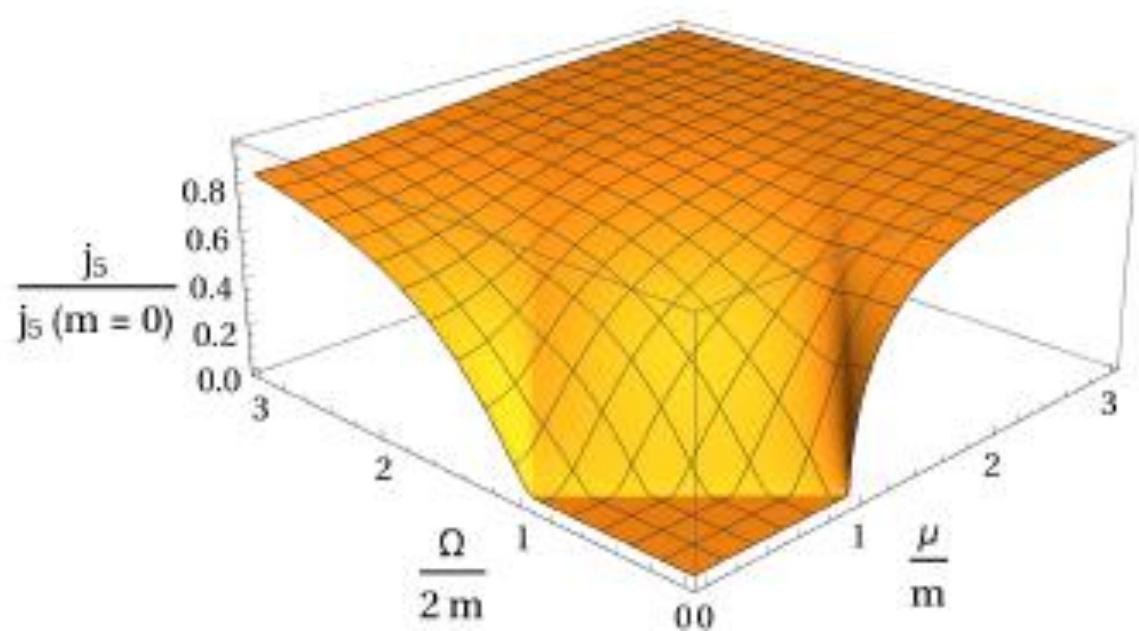


Temperature



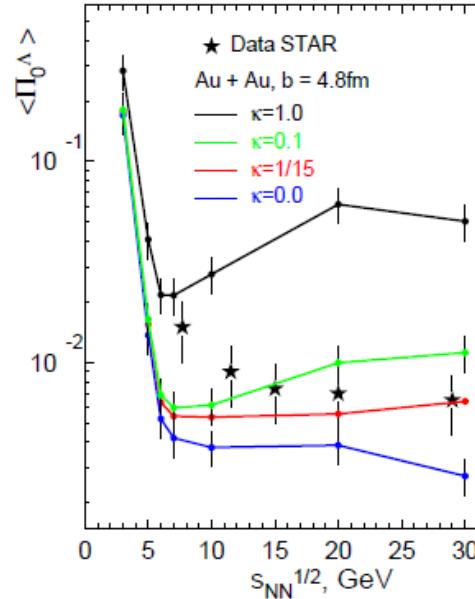
Axial current in TD approach: Role of mass effects (Prokhorov, OT, Zakharov, PRD98 (2018), 071901)

- Threshold effects in chemical potential and angular velocity; 1906.03529: acceleration (important for longitudinal polarization)



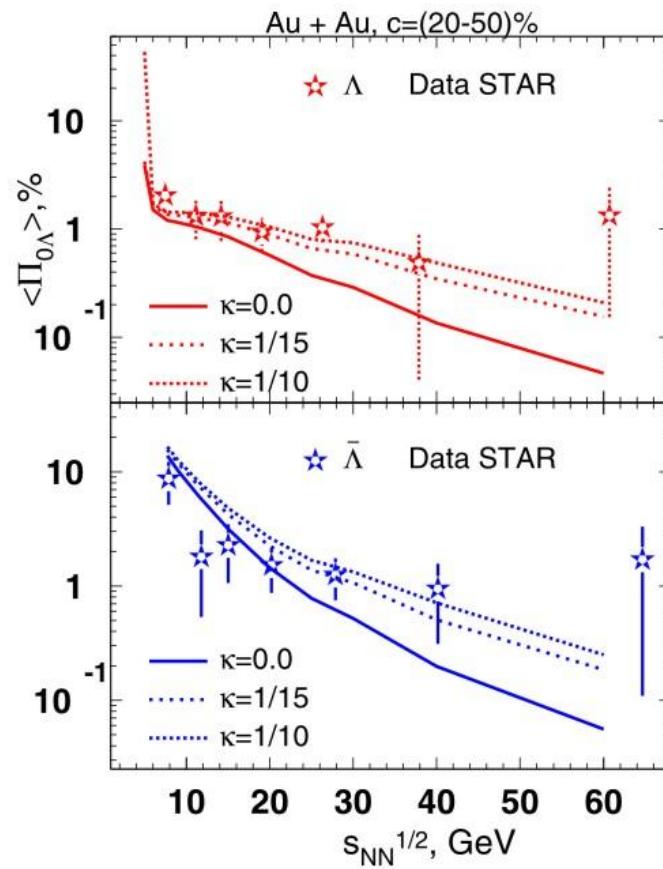
The role of (gravitational anomaly related) T^2 term

- Different values of coefficient probed



- LQCD suppression by collective effects supported

Λ vs Anti Λ



КХД и гравитация (продолжение)

- В термализованной кварк-глюонной среде возникают квантовые эффекты (**Унру: квантовое измерение в ускоренной системе отсчета**), соответствующие (**эмержентной**) конической сингулярности: связь термальной и гравитационной потери информации?
- Гравитационная **квантовая аномалия** порождает термальные эффекты, сохраняющиеся **после** ее исчезновения
- **Голография: AdS/QCD**



Gravitational chiral anomaly in hydrodynamics

G. Yu. Prokhorov, O.V. Teryaev, V.I. Zakharov, Phys. Rev. Lett. 129, 151601 (2022)

- In heavy ion collisions (in particular, at **NICA**) a relativistic quantum liquid is formed with **extremely high vorticity** ω^μ and **acceleration** a^ν . The axial current is related to the hyperon **polarization**. **New** contributions to current and **polarization**:

$$\text{KVE: } j_A^\mu = \lambda_1(\omega_\nu \omega^\nu) \omega^\mu + \lambda_2(a_\nu a^\nu) \omega^\mu$$

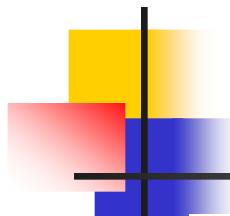
Transport coefficients

- The novel kinematical vortical effect (KVE) depends only on vorticity and acceleration, but is independent of **temperature** and **chemical potential**, and is determined by a **quantum anomaly** in **curved** space.
- The effect exists even when there are **no gravitational** fields ('Cheshire cat' or "Structural stability of hydrodynamics")

Connection with anomaly and gravity:

$$\lambda_1 - \lambda_2 = 32\mathcal{N}$$

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}^{\quad\lambda\rho}$$



Gravitational Formfactors

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha / 2M \right] u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Distributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x, b) = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q} \cdot \vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1 + \tau}$$

$$\rho_0^{\text{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

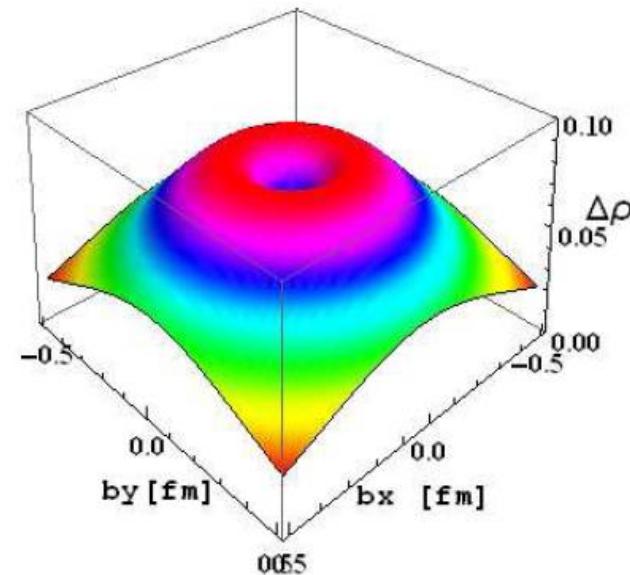
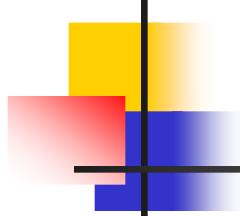


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)



Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \quad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

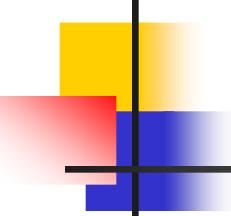
- Static limit

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$
$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q) \quad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

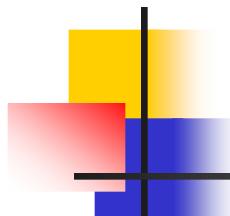
- Mass as charge – equivalence principle
(Einstein '10-11, Praha)



Equivalence principle

- Newtonian – “Falling elevator” – well known and checked with high accuracy (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun' ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservation laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way (On the Earth- quantum measurement! Not just pendulum)

- For GEDM –checked with sometimes controversial results
- For AGM not checked on purpose but in fact checked in the same atomic spins experiments at % level (Silenko, OT'07)



Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

$$\vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i}$$

spin dragging twice
smaller than EM

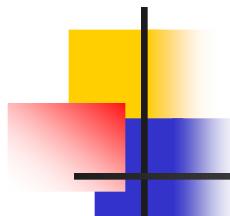
- Lorentz force – similar to EM case: factor 1/2
cancelled with 2 from frequency same as EM

$$h_{00} = 2\phi(x)$$

Larmor

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \quad \vec{H}_L = \text{rot} \vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle



Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

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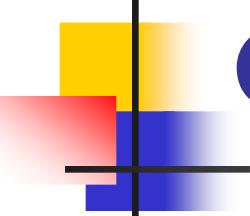
Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

- If (CP-odd!) GEDM=0 \rightarrow constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \mathbf{B} \cdot \mathbf{S} - \zeta \hbar \boldsymbol{\omega} \cdot \mathbf{S}, \quad \zeta = 1 + \chi$$

$$|\chi(^{201}\text{Hg}) + 0.369\chi(^{199}\text{Hg})| < 0.042 \quad (95\%\text{C.L.})$$



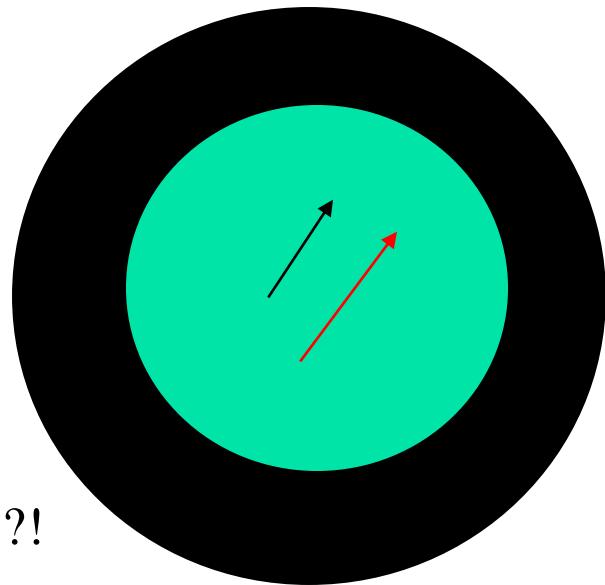
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $\langle P+q | T^{mn} | P-q \rangle = P^m \langle P+q | J^n \rangle | P-q \rangle / e$ up to the terms linear in q
- Special role of $g=2$ for any spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also $g=2$ for Black Holes. Indication of “quantum” nature?!

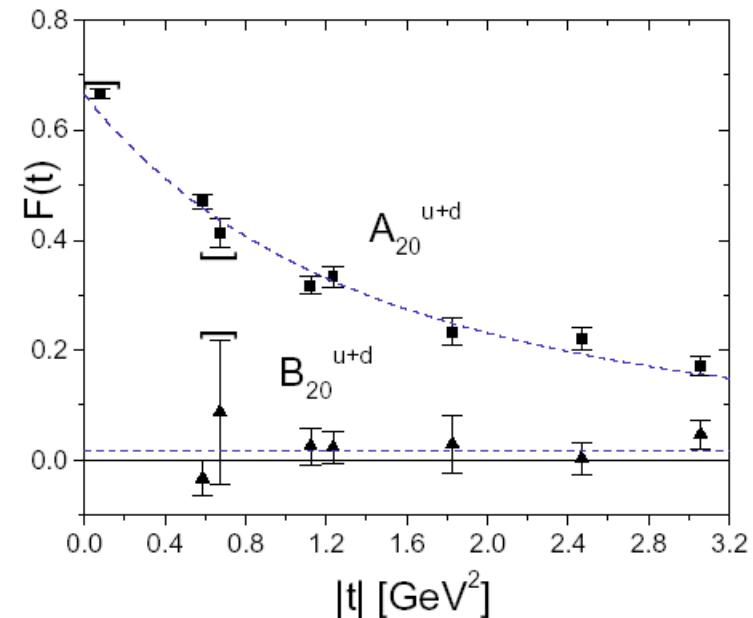
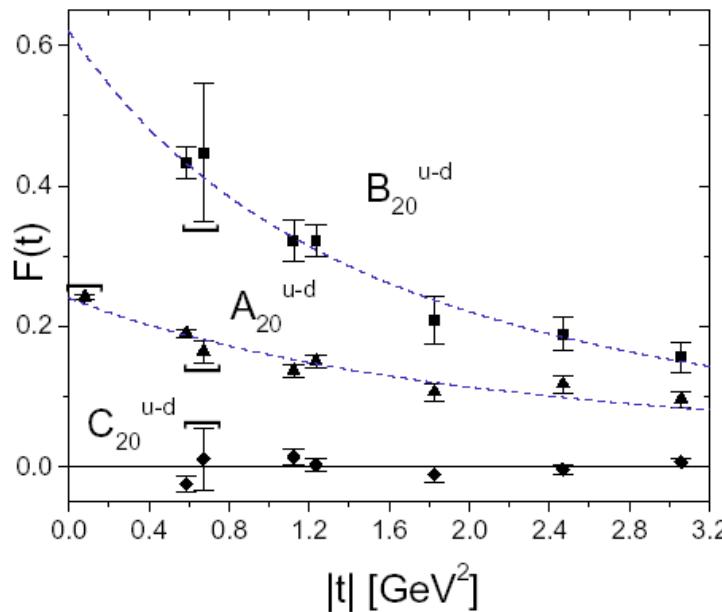
Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's&MTW textbooks) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For $E_G = E_M$ (**inflated, produced from vacuum, flat?**) “Universe” - equal to that of shell rotation
- Simple observation-Must be the and **quantum** rotators – PNEP!
- More elaborate models - Tests for cosmology ??!



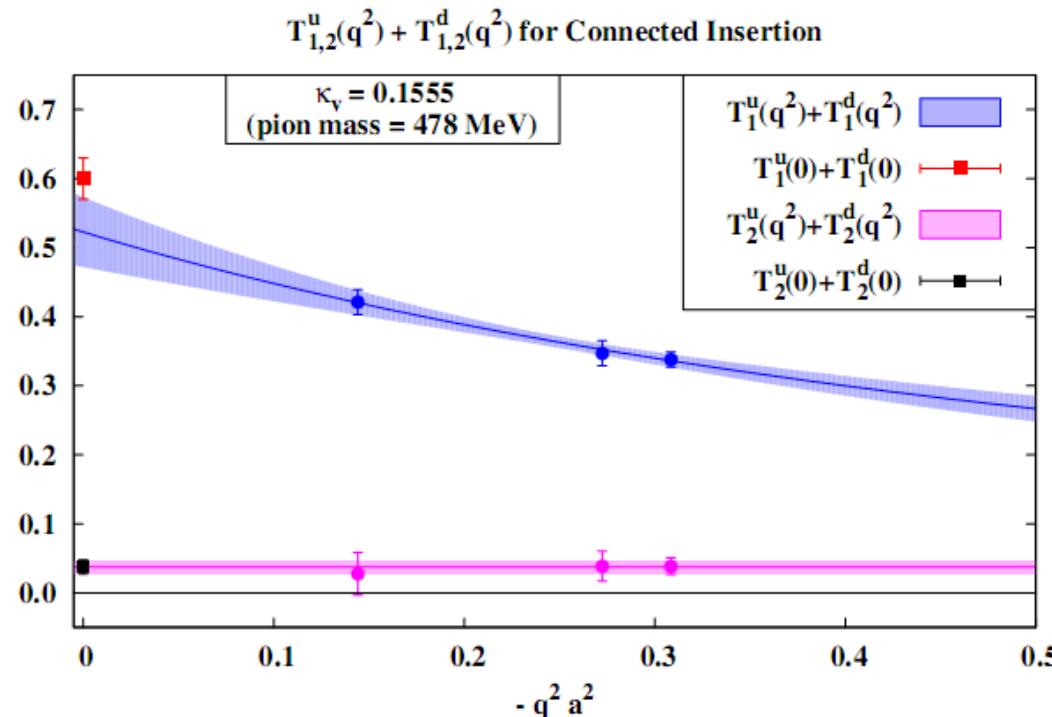
Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



Recent lattice study (M. Deka et al. [arXiv:1312.4816](https://arxiv.org/abs/1312.4816))

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs

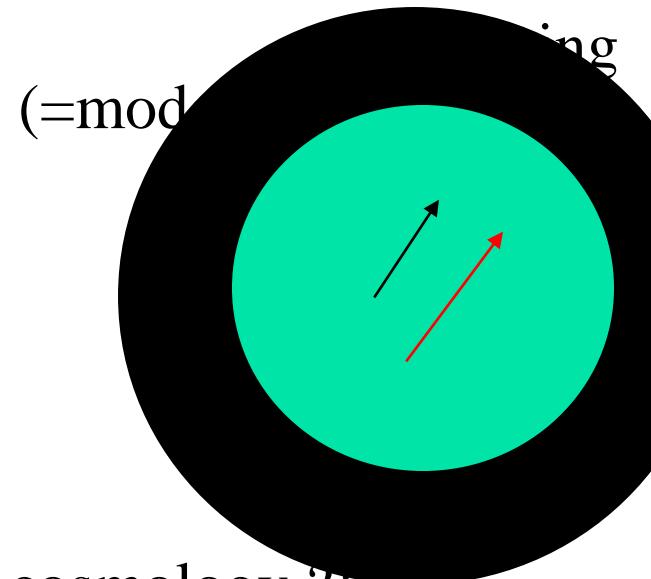


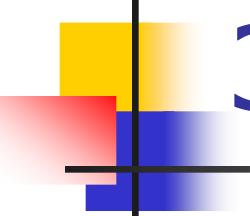
Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive entering Black Hole?)?!

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive empty shell
- For **flat** “Universe” - frequency rotation
- Simple observation-Must be the for classical and **quantum** PNEP!
- More elaborate models - Tests for cosmology ?!

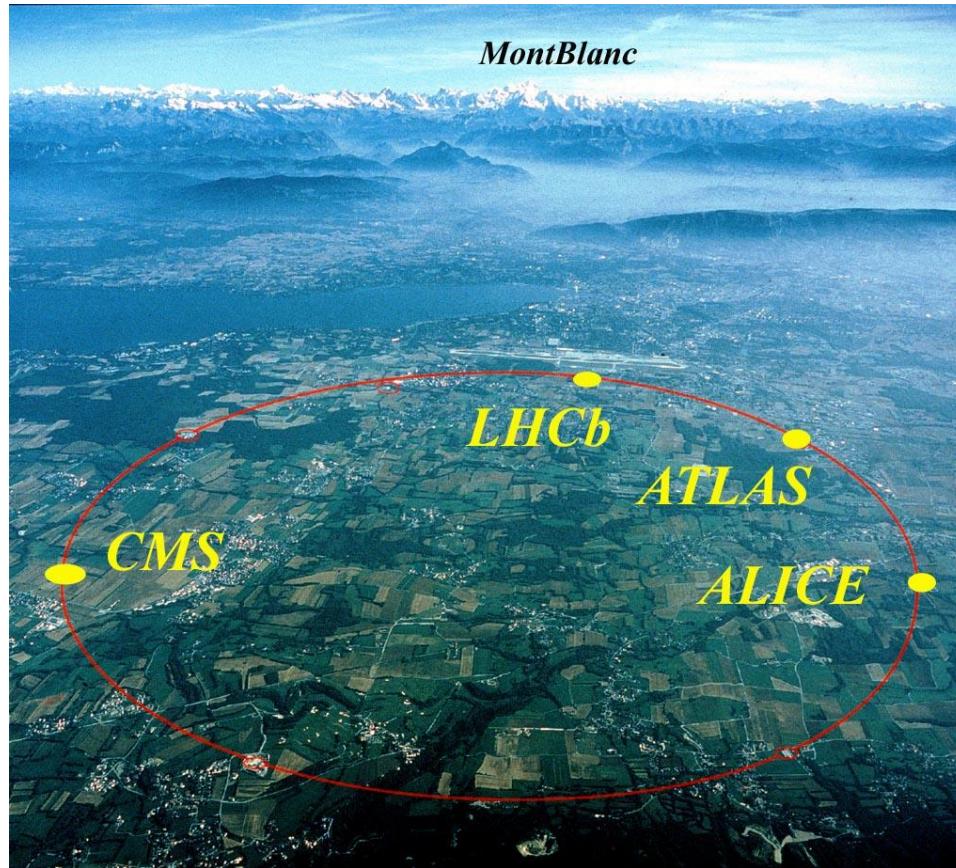




Заключение

- Аномалии и гравитация – проявляются в **спиновой** структуре адронов и **поляризации** в столкновениях тяжелых ионов
- **NICA** – **поляризация** в соударениях тяжелых ионов и адронов - зависящие от поперечного импульса (**косвенно** связаны с гравитационными формфакторами) глюонные распределения
- Множество нерешенных вопросов и в теории, и в эксперименте ждут вас!

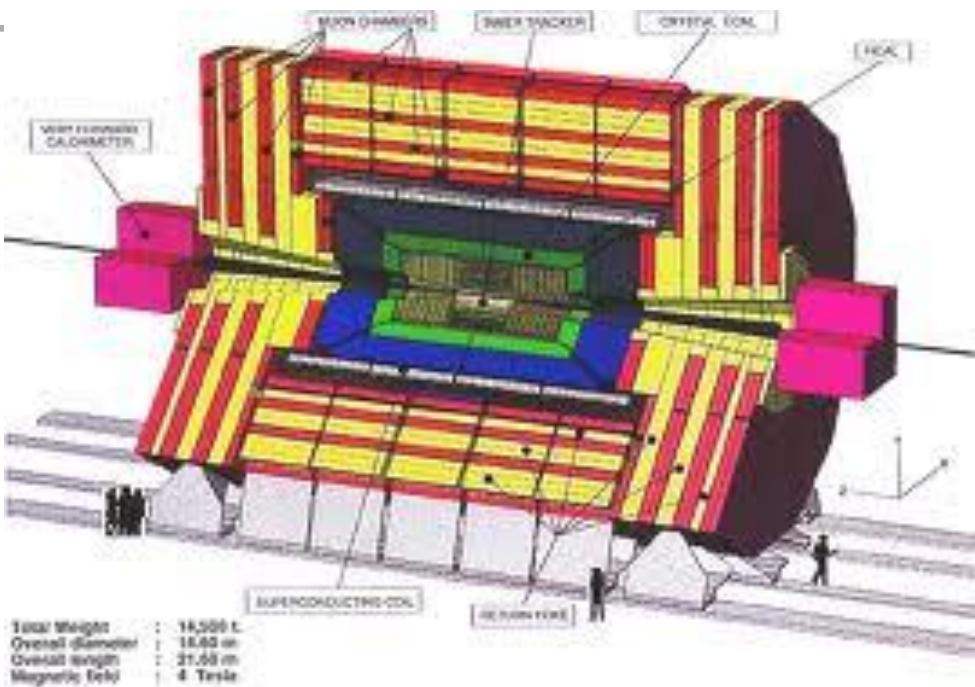
Большой Адронный Коллайдер



Ускоритель



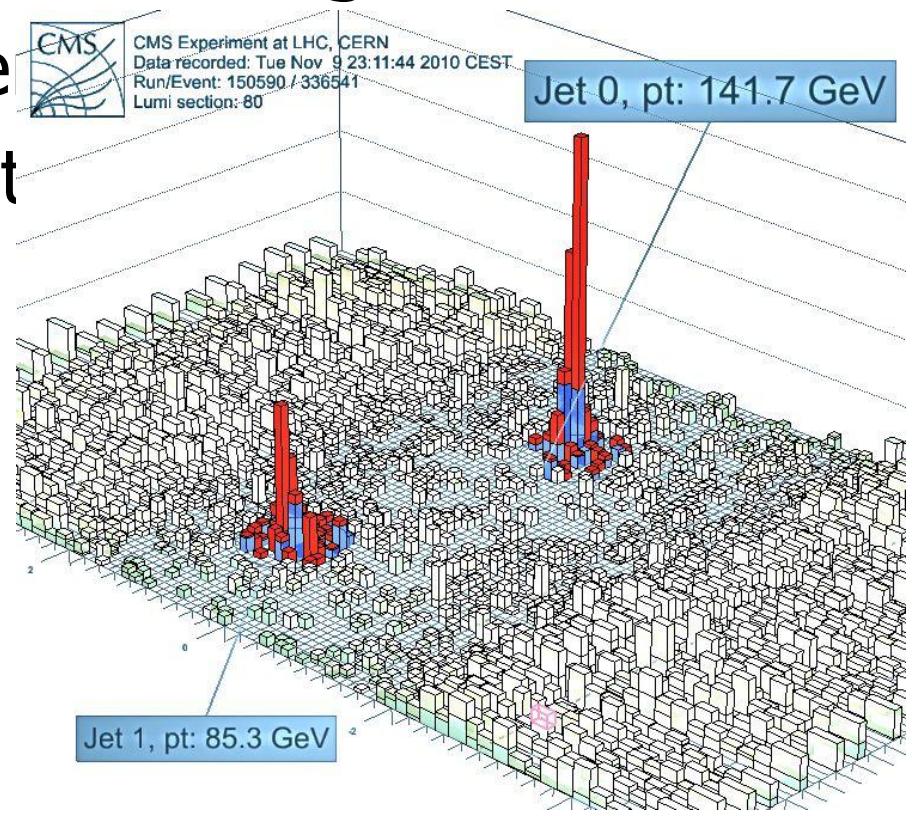
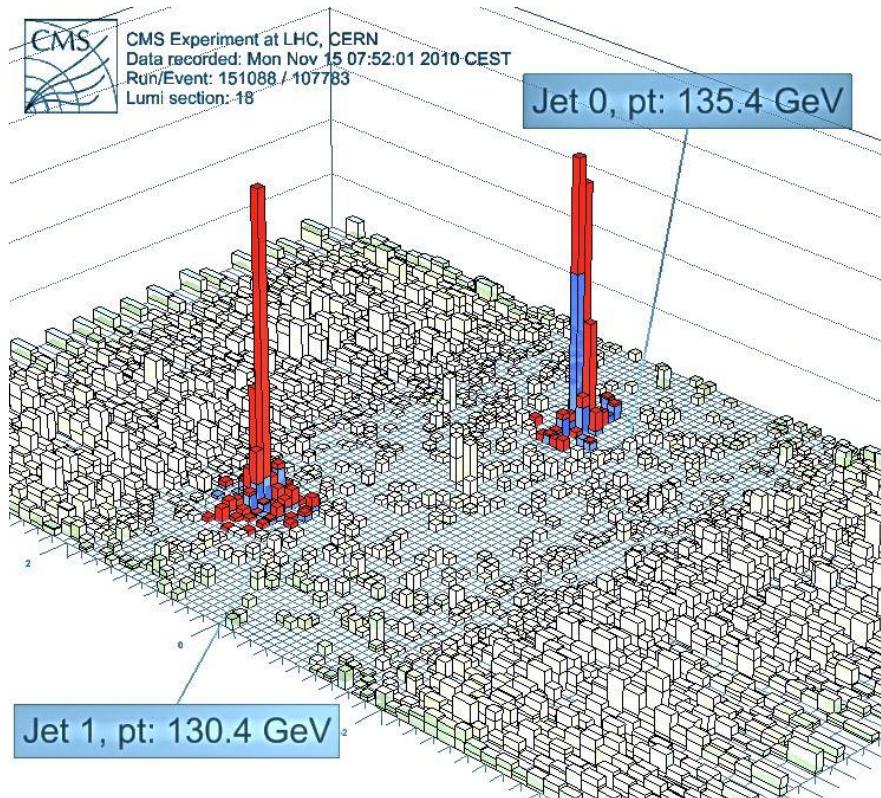
Детектор (CMS)



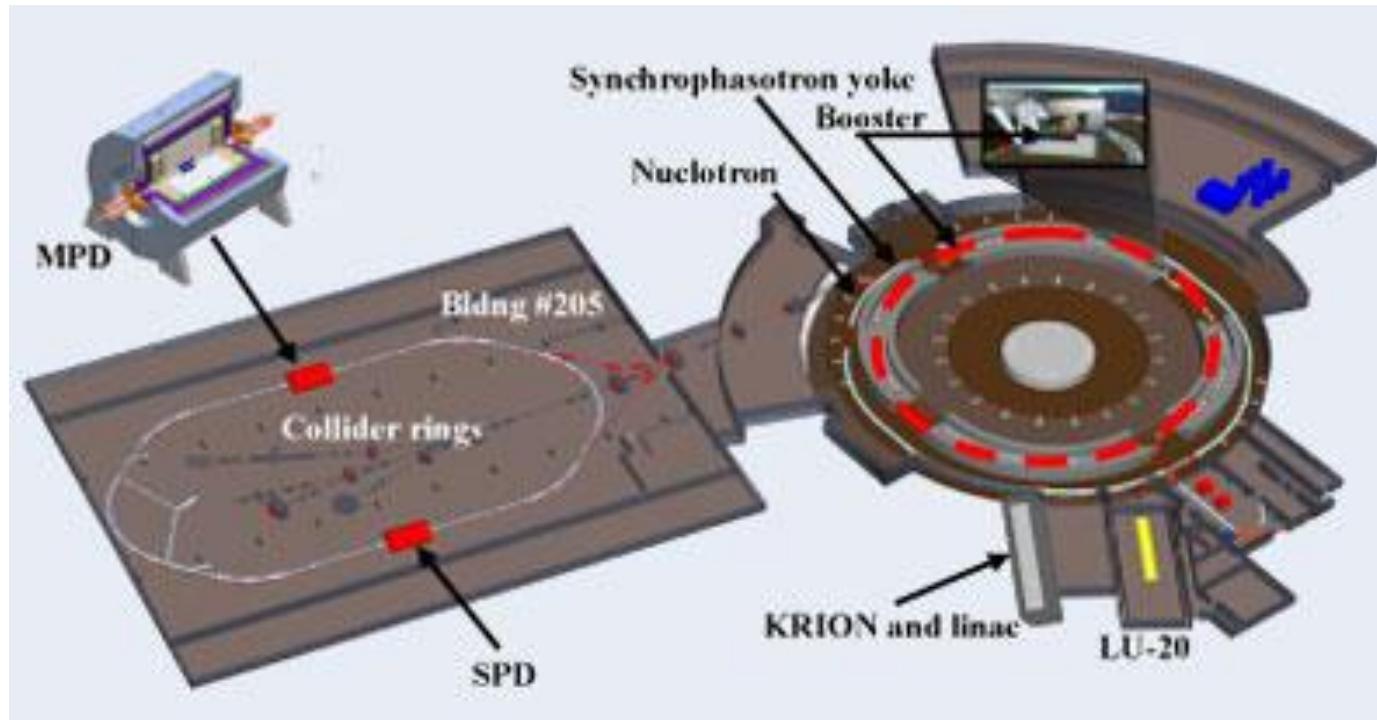
Total weight : 16,500 t.
Overall diameter : 18.60 m
Overall length : 21.60 m
Magnetic Field : 4 Tesla

Dijet event candidates in CMS

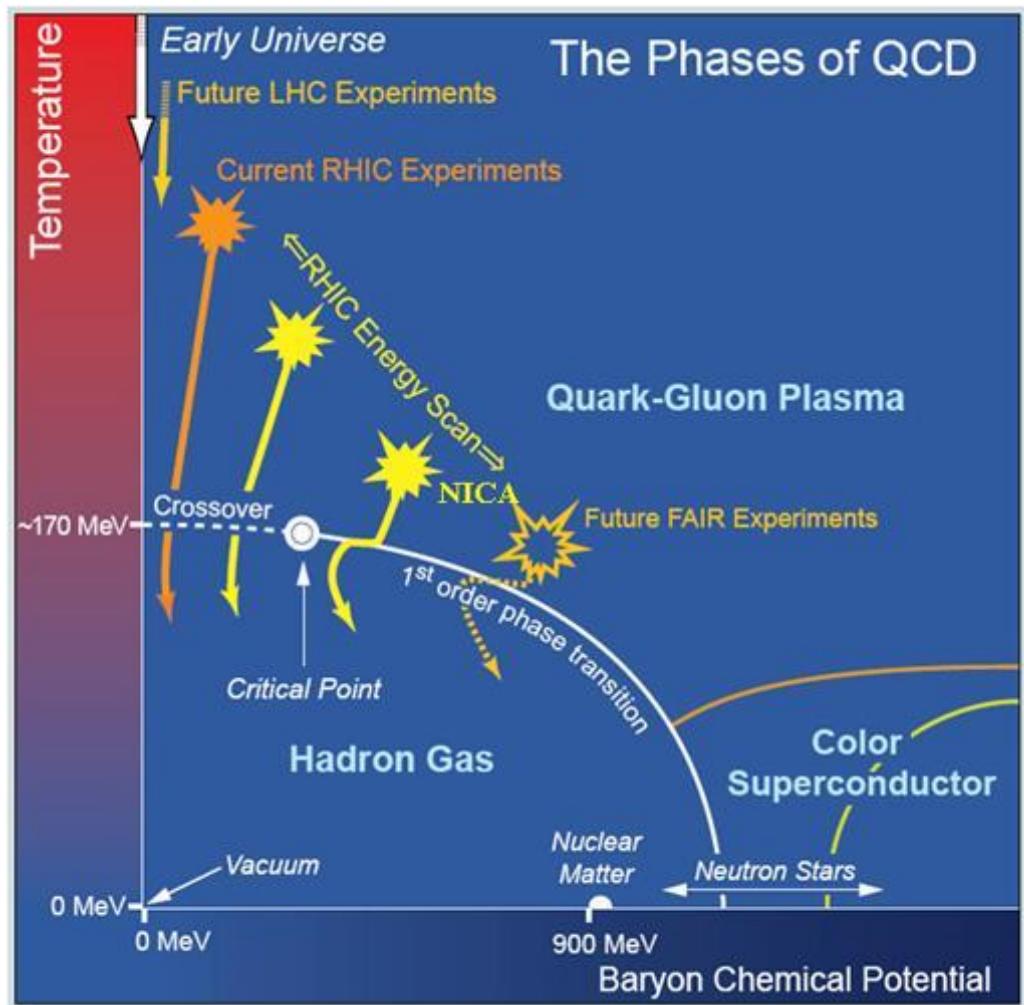
■ First hours of LHC running



ОИЯИ – NICA=Nuclotron-based Ion Collider fAcility



Фазы кварт-адронной материи

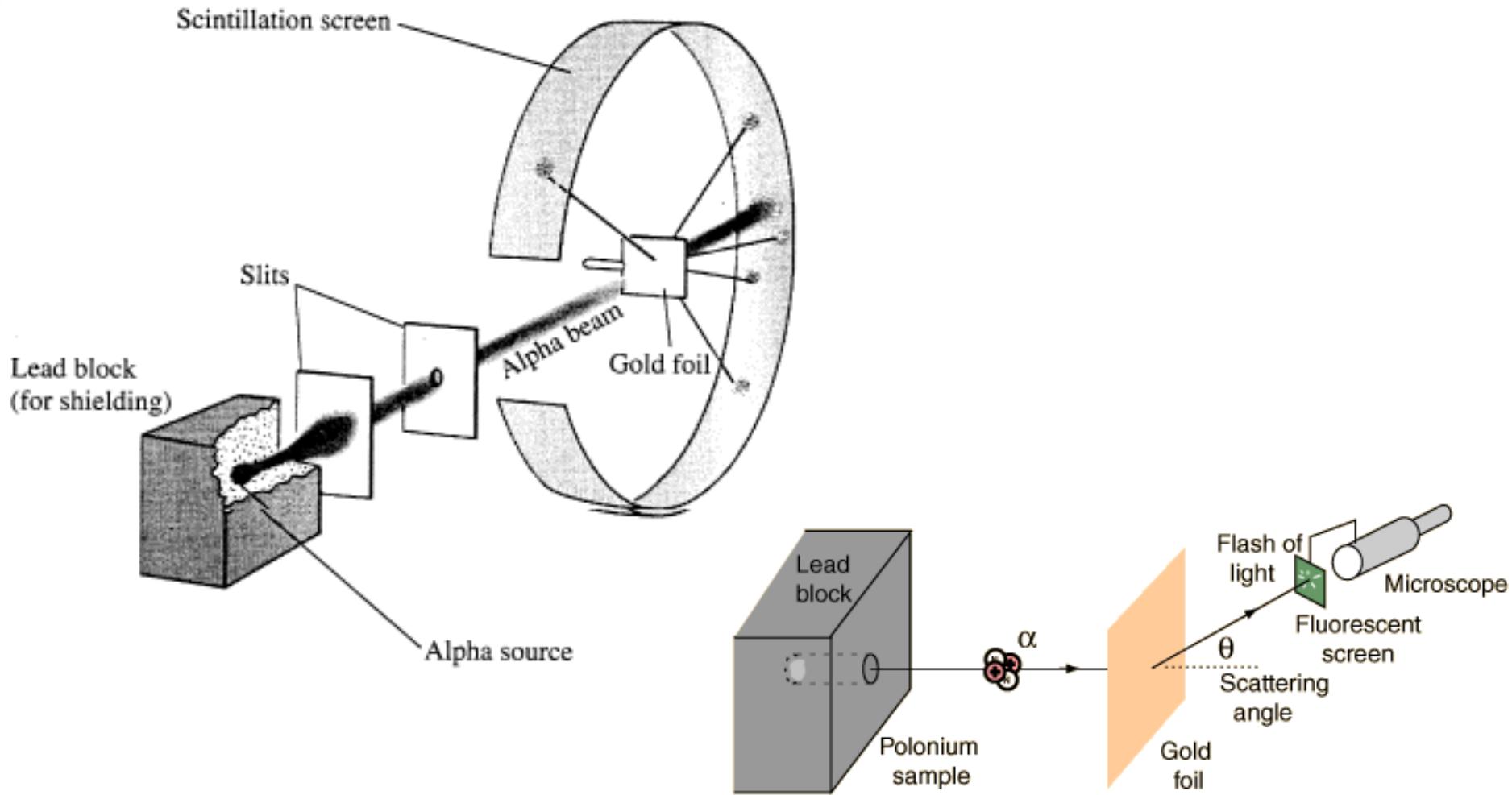


Начало: Резерфорд (1911)

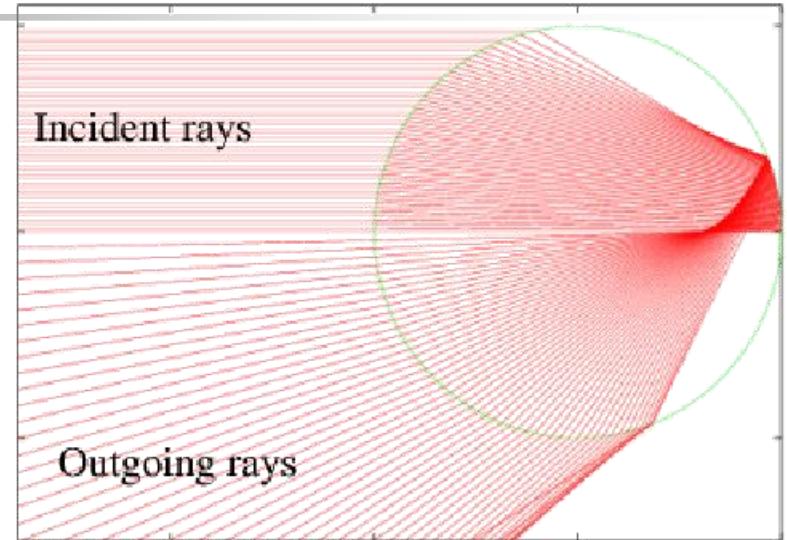
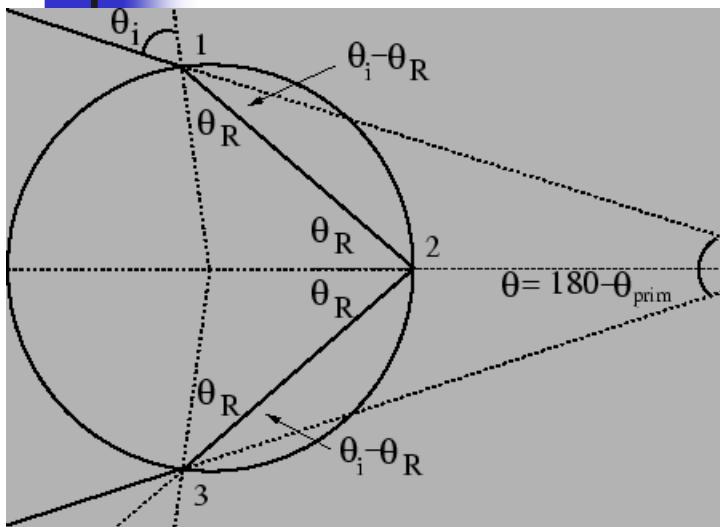
Протоны \leftrightarrow альфа-частицы

Ускоритель \leftrightarrow их источник

Детектор \leftrightarrow флюоресцентный экран



Радуга как бесконечное сечение

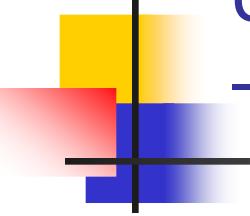


$$\theta = (\theta_i - \theta_r) + (180 - 2\theta_r) + (\theta_i - \theta_r) = 180 + 2\theta_i - 4\theta_r$$

$$0 = \frac{d\theta}{d\theta_i} = 2 - \frac{4\cos\theta_i}{n_{\text{water}} \sqrt{1 - \frac{\sin^2\theta_i}{n_{\text{water}}^2}}}$$

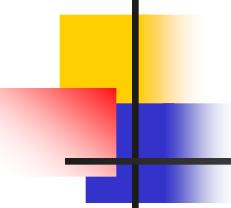
$$\theta_i = \cos^{-1} \left[\frac{1}{3} (n_{\text{water}}^2 - 1) \right]^{\frac{1}{2}}$$

$$\theta_{\text{primary}} = 180 + 2\cos^{-1} \left[\frac{1}{3} (n_{\text{water}}^2 - 1) \right]^{\frac{1}{2}} - 4\sin^{-1} \left[\frac{\sin \left[\cos^{-1} \left[\frac{1}{3} (n_{\text{water}}^2 - 1) \right]^{\frac{1}{2}} \right]}{n_{\text{water}}} \right]$$



Видимость в тумане и вероятность столкновения с айсбергом при ясной погоде – сечение при разном числе измерений

- Видимость в тумане (задача П.Л. Капицы):
 - $L \propto s/d^3 \sim 1$
 - s – (2-мерное) сечение капли
 - d – среднее расстояние между каплями
 - L – видимость = длина свободного пробега
- “Титаник” и айсберги:
 - $L \propto s/d^2 \sim 1$
 - ...
 - s (1-мерный) размер льдины
 - $s=100\text{m}$, $d=100\text{km} \rightarrow L= 100.000\text{km} (\sim 100 \text{ рейсов})$



Hard exotics: Theoretical Interpretation of L3 data on neutral and charged rho's production in collisions of virtual and real photons

Observation: Q dependence of charged/
neutral ratio

Large Q

$$\frac{d\sigma}{dQ^2}(\rho^+ \rho^-) : \frac{d\sigma}{dQ^2}(\rho^0 \rho^0) = 2 : 1$$

Small Q

$$\frac{d\sigma}{dQ^2}(\rho^0 \rho^0) > \frac{d\sigma}{dQ^2}(\rho^+ \rho^-)$$

The L3 data can be described by exotical 4 –
quarks isotensor resonance – higher twist
4 – explains Q suppression

Exploring hard exotics in QCD

- The QCD fit of L3 data is compatible with the existence of isotensor ($I=2$) four quark exotic meson with mass: 1.5 GeV width: 0.4 GeV
 - neutral – red
 - charged - blue

