Femtoscopy of High Energy Collisions

- History
- QS correlations
- FSI correlations & Coalescence
- Correlation asymmetries
- Strong interaction from momentum correlations
- Summary

History of Correlation femtoscopy

measurement of space-time characteristics R, $c\tau \sim$ fm
of particle production using particle correlations

Fermi'34, Watson'52, Migdal'55, GGLP'60, Dubna (GKPLL..'71-) …

FSI in β-decay (Fermi): Coulomb FSI between $e^{\pm}$ and Nucleus in β-decay modifies relative momentum (**k**) distribution → Fermi function

$$F(k,Z,R) = \langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle \sim (2kR)^{-2\varepsilon},\ \varepsilon = 1 - \sqrt{1 - \left(\frac{Z}{137}\right)^2}$$

is sensitive to Nucleus radius R if $Z \gg 1$ $2\epsilon = 0.41$ for Bi (Z=83)

$\psi_{-\mathbf{k}}(\mathbf{r})$ = electron – residual Nucleus WF (Δt=0)

FSI & QS in Production of low-energy nucleons (Watson, Migdal): $\sim|f^S(k)|^2/(r_0^2+d^2)$
& deuterons (Migdal): $\sim 1/(r_0^2+R_d^2)^{3/2}$

r_0 = production radius assumed $< \sim d$ ⇒ no r_0-sensitivity of NN & d
d = FSI radius (1-2 fm) production at so small r_0
R_d = deuteron radius (2.2 fm)
$f^S(k)$= NN s-wave scattering amplitude for pair spin S (QS forbidds S = 1)

Goldhaber, Goldhaber , Lee & Pais

QS in $\bar{p}\,p \rightarrow 2\pi^{+}\,2\pi^{-}\,n\pi^{0}$

GGLP'60 showed within Stat.Model that QS can explain enhanced number of like pion pairs at small opening angles if assuming a small production radius r_0 ~0.5 fm

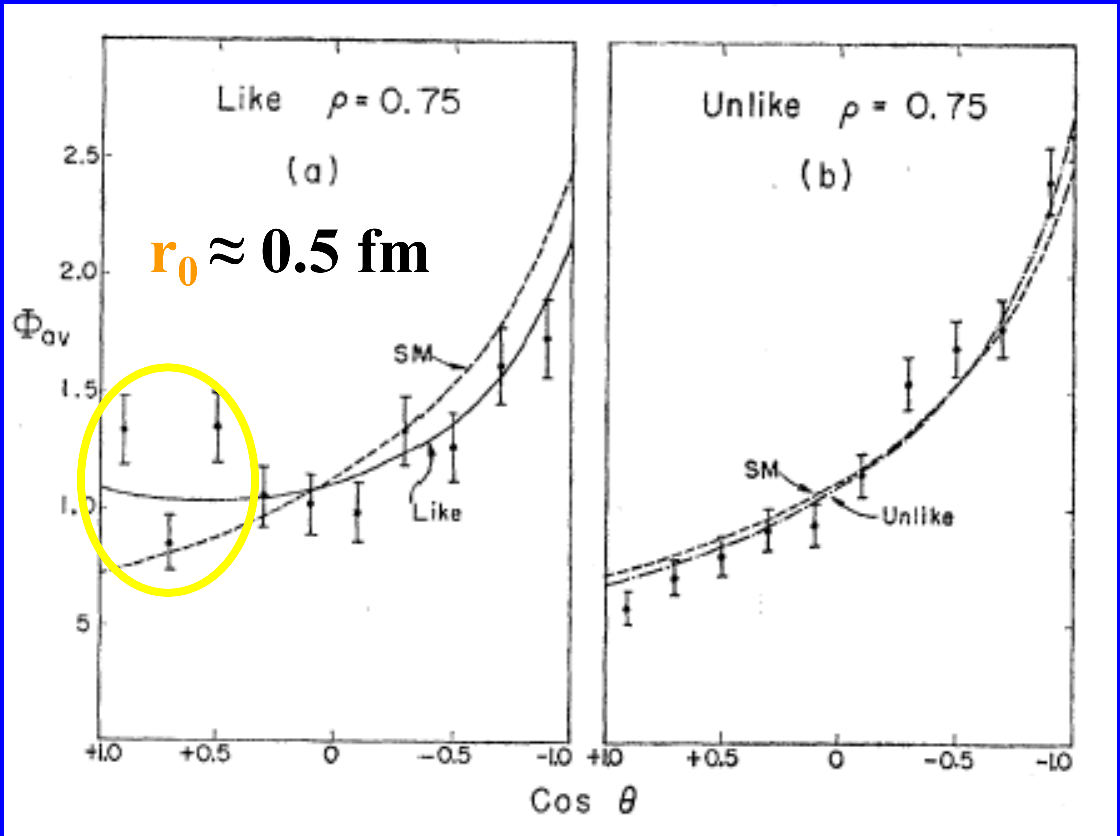
- Stat.Model multiplicity requires radius r_0 3x larger !
- In fact, later femtoscopy of $\bar{p}p$ collisions lead to r_0 ~ 1.5 fm

⇓

- cosθ-distribution integrates out the narrow BE enhancement from large space-time separations → source radius r_0 is underestimated !

⇓

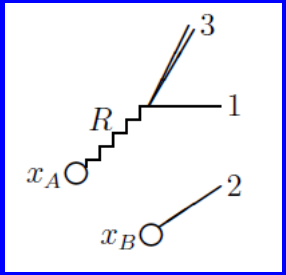
- GGLP effect is insensitive to r_0 increase due to path lengths of resonances



Resonances as emitters

Grishin, Kopylov, Podgoretsky'71, Grassberger'77, RL'78, RL-Progulova'92

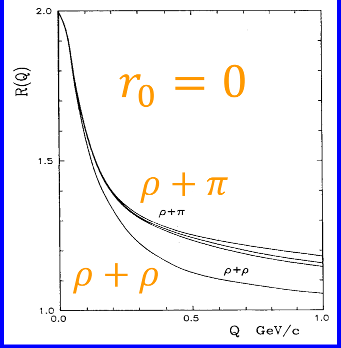
most of the produced pions come from resonance decays ⇒ symmetrization accounting for resonance propagator $(M^2-k_r^2-iM\Gamma)^{-1}$ yields in the limit $\Gamma << M$



$$CF = 1 + \left\langle \frac{\cos(q\Delta x)}{1+y^2} \right\rangle \qquad y = \frac{k_r q}{M\Gamma} \equiv (q\, l) = -\mathbf{Q}\, \mathbf{l}^* = \frac{\mathbf{Q p}_D}{m\Gamma}$$

$$= 1 + \frac{\exp(-r_0^2 Q^2)}{\langle 1+y^2 \rangle} \qquad \text{in PRF } q = q^* = (0, \mathbf{Q})$$

↳ **Pair Rest Frame**



- BE enhancement width is determined by the source size enhanced by the resonance decay length in PRF $\mathbf{l}^* = \mathbf{k}_r/M\Gamma = -\mathbf{p}_D/m\Gamma$ $\quad \mathbf{p}_D$ = decay momentum
- ρ-meson as a typical resonance → decay length in pion PRF $l^* \sim 3.3$ fm
- in contrast with pion correlation radii 1-1.5 fm measured from CF(Q) in πp, pp, e^+e^-
- explained by a rapid decrease of the slope of the resonance factor $1/(1+y^2)$ so that at Q > 0.1 GeV/c the Q-dependence is dominated by $\exp(-r_0^2 Q^2)$
- such analysis of $\pi\pi$ CF in pp & $\bar{p}$p at ISR: r_0 = 0.5-0.6 fm direct production radius RL-Progulova'92

Modern correlation femtoscopy formulated by Kopylov & Podgoretsky

KP'71-75: **in > 20 papers settled basics of QS correlation femtoscopy**

- **proposed CF= N^{corr}/N^{uncorr} vs relative relative momentum q & mixing techniques to construct N^{uncorr}**
- **justified two-body approximation (instead of GGLP multi-particle WFs) to calculate theor. CF**
- **substituted WFs by time-dependent Bethe-Salpeter amplitudes (for free particles) & clarified role of space-time production characteristics: shape & time picture from various q-projections**
- **showed that sufficiently smooth momentum spectrum allows one to neglect space-time coherence at small q: smoothness approximation**

$$|\int d^4x_1 d^4x_2 \psi_{p1p2}(x_1,x_2)...|^2 \rightarrow \int d^4x_1 d^4x_2 |\psi_{p1p2}(x_1,x_2)...|^2$$

|Sum of Ampl.|^2 → Sum of Probab. (like in Stat. Model)

QS symmetrization of production amplitude

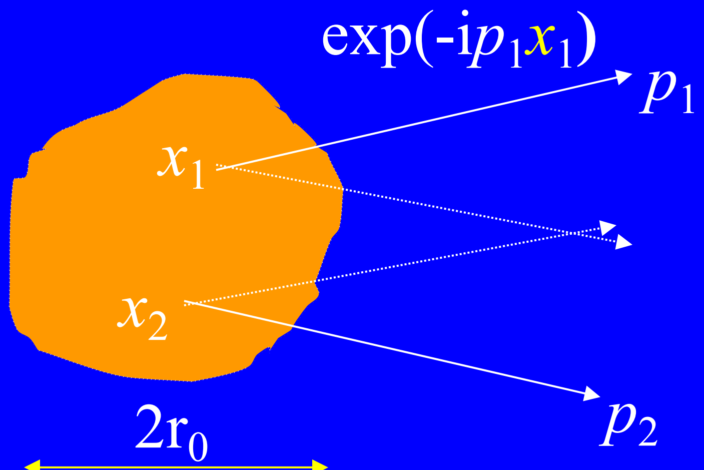
→ *momentum correlations of identical particles are sensitive to space-time structure of the source*

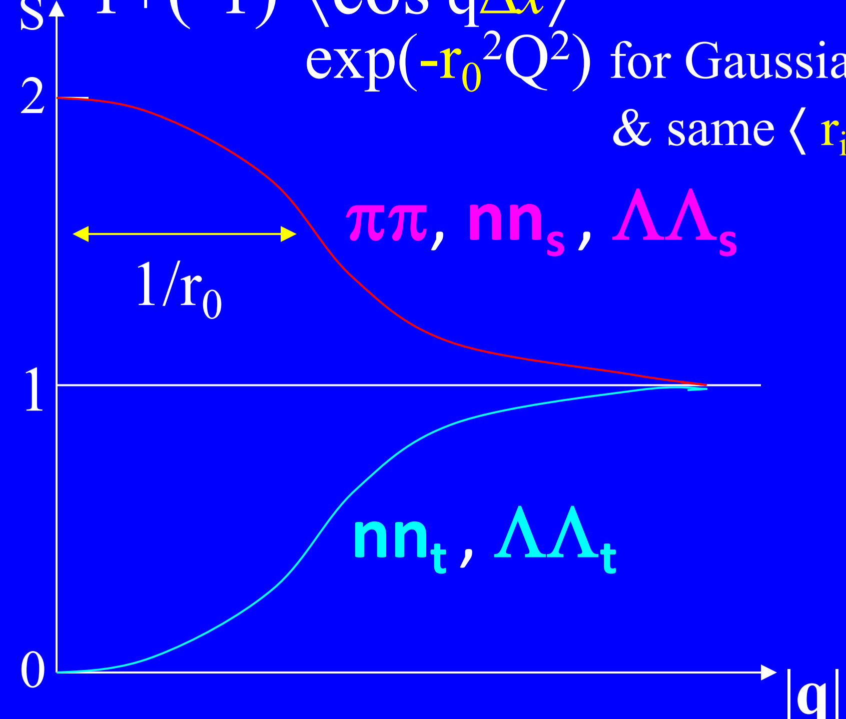
KP'71-75

total pair spin

$CF_S = 1 + (-1)^S \langle \cos q\Delta x \rangle$

$\exp(-r_0^2 Q^2)$ for Gaussian r_i^* & same $\langle r_i^{*2} \rangle = 2r_0^2$





PRF **Q**

$q = p_1 - p_2 \rightarrow \{0, 2\mathbf{k}^*\}$

$\Delta x = x_1 - x_2 \rightarrow \{t^*, \mathbf{r}^*\}$

$CF_S \rightarrow \langle |\psi^{S(sym)}_{-\mathbf{k}^*}(\mathbf{r}^*)|^2 \rangle = \langle | [e^{-i\mathbf{k}^*\mathbf{r}^*} + (-1)^S e^{i\mathbf{k}^*\mathbf{r}^*}]/\sqrt{2} |^2 \rangle$

! CF of noninteracting identical particles (on mass-shell) is independent of t^* in PRF

⇒ the unique Fourier space-time reconstruction of particle emission is impossible

"General" parameterization at $|\mathbf{q}| \to 0$

Particles on mass shell & azimuthal symmetry $\Rightarrow$ 5 pair variables:

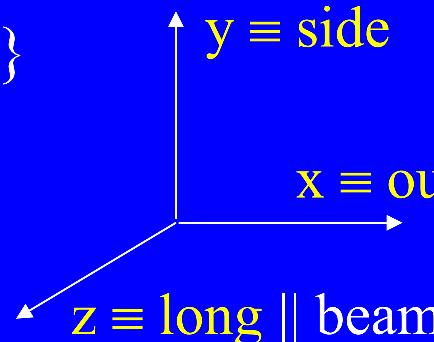
$\mathbf{q}=\{q_x, q_y, q_z\} \equiv \{q_{out}, q_{side}, q_{long}\}$, pair velocity $\mathbf{v}=\{v_x,0,v_z\}$

A modification of Kopylov variables:

$\mathbf{q}=\{q_L, q_T, \varphi_q\}$, $\mathbf{v}=\{v\sin\theta, 0, v\cos\theta\}$

$q_x=q_L\sin\theta+q_T\cos\theta\cos\varphi_q$, $q_y=q_T\sin\varphi_q$

$q_z=q_L\cos\theta-q_T\sin\theta\cos\varphi_q$



Grassberger'77

RL'78

Kopylov relation:

$$q_0=\mathbf{qp}/p_0\equiv\mathbf{qv}=q_L v=q_x v_x+q_z v_z$$

$\Rightarrow$ Δt enters CF through $(\Delta x_L-v\Delta t)q_L=(\Delta x-v_x\Delta t)q_x+(\Delta z-v_z\Delta t)q_z$

$$\langle\cos q\Delta x\rangle=1-\tfrac{1}{2}\langle(q\Delta x)^2\rangle+..\approx\exp(-R_x^2q_x^2-R_y^2q_y^2-R_z^2q_z^2-2R_{xz}^2q_x q_z)$$

the only cross term

Podgoetskyr'83

Femtoscopic radii:

$$R_x^2=\tfrac{1}{2}\langle(\Delta x-v_x\Delta t)^2\rangle,\ R_y^2=\tfrac{1}{2}\langle(\Delta y)^2\rangle,\ R_z^2=\tfrac{1}{2}\langle(\Delta z-v_z\Delta t)^2\rangle$$

Podgoretsky'83, Bertsch, Pratt'95; so called out-side-long parameterization

Csorgo, Pratt'91: | LCMS $v_z=0$ |

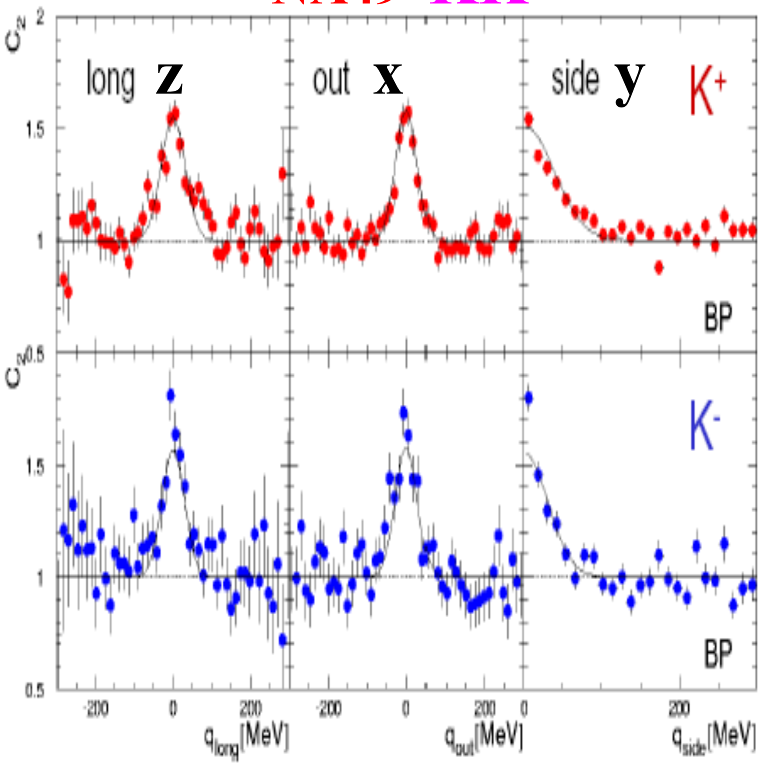
3-dim fit: $CF = 1 + \lambda \exp(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2 - 2R_{xz}^2 q_x q_z)$

Correlation strength or chaoticity (λ) — **Femtoscopic radii** (R_i)

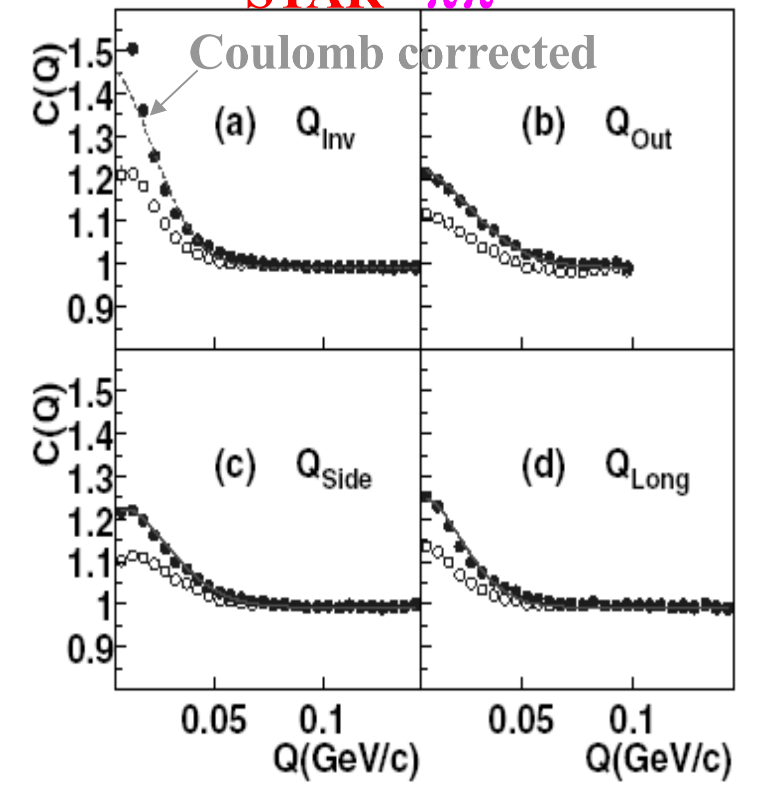
Examples of 3d-CFs in LCMS: NA49 Pb+Pb & STAR Au+Au

Agreement with Gaussians in q-projections is worse in $Q = \sqrt{(q_x^2/\gamma_t^2 + q_y^2 + q_z^2)}$ due to non-equal LCMS radii R_i and the pair transverse Lorentz factor $\gamma_t = m_t/m$; non-Gaussian tail from resonance decays is less important in HICs

NA49 KK



STAR ππ



Probing source shape and emission duration

KP (71-75) …

Static Gaussian model with space and time dispersions $R_\perp^2, R_\parallel^2, \Delta\tau^2$ $\rightarrow$

$$R_x^2 = R_\perp^2 + v_\perp^2 \Delta\tau^2$$
$$R_y^2 = R_\perp^2$$
$$R_z^2 = R_\parallel^2 + v_\parallel^2 \Delta\tau^2$$

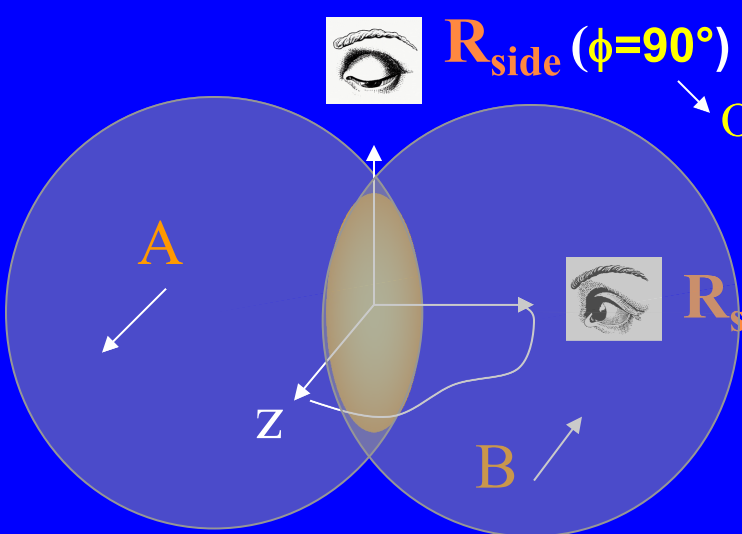
$\rightarrow$ **Emission duration**

$$\Delta\tau^2 = (R_x^2 - R_y^2)/v_\perp^2$$

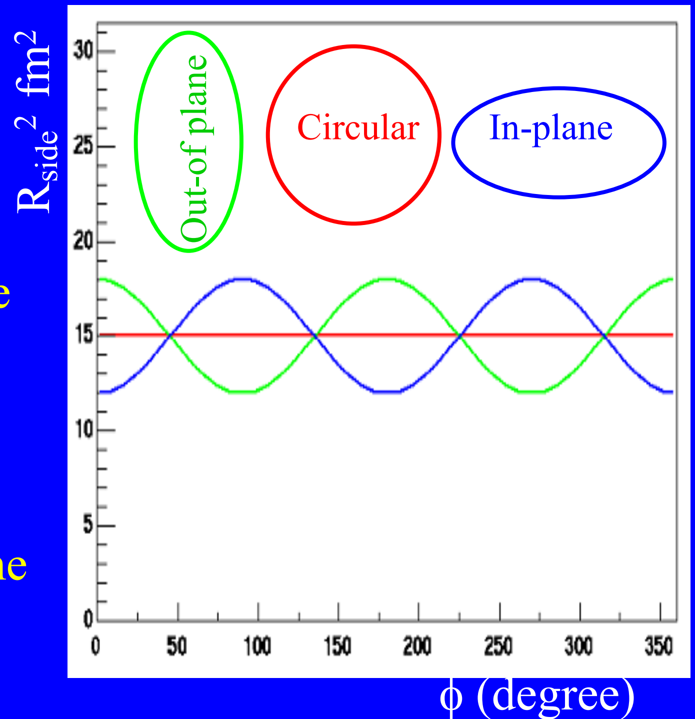
If elliptic shape also in transverse plane

$\Rightarrow R_y \equiv R_{side}$ oscillates with pair **azimuth ϕ**

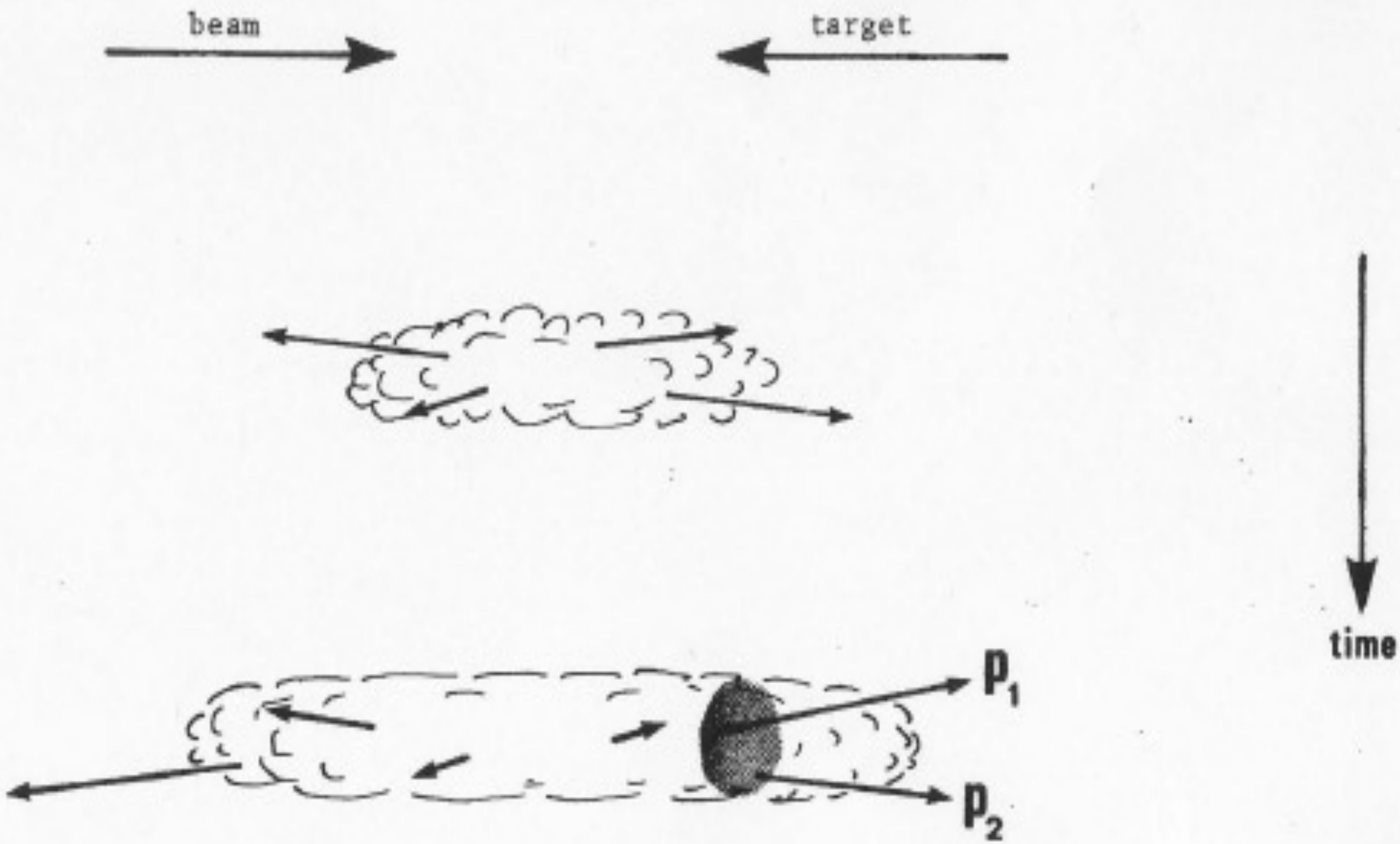
R_{side} (ϕ=90°) small ↘ Out-of reaction plane



R_{side} (ϕ=0°) large ↘ In reaction plane



Grassberger'77: fire sausage
x-p correlation



Probing source dynamics - expansion

Dispersion of emitter velocities & limited emission momenta (T) ⇒

***x-p* correlation**: interference dominated by pions from nearby emitters

→ **Interference probes only a part of the source**

→ **Interferometry radii decrease with pair velocity**

Resonances GKP'71 ..

Strings Bowler'85 ..

Hydro Pratt'84,86

Kolehmainen, Gyulassy'86

Makhlin-Sinyukov'87

Bertch, Gong, Tohyama'88

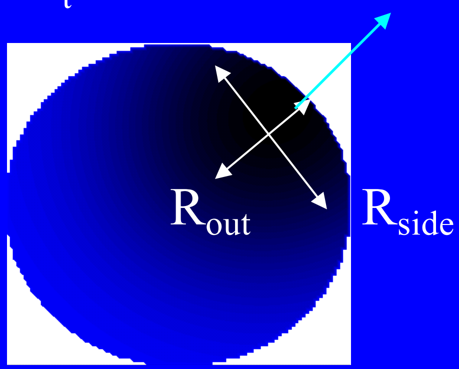
Hama, Padula'88

Pratt, Csörgö, Zimanyi'90

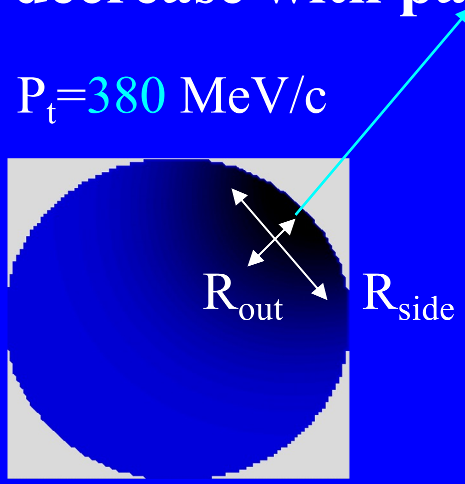
Mayer, Schnedermann, Heinz'92

.....

P_t=160 MeV/c



P_t=380 MeV/c



Transverse radial expansion with tr. flow rapidity $\rho=\rho_0\, r/r_0$ & Gaussian tr. radius r_0 → $R_{side} \approx r_0/(1+\rho_0^2 m_t/T)^{1/2}$

in **LCMS**: **1**

Longitudinal boost invariant expansion for a proper evolution time τ, $\tau_0^2 = \langle\tau^2\rangle$ → $R_{long} \approx (T/m_t)^{1/2}\tau_0/\widetilde{\cosh y}$

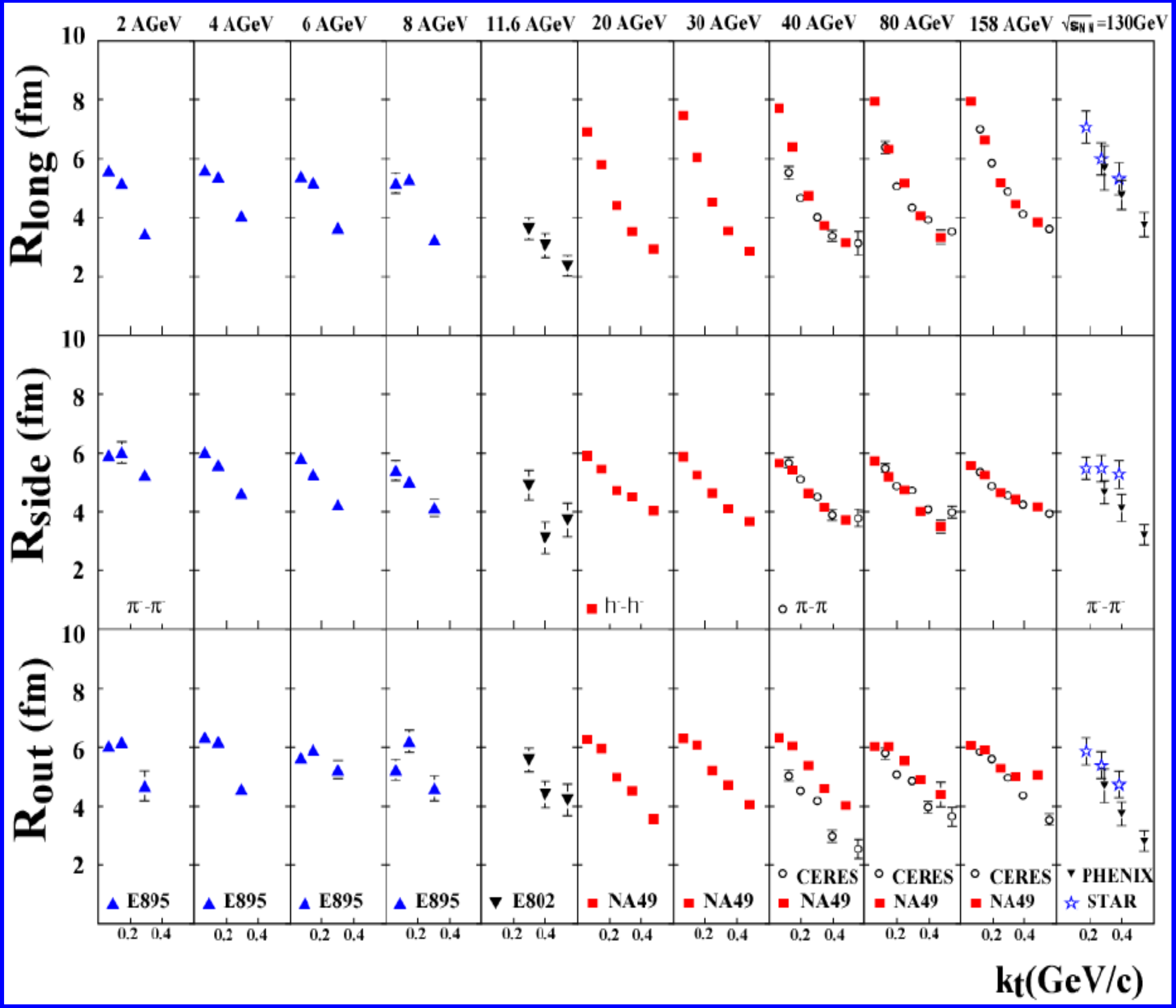
AGS→SPS→RHIC: ππ radii vs E_{lab} & p_t

Central Au+Au or Pb+Pb

$R_{long} \approx (T/m_t)^{1/2}\tau_0$
↑ with energy
& points to short evolution time
$\tau_0 \sim$ 8-10 fm/c

$R_{side} \approx R_{out}$
~ const up to RHIC energy
& short emission duration
$\Delta\tau \sim$ 2 fm/c

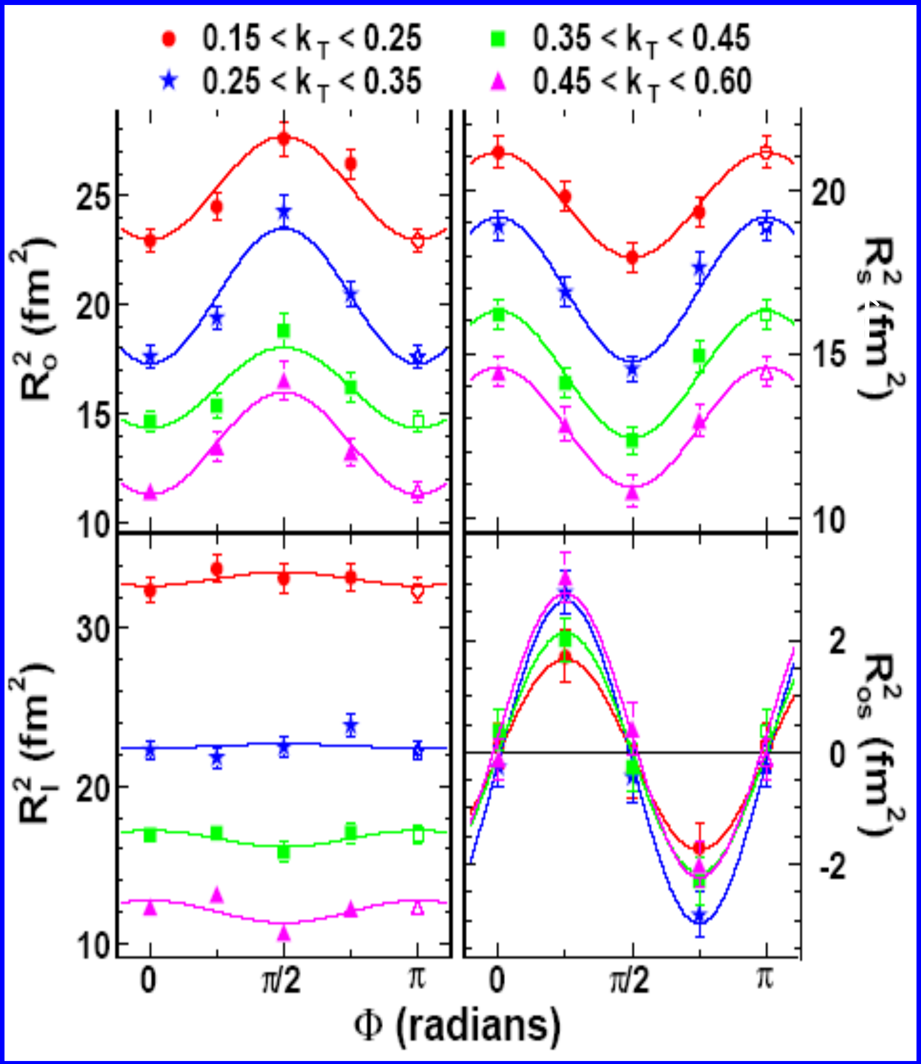
& ↓ $p_t \Rightarrow$ strong transverse flow
$\rho_0 \sim$ 0.4-0.6



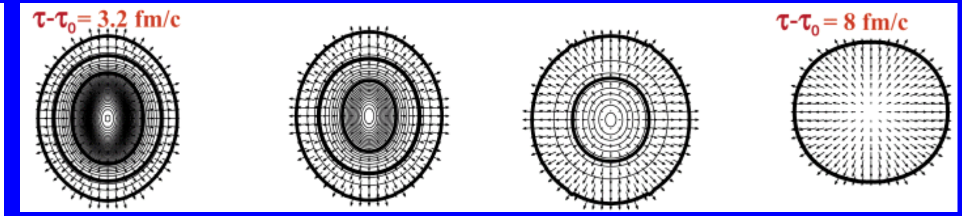
Interferometry wrt reaction plane

STAR’04 Au+Au 200 GeV 20-30%

$\pi^+\pi^+$ & $\pi^-\pi^-$



Typical hydro evolution



Out-of-plane Circular In-plane

Time

↑

STAR data:

ϕ oscillations like for a static out-of-plane source

⇓

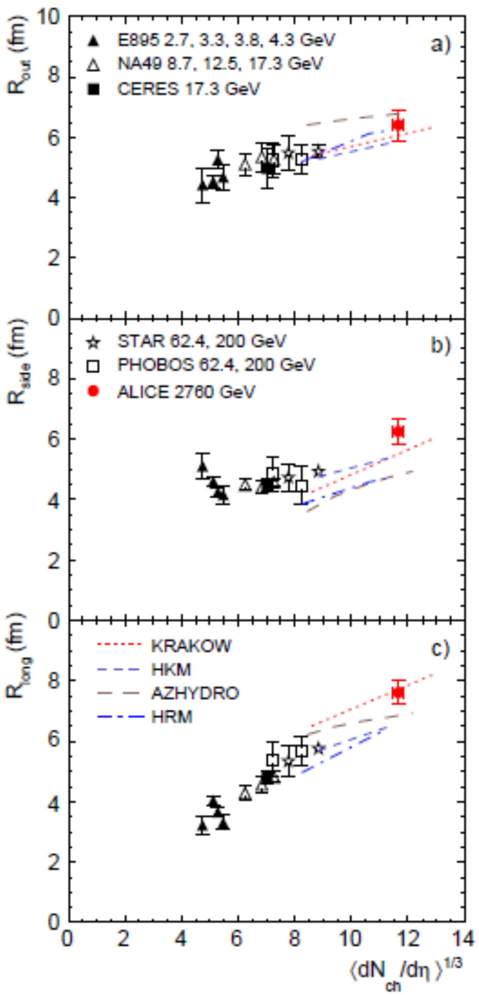
confirms

Short evolution time

Femtoscopy of Pb+Pb at LHC

ALICE, arXiv:1012.4035

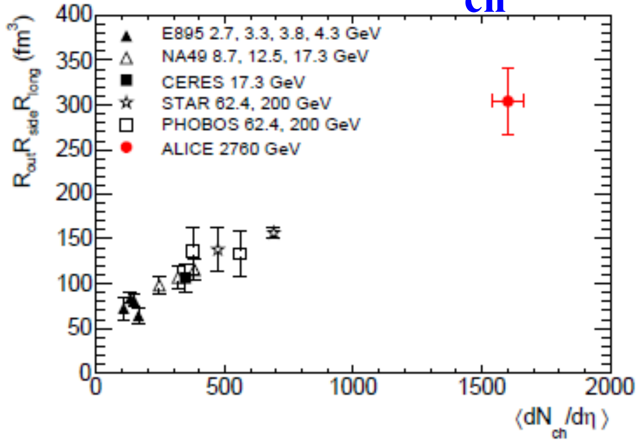
All radii increase with N_{ch} from RHIC to LHC



multiplicity N_{ch} scaling of the correlation volume V

→ universal freeze-out density N_{ch}/V

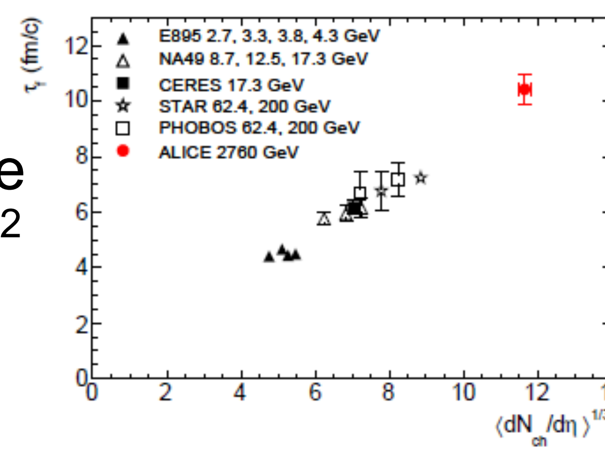
Volume vs N_{ch}



The LHC vs RHIC fireball:

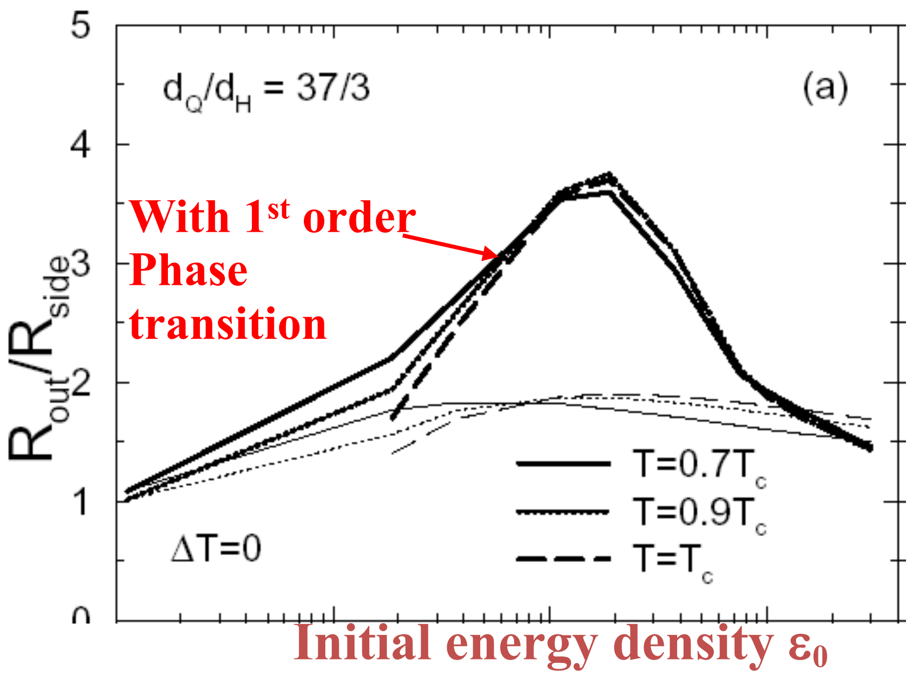
- hotter
- lives longer: $\tau_f^{LHC} > \tau_f^{RHIC}$
- expands ~ same $\rho_0 \approx$ ½c to a larger size
- decays similarly with a short emission duration $\Delta\tau \ll \tau_f$

Freeze-out (evolution) time $\tau_f \equiv \tau_0$ from $R_{long} = \tau_f \, (T/m_t)^{1/2}$



Femtoscopic signature of QGP onset

3D 1-fluid Hydrodynamics
Rischke & Gyulassy, NPA 608, 479 (1996)



A large statistics required to reconstruct the $\Delta\tau$ tail due to a small partonic fraction at the onset region

Long-standing signature of QGP onset:

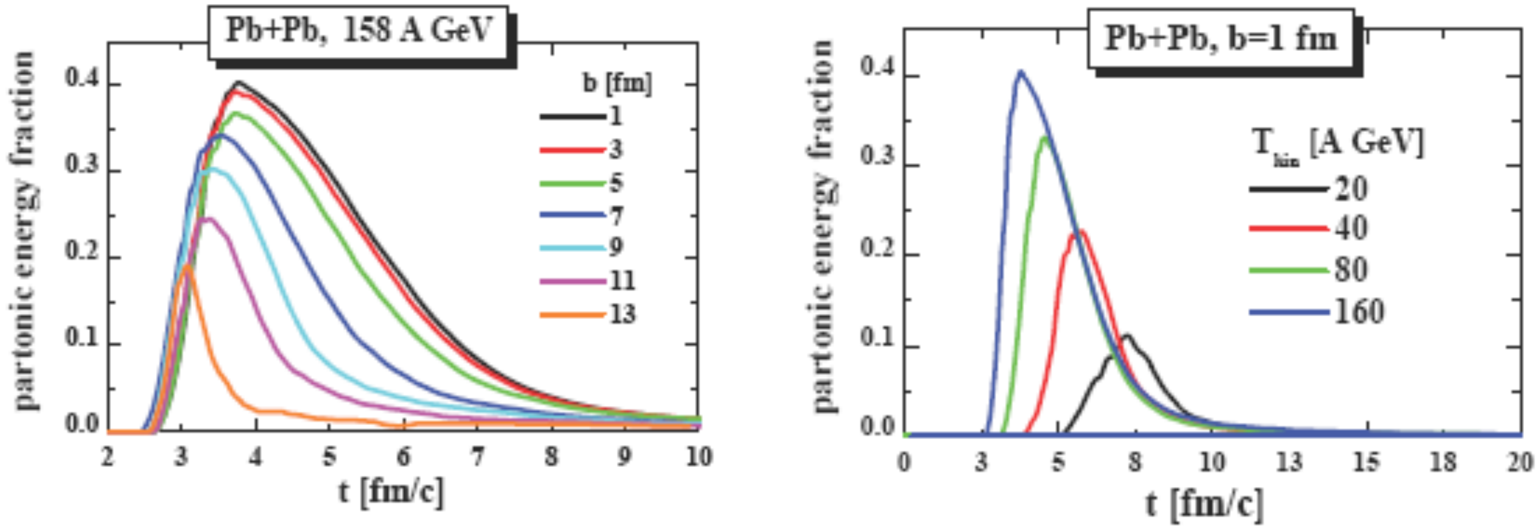
- **increase in emission duration $\Delta\tau$ (reflected in R_{OUT}/R_{SIDE}) due to the 1st order Phase transition**
- **hoped-for "turn on" as QGP threshold in ε_0 is reached**
- **$\Delta\tau$ decreases with decreasing Latent heat & increasing tr. Flow**

(high ε_0 or initial tr. Flow)

Cassing – Bratkovskaya: Parton-Hadron-String-Dynamics

Perspectives at FAIR/NICA energies

partonic energy fraction vs centrality and energy



→Dramatic decrease of partonic phase with decreasing energy and centrality !

vHLLE+UrQMD model

Pre-thermal phase → hydrodynamic phase → hadronic cascade

UrQMD | vHLLE (3+1)-D viscous hydrodynamics | UrQMD

Iu. Karpenko, P. Huovinen, H.Petersen, M. Bleicher, Phys.Rev. C 91, 064901 (2015), arXiv:1502.01978,1509.3751 , talk QM2015

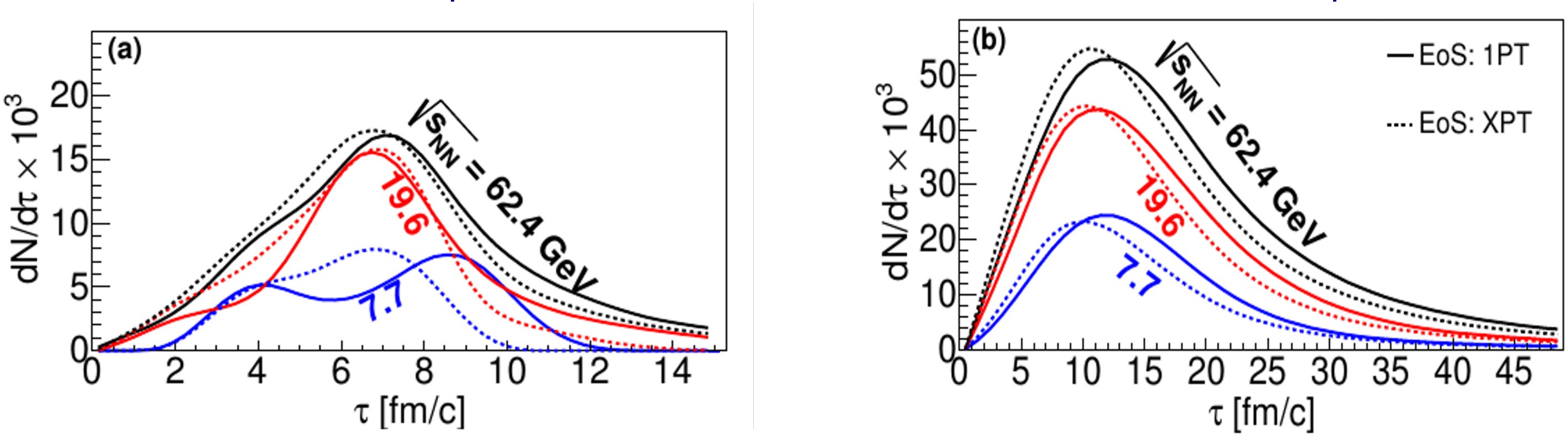
vHLLE code: free and open source, https://github.com/yukarpenko/vhlle, Comput. Phys. Commun. 185 (2014), 3016

Model with XPT EoS is tuned by matching with the experimental data of SPS and BES RHIC.

Chiral EoS → crossover PT (XPT)
J. Steinheimer et al, J. Phys. G 38, 035001 (2011)

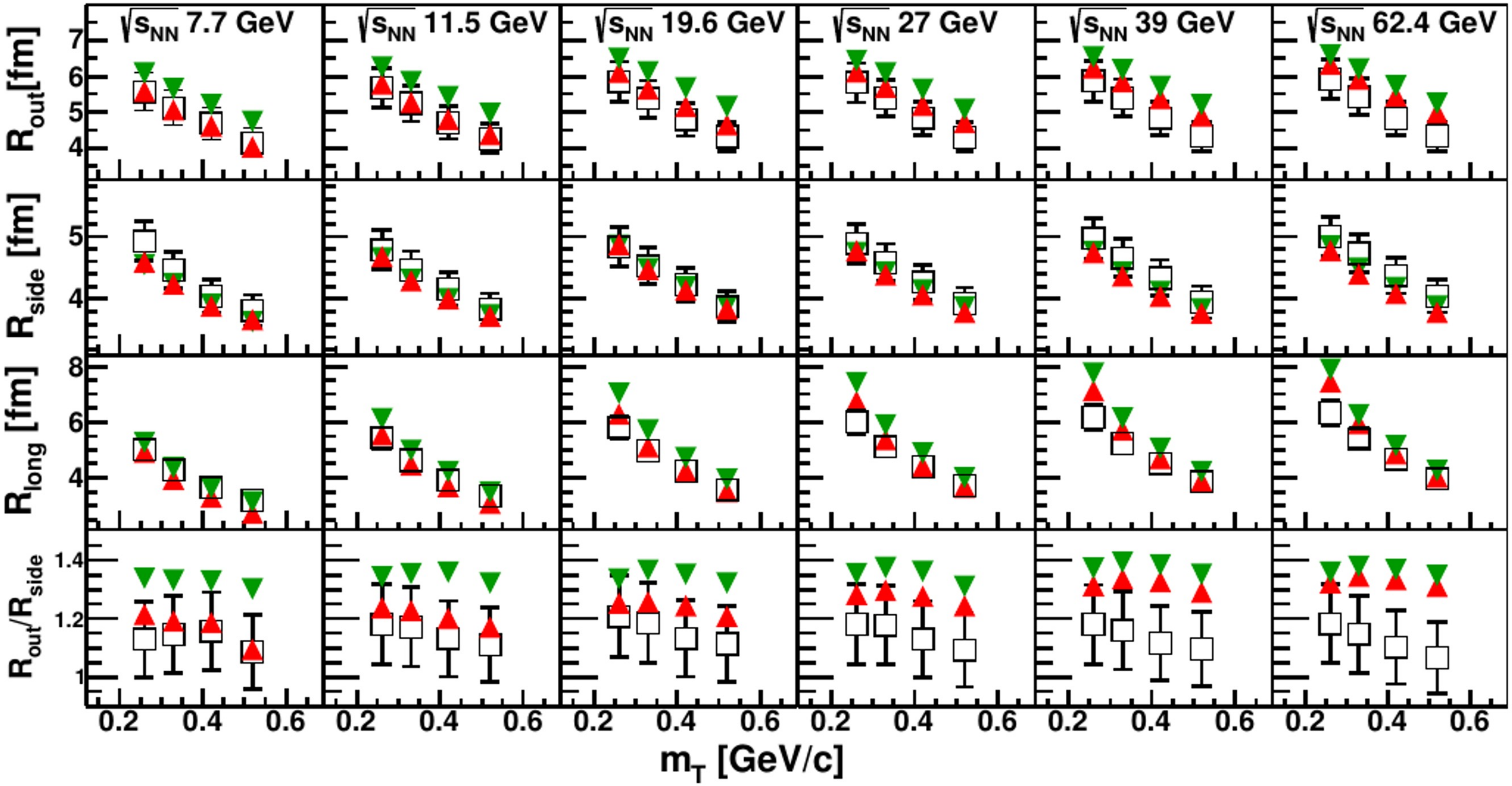
HadronGas + Bag Model → 1st order PT (1PT)
P.F. Kolb et al, Phys.Rev. C 62, 054909 (2000)

Pion emission times at the particlization surface and at the last interaction points



3D Pion Gaussian radii @ 0-5%: STAR, $vHLLE_{1PT, XPT}$+UrQMD

Batyuk ... PRC 96 (2017)



↘ Shorter emission duration required

Green triangles - 1PT EoS, Red triangles - XPT EoS, Open black squares STAR data BES

- R_{out}(XPT) at high energies and R_{out}(1PT) at all energies are overestimated
- R_{long}(XPT) at low energies are slightly underestimated
  R_{long}(1PT) at high energies are slightly overestimated
- R_i(1PT) > R_i(XPT) by 0.5-1 fm for i = out, long
- R_{side}(1PT) ≈ R_{side}(XPT) are slightly underestimated

Conclusions from vHLLE+UrQMD model

- Hydro phase lasts longer with 1st order PT.
- Hadronic cascade diminishes the difference between 1PT and XPT source functions, though there is still a possibility to distinguish them using the femtoscopy.
- Femtoscopic Gaussian radii from STAR BES require about 1 fm/c shorter pion emission duration in the present vHLLE+UrQMD model, which may result in agreement of the model 1PT/XPT at low/high energies.
- A 3D CF analysis with heavier particles (Kaons and Baryons) could be useful to discriminate 1PT and XPT source functions because of their more Gaussian shapes and less influence of resonances.

Final State Interaction

Similar to Coulomb distortion of β-decay Fermi'34: $\langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle$? t

Migdal, Watson, Sakharov, … Koonin, GKW, LL, ...

$f_c\sqrt{A_c}\cdot(G_0+iF_0)$

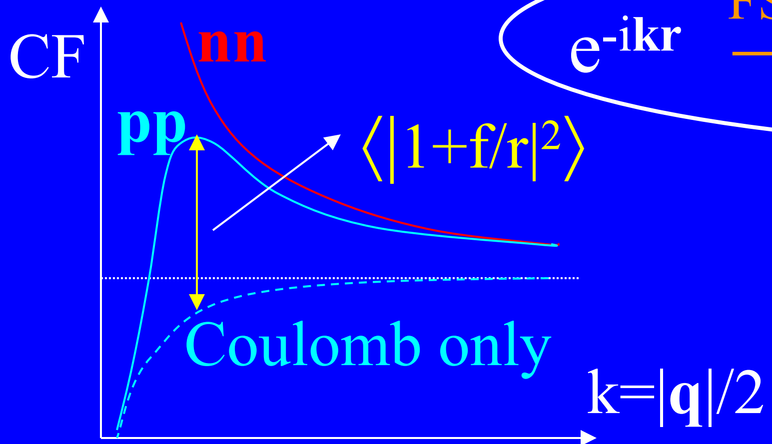
s-wave

$$e^{-ikr} \xrightarrow{\text{FSI}} \psi_{-\mathbf{k}}(\mathbf{r}) \xrightarrow{\text{strong FSI}} [\, e^{-ikr} + f(\tilde{k})e^{ikr}/r \,]$$

Coulomb

$e^{i\delta_c}\sqrt{A_c}$ — Point-like Coulomb/Gamow factor

$F = 1 + \frac{\mathbf{kr}+kr}{ka} + \ldots$ — Bohr radius



⇒ FSI is sensitive to source size r and scattering amplitude f

FSI **complicates CF analysis** but makes possible:

→ **Femtoscopy with nonidentical particles** πK, πp, ..

including relative space-time asymmetries delays, flow

→ **Femtoscopy using Coalescence deuterons, ..**

→ **Study "exotic" scattering** ππ, πK, KK, πΛ, pΛ, ΛΛ, $\bar{p}\bar{p}$ ..

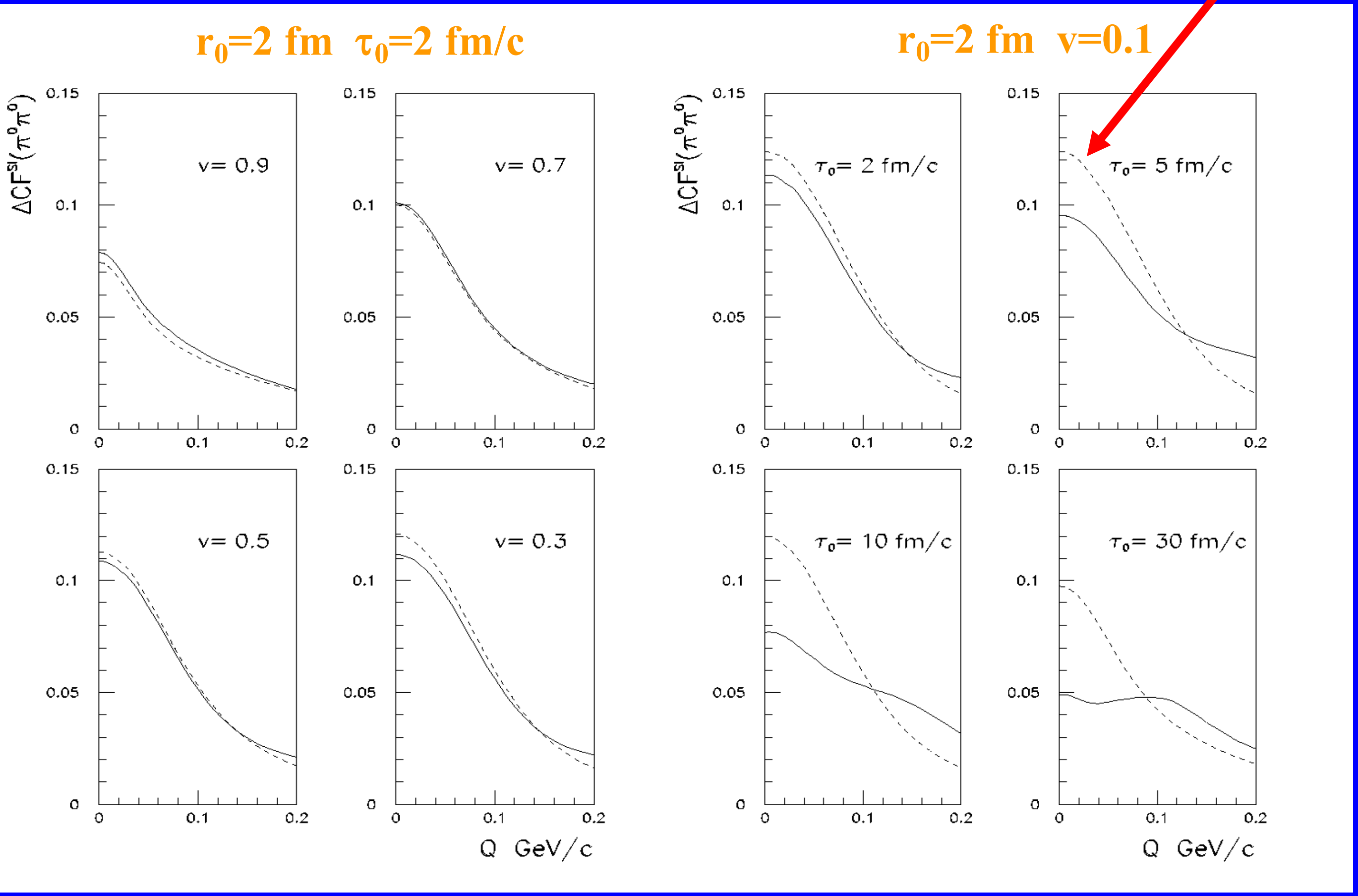
the measurement of strange particle interaction is highly required to understand the properties (EoS) of neutron stars

Effect of nonequal times in pair cms

RL, Lyuboshitz SJNP 35 (82) 770; RL nucl-th/0501065 $\Psi^{S(+)}_{p_1,p_2}(x_1,x_2) \rightarrow e^{iPX}\psi^{S}_{-\mathbf{k}^*}(\mathbf{r}^*)$

Applicability condition of equal-time approximation: $|t^*| \ll m_{1,2}r^{*2}$

$|k^*t^*| \ll m_{1,2}r^*$



↓

OK for heavy particles & small k*

→ OK within 10% even for pions if $\Delta\tau=\tau_0 \sim r_0$ or lower

Analytical dependence of CF on s-wave scatt. amplitudes $f(k)$ and source radius r_0 LL'81

Using spherical wave in the outer region ($r>\varepsilon$) & inner region ($r<\varepsilon$) correction,
assuming Gaussian separation distribution $W(r)=\exp(-r^2/4r_0^2)/(2\sqrt{\pi}\, r_0)^3$
& single channel & no Coulomb

$\Longrightarrow$

FSI contribution to the CF of nonidentical particles

at $kr_0 \ll 1$: $$\Delta CF^{FSI} = \tfrac{1}{2}|f_0/r_0|^2[1-d_0/(2r_0\sqrt{\pi})]+2\mathrm{Re}f_0/(r_0\sqrt{\pi})$$

f_0 & d_0 are the s-wave scatt. length and eff. radius determining the scattering amplitude in the effective range approximation:

$$f(k) = \sin\delta_0\exp(i\delta_0)/k \approx (1/f_0+\tfrac{1}{2}d_0k^2 - ik)^{-1}$$

f_0 and d_0 : characterizing the nuclear force

$u(r) = e^{i\delta} r\psi(r)$

$f_0 = -a$

$d_0 \approx r_0$

at $k \rightarrow 0$

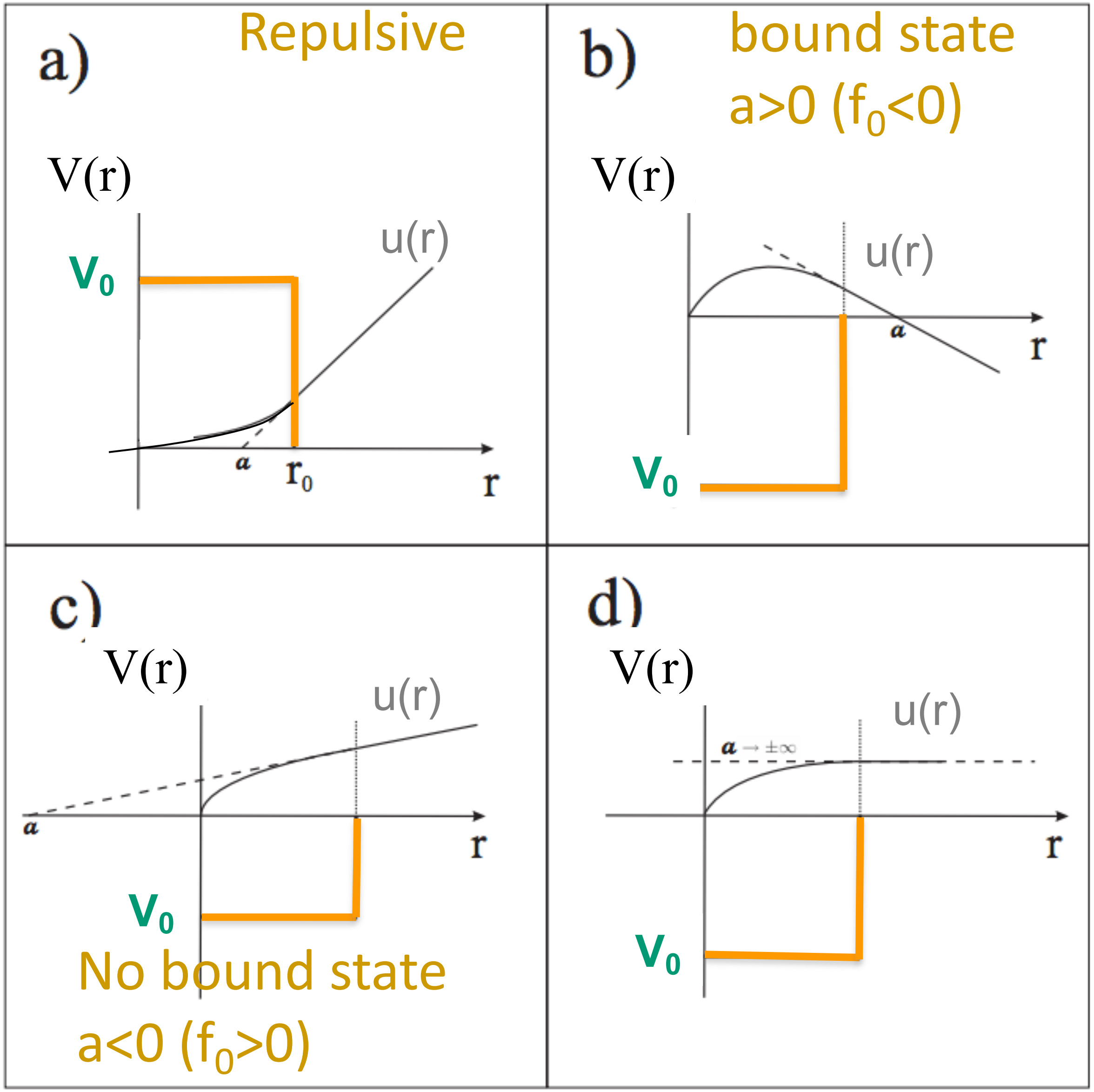
$r > r_0$

$u(r) \sim (r - a)$

Resonance:

$f_0 > 0$

$d_0 < 0$



Correlation asymmetries

RL, Lyuboshitz, Erazmus, Nouais PLB 373 (1996) 30

CF of **identical particles** sensitive to terms **even** in $\mathbf{k}^*\mathbf{r}^*$ (e.g. through $\langle \cos 2\mathbf{k}^*\mathbf{r}^* \rangle$)
$\rightarrow$ measures **dispersion** of the components of relative PRF separation $\mathbf{r}^* = \mathbf{r}_1{}^* - \mathbf{r}_2{}^*$

$$\langle (\Delta x^*)^2 \rangle, \langle (\Delta y^*)^2 \rangle, \langle (\Delta z^*)^2 \rangle$$

CF of **nonidentical particles** sensitive also to terms **odd** in $\mathbf{k}^*\mathbf{r}^*$
$\rightarrow$ measures also relative **space-time asymmetries** - shifts $\langle \mathbf{r}^* \rangle$:

$$\langle \Delta x^* \rangle, \langle \Delta y^* \rangle, \langle \Delta z^* \rangle$$

$\rightarrow$ Construct $\mathbf{CF}_{+x}$ and $\mathbf{CF}_{-x}$ with **positive** and **negative** $\mathbf{k}^*$**-projection** k^*_x on a given **direction** $\mathbf{x}$ and study CF-ratio $\mathbf{CF}_{+x}/\mathbf{CF}_{-x}$

In LCMS ($v_z=0$) or $\mathbf{x} \parallel \mathbf{v}$: $\Delta x^* = \gamma_t(\Delta x - v_t \Delta t)$

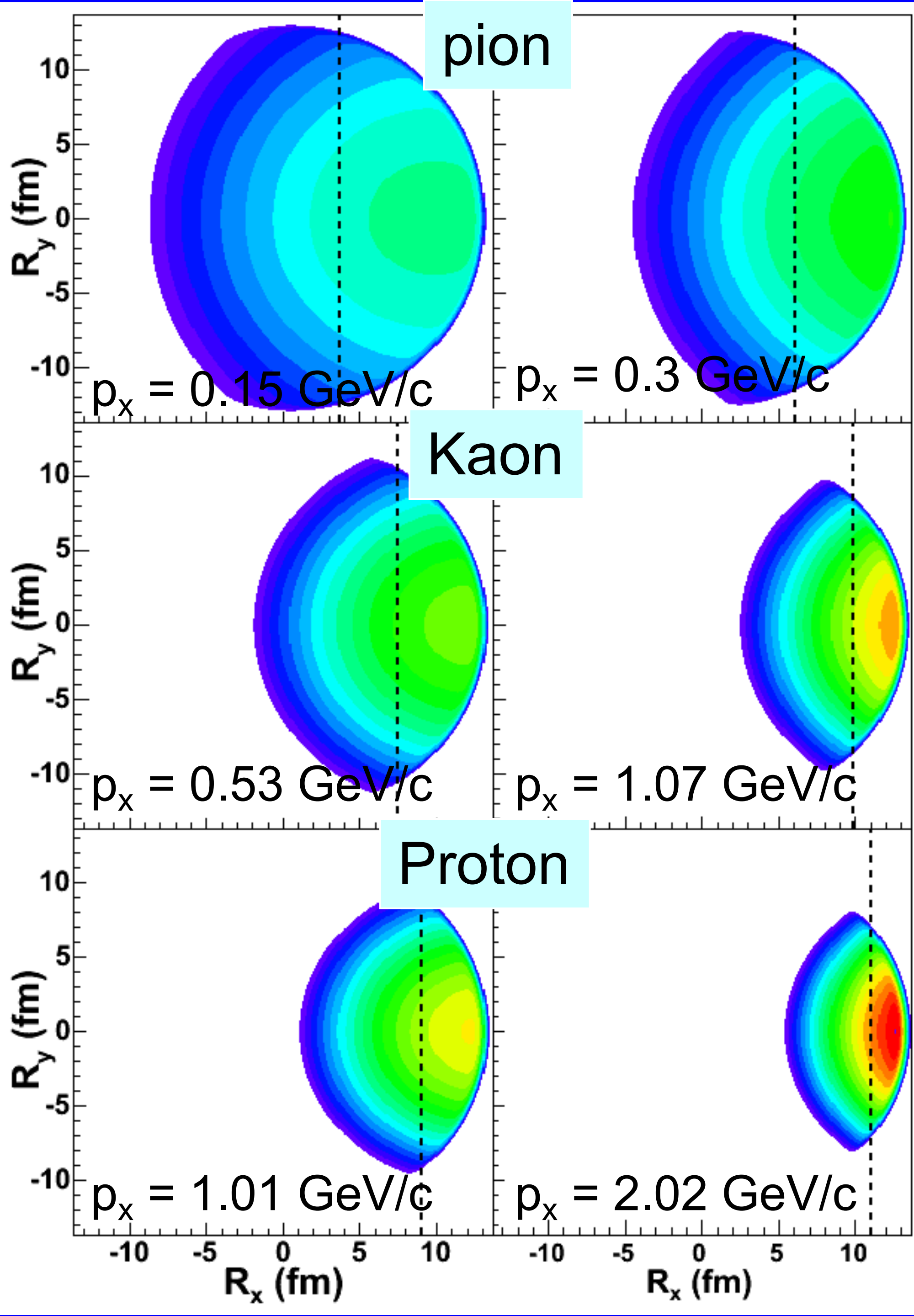
$\Rightarrow$ **CF asymmetry is determined by space and time asymmetries**

at $k^* \rightarrow 0$, asymmetry for charged particles arises mainly from Coulomb FSI:

$$CF_{+x}/CF_{-x} \rightarrow 1+2 \langle \Delta x^* \rangle / a$$

a — Bohr radius = ± 226 fm for $\pi^\pm p$
± 388 fm for $\pi^+\pi^\pm$

$\Rightarrow$ Mirror symmetry in case of ~ same Δx^* for $\pm$charged particles



BW Retiere@LBL'05

Distribution of emission points at a given equal velocity:

- Left, $v_x = 0.73c$, $v_y = 0$
- Right, $v_x = 0.91c$, $v_y = 0$

Dash lines: average emission R_x

$\Rightarrow \langle R_x(\pi)\rangle < \langle R_x(K)\rangle < \langle R_x(p)\rangle$

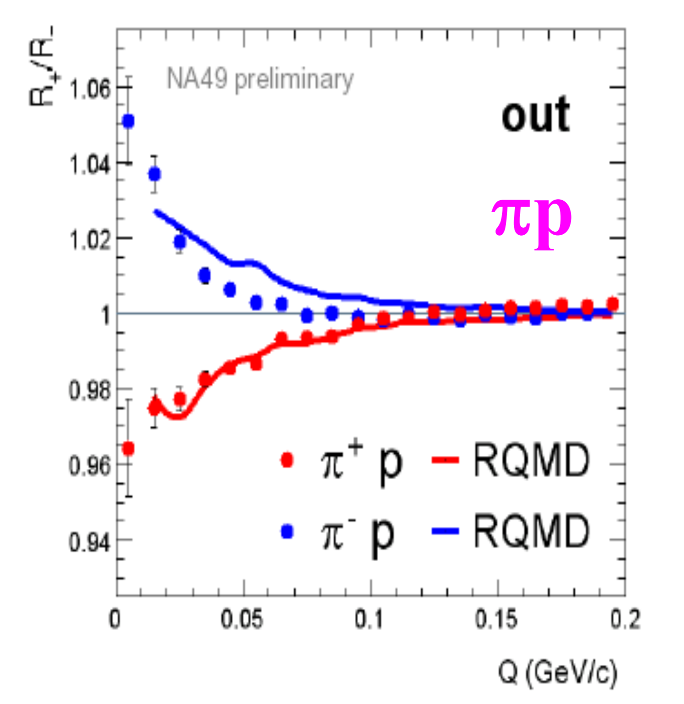
For a Gaussian tr. density profile with a radius r_0 and tr. flow rapidity profile $\rho(r) = \rho_0\, r/\, r_0$ **RL'04, Akkelin-Sinyukov'96** :

$$\langle x\rangle = r_0 v_t\, \rho_0 / [\rho_0^2 + T/m_t]$$

NA49 & STAR out-asymmetries

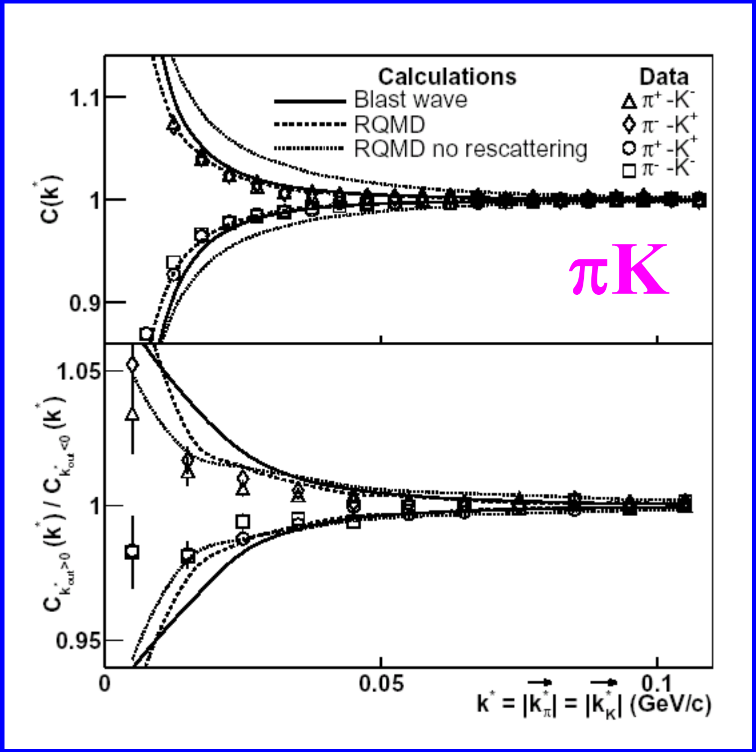
Pb+Pb central 158 AGeV

not corrected for ~ 25% impurity
r* RQMD scaled by 0.8



Au+Au central $\sqrt{s_{NN}}$=130 GeV

corrected for impurity



→ **Mirror symmetry** (~ same mechanism for + and – mesons)

→ **RQMD, BW ~ OK ⇒ points to strong transverse flow**

(⟨Δt⟩ gives only ~ ¼ of CF asymmetry)

Bound state production

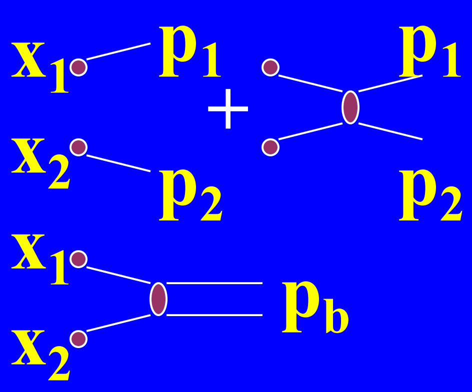
- **Dominated by FSI provided a small binding energy ε_b**
- **Closely related to production of free particles at $k^* \to 0$**

FSI theory: Fermi β-decay

Migdal, Watson, Sakharov, .. hadronic processes

- **Continuum:**

$$d^6N/(d^3p_1\, d^3p_2) = d^6N_0/(d^3p_1\, d3p2)\langle|\psi_{-k^*}(r^*)|^2\rangle$$



- **Discrete spectrum:**

$$d^3N/d^3p_b = (2\pi)^3\gamma_b d^6N_0/(d^3p_1\, d^3p_2)\langle|\psi_b(r^*)|^2\rangle$$

$$p_1/m_1 \approx p_2/m_2 \approx p_b/m_b$$

$$\langle|\psi(r)|^2\rangle = \int d^3r\, W_P(r)\, |\psi(r)|^2$$

$W_P(r^*)$ = normalised (to 1) distribution of
$r^* = x_1^* - x_2^*$ = emitter separation in pair cms

Basis of bound state coalescence femtoscopy

Coalescence: deuterons ..

$$E_d d^3N/d^3p_d = B_2\, E_p d^3N/d^3p_p\; E_n d^3N/d^3p_n$$ $p_p \approx p_n \approx \frac{1}{2} p_d$

Coalescence factor: $B_2 = (2\pi)^3 (m_p m_n / m_d)^{-1} \rho_t \langle|\psi_b(r^*)|^2\rangle$

Triplet fraction = ¾ ↲ unpolarized Ns

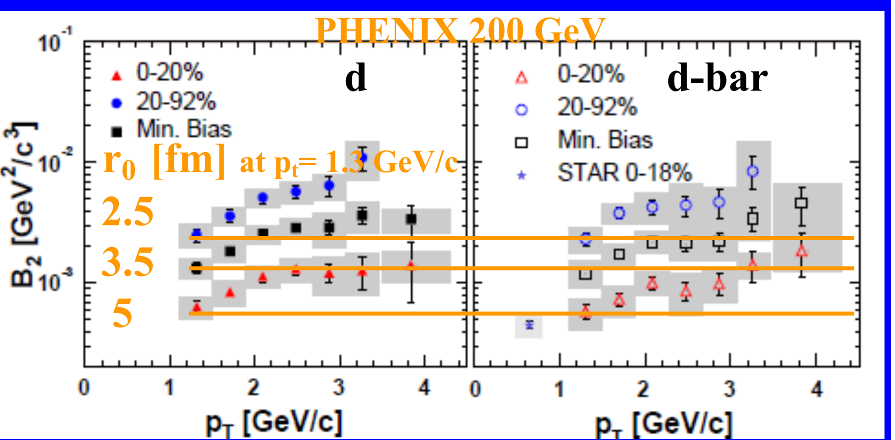
Sato-Yazaki'81, Mrowczynski'87 Lyuboshitz'88 ..

Assuming Gaussian r* distribution $\exp(-r^2/4r_0^2)$, $|\psi_b(r)|^2 \sim \exp[-r^2/(8R_d^2/3)]$, where R_d= 2.2 fm is r.m.s. deuteron radius, and accounting for a boost $\gamma_t = m_{pt}/m_p$ from LCMS to PRF:

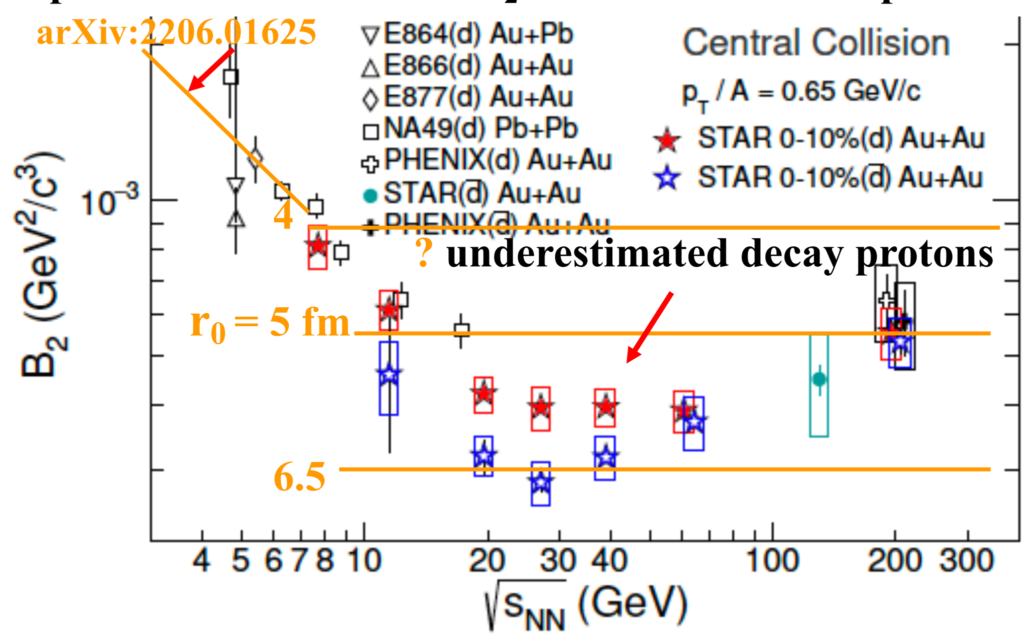
$B_2 \approx {}^3/_4 \pi^{3/2} 2/[m_p(r_0^2 + {}^2/_3 R_d^2)(\gamma_t^2 r_0^2 + {}^2/_3 R_d^2)^{1/2}]$

$\xrightarrow{r_0^2 \gg R_d^2}$ $\frac{3}{4}\pi^{3/2} 2/(m_{pt} r_0^3)$ CoalM

$B_2 = \frac{3}{4}(2\pi)^3 2/(m_p V)$ TherM



? primordial nucleons in $B_2 \neq$ final nucleons & p's≠n's



- r_0(pp) ~ 4 fm from AGS to RHIC central HICs
- Exp. B_2 ↑ with ↑p_t and ↓ centrality (↓ r_0) OK but B_2 too small (r_0 too high) at $\sqrt{s_{NN}} > 10$ GeV : ? residual decay protons & B_2 vs p_t too strong : ? box-like density Scheibl-Heinz'99

tp/d² ratio → density fluctuation measure

$$N_tN_p/N_d^2 \approx 0.29\left(1+\frac{\langle\delta n^2\rangle}{\langle n\rangle^2}\right) \rightarrow \frac{4}{9}\left[1+\left(\frac{1.14\text{fm}}{r_0}\right)^2\right]^3$$

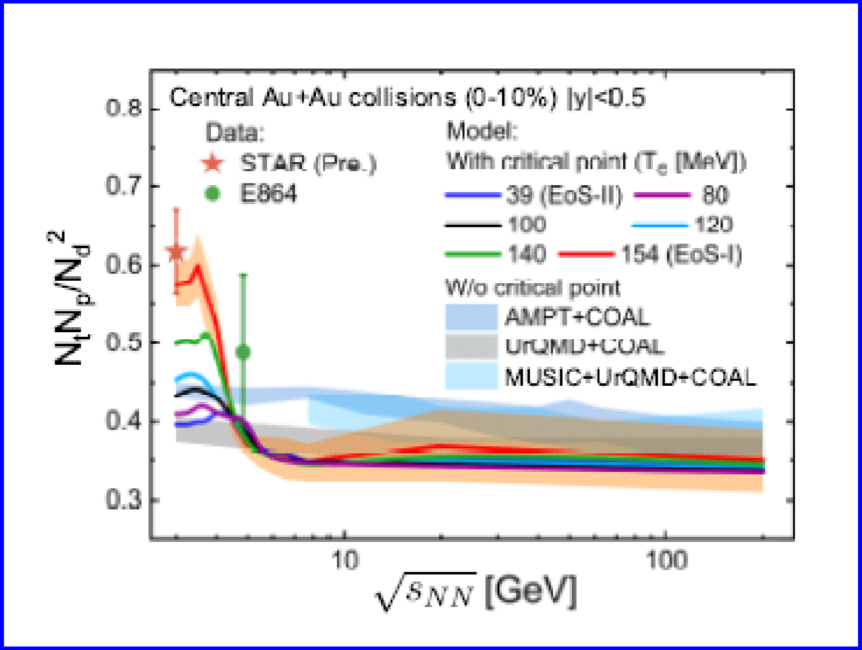
if $\langle\delta n^2\rangle > 0$ due to **Gaussian source density only** & Gaussian radius $r_0 \gg 1$fm

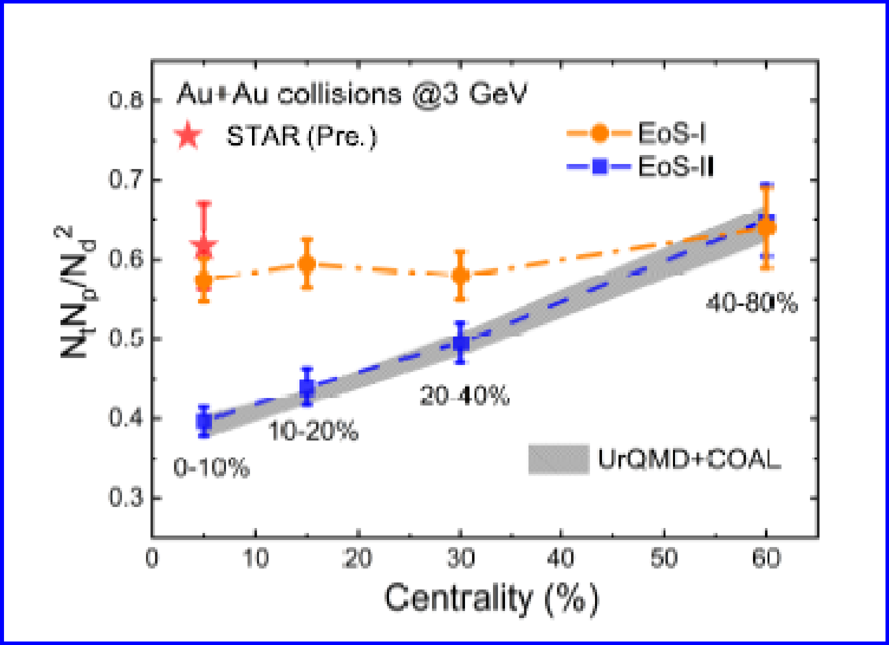
⇒ then tp/d² ratio ↓ for central events (r_0 ↑)

The 1st order phase transition leads to [Sun et al, arXiv:2205.11010]

increased fluctuations at 3-4 GeV & flat centrality dependance at 3-4 GeV

OK: STAR 3 GeV arXiv:2208.04650, 2311.11020 [nucl-ex]





Correlation femtoscopy with nonid. particles

pΛ CFs at AGS & SPS & STAR

Goal: No Coulomb suppression as in pp CF &
Wang-Pratt'99 Stronger sensitivity to r_0

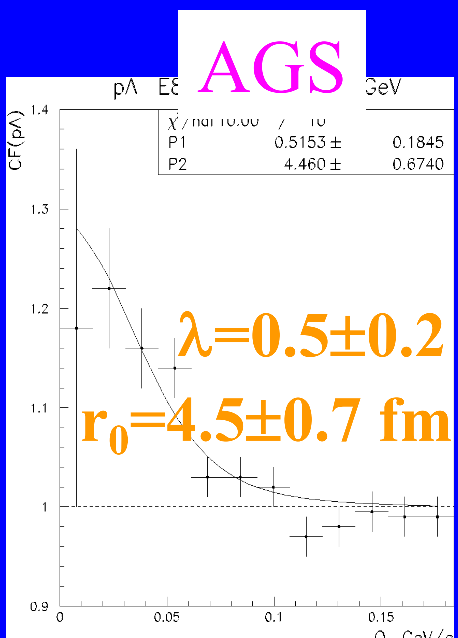
Fit using RL-Lyuboshitz'82 with

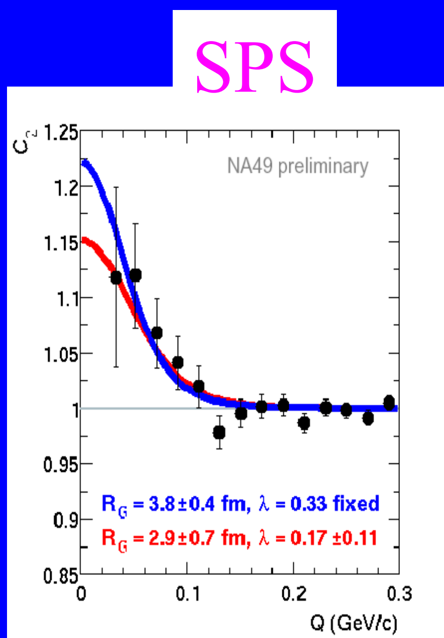
	singlet	triplet
Scattering lengths, fm:	2.31	1.78
Effective radii, fm:	3.04	3.22

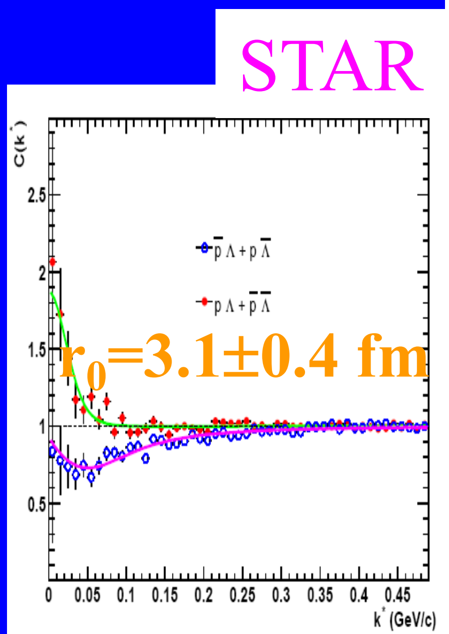
r_0~ 3-4 fm consistent with the radius from pp CF & m_t scaling

λ consistent with estimated pair purity 15-40%

? neglected residual parent correlations







Pairs	Fractions (%)
p_{prim}-Λ_{prim}	15
p_{Λ}-Λ_{prim}	10
p_{Σ^+}-Λ_{prim}	3
p_{prim}-Λ_{Σ^0}	11
p_{Λ}-Λ_{Σ^0}	7
p_{Σ^+}-Λ_{Σ^0}	2
p_{prim}-Λ_{Ξ}	9
p_{Λ}-Λ_{Ξ}	5
p_{Σ^+}-Λ_{Ξ}	2

NA49 central Pb+Pb 158 AGeV vs RQMD: FSI theory OK

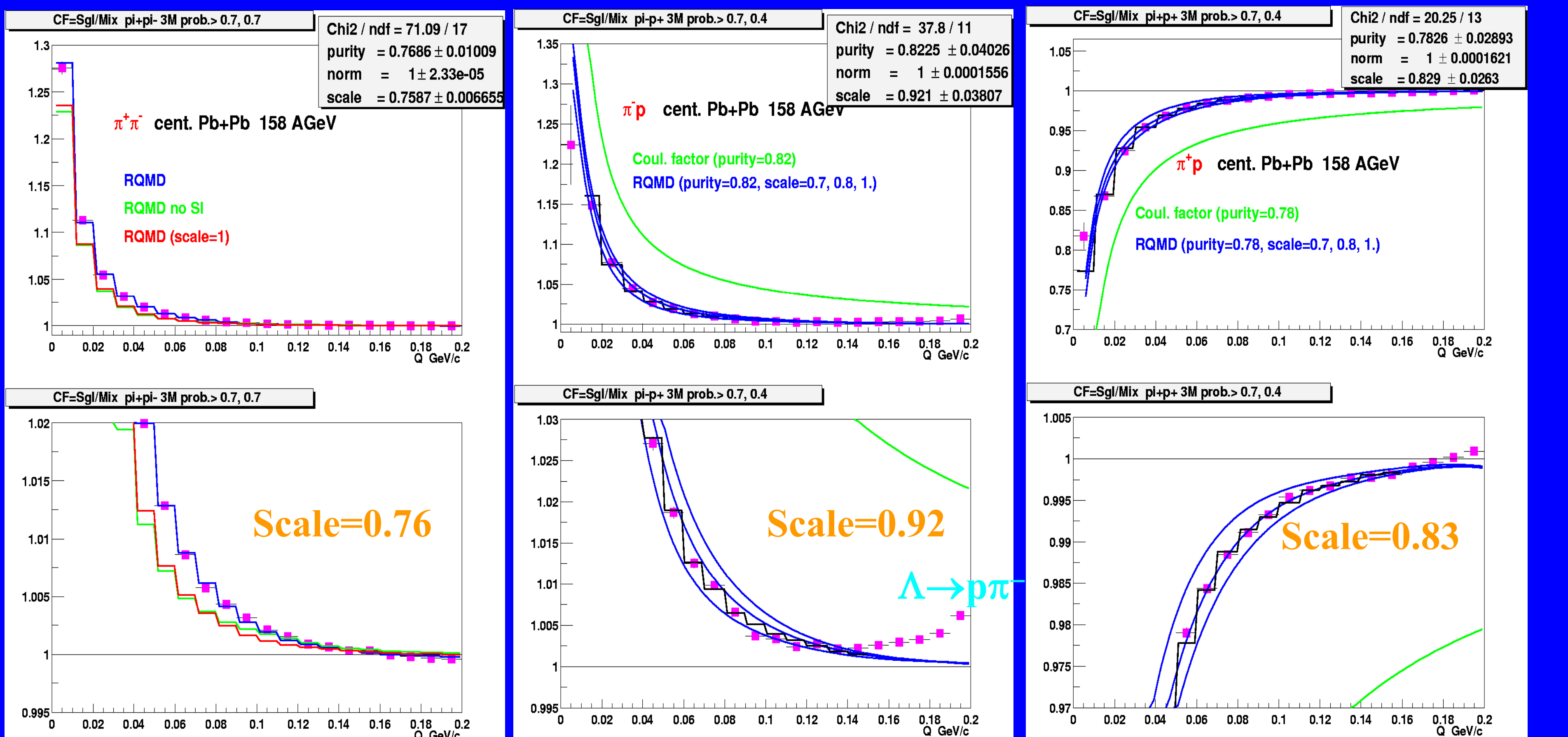
Long tails in RQMD: $\langle r^* \rangle$ = **21 fm** for r^* < **50 fm**

29 fm for r^* < **500 fm**

Fit $CF = \text{Norm}\,[\text{Purity}\ \text{RQMD}(r^* \rightarrow \text{Scale}\cdot r^*) + 1 - \text{Purity}]$

⇒ **RQMD overestimates** r^* **by 10-20% at SPS** cf ~ **OK at AGS**

worse at RHIC



Correlation study of strong interaction

$\pi^+\pi^-$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ s-wave scattering parameters

from NA49, STAR and ALICE (fits using LL'82 & Pot. models)

$\pi^+\pi^-$: NA49 Pb+Pb vs RQMD with SI scale: $f_0 \rightarrow$ sisca f_0 (=0.232fm)
sisca = 0.57±0.07 compare with
~0.8 from SχPT & K-decays & pionium lifetime

a suppression of $\pi\pi$ amplitude can be due to eq. time approx.

$\bar{p}\Lambda$,$\bar{B}B$: STAR & ALICE data accounting for residual correlations
- **Kisiel et al, PRC 89 (2014): assuming a universal Imf_0**
- **Shapoval et al PRC 92 (2015): Gauss. par. of residual CF**
- **ALICE arXiv:1903.06149: similar to Kisiel + free d_0**

$Ref_0 \approx$ 0.5 fm,	**$Imf_0 \approx$ 1 fm,**	**d_0**	**fixed 0 fm**	**STAR**
-1 fm	**0.5 fm**		**$\approx$ 2.7 fm**	**ALICE**

$\Lambda\Lambda$: STAR, PRL 114 (2015) Au+Au: $f_0(\Lambda\Lambda) \approx$ -1 fm, $d_0(\Lambda\Lambda) \approx$ 8 fm
ALICE, PLB 797(2019) p+p, p+Pb: prefer $f_0(\Lambda\Lambda) > 0$

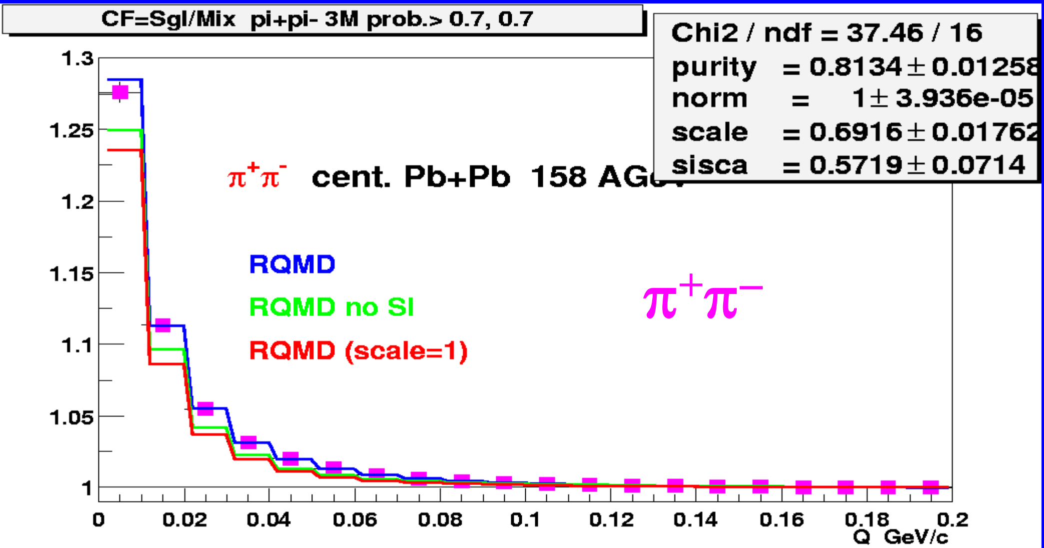
pΞ,pΛ: ALICE (2019) p+Pb: substantial pΞ SI
(2021) p+p : pΛ
both agree with χEFT potentials

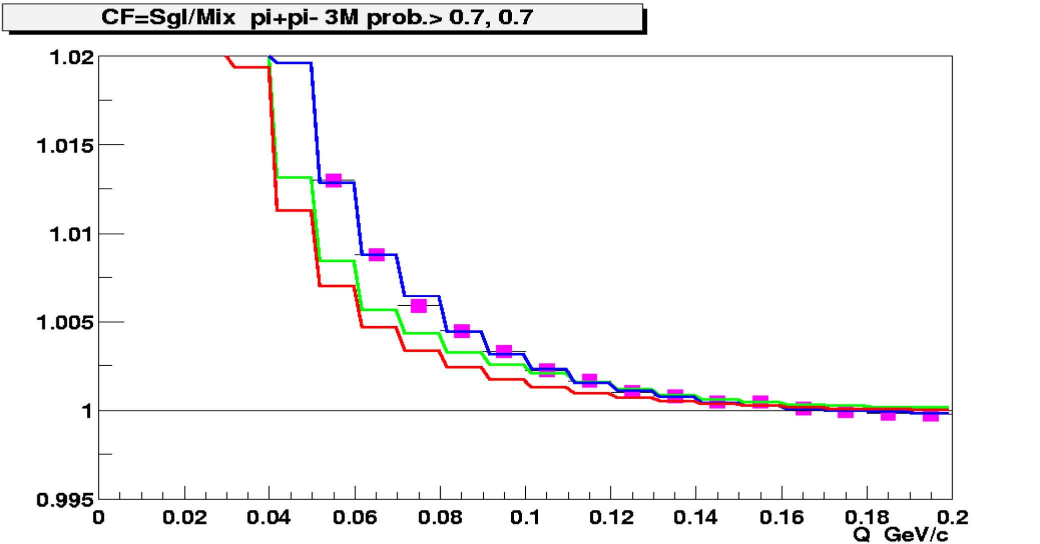
pϕ: ALICE (2021) p+p: spin-aver. $Ref_0 \approx$ 0.9 fm, $Imf_0 \approx$ 0.2 fm, $d_0 \approx$ 8 fm

$\bar{p}\ \bar{p}$: STAR Au+Au, Nature (2015): f_0 and d_0 coincide with table pp-values

Correlation study of strong interaction

CF = Norm [Purity RQMD(r* → Scale•r*)+1-Purity]





$\pi^+\pi^-$ scattering length f_0 from NA49 CF

Fit CF($\pi^+\pi^-$) by RQMD with SI scale:

$f_0 \rightarrow$ **sisca** f_0^{input}

$f_0^{input} = 0.232$ fm

sisca = 0.57±0.07

differs 6σ from 1 and

3σ from ~0.8

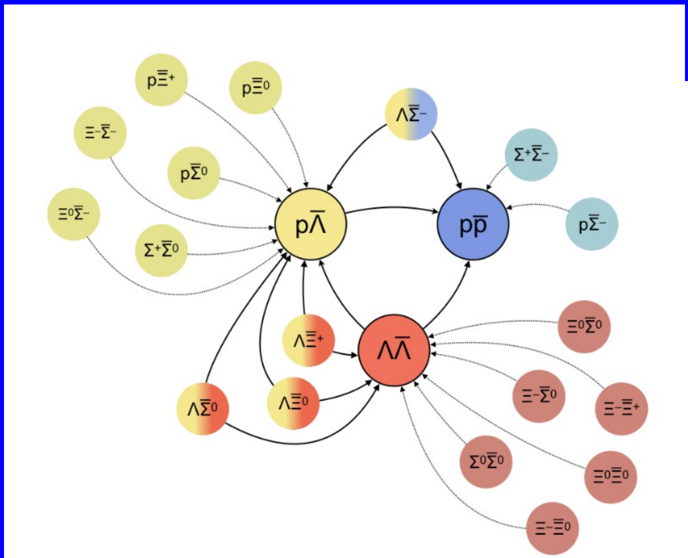
(f_0 = 0.186 fm from SχPT &

K → eνππ, ππ^{0}π^0 & pionium lifetime)

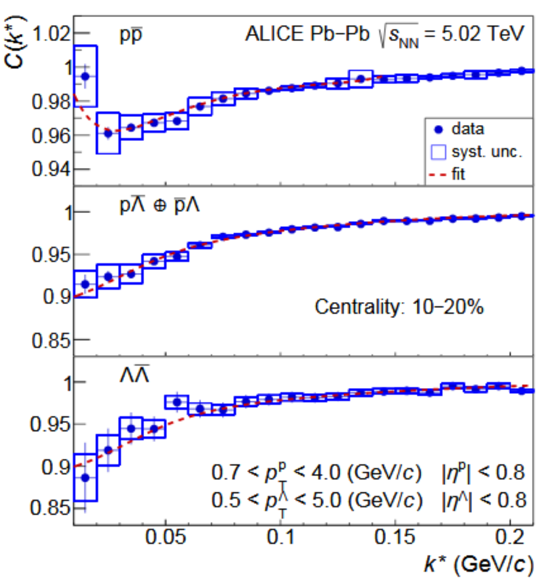
→ 3σ indication of f_0 suppression – likely due to non-equal emission times in PRF

ALICE $\overline{\mathrm{B}}\mathrm{B}$ in Pb+Pb   arXiv:1903.06149

Pair fractions λ from AMPT (HIJING)



$p\overline{p}$		$p\overline{\Lambda}\oplus\overline{p}\Lambda$		$\Lambda\overline{\Lambda}$	
Pair	λ	Pair	λ	Pair	λ
$p\overline{p}$	0.25 (0.32)	$p\overline{\Lambda}$	0.29 (0.28)	$\Lambda\overline{\Lambda}$	0.37 (0.24)
$p\overline{\Lambda}$	0.12 (0.19)	$\Lambda\overline{\Lambda}$	0.08 (0.09)	$\Lambda\overline{\Xi}^+$	0.04 (0.06)
$p\overline{\Sigma}^-$	0.04 (0.04)	$\Lambda\overline{\Sigma}^-$	0.03 (0.02)	$\Lambda\overline{\Xi}^0$	0.03 (0.05)
$\Lambda\overline{\Lambda}$	0.02 (0.03)	$p\overline{\Xi}^{0/+}$	0.02 (0.03)	$\Lambda\overline{\Sigma}^0$	<0.01 (0.20)
$\Lambda\overline{\Sigma}^-$	0.01 (0.01)	$p\overline{\Sigma}^0$	<0.01 (0.12)	$\Sigma^0\overline{\Sigma}^0$	<0.01 (0.05)
$\Sigma^+\overline{\Sigma}^-$	<0.01 (<0.01)	$\Lambda\overline{\Sigma}^0$	<0.01 (0.04)	$\Xi^{0/-}\overline{\Sigma}^0$	<0.01 (0.02)
		$\Lambda\overline{\Xi}^{0/+}$	<0.01 (0.01)	$\Xi^{0/-}\overline{\Xi}^{0/+}$	<0.01 (<0.01)
		$\Sigma^+\overline{\Sigma}^0$	<0.01 (<0.01)		
		$\Xi^{0/-}\overline{\Sigma}^+$	<0.01 (<0.01)		



$$C_{xy}(k^*) = 1 + \sum_i \lambda_i \left[C_i(k^*) - 1\right],$$

Spin averaged scattering parameters

Parameter	$\overline{p}\Lambda\oplus p\overline{\Lambda}$	$\Lambda\overline{\Lambda}$	$B\overline{B}$
$\Re f_0$ (fm)	$-1.15^{\pm 0.23\text{ (syst.)}}_{\pm 0.05\text{ (stat.)}}$	$-0.90^{\pm 0.16\text{ (syst.)}}_{\pm 0.04\text{ (stat.)}}$	$-1.08^{\pm 0.11\text{ (syst.)}}_{\pm 0.20\text{ (stat.)}}$
$\Im f_0$ (fm)	$0.53^{\pm 0.15\text{ (syst.)}}_{\pm 0.04\text{ (stat.)}}$	$0.40^{\pm 0.18\text{ (syst.)}}_{\pm 0.06\text{ (stat.)}}$	$0.57^{\pm 0.25\text{ (syst.)}}_{\pm 0.19\text{ (stat)}}$
d_0 (fm)	$3.06^{\pm 0.98\text{ (syst.)}}_{\pm 0.14\text{ (stat.)}}$	$2.76^{\pm 0.73\text{ (syst.)}}_{\pm 0.29\text{ (stat.)}}$	$2.69^{\pm 0.46\text{ (syst.)}}_{\pm 0.74\text{ (stat.)}}$

Correlation study of strong interaction

ΛΛ scattering lengths f_0 from STAR(Au+Au)/ALICE (p+p, p+Pb) data

STAR fit using RL-Lyuboshitz '82:

$\lambda \approx 0.18$, $r_0 \approx 3$ fm,

$a_{res} \approx -0.04$, $r_{res} \approx 0.4$ fm

$f_0 \approx -1$ fm, $d_0 \approx 8$ fm

- **no s-wave resonance**
- **deeply bound state possible: $\varepsilon_b \sim 0.5\ GeV$**
- **ALICE'19 data allow for a shallow bound state, assuming however a flat residual correlation**
- **a more correct treatment of residual correlations ($\Lambda\Sigma^0$, $\Lambda\Xi$ . .) is required**

$$CF=1+\lambda\,[\,\Delta CF^{FSI}$$

$$+\Sigma_S \rho_S (-1)^S \exp(-r_0^2 Q^2)\,] + a_{res} \exp(-r_{res}^2 Q^2)$$

$\rho_0 = ¼(1-P^2)$ $\rho_1 = ¼(3+P^2)$ P=Polar.=0

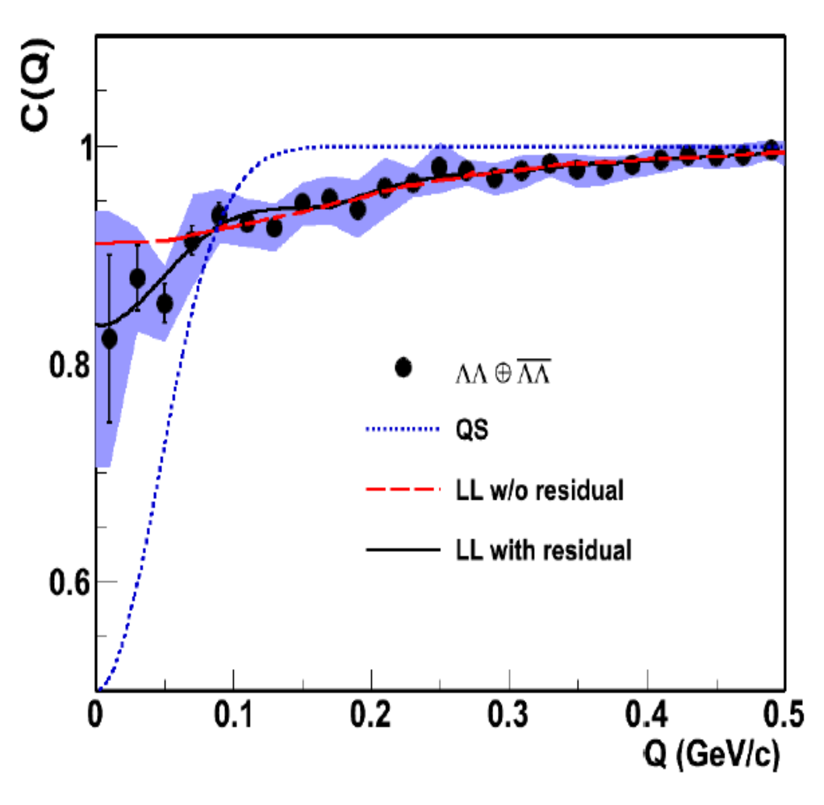
$$\Delta CF^{FSI} = 2\rho_0[½|f^0(k)/r_0|^2(1-d_0^0/(2r_0\sqrt{\pi}))$$

$$+2\mathrm{Re}(f^0(k)/(r_0\sqrt{\pi}))F_1(r_0Q)$$

$$-\ \mathrm{Im}(f^0(k)/r_0)F_2(r_0Q)]$$

$f^S(k)=(1/f_0^S+½d_0^S k^2 - ik)^{-1}$, $k=Q/2$

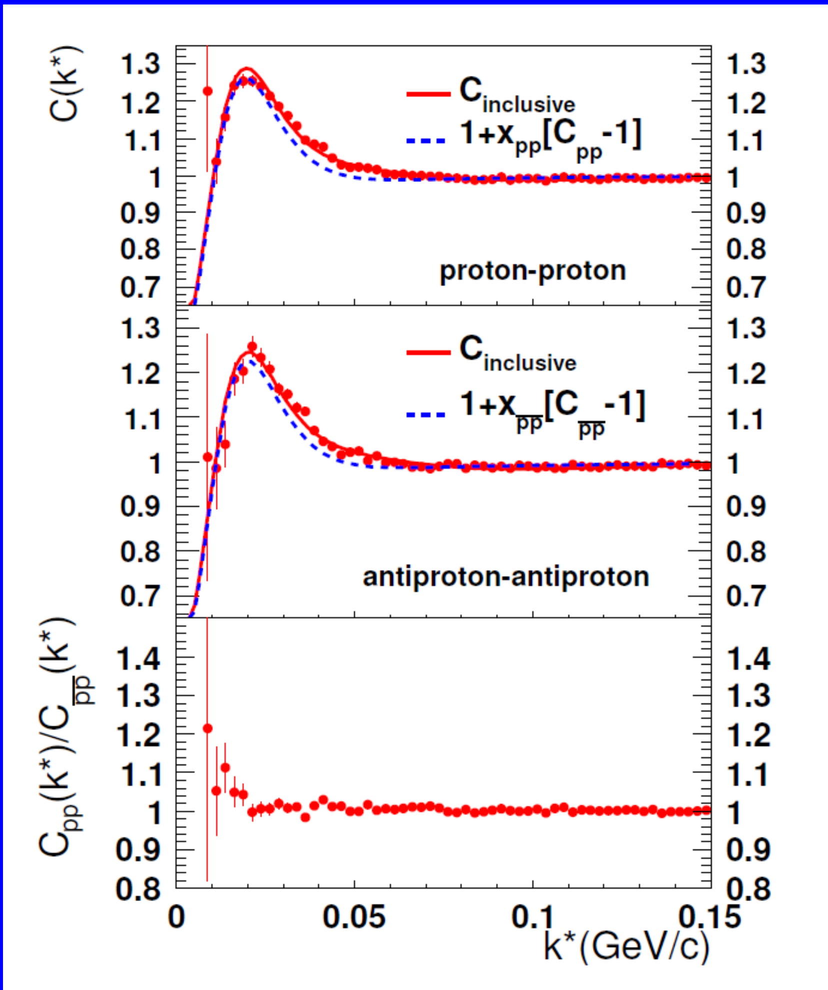
$F_1(z)=\int_0^z dx\, \exp(x^2-z^2)/z$, $F_2(z)=[1-\exp(-z^2)]/z$



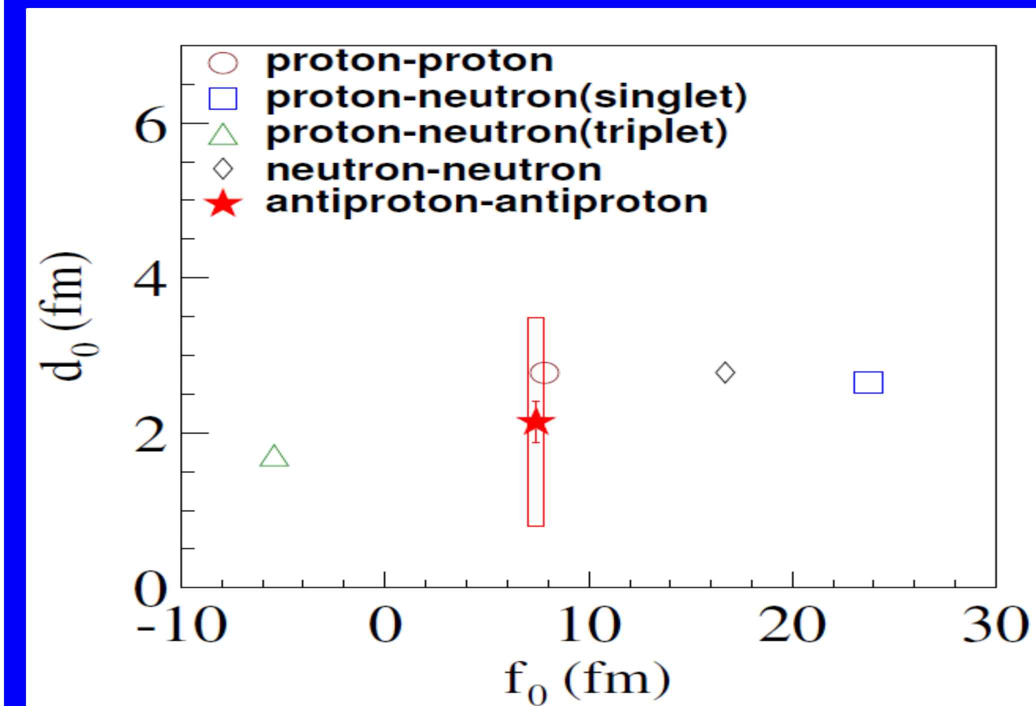
Correlation study of strong interaction

$\bar{p}\bar{p}$ s-wave scattering parameters from STAR Au+Au 200 GeV

$$C_{\text{inclusive}}(k^*) = 1 + x_{pp}[C_{pp}(k^*; R_{pp}) - 1] + x_{p\Lambda}[\tilde{C}_{p\Lambda}(k^*; R_{p\Lambda}) - 1] + x_{\Lambda\Lambda}[\tilde{C}_{\Lambda\Lambda}(k^*) - 1]$$



	DCA	x_{pp}	$x_{p\Lambda}$	$x_{\Lambda\Lambda}$
p-p	$2cm$	0.45	0.375	0.077
	$1cm$	0.51	0.335	0.055
pbar-pbar	$2cm$	0.42	0.385	0.092
	$1cm$	0.485	0.35	0.063

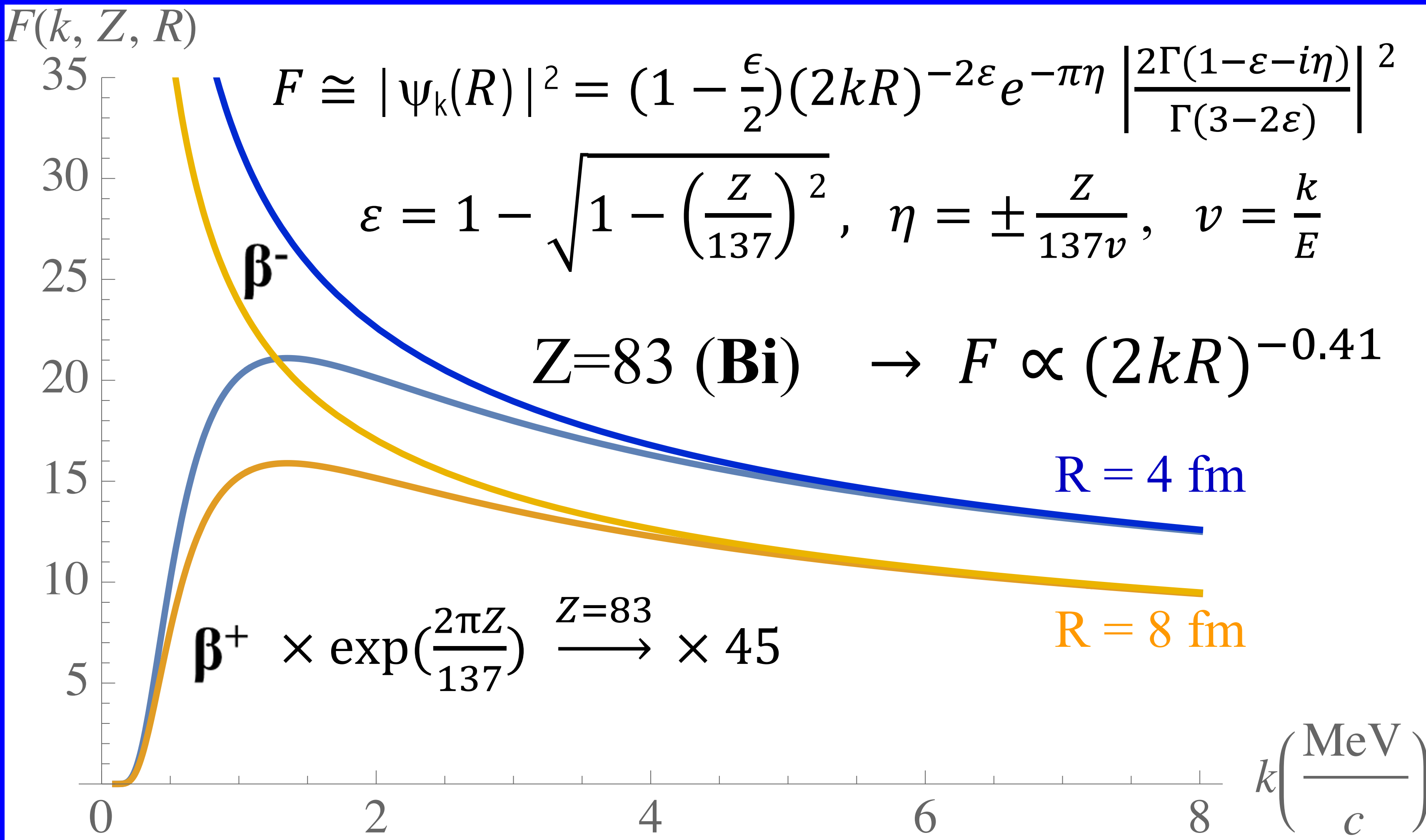


Summary

- The femtoscopy theory is validated in HICs;
  ? violation of equal-time FSI approximation for CFs involving pions.
- Wealth of data on correlations of various particle species $\pi^{\pm}$ $K^{\pm 0}$ ϕ $p^{\pm}$ Λ Ξ .. is available & gives unique space-time info on the production characteristics including collective flows.
- Of particular importance is the BES femtoscopy in the search for the CEP, softening of EoS and corresponding increase of emission duration. The nonidentical particle correlations, in addition to the correlations of identical particles, yield important info on the space and time shifts.
- The momentum correlations yield valuable info on two-particle strong interaction: scattering lengths & effective radii, often hardly available by other means; the measurement of strong interaction involving strange particles is highly required to understand neutron stars.
- The coalescence femtoscopy provides important information on the properties of the excited hadronic matter; subtilities: nucleons from hyperon decays & neutron spectra & account of the non-FSI correlations.
- A good perspective: high statistics correlation and coalescence data from running & future experiments at RHIC, LHC & NICA, FAIR.

Thank you for the attention

Fermi function in β-decay



Assumptions to derive KP formula

$$CF - 1 = \langle \cos q\Delta x \rangle$$

- two-particle approximation (small freeze-out PS density f)

~ OK, $\langle f \rangle \ll 1$? low p_t fig.

- smoothness approximation: $R_{emitter} \ll R_{source} \Leftrightarrow \langle|\Delta p|\rangle \gg \langle|q|\rangle_{peak}$

~ OK in HIC, $R_{source}^2 \gg 0.1\ fm^2$ $\approx p_t^2$-slope of direct particles

- neglect of FSI

OK for photons, ~ OK for charged pions up to Coul. repulsion

- incoherent or independent emission

2π and 3π CF data consistent with KP formulae:

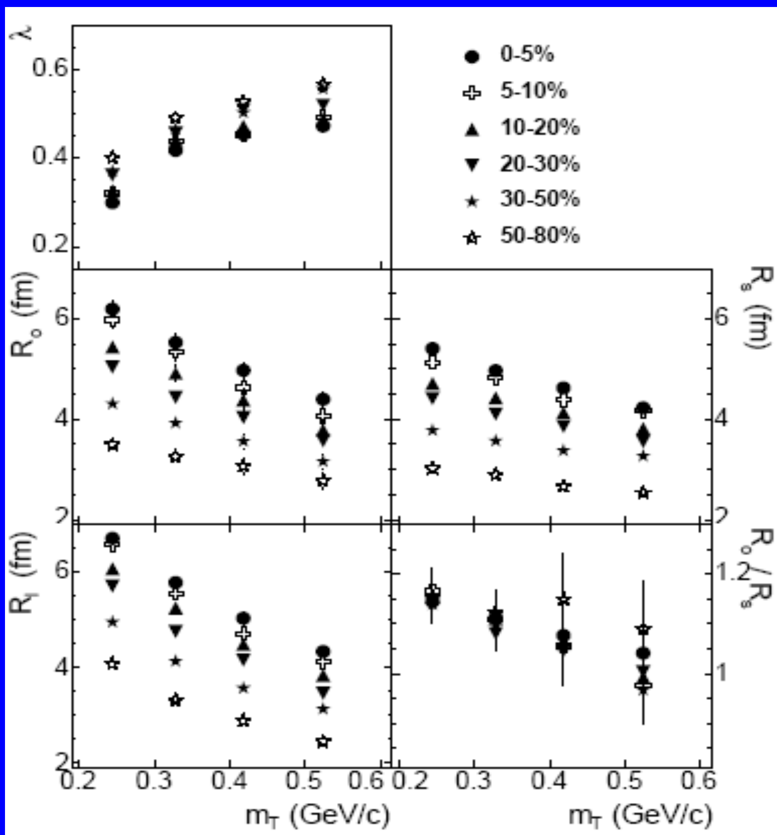
$$CF_3(123) = 1+|F(12)|^2+|F(23)|^2+|F(31)|^2+2\mathrm{Re}[F(12)F(23)F(31)]$$

$$CF_2(12) = 1+|F(12)|^2, \quad F(q) = \langle e^{iqx} \rangle$$

AGS→SPS→RHIC: $\pi\pi$ radii

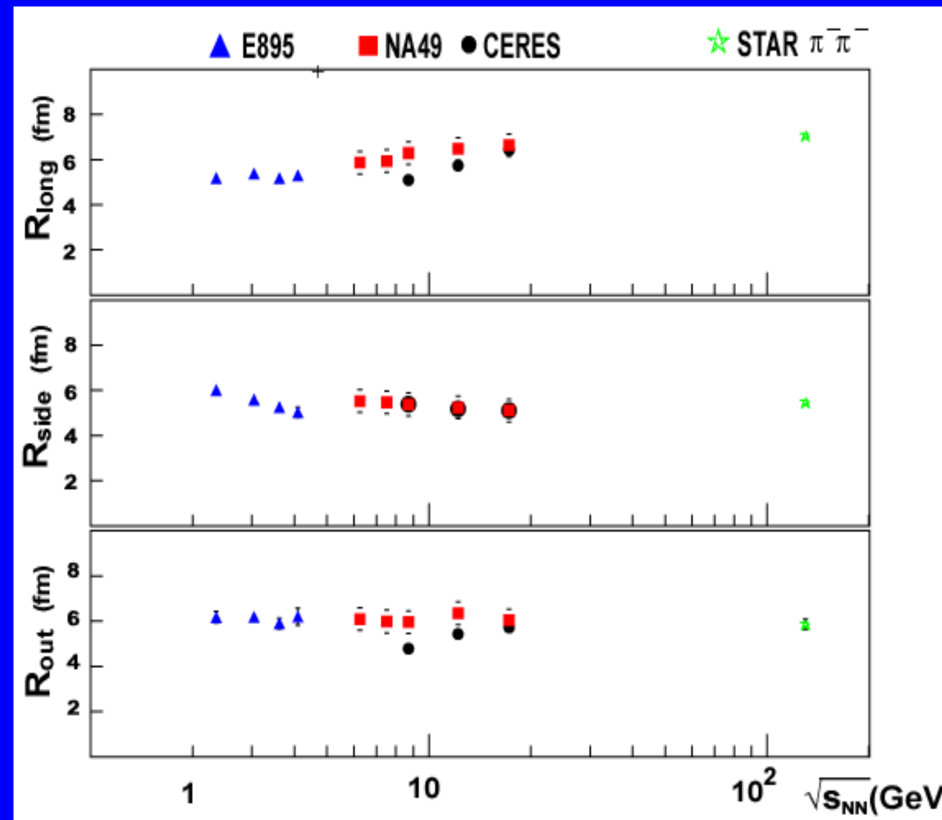
Clear centrality & m_t dependence

STAR Au+Au at 200 AGeV



Weak energy dependence

0-5% central Pb+Pb or Au+Au



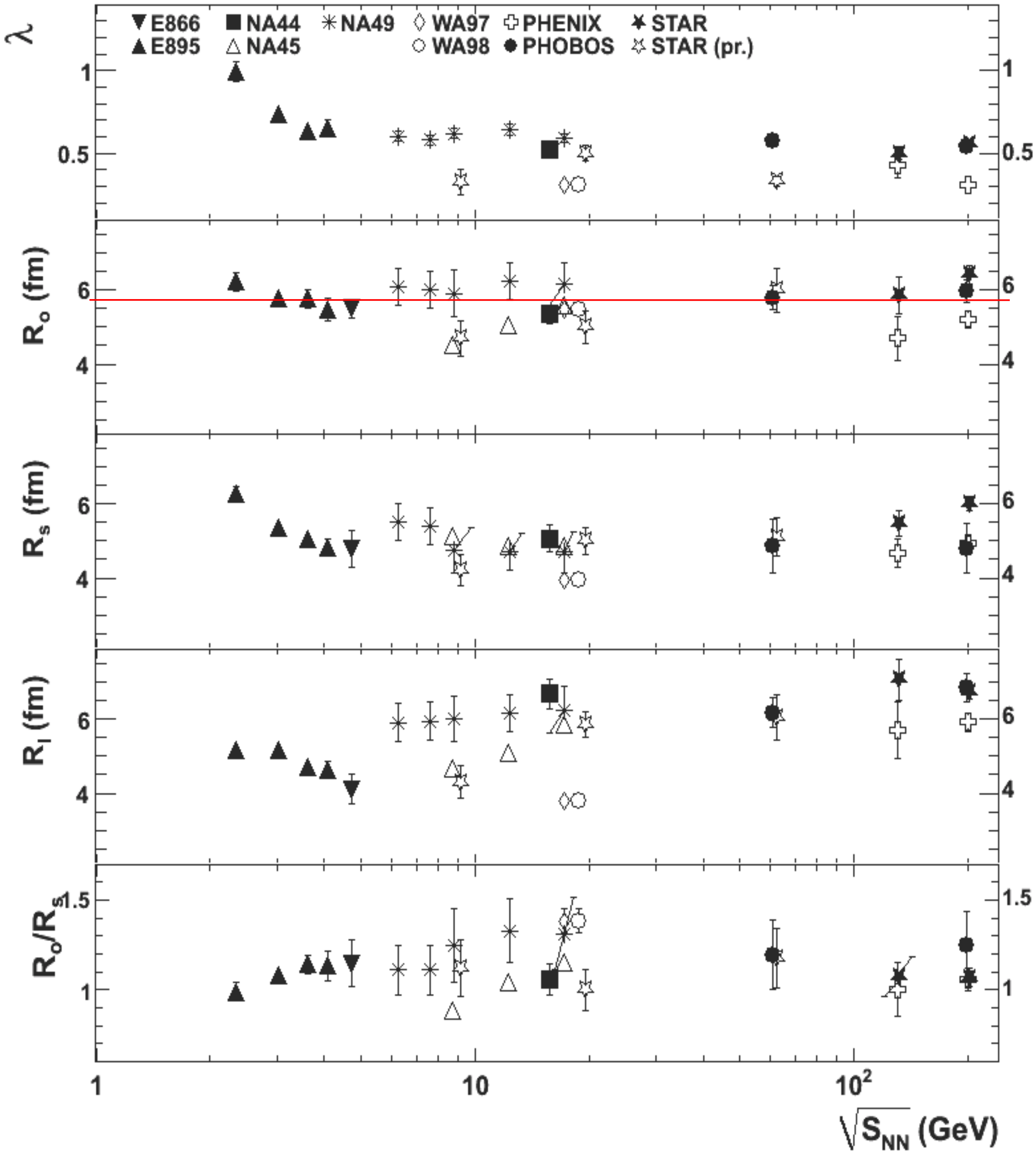
Radii ↑ with centrality & ↓ with m_t

only R_{long} ↑ with energy up to RHIC

→ $R_{long} \approx \tau_0\,(T/m_t)^{1/2}$ → evolution (freeze-out) time $\tau_0 \sim$ 8-10 fm/c

$R_{out}^2 \approx \ldots + v_t^2(\Delta\tau)^2$ → emission duration $\Delta\tau \sim$ 2 fm/c

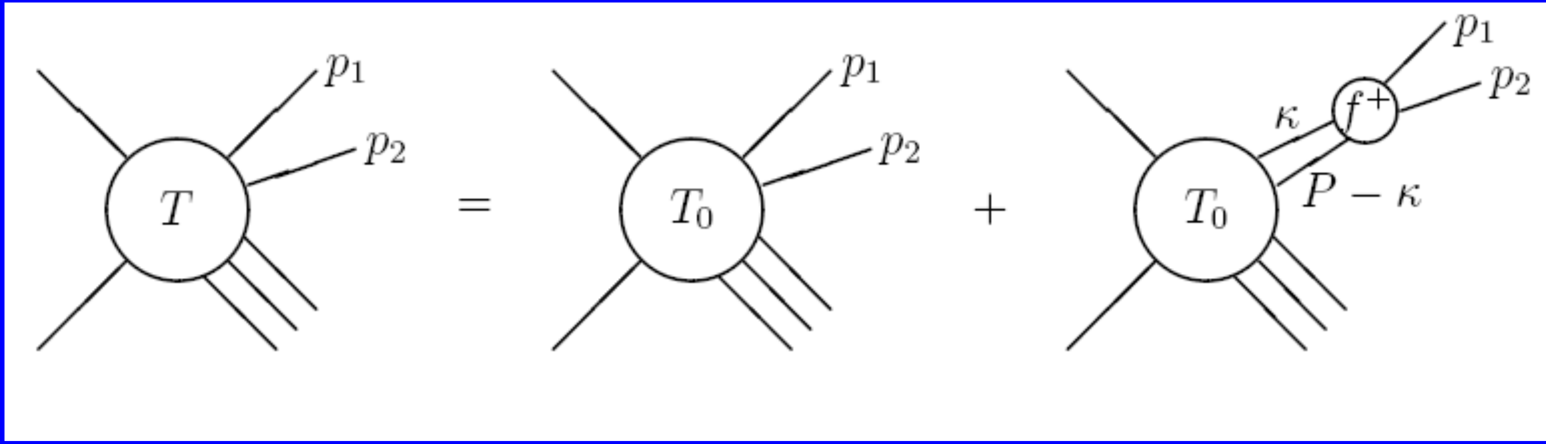
$R_{side} \approx r_0/(1+m_t\,\rho_0^2/T)^{1/2}$ → tr. collective flow rapidity $\rho_0 \sim$ 0.4-0.6



Femto-puzzle II

No signal of a bump in R_{out} near the QGP threshold (expected at AGS-SPS energies) !? – likely solved due to a decrease of partonic phase at these energies

BS-amplitude Ψ



$$T(p_1,p_2;\alpha) = T_0(p_1,p_2;\alpha) + \Delta T(p_1,p_2;\alpha)$$

$$\Delta T(p_1,p_2;\alpha) = \frac{\mathrm{i}\sqrt{P^2}}{2\pi^3}\int \mathrm{d}^4\kappa \frac{T_0(\kappa, P-\kappa;\alpha) f^{S*}(p_1,p_2;\kappa,P-\kappa)}{(\kappa^2 - {m_1}^2 - \mathrm{i}0)[(P-\kappa)^2 - {m_2}^2 - \mathrm{i}0]}$$

Inserting KP amplitude $T_0(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)exp(-ip_1x_A-ip_2x_B)$ in ΔT and taking the amplitudes $u_A(\kappa)u_B(P-\kappa)$ out of the integral *at* $\kappa \approx p_1$, $P-\kappa \approx p_2$ (again "smoothness assumption") ⇒

Plane waves $exp(-ip_1x_A-ip_2x_B)$ → BS-amplitude $\Psi p_1 p_2(x_A, x_B)$:

$$T(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)\,\Psi p_1 p_2(x_A, x_B)$$

“Fermi-like eq. time” CF formula

$$CF = \langle\, |\psi_{-k^*}(r^*)|^2 \,\rangle$$

Koonin’77: heuristic generalization of KP CF for identical noninteracting particles to interacting nonrelativistic protons

RL, Lyuboshitz’82: derivation of the smoothess approx. to CF of any interacting particles relativistic & polarized & nonidentical & accounting for nonequal times

Assumptions:

- same as for KP formula in case of pure QS &
- equal time approximation in PRF

RL, Lyuboshitz’82 → eq. time conditions: $|t^*| \ll m_{1,2} r^{*2}$, $|k^* t^*| \ll m_{1,2} r^*$

OK (usually, to several % even for pions) fig.

- $t_{FSI} = d\delta/dE \gg t_{prod}$

t_{FSI} (s-wave) $= \mu f_0/k^* \rightarrow |k^*| = \frac{1}{2}|q^*| \ll$ hundreds MeV/c ≈ typical momentum transfer in production

RL, Lyuboshitz ..’98:

& account for coupled channels (within the same isomultiplet only):

$\pi^+\pi^- \leftrightarrow \pi^0\pi^0$, $\pi^- p \leftrightarrow \pi^0 n$, $K^+K^- \leftrightarrow K^0\bar{K}^0$, ...

Pair purity problem for pΛ CF @ STAR

Particle	Identification	Fraction Primary
p	$76 \pm 7\%$	$52 \pm 4\%$
$\overline{p}$	$74 \pm 7\%$	$48 \pm 4\%$
Λ	$86 \pm 6\%$	$45 \pm 4\%$
$\overline{\Lambda}$	$86 \pm 6\%$	$45 \pm 4\%$

⇒ **PairPurity ~ 15%**

Assuming no correlation for misidentified particles and particles from weak decays

→ $$C^{corr}_{measured}(k^*) = \frac{C_{measured}(k^*) - 1}{\mathrm{PairPurity}} + 1$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0} \right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(Qr_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Qr_0) \right],$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$.

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^* \right)^{-1}$$

← Fit using RL-Lyuboshitz'82 (for np)

Pairs	Fractions (%)
p_{prim}-Λ_{prim}	15
p_{Λ}-Λ_{prim}	10
p_{Σ^+}-Λ_{prim}	3
p_{prim}-Λ_{Σ^0}	11
p_{Λ}-Λ_{Σ^0}	7
p_{Σ^+}-Λ_{Σ^0}	2
p_{prim}-Λ_{Ξ}	9
p_{Λ}-Λ_{Ξ}	5
p_{Σ^+}-Λ_{Ξ}	2

← but, there can be residual correlations for particles from weak decays requiring knowledge of $\Lambda\Lambda$, pΣ, $\Lambda\Sigma$, $\Sigma\Sigma$, pΞ, $\Lambda\Xi$, $\Sigma\Xi$ correlations

Thermal Model

Thermal equilibrium in nonrel. limit: $T, \mu_I << E \sim m_i$

$$d^3N_i/d^3\mathbf{p} = V g_i (2\pi)^{-3} \exp[-(E-\mu_i)/T]$$

$g_i = 2S_i + 1$ = spin factor,
μ_i = chemical potential,
T = temperature

$g_p = g_n = 2$, $g_d = 3$, $\mu_d = \mu_p + \mu_n$, $E_d = E_p + E_n$

↓

$B_2 = E_d d^3N/d^3p_d / (E_p d^3N/d^3p_p \, E_n d^3N/d^3p_n) = \frac{3}{4}(2\pi)^3 2/(m_p V)$

coincides with Coalescence Model B_2 in large volume limit

$V^{1/3} = 2\sqrt{\pi}\, r_0 \gg R_d$

Coalescence Model

B_A is decreased due to finite bound state r.m.s. radius R_A:

$\mathbf{r_0^2 \to r_0^2 + {}^2/_3 R_d^2}$ $\quad$ **A= ^{2}H = d**

$\mathbf{r_0^2 \to r_0^2 + {}^4/_9 R_\alpha^2}$ $\quad$ **^{4}He = α**

⟹ B_A ratios for nuclei with different radii depend on centrality (incr. with $\mathbf{r_0}$)

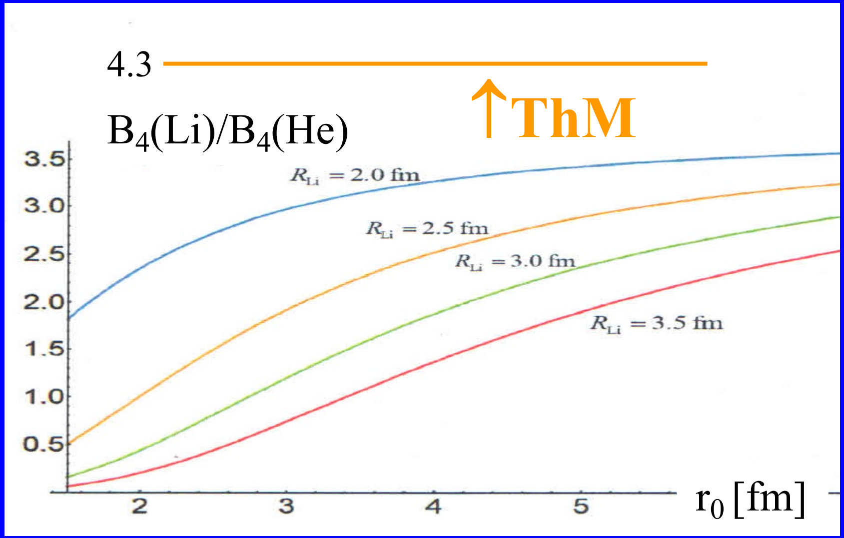
Bazak, Mrowczynski'2020

loose ^{4}Li vs compact ^{4}He: $R_{Li} > R_\alpha = 1.68$fm

Ratio= $B_4(^4Li)/B_4(^4He)$

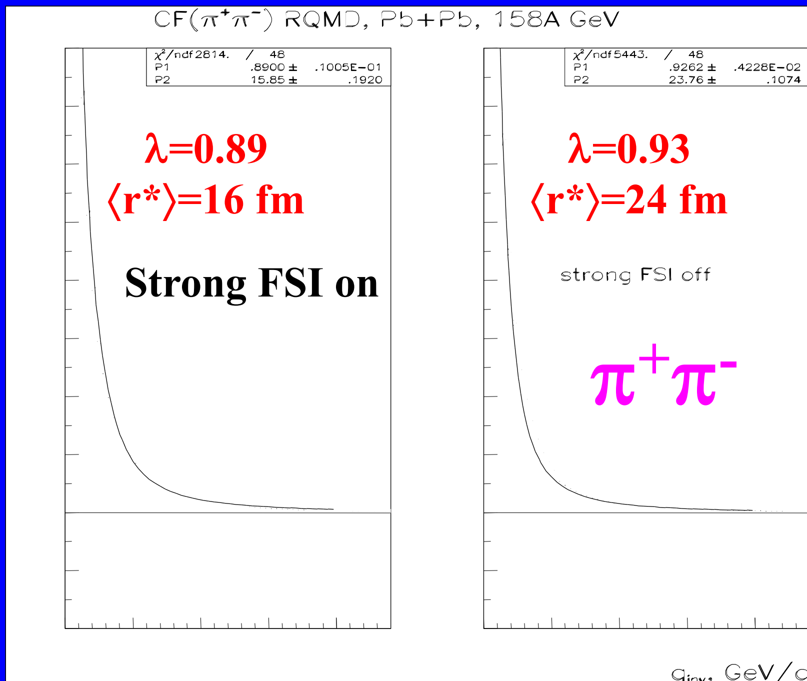
~ **5** in ThM (**$S_{Li}=2$, $S_\alpha = 0$**) Ratio indep. of r_0

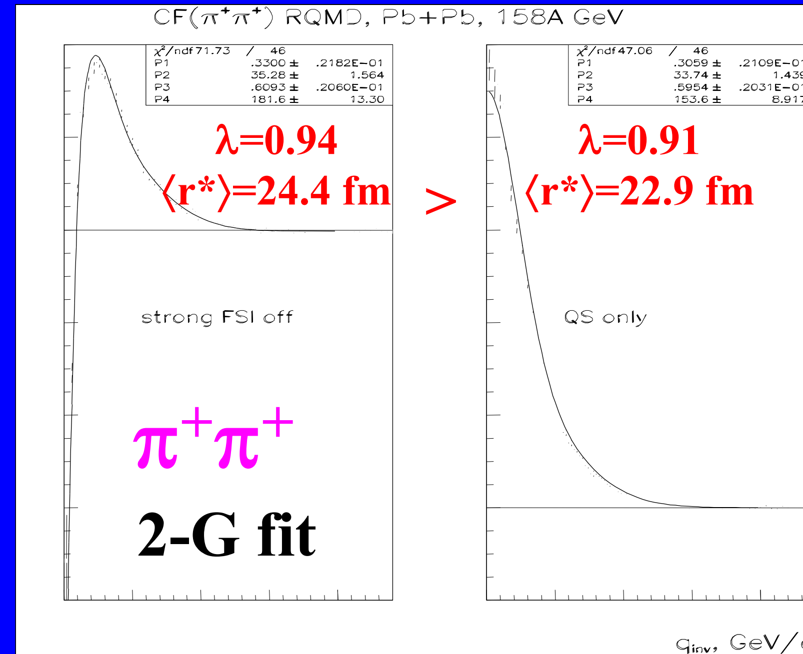
< 5 in CoM Ratio ↑ with ↑r_0 (centrality)

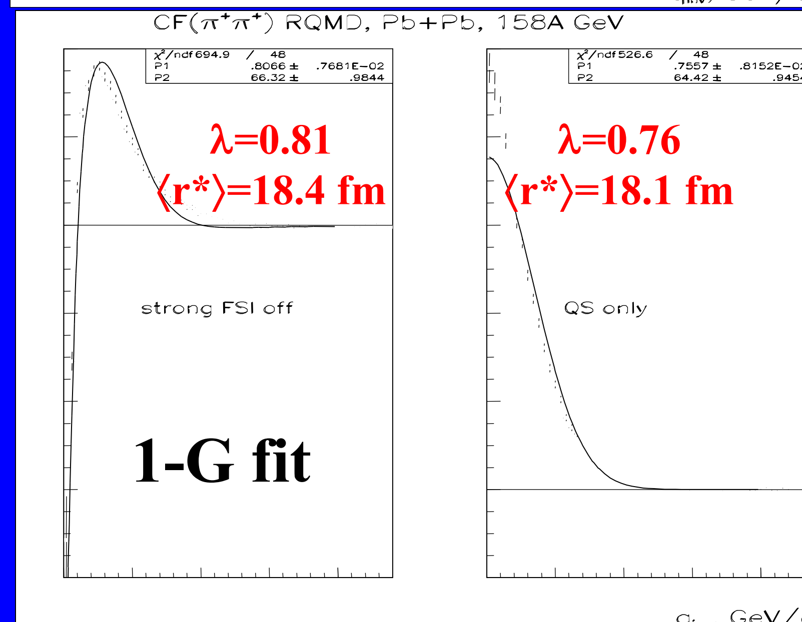


Tails in RQMD:

$\langle r^* \rangle = 21$ fm for $r^* < 50$ fm
29 fm for $r^* < 500$ fm







- Strong FSI important for $\pi^+\pi^-$
- 1-G fit: $\lambda(\pi^+\pi^+) \approx 0.8$, $\langle r^* \rangle$ 25% ↓
- 2-G fit: $\pi^+\pi^+ \approx \pi^+\pi^-$

  $\langle r^* \rangle_{QS} < \langle r^* \rangle_{Coul}$

Femtoscopy with nonidentical particles

$$CF = \langle|\psi_{-k^*}(r^*)|^2\rangle$$

Be careful when comparing
QS ($\pi^+\pi^+$ ..) and FSI correlations ($\pi^+\pi^-$..) → different sensitivity to r^*-distribution tails

→ **QS & strong FSI: non-Gaussian r^*-tail influences only first few bins in $Q=2k^*$ and its effect is mainly absorbed in suppression parameter λ**

→ **Coulomb FSI: sensitive to r^*-tail up to r^* ~ Bohr radius**

$|a|=|z_1z_2e^2\mu|^{-1}$	$\pi\pi$	πK	πp	KK	pp
fm	388	249	223	110	58

⇒ **In Gaussian fits one may expect $r_0(\pi^+\pi^+) < r_0(\pi^+\pi^-)$**

→ **Use realistic models like transport codes**