Recent progress in heavy-ion physics: A (brief) theoretical review

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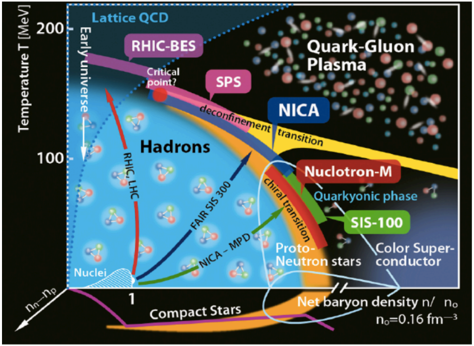
Workshop on physics performance studies at NICA, November 2024

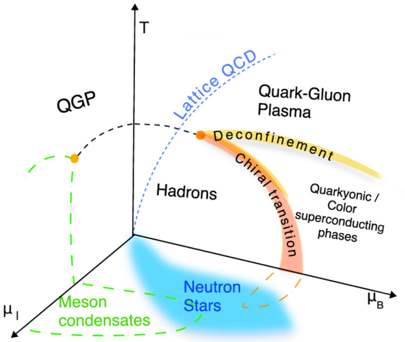
QCD phase diagram

- Synthesises our current knowledge (and ignorance) about the phases of strongly interacting matter.
- Phase transition from a hadron to a quark-gluon plasma with three active quark flavors, at $\mu_B = 0$, $\mu_s = 0$, smooth crossover at $T_{pc} \sim 158$ MeV.
- Possible hints but not conclusive experimental signature of the existence of CEP.



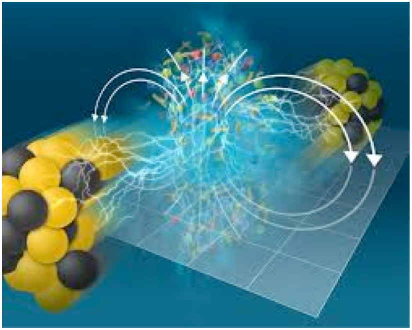
QCD phase diagram $\mu_I \neq 0$

- LQCD simulations cannot reliably explore deeper into the $\mu_B - T$ phase diagram due to the sign problem.
- On the other hand, LQCD calculations with $\mu_I \neq 0, \ \mu_B = \mu_s = 0$ can be safely performed since they are not hindered by the sign problem.
- LQCD finds a transition from the hadron to the pion condensed phase at a critical $\mu_I^c = m_\pi$



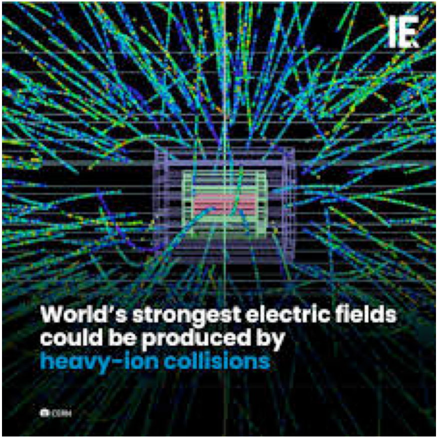
Magnetic fields

- Magnetic fields of a sizable intensity can be produced in peripheral HICs.
- A possible signature of the presence of these fields in the interaction region may be the chiral magnetic effect.
- Other manifestations of the presence of these fields?



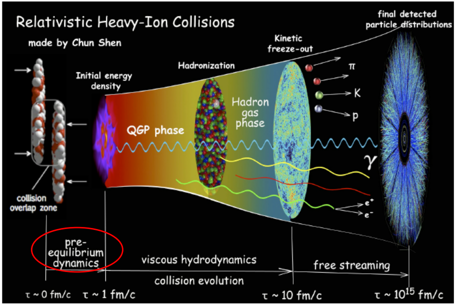
Electric fields

- Electric fields of a large intensity, $eE \sim (50\text{ MeV})^2$ can also be produced both in peripheral and central HICs.
- Fields last the longest for intermediate energies, $\sqrt{s_{NN}} \lesssim 5$ GeV.
- Possibility to study non-linear effects for pair production.



Time evolution of a heavy-ion collision

- Novel effects on observables induced by electromagnetic fields.
- Influence during different stages of the collision evolution.
- Fields are stronger at the beginning of the reaction. Probes from pre-equilibrium.



Outline

- CEP position: Lee-Yang zeroes and SDE's.
- Universality arguments: mapping of the 3d Ising model to QCD.
- Expected signatures of CEP existence in cumulant fluctuations.
- Isospin imbalanced QCD matter.
- The road towards equilibrium.
- The strongest ever produced electromagnetic fields.
- Summary and Conclusions.

Search for the CEP: Lee-Yang zeroes

- A phase transition occurs when the partition function vanishes and the free energy is singular.
- The partition function may depend on a control parameter q and a corresponding conjugte stochastic variable Φ (for example, for a spin system, q may be a magnetic field h and Φ the magnetization M).

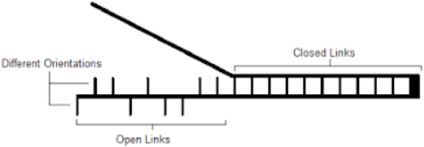
$$Z = \sum_i e^{-\beta(E_i - q\Phi_i)}$$

- For finite-size systems, the partition function is a finite sum of exponential functions and is always positive for real values of q. **Therefore, the free energy is always well-behaved and analytic for finite-size systems.**
- In contrast, **in the thermodynamic limit, the free energy may exhibit a non-analytic behavior.**
- Using that the partition function is an entire function for finite-size systems, **Lee–Yang theory** uses that the partition function can be fully characterized by its zeros in the complex q-plane.

Example: Molecular zipper

- N links that can be open or closed. **A fully closed zipper has zero energy.**
- Each open link increases the energy by ϵ
- A link can be open if the preceding one is open.
- If g is the number of ways that a link can be open, the partition function with N links is

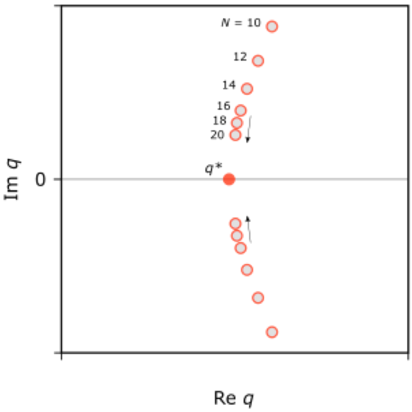
$$
\begin{aligned}
Z &= \sum_{n=0}^{N} g^n e^{-\beta n \epsilon} \\
&= \frac{1-(ge^{-\beta\epsilon})^{N+1}}{1-ge^{-\beta\epsilon}}
\end{aligned}
$$



- The partition function has the complex zeroes $\beta_k = \beta_c + i\frac{2\pi k}{\epsilon(N+1)}$.
- The critical temperature is $T_c = \beta_c^{-1} = \epsilon/\ln(g)$. For $g=1$, $T_c = \infty$ (no phase transition. For $g > 1$ a phase transition happens at a finite T_c.
- **In the limit $N \to \infty$, the zeroes closest to the real axis approach the critical value β_c.**

Search for the CEP: Lee-Yang zeroes

- The main idea of Lee–Yang theory is to study how the positions and the behavior of the zeros change as the system size grows.
- If the zeros move onto the real axis of the control parameter in the thermodynamic limit, this signals the presence of a phase transition at the corresponding real value of q, namely $q*$.



Estimation of CEP location form Ising model universality

- The QCD CEP is expected to belong to the $3-d$, $Z(2)$ universality class (well-known critical exponents $\beta = 0.326419$ and $\delta = 4.7898$).
- The mapping from the control parameters T and μ_B to the temperature-like t and magnetization-like h is not known.
- A frequently used ansatz is a linear maping

$$\begin{aligned} t &= A_t \Delta T + B_t \Delta\mu_B \\ h &= A_h \Delta T + B_h \Delta\mu_B, \;\; \text{with } \; \Delta T = T - T^{\text{CEP}}, \;\; \Delta\mu_B = \mu_B - \mu_B^{\text{CEP}} \end{aligned}$$

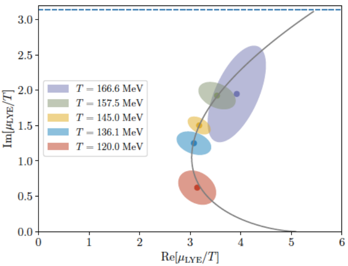
- The extrapolation of the poles to the real axis follows a path in terms of the scaling variable $z \equiv \left(\frac{t}{|h|}\right)^{\frac{1}{\beta\delta}}$ of constant magnitude: $z_{\text{LYE}} = |z_{\text{LYE}}| e^{i\pi/2\beta\delta}$ ($|z_{\text{LYE}}|$ is a universal constant)
- The above means that μ_{LYE} should scale as

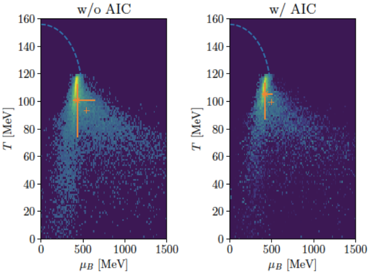
$$\begin{aligned} \text{Re}\mu_{\text{LYE}} &= \mu_B^{\text{CEP}} + c_1 \Delta T + c_2 \Delta T^2 \\ \text{Im}\mu_{\text{LYE}} &= c_3 (\Delta T)^{\beta\delta} \end{aligned}$$

- Five parameter fit: c_1, c_2, c_3, μ_B^{CEP}, T^{CEP}

Lee-Yang zeroes: $(T^{CEP}, \mu_B^{CEP}) = (105^{+8}_{-18}, 422^{+80}_{-35})$ MeV

- Use information from simulations at **purely imaginary μ_B (where no sign problem exists)** to construct a Padé approximation to the logarithm of the QCD grand partition function for complex μ_B.
- Determine singularities of the approximant to estimate the CEP location.



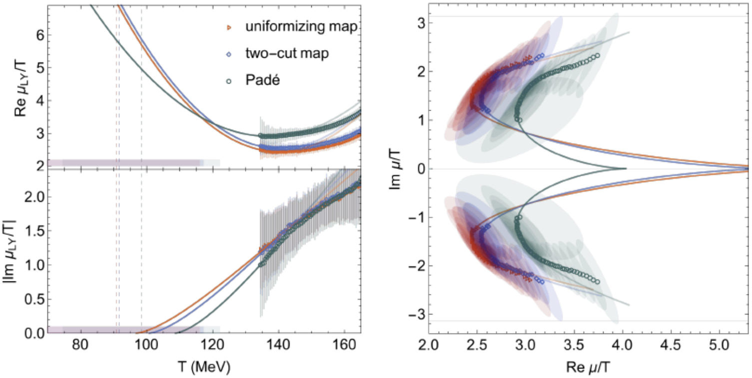


D. A. Clarke, P. Dimopoulos, F. Di Renzo, J. Goswami, C. Schmidt, S. Singh, and K. Zambello,

arXiv:2405-10196 [hep-lat]

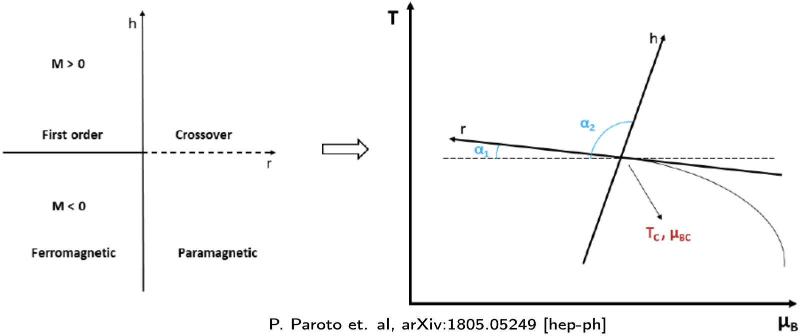
Lee-Yang zeroes: $(T^{CEP}, \mu_B^{CEP}) \sim (100, 600)$ MeV

- Using recent LQCD results for the Taylor expansion coefficients up to eighth order in μ_B and resummation techniques that combine Padé expansions and conformal maps.



G. Basar, arXiv:2312.06952. [hep-th]

Maping 3d Ising model to QCD

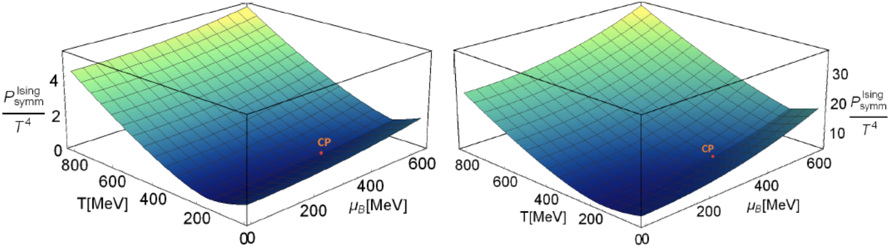


P. Paroto et. al, arXiv:1805.05249 [hep-ph]

$$\frac{T - T_C}{T_C} = w\,(r\rho\,\sin\alpha_1 + h\,\sin\alpha_2)\ ,$$
$$\frac{\mu_B - \mu_{BC}}{T_C} = w\,(-r\rho\,\cos\alpha_1 - h\,\cos\alpha_2)$$

- Universality (3d Ising model) constrains EoS near the critical point up to a few unknown non-universal parameters $\Longrightarrow$ family of model EoS, each containing a critical point somewhere in the region of the phase diagram covered by BES-II respecting known features from LQCD up to $O(\mu_B^4)$.

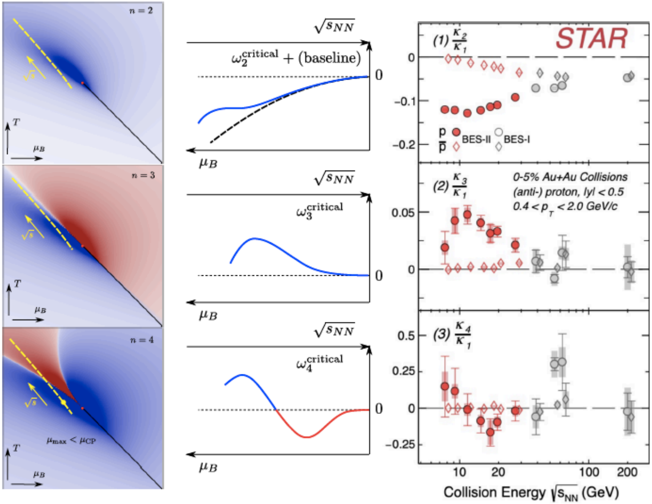
Maping 3d Ising model to QCD



P. Paroto et. al, arXiv:1805.05249 [hep-ph]

Critical pressure for the choice of two different parameter sets. The CEP is indicated by the red points and the first transition line is visible as the sharp edges for large μ_B.

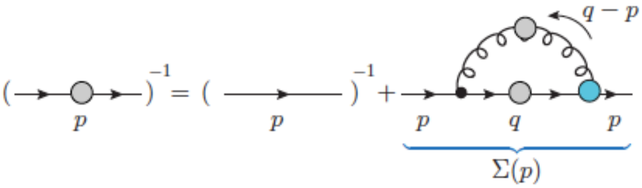
Expected signatures of CEP existence



M. Stephanov, arXiv:2410.02861 [nucl-th]

Search for CEP: Minimal approach to SDE

- Minimal computational scheme for Schwinger-Dyson equations.
- Technically simpler approximation at finite temperature and density that reproduces the phase structure results obtained with the state-of-the-art calculations



Search for CEP: Minimal approach to SDE

- Ingredients:

→ Both the full dressings of primitively divergent vertices as well as the respective bare renormalization constants approach unity at the symmetric point $p = \mu$ for asymptotically large μ. This is the MOM^2 renormalization scheme.

→ The vertex Γ_μ is taken flavor-diagonal and considers only the most dominant structures

Search for CEP: Minimal approach to SDE

$$\Gamma_\mu(q,p) = \mathcal{T}_\mu^{(1)}(q,p)\,\lambda^{(1)}(q,p) + \mathcal{T}_\mu^{(4)}(q,p)\,\lambda^{(4)}(q,p)$$

$$\lambda^{(1)}(q,p) = g_s F(k^2)\Sigma_A(q,p)$$

Ghost dressing function

Quark momentum dressing function

$$\Sigma_A(q,p) = \frac{A(p)+A(q)}{2}$$

$$\lambda^{(4)}(q,p) = \frac{g_s}{Z^{1/2}(k)}\Delta_B(q,p)$$

Quark scalar dressing function

$$\Delta_B(q,p) = \frac{B(p)-B(q)}{p^2-q^2}$$

$$G_A(k) = \frac{1}{Z_{A,\mathrm{scr}}(k)}\frac{1}{k^2+m_{\mathrm{scr}}^2}$$

Gluon dressing function

Gluon screening mass

Search for CEP: Minimal approach to SDE, finite T, μ_B

- The full quark and gluon propagators at finite temperature and baryon density are parametrized as

$$S^{-1}(\tilde{p}) = i\gamma_4\tilde{\omega}_n\, C(\tilde{p}) + i\boldsymbol{\gamma}\cdot\boldsymbol{p}\, A(\tilde{p}) + B(\tilde{p})$$

$$p^2\, D_{\mu\nu}(p) = \Pi^{\mathrm{E}}_{\mu\nu}(p)\, Z_{\mathrm{E}}(p) + \Pi^{\mathrm{M}}_{\mu\nu}(p)\, Z_{\mathrm{M}}(p)$$

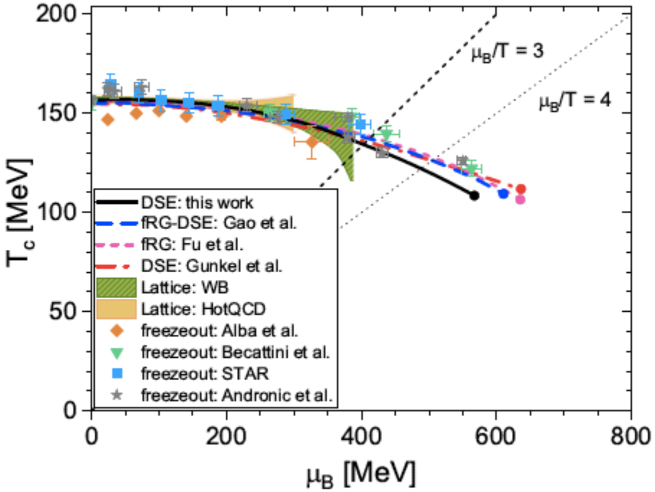
$$\tilde{\omega}_n = \omega_n + i\mu_q\,, \quad \tilde{p} = p + i\mu_q\,, \quad p = (\boldsymbol{p}, \omega_n)$$

$$\mathcal{T}^{(4)}(q,p)\,\lambda^{(4)}(q,p) \;\rightarrow \mathcal{T}^{(4)}(p,q)\left[\Pi^{\mathrm{E}}(k)\lambda^{(4)}_{\mathrm{E}}(q,p) + \Pi^{\mathrm{M}}(k)\lambda^{(4)}_{\mathrm{M}}(q,p)\right]$$

$$\lambda_4^{\mathrm{E,M}}(k;\tilde{q},\tilde{p}) = g_s\, Z^{-1/2}_{\mathrm{E,M}}(k^2)\,\Delta_B(\tilde{q},\tilde{p})$$

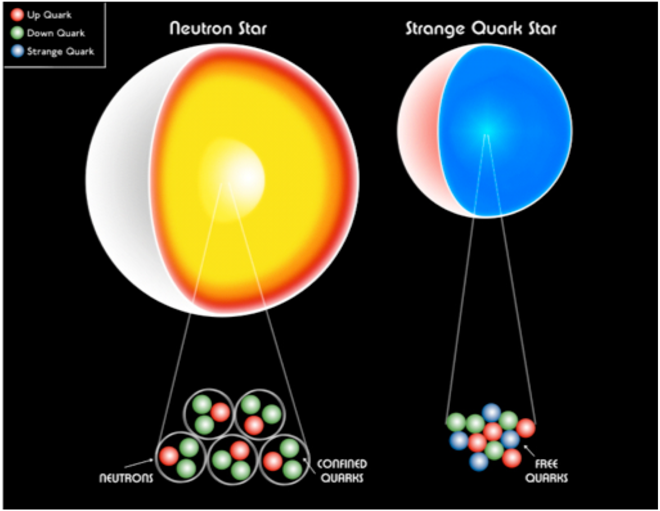
Electric and magnetic
dressing functions

Minimal approach SDE: $(T^{CEP}, \mu_B^{CEP}) \sim (108.5, 567)$ MeV

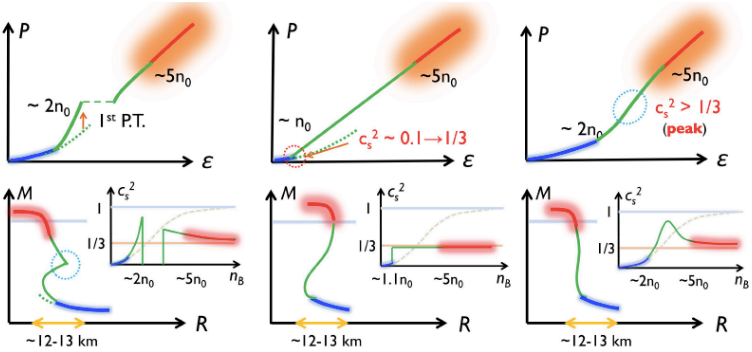


Y. Lu, F. Gao, Y.-x. Liu and J. M. Pawlowsk, arXiv:2310.18383 [hep-ph]

Isospin imbalanced strongly interacting matter: NS's



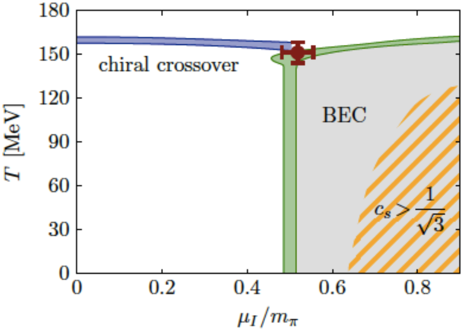
Isospin imbalanced strongly interacting matter



T. Kojo, arXiv:2011.10940 [nucl-th]

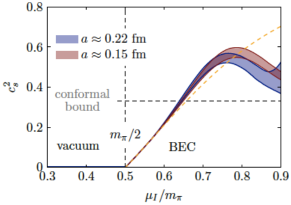
Isospin imbalanced strongly interacting matter

- LQCD calculations with $\mu_I \neq 0$ can be safely performed since they are not hindered by the sign problem.
- Bose-Einstein condensed phase at $\mu_I = m_\pi$.
- c_s^2 exceeds the conformal limit $1/3$.

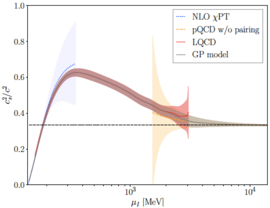


B.B. Brandt, F. Cuteri, G. Endrödi, arXiv:2212.1401 [hep-lat]

Isospin imbalanced strongly interacting matter



B.B. Brandt, F. Cuteri, G. Endrödi, arXiv:2212.1401 [hep-lat]



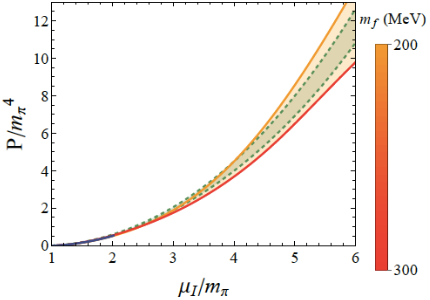
R. Abbott et. al, arXiv:2406.09273 [hep-lat]

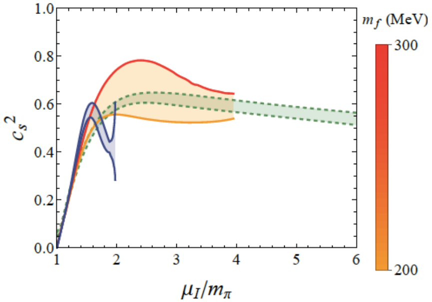
$$\begin{aligned} \pi_\pm &\rightarrow \pi_\pm + \frac{\Delta}{\sqrt{2}} \exp\{\pm i\theta\} \\ \mu_d &= -\mu_u = \mu_I/2 \end{aligned}$$

- Depending on the implementation, LQCD calculations show the peak on c_s^2 within the range $1.5\ m_\pi \lesssim \mu_I \lesssim 2.6\ m_\pi$

Isospin imbalanced strongly interacting matter

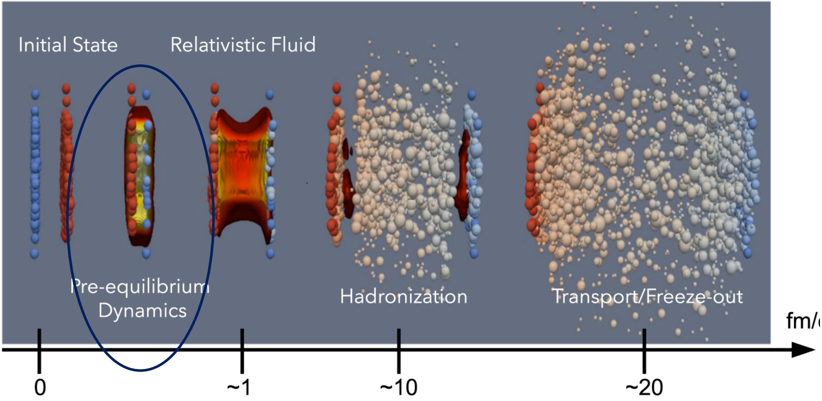
- $SU(2)$ Linear Sigma Model with quarks, $\sigma \to \sigma + v$
- Use full chiral content and let charged pions and sigma mix.
- Enforce existence of a Goldstone mode (non-trivial relation between Δ and v).





AA, R. Farias, B. Lopes, L.C. Parra, arXiv:2310.13130 [hep-ph]

Pre equilibrium: the road to isotropization

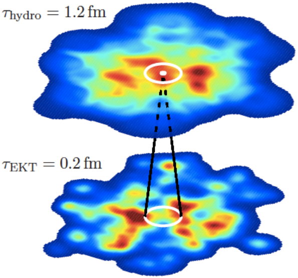


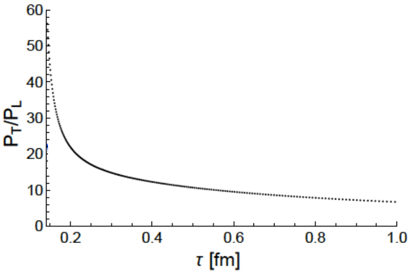
- Q: How does the system evolve from a shattered glasma to an **equilibrated relativistic fluid?**

Pre equilibrium: the road to isotropization

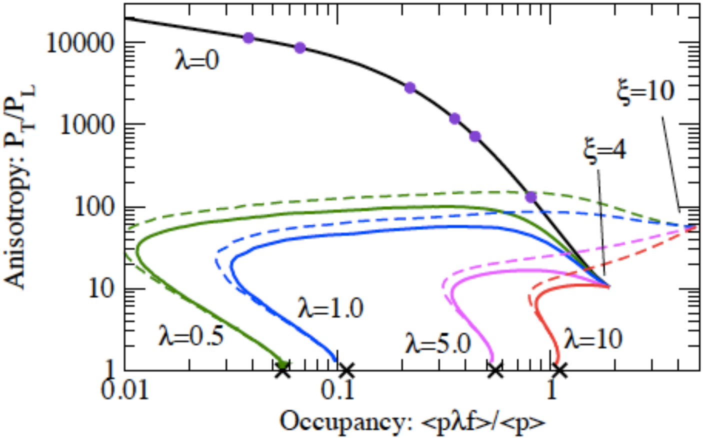
- Effective kinetic theory (Boltzmann equation for the color and spin averaged gluon distribution function at leading order in 't Hooft coupling $\lambda = 4\pi N_c \alpha_s$).
- Quantify in terms of the time when transverse and longitudinal pressures become comparable.

A. Kurkela, et. al, arXiv:1805.00961 [hep-ph].



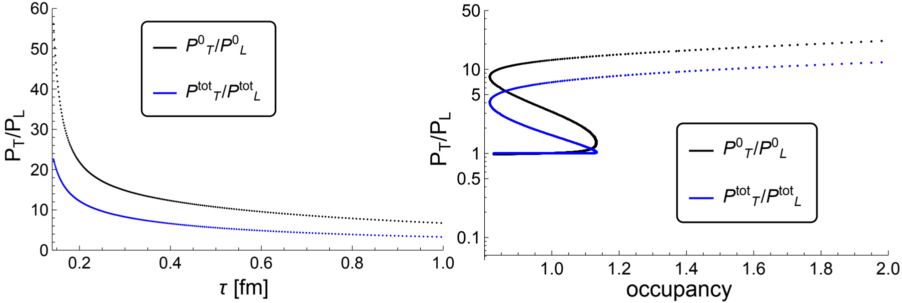


Pre equilibrium: the road to isotropization



A. Kurkela, et. al, arXiv:1805.00961 [hep-ph].

- Pure Yang-Mills gluon evolution cannot do the job.

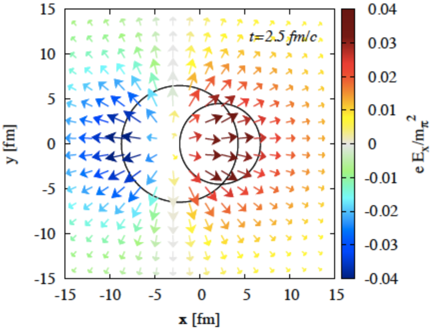


A.A., A. Mizher, arXiv:2407.09754 [hep-ph], to appear in PRD letters

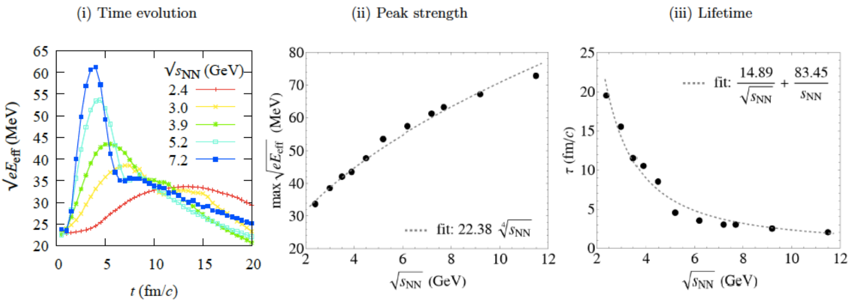
Strong electromagnetic fields (peripheral collisions)

- A strong electric field can be produced in asymmetric peripheral collisions.
- Peak intensity $eE \sim 0.04 m_\pi^2$.
- Effect can be observed comparing the directed flow of same mass but opposite charge hadrons.

V. Toneev, O. Rogachevsky, V. Voronyuk, arXiv:1604.06231 [hep-ph].



Strong electromagnetic fields (central colisions)



H. Taya, T. Nishimura, A. Ohnishi, arXiv:2402.17136 [hep-ph]

- Largest peak intensity and lifetime of the pulse for $\sqrt{s_{NN}} \lesssim 5$ GeV.
- Non-perturbative vacuum + $E \to e^+ + e^-$ allowed since $eE\tau/m_e$ and $eE\tau^2$ are large.
- Interesting to study possible consequences for dilepton production.

Summary and conclusions

- Large theoretical progress on the efforts to understand the nature and the location of the CEP.
- EoS can become better constrained from studying isospin imbalanced matter (nature of the peak in the speed of sound)
- Pre equilibrium is still poorly understood. Mechanism to overcome the strong initial pressure anisotropy needs to be better explored.
- Huge electromagnetic fields can leave imprints in observables that need to be quantified.

THANKS!